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Fracture under Complex Stress – The Angled Crack Problem

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ABSTRACT

Experiments are described in which thin plates of polymethylmethacrylate were fractured with cracks set at various angles to an applied uniaxial stress. While there is substantial agreement with previous analytical predictions, it is shown that inclusion of the stress component parallel to the crack can improve the correlation between linear theory and experiment, using a critical stress at a critical distance interpretation of the stress intensity factor criterion.

1. Introduction

Erdogan and Sih [1] have analysed crack propagation under a general two-dimensional stress system. In particular their work includes the case of a plate under a uniaxial uniform tension p with a central crack of length $2a$ inclined at an angle β to the direction of the stress as shown in

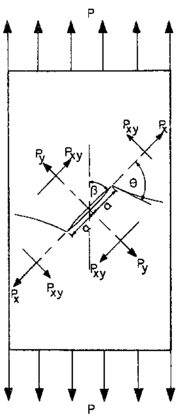

Figure 1. Stress Systems.

fig. 1. After resolution of the stresses, the crack may be regarded as subjected to a stress system of p_y normal to the crack, p_x parallel and a shear stress of p_{xy} having the values;

$$p_y = p \sin^2 \beta\,, \quad p_x = p \cos^2 \beta\,, \quad p_{xy} = p \sin \beta \cos \beta\,. \tag{1}$$

The analysis examines the stress distribution around the crack tip and postulates that the crack will run in a direction dictated by the maximum value of the stress p_θ normal to a radial line from the crack tip as shown in fig. 2. The result of their analysis is a curve of β *versus* θ, the angle the crack growth makes with the original crack direction as shown in fig. 3. (The angle θ is negative because the crack grows downwards.) Propagation of the crack normal to the applied stress p_y would be given by the straight line $\beta + (-\theta) = \pi/2$, also shown on fig. 3. The Erdogan–Sih theoretical solution therefore predicts that if $\beta > 30°$ the growth should be below the horizontal, whereas for $\beta < 30°$ above the horizontal.

The results of our experiments on PMMA sheets are shown in fig. 4, and are in substantial agreement with the theoretical prediction except for some portions which merit further attention.

2. Experimental Method

Tests were performed on polymethylmethacrylate (PMMA) sheet specimens of $\frac{1}{8}$ inch nominal thickness. The specimens were 6 inches wide and 12 inches long and pin loaded via bolted clamps and universal joints. The initial cracks were formed by three different methods. The first batch had a $\frac{1}{4}$ inch diameter central hole with $\frac{1}{32}$ inch wide slits sawn in at the required angles. In all cases the ends of the cracks were formed by forcing a razor blade into the end of the slits to form a natural crack. It was felt that the finite width of the slots used and the holes could influence the crack angles and for this reason a second set of specimens was made by milling in $\frac{1}{16}$ inch wide slots and forming the ends of these into cracks. Finally some specimens were made with slots machined in with a 0.01 inch wide slitting saw. Four half crack lengths were used, namely, 0.3, 0.5, 0.7 and 1.0 inches, but as it was difficult to control the lengths precisely there was some 10% variation in these lengths. No smaller lengths could be considered because of manufacturing problems and no larger ones because of finite width effects.

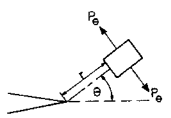

Figure 2. Co-ordinates at the Crack Tip.

The tests were performed on an Instron testing machine at 0.2 ins/min displacement rate, 20°C, and 50% relative humidity. The load at final fracture was recorded. In these tests it is possible to measure two loads; firstly the load when the crack begins to move and secondly when there is catastrophic failure. An examination of the fracture surface reveals a clearly defined "slow growth" zone with a rough surface and its limit gives the final crack length at fracture. Thus the first load and the initial crack length can be combined to give the initial or slow growth stress intensity factor;

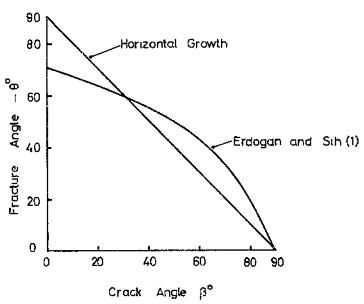

Figure 3. Plot of Erdogan and Sih's results.

$$K_{IC} = p(\pi a)^{\frac{1}{2}} \tag{2}$$

while the final load with crack length including the slow crack growth gives the instability failure value. The first of these values depends on loading rate and is often difficult to differentiate from craze growth while the second is definite crack growth and is not very rate sensitive. (Detailed discussion of the phenomena will be found in [2]).

In the tests described here the crack angle was varied between 0 and 90° in nominal increments of 5°. These values could not be fixed precisely because the natural crack could never be grown at an angle identical with the initial slot and differences of up to 2° were found. In all cases the angle of the crack, i.e. the tip, was used and it was found that this generally gave less scatter than the slot angle. This would be expected since the fracture is a localised phenomenon and thus governed by local conditions subject to there being no large differences between slot and crack angle.

The fracture angle was measured using a projection microscope with a 25 × magnification. The image of the end of the crack usually showed a band with the upper and lower edges giving different values of θ. These angles could be measured to within 1° and the variation between them was rarely more than 3°. The reason for the difference is that if the initial crack front is not precisely normal to the sheet surface then one side moves before the other and a distorted angle is produced. The larger of the two angles is the one given subsequently unless otherwise stated.

Figure 4 shows all the values of θ plotted against β with values taken from each end of the cracks. The three types of notch formation produced no noticably different results and the variation between notch length is not significant. Considerable care was taken with the tests to achieve the accuracy described and it is clear that the trend evident in the Erdogan and Sih

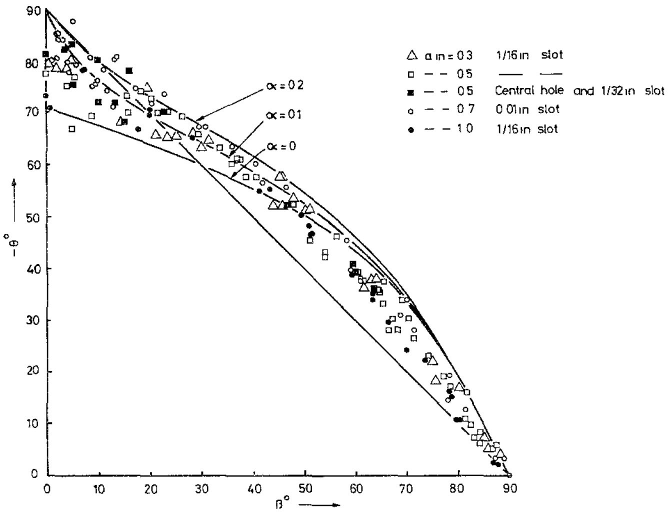

Figure 4. Fracture angle *versus* original crack angle.

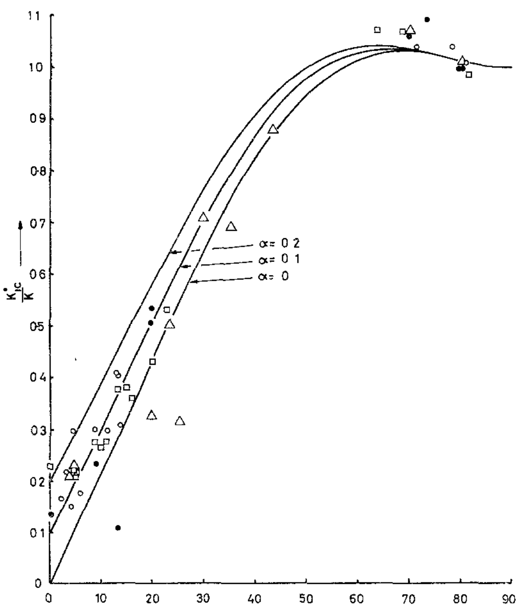

Figure 5. Ratio of $\beta=90°$ K_{1c}^{0} value divided by K at angle β as a function of β. $\alpha(2c/a)^{\frac{1}{2}}$

work is confirmed for the range they covered. However for $\beta < 40°$ the points are definitely above the theoretical line and tend to a value of $-90°$ and not $-70.5°$ at $\beta = 0°$. This difference warrants a further consideration of the analysis.

The instability fracture results are shown in fig. 5 plotted as the ratio K^0_{1c}/K *versus* β where K^0_{1c} is the value at $\beta = 90°$, i.e. a conventional fracture test, and K is the value for the angle. Crack initiation values were not used as in the case of Erdogan and Sih because of the difficulty in differentiating between craze and crack growth in these circumstances. The small scatter on the results would appear to justify this decision. Many of the results could not be used and are not shown because the K values determined with β other than 90° could not be corrected for slow growth (i.e. at final fracture there was a bent crack, a different situation). The points shown are those for specimens where the slow growth was very small (<0.01 in). For $\beta < 20°$ all the specimens were of this type and at $\beta = 90°$ all could be corrected to give an average value of $K^0_{1c} = 1247$ lbf/in$^{\frac{3}{2}}$. As K generally increases with decreasing β the lower of the two β values was used in fig. 5 as this was the controlling variable. For angles greater than 20° the incidence of small slow growth was low and apparently random.

3. Analysis

The expression for stress p_θ (fig. 2) may be written as a series of the form [3];

$$p_\theta = \left(\frac{a}{2r}\right)^{\frac{1}{2}} \cdot \cos\frac{\theta}{2}\left(p_y \cos^2\frac{\theta}{2} - \frac{3p_{xy}}{2}\sin\theta\right) + p_x \sin^2\theta + \left(\frac{2r}{a}\right)^{\frac{1}{2}} \cdot F(p_y, p_{xy}, \theta) + \ldots \quad (3)$$

The terms involving a and r express the variation of both p_y and p_{xy} due to the presence of the crack and the first term gives the singularity as $r \to 0$. p_x appears in none of these terms since it is parallel to the crack and thus unaffected by it and is simply superimposed on the stress distribution. It is usual to consider that, as fracture is a localised phenomena near the crack tip, the first term dominates the solution and all others may be ignored. Cotterell [4] has discussed the inclusion of other terms with reference to crack direction under simple tension and has concluded that the second term has a significant effect. If the fracture criterion is taken as a critical value of p_θ at a critical value of r, say p_c and c, then (3) may be used to examine the effect of higher terms on fracture. It is also of interest to note that tests on blunt notches [5] in amorphous polymers enable both p_c and c to be determined independently. In conventional tests they are combined in K^0_{IC} as this is usually all that is required but when they are separated typical values of c are around 0.002 in. For the tests described here crack half lengths have an average value of about 0.5 in giving a value of the critical parameter,

$$\alpha = \left(\frac{2c}{a}\right)^{\frac{1}{2}} \simeq 0.1$$

For values of this order terms higher than the second will produce very small effects but the omission of the second term cannot be justified. If this term is included then the direction of crack growth is given when $dp_\theta/d\theta = 0$ i.e.

$$p_y - \left[\frac{1-3\cos\theta}{\sin\theta}\right] p_{xy} - \frac{16}{3}\left[\frac{\sin\theta/2}{\tan\theta}\right][p_x\alpha] = 0\,. \quad (4)$$

For the special case of the angled crack problem we may substitute from (1) to give

$$\tan\beta - \frac{1-3\cos\theta}{\sin\theta} \cdot \tan\beta - \frac{16\alpha}{3}\,\frac{\sin\theta/2}{\tan\theta} = 0 \quad (5)$$

at the fracture condition. In simple tension ($p_x = p_{xy} = 0$) we have the fracture condition from (3);

$$p_c = \frac{1}{\alpha} \cdot p_y \cdot \cos^3\theta/2$$

and as $\theta = 0$ is the limiting condition from (4);

$$K_{1c}^{0} = p_c(2c)^{\frac{1}{2}}\pi^{\frac{1}{2}} = p_y(\pi a)^{\frac{1}{2}} \tag{6}$$

(the factor π is introduced to conform to accepted convention).

For the general case

$$K_{1c}^{0} = (\pi a)^{\frac{1}{2}} \cos\theta/2\,(p_y \cos^2\theta/2 - \tfrac{3}{2}p_{xy}\sin\theta) + (\pi 2c)^{\frac{1}{2}}\, p_x \sin^2\theta \tag{7}$$

and for the angle crack we have

$$\frac{K_{1c}^{0}}{K} = \cos\theta/2\,\sin\beta\,(\sin\beta\cos^2\theta/2 - \tfrac{3}{2}\cos\beta\,\sin\theta) + \alpha\,\cos^2\beta\,\sin^2\theta \tag{8}$$

and by substituting for θ for each value β from (5) a condition of fracture is obtained.

4. Discussion

The condition of $\alpha = 0$ is the Erdogan and Sih solution shown in fig. 4 together with $\alpha = 0.1$ and 0.2. As $\alpha \to 0$ it can be seen that the line tends rapidly to $\theta = -90°$ and not $-70.5°$ at $\beta = 0°$, and that $\alpha = 0.1$ gives an improved fit to the experimental points for $\beta < 50°$. The scatter on the points is such that it is not possible to discern any trend with crack length in this test but clearly inclusion of the additional term in the theory can produce a much better fit to the experimental points than $\alpha = 0$, without however explaining why the experimental points drop below the $\alpha = 0$ line for $\beta > 50°$. Most of the experimental points in this region are slow growth cracks and a possible explanation may lie in this fact. Slow growth is largely a craze growth phenomena and crazes behave more like plastic zones than cracks. There is some evidence to suggest that they tend to be controlled by the overall maximum tensile stress direction rather than the local stress, in which case the tendency to the straight line, the horizontal direction, would be expected.

Fig. 5 shows (8) plotted for $\alpha = 0$, 0.1 and 0.2 from which $\alpha = 0.1$ is chosen as the best fit to the experimental data. Those for $\beta > 60°$ are above unity as predicted and the scatter is within experimental accuracy. For $\beta < 25°$ the majority of the points fall between $\alpha = 0$ and 0.2 with a few exceptionally low points. These correspond to high values of K, two or three times in some cases, and their occurrence is difficult to explain. The original cracks were not exceptional and the fracture surfaces are not noticeably different from others. It seems most likely that they occur in a random manner due to local inhomogeneities in the material which inhibit crack growth.

5. Conclusions

The experimental results show that the inclusion of the second (non-regular) term in the series expansion for the stress distribution around the crack tip can produce an improved correlation with fracture predictions for the angle of fracture and the critical stress intensity factor. The criterion of a critical circumferential stress (p_c) at a critical radius (c) seems satisfactory for PMMA at moderate loading rates and ambient conditions. While there is some scatter in the results, a value of the critical radius of 0.002 inch is reasonable, although it should be emphasised that the analytical-experimental fracture correlation proposed, incorporates certain semi-empirical ingredients.

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RÉSUMÉ

On décrit des essais de rupture de tôles minces en polyméthylméthacrylate contenant des fissures orientées de diverses manières par rapport à une contrainte uniaxiale. Les prédictions analytiques sont confirmées de manière substantielle par les résultats.

On montre par ailleurs que le fait d'introduire la composante de tension parallèle au plan de la fissure permet d'améliorer la corrélation entre la théorie linéaire et l'expérience, en interprétant le critère de facteur d'intensité de contraintes selon une contrainte critique à une certaine distance critique de la fissure.

ZUSAMMENFASSUNG

Es werden Versuche beschrieben, bei denen dünne Polymethylmethacrylatplatten mit Rißen, deren Richtung mit der einachsigen Spannung verschiedenartige Winkel bildeten, geprüft wurden.

Die Ergebnisse bestätigen eindeutig die analytischen Voraussagen. Es wird jedoch weiter gezeigt, daß die Übereinstimmung von linearer Theorie und Versuch noch verbessert werden kann durch die Inbetrachtnahme der parallel zur Rißebene liegenden Spannungskomponenten, wobei eine kritische Spannung in einer kritischen Entfernung als Ausdruck des Spannungsintensitätsfaktors benutzt wurde.