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A Modified Mapping-Collocation Technique for Accurate Calculation of Stress Intensity Factors

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ABSTRACT
A technique combining the advantages of conformal mapping and boundary collocation arguments for calculating stress intensity factors for cracks in plane problems is described. The difficulty of finding the mapping function on a rigidly prescribed parameter region is avoided at the expense of using boundary collocation methods on part of the boundary. Conventional collocation arguments are modified by prescribing stress, force, and moment conditions in a least-square collocation sense. These pseudo-redundant conditions provide a reasonable basis for estimation of the effects of inaccuracy of the boundary conditions. The technique is applied to the problem of a circular disk with an internal crack under a loading of external hydrostatic tension.

Introduction

Methods for calculating stress intensity factors for cracks have important applications in fracture mechanics. For plane problems, the authors have previously developed the Muskhelishvili conformal mapping method [1] into an effective technique for a certain class of crack problems [2], [3]. At about the same time, boundary collocation methods were reintroduced and applied to crack problems by several authors [4], [5].

The boundary collocation methods depend on the selection of a class of stress functions satisfying the loading conditions on the crack and then matching boundary conditions at selected points on the remaining portion of the boundary. The computational simplicity of this approach is attractive; on the other hand, serious difficulties arise in assessment of accuracy. Convergence to the "correct" solution must be assessed on the basis of estimating the effect of the off-point residual errors in the boundary conditions. In fact, "apparent" convergence to incorrect values is quite possible if, for example, the class of stress functions chosen were incomplete.

The mapping technique has its difficulties too. It is frequently very difficult to find accurate polynomial mappings of the physical region onto a suitable parametric region. On the other hand, the ensuing stress analysis is well understood. Assessment of accuracy is related to the approximation of geometry rather than the effect of residual errors in the boundary conditions.

The technique proposed in this paper is a natural compromise of the two methods. The simple form of a mapping function carrying a circle and its exterior in the parameter plane into a crack and its exterior, respectively, will be used. The remaining portion of the boundary in the physical plane will correspond to a directly calculable curve in the auxiliary plane. The continuation arguments of Muskhelishvili are then employed to describe stress functions with, e.g., "traction-free" conditions on the crack. Collocation methods can then be introduced to satisfy the conditions on the remaining portions of the boundary.

This plane eliminates the difficulty of finding accurate polynomial approximations of the exact geometry in terms of a rigidly specified parameter domain. On the other hand, much of the mathematical insight provided by the complex variable formulation is preserved. In fact, it will be shown that an effective modification of the conventional boundary collocation procedure is suggested for the reduction of residual errors intermediate to the points of collocation.

Stress Function for a Circular Disk Containing a Pressurized Internal Crack

The model problem chosen to illustrate the technique is the plane problem corresponding to a circular disk containing an internal crack with loading shown in Fig. 1. It is obvious that the loading in Fig. 1 leads to the same K_I (stress intensity factor) as that for a pressurized internal crack. Such a problem illustrates the practical difficulties of employing the strict mapping technique. Since the region is doubly connected, a natural choice of the parameter domain is a concentric ring. Although such a mapping function is known in terms of elliptic functions, [6], the conversion to accurate polynomial approximations necessary to the stress analysis is not easy.

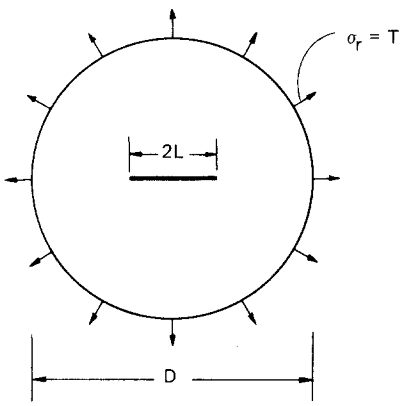

Figure 1. Circular disk with internal crack loaded by uniform external tension, T.

The first step in our procedure is to introduce a limited form of mapping. The physical region in Fig. 1 will be considered as defined in the complex Z-plane. Introducing an auxiliary ζ-plane, we consider the simple mapping

$$Z = \omega(\zeta) = (L/2)(\zeta + \zeta^{-1}) . \tag{1}$$

The unit circle, $\zeta = \sigma = e^{i\lambda}$, and its exterior in the ζ-plane map into the crack and its exterior, respectively, in the Z-plane. Clearly the image points in the physical plane can be related to the parameter plane by

$$\zeta = (Z/L) + [(Z/L)^2 - 1]^{\frac{1}{2}} . \tag{2}$$

In particular, the boundary $Z = (D/2)e^{i\theta}$ will correspond to a closed curve τ exterior to the unit circle in the ζ-plane (Fig. 2).

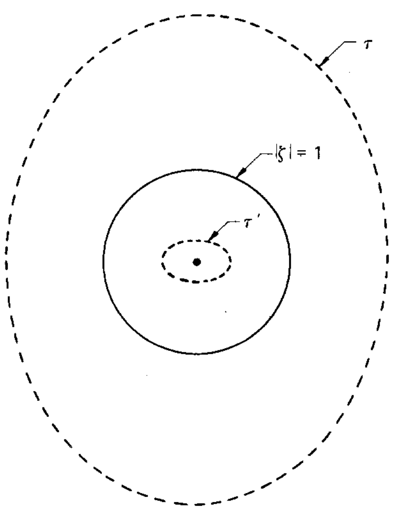

Figure 2. Region defined in the parameter plane.

The stress functions $\phi(Z)$ and $\psi(Z)$ in the Muskhelishvili notation can be considered as analytic functions of ζ. Furthermore, we adopt the notation $\phi'(Z)=\Phi(Z)$, $\psi'(Z)=\Psi(Z)$, $\phi[\omega(\zeta)]=\phi(\zeta)$, $\Phi(\zeta)=\phi'(\zeta)/\omega'(\zeta)$, *etc.*, where primes denote differentiation. Then,

$$\sigma_y+\sigma_x=4\,\mathrm{Re}\,\{\Phi(\zeta)\}=4\,\mathrm{Re}\,\{\phi'(\zeta)/\omega'(\zeta)\}$$

$$\begin{aligned}\sigma_y-\sigma_x+2i\tau_{xy}&=2\{\overline{\omega(\zeta)}\,\Phi'(\zeta)/\omega'(\zeta)+\Psi(\zeta)\}\\&=2\{\overline{\omega(\zeta)}\,[\phi'(\zeta)/\omega'(\zeta)]'+\psi'(\zeta)\}/\omega'(\zeta)\,.\end{aligned}$$

If we denote by $X_n\mathrm{d}s$ and $Y_n\mathrm{d}s$ the horizontal and vertical forces acting on an element of arc $\mathrm{d}s$ with normal n, the force resultant along the arc can be written as

$$\phi(\zeta)+\omega(\zeta)\overline{\phi'(\zeta)}/\overline{\omega'(\zeta)}+\overline{\psi(\zeta)}=i\int^{s}(X_n+iY_n)\mathrm{d}s=f_1(s)+if_2(s)\,. \tag{4}$$

Similarly, the moment (with respect to the origin) of the same forces is given by

$$M_0=\int^{s}(xY_n-yX_n)\mathrm{d}s=\mathrm{Re}\left\{\int^{\zeta}\psi(\zeta)\omega'(\zeta)\mathrm{d}\zeta-\omega(\zeta)\,\psi(\zeta)-|\omega(\zeta)|^2\,\Phi(\zeta)\right\}. \tag{5}$$

The condition that the crack be traction-free can be handled effectively by using the extension concept of Muskhelishvili. If S_ζ^+ and S_ζ^- denote the interior and exterior, respectively, of the unit circle in the ζ-plane, the function $\phi(\zeta)$ will be extended into S_ζ^+ by defining

$$\phi(\zeta)=-\omega(\zeta)\bar{\phi}'(1/\zeta)/\bar{\omega}'(1/\zeta)-\bar{\psi}(1/\zeta)\,,\qquad \zeta\in S_\zeta^+ \tag{6}$$

where the bar notation is defined by

$$\bar{f}(1/\zeta)\equiv\overline{f(1/\bar{\zeta})}\,. \tag{7}$$

The function $\psi(\zeta)$ can now be expressed as

$$\psi(\zeta)=-\bar{\phi}(1/\zeta)-\bar{\omega}(1/\zeta)\,\phi'(\zeta)/\omega'(\zeta),\qquad \zeta\in S_\zeta^-\,. \tag{8}$$

Similarly, one finds

$$\omega'(\zeta)\,\Psi(\zeta)=\zeta^{-2}\bar{\omega}'(1/\zeta)\{\Phi(\zeta)+\Phi(1/\zeta)\}-\bar{\omega}(1/\zeta)\,\Phi'(\zeta)\,,\qquad \zeta\in S_\zeta^- \tag{9}$$

From (4) and (8) evaluated on the unit circle $\zeta=\sigma$, it follows that the resultant force on the crack is identically zero as a function of σ provided the extended definition of $\phi(\zeta)$ is continuous across the unit circle. Thus, let τ' be the closed curve interior to the unit circle obtained by inversion of τ with respect to the unit circle. Then, if $\phi(\zeta)$ is considered as analytic in the doubly connected region enclosed by $\tau+\tau'$, traction-free conditions on the crack are automatically satisfied.

The determination of $\phi(\zeta)$ requires a form of representation and the satisfaction of conditions on τ corresponding to the tractions on $|Z|=D/2$. It will be assumed that $\phi(\zeta)$ can be represented in the form of a Laurent series. This appears to be a reasonable assumption although the boundaries τ' and τ are not circular. There is no *a priori* reason to suspect that the region of convergence of such a series could not extend over the desired parameter range in the present problem. Thus, taking into account obvious stress symmetries,

$$\phi(\zeta)=T\sum_{\infty}^{\infty}\alpha_n\zeta^{2n+1} \tag{10}$$

where the α_n's are real and must be determined from the boundary conditions on τ.

It is interesting to compare (10) with the class of stress functions used in [5] for the problem of an internal crack in a rectangular strip under tension. In [5] a generalized Westergaard's stress function, $\phi_w'(Z)$ was chosen of the form

$$\phi'_w(Z) = \sum_{m=0}^{N} C'_m Z^m/[Z^2 - L^2]^{\frac{1}{2}}.$$

This corresponds essentially to

$$\phi_w(\zeta) \approx \sum_{m=0}^{N} C'_m(\zeta+\zeta^{-1})^m \tag{12}$$

in the present formulation. Since (12) represents a restricted version of (10), the question of completeness can certainly be raised. This does not preclude the possibility of accuracy in an asymptotic sense in [5].

Modified Collocation and the Boundary Conditions

For the loading in Fig. 1, we have the stress boundary conditions

$$\sigma_r = T,\ \tau_{r\theta} = 0 \ \text{ on } \ |Z| = D/2 \tag{13}$$

and traction-free conditions on the crack. Thus, from (3) and (9)

$$\Phi(\zeta)+\overline{\Phi(\zeta)}-e^{2i\theta}\{\overline{\omega(\zeta)}\Phi'(\zeta)+\zeta^{-2}\bar{\omega}'(1/\zeta)[\bar{\Phi}(1/\zeta)+\Phi(\zeta)]-\bar{\omega}(1/\zeta)\Phi'(\zeta)\}/\omega'(\zeta) = T \quad \zeta\in\tau. \tag{14}$$

The boundary condition in terms of the force resultant (4) becomes

$$\phi(\zeta)-\phi(1/\bar{\zeta})+[\omega(\zeta)-\omega(1/\bar{\zeta})]\,\overline{\phi'(\zeta)}/\overline{\omega'(\zeta)} = T\omega(\zeta), \qquad \zeta\in\tau. \tag{15}$$

Only the interval $0\leq\theta\leq\pi/2$ need be considered explicitly since the symmetry of the stress function guarantees satisfaction of conditions on the remaining interval. Since boundary collocation will be used, the parametric nature of the Muskhelishvili formulation can be utilized by referring to both coordinate systems interchangeably. Thus, e.g., $\omega(\zeta)$ can be programmed in terms of either coordinates (whichever is more convenient), *etc.*

(There is an interesting parallelism here with a method by Bowie [7] for handling edge notches in a semi-infinite region. In [7], the reflection principle was used to ensure traction-free conditions along the real axis by the analyticity of the extended function $\phi(Z)$. Conditions on the notch and its reflected image paralleled the present conditions on τ and τ'. It was shown in [7] that a strict Fourier argument could be made even though explicit boundary conditions were imposed only on the notch interval. It was necessary to recognize that the assumed analytic continuation of $\phi(Z)$ implied conditions on the reflected image of the notch so that a "complete" interval was specified in a Fourier sense. In the present case, a similar argument can be made to justify the apparent inconsistency of determining the coefficients of a Laurent expansion from explicit conditions on only one of the "boundaries".)

To apply the boundary collocation argument, it is necessary to consider truncations of (10), e.g.,

$$\phi(\zeta) = T\sum_{-M}^{N} \alpha_n \zeta^{2n+1} \tag{16}$$

or

$$\omega'(\zeta)\Phi(\zeta) = T\sum_{-M}^{N} (2n+1)\alpha_n\zeta^{2n} \tag{17}$$

Then, if (17) is substituted into (14), there are $M+N+1$ independent α_n's available to satisfy (14). If the interval $0\leq\theta\leq\pi/2$ is subdivided into a set of discrete points, the matching of (14) corresponds to two real conditions at each point. If the number of conditions is matched by a consistent choice of M and N, a linear system of equations can then be solved for the α_n's. Of course, the number of conditions can exceed the degrees of freedom if one resorts to a least-square minimization of the total error summed over the discrete points.

When the conventional collocation argument is applied to the stress boundary conditions, a major difficulty becomes immediately obvious. For a fixed M/N ratio in the truncation of (17), apparent convergence to a value of K_I can be found by successively increasing the system size. However, by simply altering the M/N ratio, "convergence" to different values of K_I is found. Furthermore, an examination of the magnitudes of error intermediate to the points of collocation is often inconclusive.

A suitable error measure for boundary collocation is clearly suggested by Saint Venant's principle. The overall effects of boundary errors should be minimized if the resultant force and moment conditions are collocated. If $f_1 + if_2$ is matched at the points of collocation, then the errors in stress boundary conditions correspond to self-equilibrating distributions of loading on the intervals between successive collocation points. A measure of the moment error is found conveniently in the present formulation. Along an arc L

$$M_0 = \int_L (xY_n - yX_n)\mathrm{d}s = -\int_L (x\,\mathrm{d}f_1 + y\,\mathrm{d}f_2) = -(xf_1 + yf_2)_L + \int_L (f_1\,\mathrm{d}x + f_2\,\mathrm{d}y)\,. \tag{18}$$

It is clear from (18) that off-point errors in f_1 and f_2 can have an accumulated effect on moment accuracy due to the second term.

The following modification of the conventional boundary collocation plan is therefore recommended: *At each boundary point of collocation, we impose the five conditions corresponding to M_0, $f_1 + if_2$, and the normal and tangential components of the applied stress.* Since these conditions are conveniently expressed by the Muskhelishvili formulation, it is an easy matter to write these conditions in terms of the coefficients α_n of the truncated series (17).

In applying the procedure, it is generally advisable to utilize least-square collocation for economy and also from certain considerations of the system. If strictly continuous arguments were being used, there is obvious redundancy in the five conditions above. Although this redundancy is removed when discrete considerations are made, nevertheless, for large systems a weakness in the determinant for the full system appears probable. This difficulty can be minimized by controlling the degrees of freedom as compared with the number of conditions and minimizing the error in a least-square collocation sense.

Numerical Results for a Circular Disk with an Internal Crack

The stress intensity factor, K_I, in the conventional notation [8] reduces to

$$K_I = 2\pi^{\frac{1}{2}}\phi'(1)/\{\omega''(1)\}^{\frac{1}{2}} = 2\pi^{\frac{1}{2}}\phi'(1)/L^{\frac{1}{2}}. \tag{19}$$

Thus,

$$K_I = 2(\pi/L)^{\frac{1}{2}} \sum_{-M}^{N} (2n+1)\alpha_n\,. \tag{20}$$

Furthermore, it is well known that

$$\phi(\zeta) \to (TL/8)(\zeta - 3\zeta^{-1}), \qquad L/D \approx 0\,. \tag{21}$$

Therefore,

$$K_I \approx T(\pi L)^{\frac{1}{2}}, \quad L/D \approx 0\,. \tag{22}$$

Initially, the conventional boundary collocation approach using the stress boundary condition (14) was attempted. When arbitrary M/N ratios were chosen, very poor results were obtained even in the range $0 < L/D < 0.05$ where (22) can be expected to hold. A tedious trail and error process based on a careful variation of M and N guided by (21) yielded some reliable results in this range. However, this uneconomical approach was abandoned in favor of the process described in the preceding section.

The analysis was then set up using (4), (14), and (15). For the radial loading in Fig. 1, $M_0 = 0$. However, since (4) is valid up to a constant of integration, the matter of consistency with the assumed form (16) must be considered. One can simply carry an unknown constant C in (4) and treat it as an unknown in the matching of (4). The usual constant of integration in (15) is zero to be consistent with the assumed form of $\phi(\zeta)$ in (16).

The computations were carried out using double precision. Furthermore, for accuracy in solving the system of equations it was found advisable to scale the unknowns by the substitution

$$\alpha_n' = \alpha_n (D/2)^{|2n|}, \qquad n=0, \pm 1, \dots \,. \tag{23}$$

The interval $0 \leqq \theta \leqq \pi/2$ was usually divided into 40 stations with equal intervals and approximately 80 degrees of freedom were considered. Least-square collocation was obviously used.

The results obtained are remarkably reliable even for very deep cracks. The values listed in Table 1 for K_I can be considered as accurate to within an error of less than one percent. This estimate is based on examination of the off-point errors and the stability of the answers with variations of M/N ratios, degrees of freedom, number of stations, *etc.*

Examination of the influence of the individual conditions was also made. In contrast to the poor solutions obtained by using the stress conditions alone, it was found that very stable values of K_I were obtained by using the condition on $f_1 + if_2$ alone. This is consistent with the Saint Venant basis for our argument. The complete system is most reasonably necessary when accuracy is affected by local stress irregularities.

Comparison of Results with an Elliptical Disk with Internal Crack

In Table 1, the present results are compared with the well-known secant correction for an infinite strip [9]. These values are obtained from

$$K_I^{(a)}/T(\pi L)^{\frac{1}{2}} = [\sec(\pi L/D)]^{\frac{1}{2}} \,. \tag{24}$$

Although (24) is only an approximation, it is frequently accepted on the basis of its agreement with numerical results obtained independently by several investigators.

TABLE 1.

Stress intensity factors K_I, for a circular disk with a pressurized internal crack, $\sigma = -T_1$

D	$D/2L$	$K_I/T(\pi L)^{\frac{1}{2}}$	$K_I^{(a)}/T(\pi L)^{\frac{1}{2}}$	$K_I^{(b)}/T(\pi L)^{\frac{1}{2}}$
100.0	25.00	1.00	1.001	1.00
40.0	10.00	1.02	1.006	1.02
20.0	5.00	1.06	1.025	1.06
10.0	2.50	1.24	1.112	1.28
6.0	1.50	1.74	1.414	2.11
5.4	1.35	1.98	1.589	2.64
5.0	1.25	2.24	1.799	3.43
4.8	1.20	2.43	1.966	4.09
4.6	1.15	2.71	2.217	5.19
4.4	1.10	3.17	2.651	7.61

(a) Secant correction for infinite strip.
(b) Elliptic disk with internal crack.

As $D/2L$ approaches unity, the secant formula increasingly underestimates the K_I in the present problem. Let us consider our present configuration as obtained by cutting away the appropriate material from the infinite strip. Then it is reasonable to hypothesize that the greater K_I values are due to an increase in flexibility with greater bending across the sections on the real axis. If this hypothesis were true, then even greater values of K_I would be obtained by considering an elliptical outer boundary with a major axis of D and a minor axis less than D. This will now be substantiated.

Although a solution for a region bounded by two confocal ellipses has been given by Sheremetjier [10], a direct solution in terms of the present formulation is possible. The mapping function (1) carries the unit circle into a crack of length $2L$ and carries the circle $|\zeta| = \rho_0 > 1$, where

$$2\rho_0 = D/L + [(D/L)^2 - 4]^{\frac{1}{2}} \,, \tag{25}$$

into an ellipse with major axis, D, and a minor axis of $L(\rho_0-\rho_0{}^{-1})$. Although the eccentricity of the ellipse varies with D/L, the solution for this configuration is still of interest for our purpose.

The stress analysis for external hydrostatic tension again involves a series expansion (10). Substituting (10) into (15) yields the following conditions:

$$\begin{aligned}
&\alpha_K(\rho_0^{2K+1}-\rho_0^{-2K-1})-\alpha_{K-1}(\rho_0^{2K-3}-\rho_0^{-2K-1})\\
&-\alpha_{-K}(2K-1)(\rho_0^{-2K+1}-\rho_0^{-2K-1})+\alpha_{-K-1}(2K+1)(\rho_0^{-2K-1}-\rho_0{}^{-2K-3})\\
&=L/2\rho_0, \qquad K=-1\\
&=(L/2)(\rho_0-\rho_0^{-3}), \quad K=0\\
&=-L/2\rho_0\,, \qquad K=1\\
&=0\,, \qquad K=\pm 2, \pm 3, \ldots\,.
\end{aligned} \tag{26}$$

An unusual situation arises in the solution of (26). If N of the conditions are chosen, a linear system of N equations in $N+1$ unknowns results. On the other hand, the structure of (10) is such that no additional relationship is provided by the usual arguments of single-valuedness of stress resultant, *etc.* There remains only the condition of series convergence! For large K, it is clear from (26) that

$$\alpha_K/\alpha_{K-1} \approx \rho_0^{-4} \quad \text{for} \quad K \gg 1\,. \tag{27}$$

On the other hand,

$$\alpha_{-K-1}/\alpha_{-K} \approx 1-\alpha_{K-1}[(1-2K)+4K\rho_0^{-2}]/\alpha_{-K}, \qquad K \gg 1\,. \tag{28}$$

The convergence problem is associated with the behavior of the coefficients α_{-K} by inspection of (28).

The plan of solution by truncation is now fairly evident. To the system formed by the conditions $K=0, \pm 1, \pm 2, \ldots, \pm M$, we add the condition

$$\alpha_{-M-1}=0\,. \tag{29}$$

The resulting system is a $(2M+2)\times(2M+2)$ linear system. Convergence with respect to M is extremely rapid.

The corresponding stress intensity values are listed in Table 1 as $K_I^{(b)}$. The values exceed the corresponding K_I values as was predicted. It is apparent that for $D/2L>2.5$, the ellipse closely approximates a circular region and the agreement is good. As $D/2L\to 1$, the ellipse becomes extremely flat and there are no grounds to make any comparisons.

Observations

The modified mapping-collocation technique proposed here appears to be particularly well suited to handling the troublesome problems of internal cracks. Preservation of the Muskhelishvili concepts provides a basis for selecting a class of stress functions with a reasonable guarantee of completeness. In the present problem, for example, the *a priori* expansion in a Laurent series was clearly justified by the excellent matching of the boundary conditions intermediate to the points of collocation.

The modified collocation argument involving the use of pseudo-redundant conditions provides the analyst with a reasonable basis for estimating the effects of inaccuracies in the boundary conditions. For example, it was not surprising that collocation of f_1+if_2 in the present problem yielded excellent values for the stress intensity factors in the range considered. On the other hand it is not difficult to recognize situations when the information desired should involve full use of the five conditions outlined above.

The basic philosophy of the approach can be carried over with suitable modifications to a wide class of problems.

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RÉSUMÉ

On décrit une technique de calcul des facteurs d'intensité de contraintes pour des fissures en état plan, qui combine les avantages de la méthode de la représentation conforme et des méthodes de fixation des conditions aux limites.

La difficulté que l'on rencontre à trouver la fonction de représentation qui correspond à une région à paramètres imposés est levée par l'emploi des méthodes de fixation des conditions aux limites sur une partie d'un contour. Le traitement conventionnel de ces méthodes est modifié en imposant les conditions de contraintes, de forces et de moments en un ajustement par moindres carrés.

Les conditions pseudo-redondantes ainsi réunies procurent une base d'appréciation des effets d'une inexactitude dans la définition du contour.

La technique est appliquée au problème du disque circulaire comportant une fissure interne et soumis à l'action d'une tension extérieure uniforme.

ZUSAMMENFASSUNG

Es wird ein Verfahren zur Berechnung von Spannungsintensitätsfaktoren für Riße in einem ebenen Zustand beschrieben, welches sowohl die Vorteile der Methode der konformen Darstellung als auch die der Verfahren zur Bestimmung der Grenzbedingungen miteinander verbindet.

Die Anwendung der Verfahren der Festlegung von Grenzbedingungen für einen Teil der Außenlinie ermöglicht es die Schwierigkeiten zu umgehen, welche sich dann ergeben wenn man versucht die darstellende Funktion für einen Bereich mit streng auferlegten Parameter zu bestimmen. Die konventionelle Behandlung dieser Verfahren wird dadurch abgeändert, daß die Bedingungen für Spannungen, Kräfte und Momente im Sinne der kleinsten Quadratzahlen auferlegt werden.

Diese pseudo-überflüssige Bedingungen ergeben eine Basis zur Beurteilung der Auswirkung einer Ungenauigkeit in der Definition des Umrisses.

Diese Methode wird auf das Problem einer runden Scheibe mit inneren Rissen, welche der Wirkung von äußeren Spannungen unterworfen ist, angewendet.

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