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The effect of astigmatism due to beam refractions on the formation of the measurement volume in LDA measurements

Z. Zhang, K. Eisele

Abstract The accuracy of LDA measurements depends on the optical alignment of the laser beams. Improperly designed optical systems lead to fringe distortion in the measurement volume and in earlier investigations this effect has always been taken as the main cause of optical inaccuracy in LDA measurements. In the present work a different cause of fringe distortion is considered: astigmatism due to beam refractions. A quantitative theory for the astigmatism of laser beams is derived for both single and multiple refractions. Parameter calculations with regard to the size of the astigmatism effect have been carried out. It is shown that astigmatism is a relevant parameter which influences the fringe uniformity and fringe distortion in an LDA measurement volume and affects the measurement accuracy of measurements in internal flow. The equations derived enable the change in cross sections of the refracted laser beams to be determined. The spatial deviations of the diverse focusing points of refracted laser beams relative to the position of the LDA measurement volume are found to depend strongly on the incident angle of the beams and therefore on the off-axis alignment angle of the LDA probe (off-axis from the normal to the flow-wall-interface).

1 Introduction

Laser Doppler Anemometry (LDA) is a technique to measure flow velocities which requires no previous calibration of the system. The flow velocity is determined directly from the known fringe spacing in the LDA measurement volume and from the Doppler frequency of the light scattered by a particle passing through the measurement volume. The fringe spacing in the measurement volume may be calculated from the wavelength of laser light and the half intersection angle between the laser beams. In the case where the measurement volume is assumed to be formed exactly at the waists of laser beams with plane waves then the fringe spacing in the measurement volume may be considered as uniform. This property of the measurement volume ensures the correct measurement both of the mean value of the flow velocity and its fluctuations. In the optical layout of laser beams of an LDA-system it is always stressed that the beam waist positions are coincident with the beam crossing position. Otherwise the alignment errors of laser beams and the curved wave of the Gaussian beams at the beam crossing cause the fringe field in the measurement volume to be distorted. This type of fringe distortion has been discussed in earlier investigations (Hanson 1975; Durst and Stevenson 1975). Recently Miles and Witze (1994) used the process of imaging the fringe field in measurement volume onto a far screen and investigated the nonuniformity of the interference fringe induced by improper beam alignment with fixed deviations of beam waists from the beam crossing. Another type of fringe distortion in the measurement volume is known to be caused by the effect of laser light diffraction through particles shortly before the measurement volume.

Although the coincidence of the laser beam waists with the beam crossing position is ensured by the layout of LDA systems, non-coincidence will occur if the laser beams suffer from refractions at windows. This occurs often in the measurement of velocities in internal flow, such as the flow in turbomachines and I.C. engines. In this case, the refraction of laser beams with unique focusing points (waists) always results in astigmatism, a third-order aberration with the change in the beam cross section. Indeed, for a single refracted laser beam, there are two focusing points corresponding to the meridional and sagittal plane of the beam. The distance between both focusing points, called the astigmatic difference, depends on the refraction angle of the laser beam. For internal flow measurements with an LDA-probe in the on-axis alignment the incident angle of laser beams is equal to the half intersection angle between the beams. Because this angle is usually small, astigmatism is usually not evident. However, if the LDA-probe has to be aligned off-axis, as is the case when a 2D- and a 1D-probe are combined for simultaneous three-component measurements, then astigmatism due to beam refraction may become considerable. The existence of astigmatism in relation to the beam refraction has the similar effect on the fringe distortion as the improper optical layout of laser beams. This work deals with the significant parameters of astigmatism in LDA measurements.

The appearance of astigmatism in LDA and its influence on measurements have been investigated by Zhang and Eisele (1995a) especially for the off-axis alignment of an LDA-probe.

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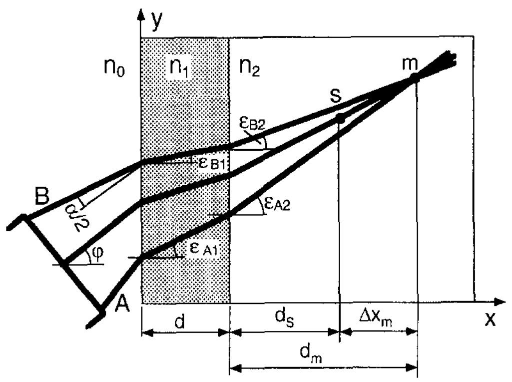

Fig. 1. Beam transmittance through a glass window into the flow and formations of two intersection points *m* (for meridional) and *s* (for sagittal)

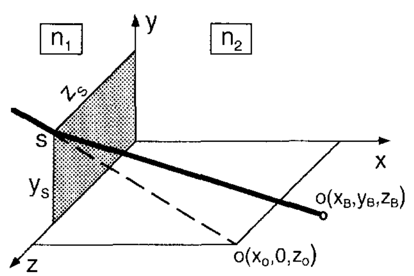

Fig. 2. Single refraction of a spatial focusing beam bundle with its initial focusing point of $o(x_0, 0, z_0)$

The astigmatism was considered there to exist at the macro focusing light bundle which is imagined to comprise all 4 laser beams from a 2D-probe. Because of astigmatism both intersection points of laser beams do not appear at the same position in the test medium. The distance between them is given according to Fig. 1 (Zhang 1993; Zhang and Eisele 1995a):

$$\Delta x_m = \frac{1}{\Phi_m}(d \cdot \Psi_1 + d_s \cdot \Psi_2) \tag{1}$$

where

$$\Phi_m = \tan \varepsilon_{A2} - \tan \varepsilon_{B2} \tag{2}$$

$$\Psi_1 = \frac{K}{\sqrt{n_1^2 - n_0^2(1-\cos^2\varphi \cdot \cos^2\alpha/2)}} - (\tan\varepsilon_{A1} - \tan\varepsilon_{B1}) \tag{3}$$

$$\Psi_2 = \frac{K}{\sqrt{n_2^2 - n_0^2(1-\cos^2\varphi \cdot \cos^2\alpha/2)}} - (\tan\varepsilon_{A2} - \tan\varepsilon_{B2}) \tag{4}$$

$$K = n_0 \cdot \cos\varphi \cdot \cos(\alpha/2) \cdot \left[\tan\left(\varphi + \frac{\alpha}{2}\right) - \tan\left(\varphi - \frac{\alpha}{2}\right)\right] \tag{5}$$

with ε_{A1}, ε_{A2}, ε_{B1} and ε_{B2} as the refraction angles of laser beams (A and B) in media 1 and 2 respectively, as defined in Fig. 1.

Because of non-coincidence between both intersection points two-dimensional velocity measurement using a 2D-probe at the off-axis alignment becomes impossible. In addition, astigmatism has been taken as the main reason for the reduction of the data rate. In order to compensate for the overall effect of astigmatism on the measurement a water-filled prism has been recommended. For the design and application of such a prism, calculation procedure based on Eq. (1) has been given by Zhang and Eisele (1995a).

In the present work the astigmatism will be considered for individual spatial laser beams. Although the laser beams of an LDA-system are usually thin, as focusing light bundles they all suffer from astigmatism whenever they are refracted. At each laser beam there are two focusing points corresponding to the meridional and sagittal plane of the beam, respectively. For an LDA-system with two laser beams the deviations of all four focusing points from the intersection point of laser beams, that is from the measurement volume, will be given as a function of the off-axis alignment angle of the LDA-probe. Both single refraction and multiple refractions of laser beams will be treated in this work.

2
Single refraction of the focusing beam bundle

A spatial focusing beam bundle will be considered which is refracted at the interface of media 1 and 2 (Fig. 2). For this configuration a coordinate system is chosen with the y–z plane on the interface between the media. The x-axis is then directed from medium 1 to medium 2. The virtual focusing point of the incident beam bundle lies on the x–z plane. Two characteristic planes related to the incident beam should be noted: the plane with the beam axis and the normal to the interface is called the meridional plane; the plane perpendicular to this and through the beam axis is called the sagittal plane.

The incident beam bundle considered has its kown initial focusing point in medium 2 at point $o(x_0, 0, z_0)$. To simplify the calculations the beam bundle will be considered as a single ray which may be described by a straight line with the parallel unit vector $\mathbf{r}_1$ as follows:

$$\frac{x-0}{r_{1x}} = \frac{y-y_s}{r_{1y}} = \frac{z-z_s}{r_{1z}} \tag{7}$$

with point $s(0, y_s, z_s)$ as the intersection point of the straight line on the interface.

The coordinates of the intersection point s may be determined by inserting the coordinates of the known initial focusing point of the incident beam $o(x_0, 0, z_0)$ in Eq. (7). For y_s, for instance, it becomes:

$$y_s = -\frac{r_{1y}}{r_{1x}} x_0 \tag{8}$$

Analogous to Eq. (7) the refracted ray which departs from the intersection point s may also be presented by a straight line with $\mathbf{r}_2$ as a parallel unit vector:

$$\frac{x-0}{r_{2x}} = \frac{y-y_s}{r_{2y}} = \frac{z-z_s}{r_{2z}} \tag{9}$$

The unit vector $\mathbf{r}_2$ depends on the unit vector $\mathbf{r}_1$ and may be determined by the application of the refraction law as:

$$r_{2y} = \frac{n_1}{n_2} \cdot r_{1y} \tag{10}$$

$$r_{2z}=\frac{n_1}{n_2}\cdot r_{1z} \tag{11}$$

with n_1 and n_2 as the refractive index of the respective medium.

If there is an infinitesimal change in the direction $\mathbf{r}_1$ of the incident ray whilst still passing through the point o (such that the incident ray still represents an infinitesimal beam bundle with the focusing point o), then the refracted ray will generally change both its direction $\mathbf{r}_2$ and its spatial position (because of the change of the intersection point s). On the refracted ray, however, a position x_B may be found at which the ray suffers from no changes in a certain coordinate (y_B or z_B) despite the change in the incident ray. This position on the refracted ray may be taken as the conditional focusing point of the corresponding refracted beam bundle. It is obvious that x_B depends on whether there is no change in the y- or z-coordinate on the refracted ray. Clearly, in consideration of both of the y and z coordinates, there exist two focusing points on the refracted beam bundle. They correspond, for a specific coordinate system (which will be shown below), to the focusing points of the meridional and sagittal plane of the incident beam. These points will be calculated in following section.

On the refracted ray given by Eq. (9) the condition for an unchangeable y-coordinate despite the change in incident ray is as follows:

$$\frac{\partial y}{\partial r_{1y}}=0 \tag{12}$$

where the change in the direction of the incident ray is accomplished by varying the y-component of the unit vector $\mathbf{r}_1$. In the following derivations the z-component of the unit vector $\mathbf{r}_1$ will be assumed to be constant. The change in r_{1y} will then cause a simultaneous change in r_{1x}.

Inserting Eq. (9) for $y=f(x)$ in Eq. (12) yields:

$$\frac{\partial y_s}{\partial r_{1y}}+x_B\cdot\frac{\partial}{\partial r_{1y}}\left(\frac{r_{2y}}{r_{2x}}\right)=0 \tag{13}$$

From Eq. (8) we obtain

$$\frac{\partial y_s}{\partial r_{1y}}=-x_0\cdot\frac{\partial}{\partial r_{1y}}\left(\frac{r_{1y}}{\sqrt{1-r_{1y}^2-r_{1z}^2}}\right)=-x_0\cdot\frac{1-r_{1z}^2}{r_{1x}^3} \tag{14}$$

Now it is known that

$$\frac{r_{2y}}{r_{2x}}=\frac{n_1}{n_2}\cdot\frac{r_{1y}}{\sqrt{1-(n_1/n_2)^2r_{1y}^2-(n_1/n_2)^2r_{1z}^2}} \tag{15}$$

and differentiation of this leads to:

$$\frac{\partial}{\partial r_{1y}}\left(\frac{r_{2y}}{r_{2x}}\right)=\frac{n_1}{n_2}\cdot\frac{1-r_{2z}^2}{r_{2x}^3} \tag{16}$$

If Eqs. (14) and (16) are now inserted into Eq. (13), the solution for x_B follows as:

$$x_B=x_0\cdot\frac{n_2}{n_1}\cdot\frac{r_{2x}^3}{r_{1x}^3}\cdot\frac{1-r_{1z}^2}{1-r_{2z}^2} \tag{17}$$

This is the position on the refracted ray, at which there is no change in y-coordinate despite the change in direction of the incident ray. The point x_B may be seen as the focusing point of the corresponding refracted laser beam bundle, however, conditionally for y-coordinate. As seen from Eq. (17), x_B obviously depends on the spatial orientation of the incident beam bundle. For the case where the x–y plane is coincident with or is parallel to the meridional plane of the incident beam, then, because of the condition $r_{1z}=r_{2z}=0$, Eq. (17) becomes (for the meridional plane):

$$x_{BM}=x_0\cdot\frac{n_2}{n_1}\cdot\frac{r_{2x}^3}{r_{1x}^3} \tag{18}$$

For the case where the x–y plane is rotated around the x-axis by 90°, then Eq. (17) can be used to calculate another focusing point of the laser beam, that is the focusing point in the sagittal plane. Because of $r_{1y}=r_{2y}=0$, i.e. $1-r_{1z}^2=r_{1x}^2$ and $1-r_{2z}^2=r_{2x}^2$, Eq. (17) becomes (for the sagittal plane):

$$x_{BS}=x_0\cdot\frac{n_2}{n_1}\cdot\frac{r_{2x}}{r_{1x}} \tag{19}$$

The distance between both focusing points is then given by:

$$\Delta x_m=x_{BM}-x_{BS}=x_0\frac{n_2}{n_1}\left(\frac{r_{2x}^3}{r_{1x}^3}-\frac{r_{2x}}{r_{1x}}\right) \tag{20}$$

The astigmatic difference (or the spatial distance between both points) is calculated to be:

$$\Delta s_m=\frac{x_{BM}-x_{BS}}{r_{2x}}=x_0\cdot\frac{n_2}{n_1}\cdot\frac{r_{2x}^2-r_{1x}^2}{r_{1x}^3} \tag{21}$$

Because the focusing points of the sagittal and meridional planes no longer occur at the same place, there exists at each focusing point a focusing line. This focusing line changes its orientation as observed from one focusing point (x_{BM}) to the another (x_{BS}). For a finite solid beam bundle, such as the laser beam of an LDA-system, the elliptical cross-section of the beam between x_{BM} and x_{BS} may be observed. Details of this may be calculated from the third-order theory (Ghatak and Thyagarajan 1978). The existence of an elliptical cross-section, instead of the single focusing line considered above, derives from the fact of the solid laser beam (laser lights do not only occur in the meridional and sagittal planes), the effect of "coma", and other third-order aberrations. The aberration caused by coma, however, is much smaller than the aberration caused by astigmatism and therefore will not influence the focusing points calculated above.

In LDA measurements the change in cross section of the refracted laser beam from one focusing point to another influences the uniformity of the fringe spacing in the measurement volume which is formed by the laser beams. This relies on the deviations of the focusing points x_{BM} and x_{BS} from the measurement volume. Detailed calculations of such deviations will be given in Sect. 4.

x_0 can be resolved From Eq. (19) and inserted in Eq. (20). With respect to Eqs. (10) and (11) Eq. (20) for the distance between meridional and sagittal focusing points becomes:

$$\Delta x_m=x_{BS}\frac{r_{2x}^2-r_{1x}^2}{r_{1x}^2}=x_{BS}\cdot\frac{n_2^2-n_1^2}{n_2^2}\cdot\frac{1-r_{1x}^2}{r_{1x}^2} \tag{22}$$

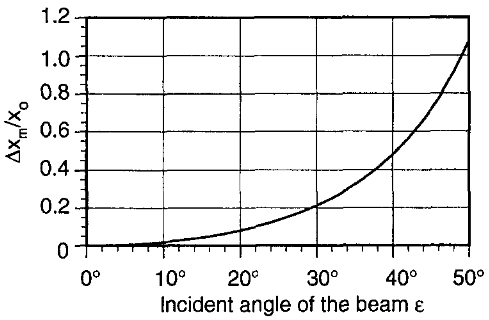

Fig. 3. Displacement between meridional and sagittal focusing points of a beam bundle in dependence on the incident angle of the beam by single refraction ($n_2/n_1 = 1.33$)

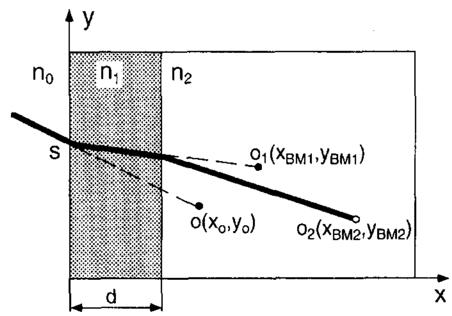

Fig. 4. Multiple refractions of a focusing beam bundle in meridional plane

In the above equations the unit vector components r_{1x} and r_{2x} may be replaced by:

$$r_{1x} = \cos \varepsilon_1 \tag{23}$$

$$r_{2x} = \cos \varepsilon_2 \tag{24}$$

with ε_1 and ε_2 as the incident and refracted angle of the laser beam, respectively. Equation (22) then becomes:

$$\Delta x_m = x_{BS} \cdot \frac{n_2^2 - n_1^2}{n_2^2} \tan^2 \varepsilon_1 \tag{25}$$

that is:

$$\Delta x_m = (x_{BM} - \Delta x_m) \cdot \frac{n_2^2 - n_1^2}{n_2^2} \tan^2 \varepsilon_1 \tag{26}$$

and

$$\Delta x_m = x_{BM} \cdot \frac{(n_2^2 - n_1^2) \cdot \tan^2 \varepsilon_1}{n_2^2 + (n_2^2 - n_1^2) \cdot \tan^2 \varepsilon_1}. \tag{27}$$

These equations may also be derived directly from Eq. (1) with $d = 0$ (for single refraction) and $\alpha/2 \to 0$ (Zhang 1995). The condition $\alpha/2 \to 0$ suggests that the macro light bundle comprising all 4 laser beams of the probe and described by Eq. (1) will turn over to a infinitesimal focusing beam bundle. The comparison between Eq. (1) and Eq. (27) has been given by Zhang and Eisele (1995b) to show the influence of the beam crossing angle ($\alpha/2$) on Δx_m.

Figure 3 shows the dependence of the displacement between the meridional and sagittal focusing points, relative to x_0, on the incident angle of a focusing beam bundle for single refraction on an air–water interface ($n_2/n_1 = 1.33$). It can be seen that at large incident angle of the laser beam the displacement between both focusing points is of the same order as the depth of the initial focusing point x_0.

3
Multiple refraction of the focusing beam bundle

Multiple refraction of the focusing beam bundle may often be found in LDA measurements in internal flow, where the laser beams have always to pass through glass windows. For such cases the focusing points corresponding to the meridional and sagittal planes of the beam bundle will be calculated in line with Fig. 4. In order to simplify the presentation only the meridional plane of the beam bundle is sketched in Fig. 4. The initial focusing point of the incident beam is fixed at point $o(x_0, y_0)$. To calculate the focusing point o_2 of the refracted beam bundle in the medium 2 the focusing point o_1 of the refracted beam bundle in medium 1 has first to be calculated.

From Eqs. (18) and (19) the meridional and sagittal focusing points of the beam bundle in medium 1 can be calculated as:

$$x_{BM1} = x_0 \cdot \frac{n_1}{n_0} \cdot \frac{r_{1x}^3}{r_{0x}^3} \tag{28}$$

$$x_{BS1} = x_0 \cdot \frac{n_1}{n_0} \cdot \frac{r_{1x}}{r_{0x}} \tag{29}$$

These points are virtual. They will be utilised to calculate the focusing points of refracted beam bundle in medium 2.

Equations (18) and (19) will be applied again to the beam which is refracted on the interface 1–2:

$$x_{BM2} = (x_{BM1} - d) \cdot \frac{n_2}{n_1} \cdot \frac{r_{2x}^3}{r_{1x}^3} + d \tag{30}$$

$$x_{BS2} = (x_{BS1} - d) \cdot \frac{n_2}{n_1} \cdot \frac{r_{2x}}{r_{1x}} + d \tag{31}$$

with d as the thickness of the medium 1.

The displacement between both focusing points is then:

$$\begin{aligned} \Delta x_m = x_{BM2} - x_{BS2} &= \frac{n_2}{n_1}\left(x_{BM1} \cdot \frac{r_{2x}^3}{r_{1x}^3} - x_{BS1} \cdot \frac{r_{2x}}{r_{1x}}\right) - d \cdot \frac{n_2}{n_1}\left(\frac{r_{2x}^3}{r_{1x}^3} - \frac{r_{2x}}{r_{1x}}\right) \\ &= x_0 \frac{n_2}{n_0}\left(\frac{r_{2x}^3}{r_{0x}^3} - \frac{r_{2x}}{r_{0x}}\right) - d \cdot \frac{n_2}{n_1}\left(\frac{r_{2x}^3}{r_{1x}^3} - \frac{r_{2x}}{r_{1x}}\right) \end{aligned} \tag{32}$$

The same calculation procedure will be given, if a beam bundle is refracted more than two times. In particular, if $d = 0$, Eq. (32) takes the form of Eq. (20) for the single refraction of a focusing beam bundle.

Equation (32) may also be derived from Eq. (1) by assuming $\alpha/2 \to 0$ (Zhang 1995).

4
Determination of measurement volume

In LDA-measurements there are at least two laser beams that form the measurement volume through their intersection. Each laser beam will suffer from astigmatism, if it has to be refracted for measuring an internal flow. The existence of astigmatism

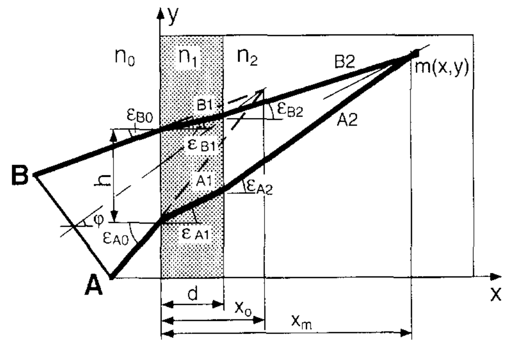

Fig. 5. Formation of the measurement volume by laser beams with multiple refractions

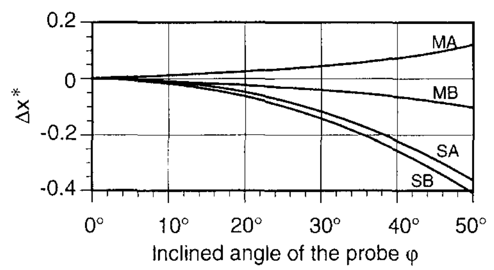

Fig. 6. Relative deviations of meridional and sagittal focusing points from the measurement volume by single refraction of laser beams at the probe with $\alpha/2 = 2.72°$ (MA: meridional focusing point of the laser beam A; MB: meridional focusing point of the laser beam B; SA: Sagittal focusing point of the laser beam A; SB: Sagittal focusing point of the laser beam B)

results in spatial deviations of beam focusing points (meridional and sagittal) from the measurement volume. To calculate these deviations the measurement volume has to be related to the initial focusing point of incident beams x_0. For the general case, a one-dimensional LDA system with two laser beams at off-axis alignment will be considered. To ensure the intersection between both laser beams the off-axis alignment of the probe has to be executed exactly in the plane of both laser beams (Zhang and Eisele 1995). This determines a two-dimensional laser beam transmittance as shown in Fig. 5 specifically for a configuration with double beam refractions. The refracted laser beams in the first medium (often a glass window) are denoted there by A_1 and B_1, respectively, while they are denoted in the test medium by A_2 and B_2.

For the first beam refractions (on the interface 0–1) with a beam distance h the following relationship may be written directly:

$$x_0(\tan\varepsilon_{A0} - \tan\varepsilon_{B0}) =$$
$$d(\tan\varepsilon_{A1} - \tan\varepsilon_{B1}) + (x_m - d)\cdot(\tan\varepsilon_{A2} - \tan\varepsilon_{B2}) = h \qquad (33)$$

This equation is based on the fact that from the first interface (0–1) to the measurement volume in the test medium both laser beams (A and B) converge a distance equal to h.

The possibility of calculating the deviations of diverse focusing points of laser beams from the measurement volume is of great relevance for further investigations of the fringe distortion in the measurement volume. In all previous investigations, improper design of the optical systems has been taken as the main reason for fringe distortion. Clearly astigmatism in relation to the beam refractions also influences the fringe uniformity in the measurement volume. Calculations of this effect will be given below by considering single and multiple refractions of laser beams, respectively.

4.1
Measurement volume for single refraction of laser beams

A single refraction of laser beams is often found in measurements in open channel flow or in channel flow where the refractive index of the fluid matches that of the window. In such cases Eq. (33) can be simplified (with 1 and 2 as the medium index) to:

$$\frac{x_0}{x_m} = \frac{\tan\varepsilon_{A2} - \tan\varepsilon_{B2}}{\tan\varepsilon_{A1} - \tan\varepsilon_{B1}} \qquad (34)$$

which represents exactly the ratio between the movement of the probe and the movement of the measurement volume in the flow (Zhang and Eisele 1995a).

Insertion of Eq. (34) into Eqs. (18) and (19), respectively, yields:

$$\Delta x_M^* = \frac{x_{BM} - x_m}{x_m} = \frac{n_2}{n_1}\cdot\frac{r_{2x}^3}{r_{1x}^3}\cdot\frac{\tan\varepsilon_{A2} - \tan\varepsilon_{B2}}{\tan\varepsilon_{A1} - \tan\varepsilon_{B1}} - 1 \qquad (35)$$

$$\Delta x_S^* = \frac{x_{BS} - x_m}{x_m} = \frac{n_2}{n_1}\cdot\frac{r_{2x}}{r_{1x}}\cdot\frac{\tan\varepsilon_{A2} - \tan\varepsilon_{B2}}{\tan\varepsilon_{A1} - \tan\varepsilon_{B1}} - 1 \qquad (36)$$

These two expressions depict, respectively, the relative deviations of the meridional and the sagittal focusing points of one laser beam (specified by r_{1x} before and r_{2x} after the refraction) from the measurement volume. For two laser beams forming the measurement volume these deviations are shown in Fig. 6. As can be seen, the sagittal focusing points of both laser beams are further away from the measurement volume than the meridional focusing points.

4.2
Measurement volume for multiple refractions of laser beams

Multiple refractions of laser beams are found in measurements in internal flow where at least one glass window is present. Following a similar procedure as shown in Sect. 4.1, however, with the window thickness $d \neq 0$ (Fig. 5), the initial focusing point x_0 from Eq. (33) will be inserted in Eqs. (28) and (29), respectively. From Eq. (30) as well Eq. (31) the relative deviations of respective focusing points from the measurement volume can be derived to be:

$$\Delta x_M^* = \frac{(x_{BM2} - d) - (x_m - d)}{x_m - d}$$
$$= \frac{n_2}{n_0}\cdot\frac{r_{2x}^3}{r_{0x}^3}\cdot\frac{\tan\varepsilon_{A2} - \tan\varepsilon_{B2}}{\tan\varepsilon_{A0} - \tan\varepsilon_{B0}} - 1$$
$$+ \left(\frac{n_2}{n_0}\cdot\frac{r_{2x}^3}{r_{0x}^3}\cdot\frac{\tan\varepsilon_{A1} - \tan\varepsilon_{B1}}{\tan\varepsilon_{A0} - \tan\varepsilon_{B0}} - \frac{n_2}{n_1}\cdot\frac{r_{2x}^3}{r_{1x}^3}\right)\frac{d}{x_m - d} \qquad (37)$$

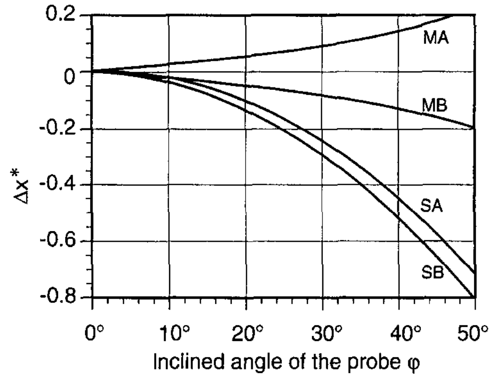

Fig. 7. Relative deviations of diverse focusing points from the measurement volume by multiple refractions ($x_m/d=2$) of laser beams with $\alpha/2=2.72°$ (for MA, MB, SA, SB see Fig. 6)

$$\Delta x_S^* = \frac{(x_{BS2}-d)-(x_m-d)}{x_m-d}$$

$$= \frac{n_2}{n_0}\cdot\frac{r_{2x}}{r_{0x}}\cdot\frac{\tan\varepsilon_{A2}-\tan\varepsilon_{B2}}{\tan\varepsilon_{A0}-\tan\varepsilon_{B0}} - 1$$

$$+\left(\frac{n_2}{n_0}\cdot\frac{r_{2x}}{r_{0x}}\cdot\frac{\tan\varepsilon_{A1}-\tan\varepsilon_{B1}}{\tan\varepsilon_{A0}-\tan\varepsilon_{B0}} - \frac{n_2}{n_1}\cdot\frac{r_{2x}}{r_{1x}}\right)\frac{d}{x_m-d} \quad (38)$$

with x_m-d as the depth of the measurement volume in the test medium (Fig. 5).

These deviations all depend on the inclined angle of the probe and on the depth of the measurement volume in the test medium. In the case of very small window thickness ($d/x_m \ll 1$) the Eqs. (37) and (38) may be simplified, respectively, to Eqs. (35) and (36) for single refractions.

Taking into account both laser beams forming the measurement volume, the relative deviations of 4 focusing points from the measurement volume are shown in Fig. 7 for the configuration with $\alpha/2=2.72°$ and $x_m/d=2$.

4.3
Astigmatism at the on-axis alignment of the probe

Most LDA-measurements in channel flow are carried out with on-axis alignment of the probe ($\varphi=0$). In such cases, there are negligible deviations of focusing points from the measurement volume. From Eqs. (35) and (36) the following quantities may be calculated (for beam crossing angle $\alpha/2=2.72°$ and $n_1/n_0=1.33$):

$$\Delta x_M^* = 0.001, \qquad \Delta x_S^* = 0$$

This signifies that there is no fringe distortion in the measurement volume to be expected, if laser beams are laid out symmetrically to the media interface.

5
Conclusions

Astigmatism caused by beam refractions in LDA measurements has been examined and shown to strongly influence the formation of the measurement volume and hence to affect the fringe uniformity. The general existence of astigmatism in the LDA technique leads to the deformations in the cross section of the refracted laser beams. Consequently there are two focusing points on each refracted laser beam corresponding to its meridional and sagittal planes. Because of this the measurement volume is not expected to form at the waists of the laser beams. The accuracy of flow measurements may then be affected, especially for the case with off-axis alignment of an LDA probe to the internal flow. The meridional and sagittal focusing points of each laser beam of an LDA system have been calculated and related to the position of the measurement volume. Both single and multiple refractions of the laser beams have been taken into account. It is shown that the effect of astigmatism can be as important as optical alignment errors in leading to fringe distortion. The exact calculations to determine the deviations of diverse focusing points from the measurement volume demonstrates the real conditions for further investigations with fringe distortions and their influence on the measurement accuracy.

References

Durst F; Stevenson WH (1975) Moiré patterns to visually model laser-Doppler signals. The Accuracy of Flow Measurements by Laser Doppler Methods. Proceeding of the LDA-Symposium Copenhagen, 183–205

Ghatak AK; Thyagarajan K (1978) Contemporary Optics. Plenum Press. New York and London

Hanson S (1975) Visualization of alignment errors and heterodyning constraints in laser Doppler velocimeters. The Accuracy of Flow Measurements by Laser Doppler Methods. Proceeding of the LDA-Symposium Copenhagen, 176–182

Miles PC; Witze PO (1994) Fringe field quantification in an LDA probe volume by use of a magnified image. Exp Fluids 16: 330–335

Ruck B (1991) Distortion of LDA fringe pattern by tracer particles. Exp Fluids 10: 349–354

Zhang Z (1993) Theoretische Untersuchungen der Einsatzmöglichkeit für 3-dimensionale Laser-Doppler-Messungen in Kanalströmungen. Sulzer Innotec Bericht. Nr.: STT. TB93.022, Winterthur, Schweiz

Zhang Z (1995) Einfluß des Astigmatismus auf Laser Doppler Messungen. Sulzer Innotec Bericht. Nr.: STT. TB95.022, Winterthur, Schweiz

Zhang Z; Eisele K (1995a) Off-axis alignment of an LDA-probe and the effect of astigmatism on the measurements. Exp Fluids 19: 89–94

Zhang Z; Eisele K (1995b) Einfluß des Astigmatismus auf LDA-Messungen. Gala-Tagung Rostock, Deutschland