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THE CRITICAL TENSILE STRESS CRITERION FOR CLEAVAGE

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ABSTRACT

An elastic–plastic analysis was used to accurately measure the microscopic cleavage strength σ_f^* of notched bars of high nitrogen steel in bending. It was found that σ_f^* increases as the root radius of the notch ρ decreases. For $\rho > 0.010''$, the variation of σ_f^* with ρ, and the difference between σ_f^* and the cleavage fracture strength of a plane tensile specimen, σ_f, may result from a statistical effect, due to differences in the volume of highly stressed material in the plastic zone. For $\rho < .010''$, the primary reason for the apparent increase in σ_f^* with decreasing ρ, is the steep stress gradient at the notch tip, which forces the critical plastic zone size to extend further to insure that unstable microcrack propagation can occur. Both the statistical and stress gradient effects have been quantitatively evaluated and found to be in good agreement with the experimental data.

INTRODUCTION

When a notched bar is loaded in plane strain bending, plastic zones form at the notch root and spread into the ligament. These zones have the form of logarithmic spirals[1,2] predicted by the slip line field theory of Hill[3]. For perfectly plastic solids the longitudinal stress within the plastic zone (x < R, Figure 1a) is given by

$$\sigma_{yy} = \sigma_Y \left[1 + \ln \left(1 + \frac{x}{\rho}\right) \right] \qquad \begin{matrix} 0 < x < R \\ R \leqslant R_\beta \end{matrix} \tag{1}$$

where ρ is the root radius and σ_Y is the tensile yield strength, assuming a Tresca yield criterion. The maximum value of σ_{yy}, (σ_{yy}^{max}), occurs at the elastic–plastic interface (x = R)

$$(\sigma_{yy}^{max}) = \sigma_Y \left[1 + \ln \left(1 + \frac{R}{\rho}\right) \right] \qquad (0 < R < R_\beta) \tag{2}$$

Equations (1) and (2) are only valid for $R < R_\beta$. For $R \geqslant R_\beta$, σ_{yy}^{max} remains constant, at its maximum possible value (Figure 1b) $(\sigma_{yy}^{max})^{max}$, which depends only on the flank angle of the notch ω (Figure 1b)

$$(\sigma_{yy}^{max})^{max} = \sigma_Y \left[1 + \frac{\pi}{2} - \frac{\omega}{2} \right] \qquad (R_\beta < x < R) \tag{3}$$

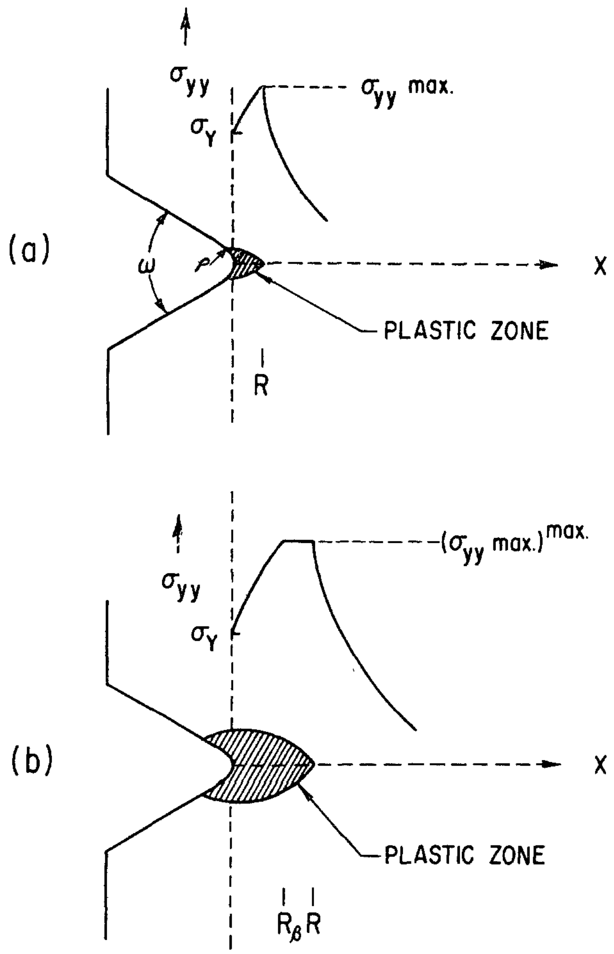

Figure 1. The variation of longitudinal stress σ_{yy} ahead of a notch for (a) plastic zone size $R < R_\beta$ and (b) $R > R_\beta$.

For $x > R$ the material is elastic and σ_{yy} decreases.

For a given notch geometry, R and σ_{yy} are unique functions of the applied bending moment M. Recently, we have developed(2) a relatively simple procedure for determining the relation between R (more exactly, R/ρ) and M/σ_Y. This is accomplished by equating the applied bending moment to the sum of the internal elastic and plastic bending moments, since the latter are unique functions of R. Figure 2 shows the excellent agreement between theory and experiment for plastic zone measurements performed on Charpy type specimens of mild steel, having variable root radii.

There is substantial evidence(2,4–7) that at cryogenic temperatures cleavage fracture occurs in notched specimens of mild steel when σ_{yy}^{max} builds up to some critical value, σ_f^*, that is determined primarily by microstructure and hence is a 'microscopic' strength. At this stage, the plastic zone size has reached a critical value R_F which is obtained from equation (2) by setting $\sigma_{yy}^{max} = \sigma_f^*$ and $R = R_F$.

$$\frac{R_F}{\rho} = \exp\,[\sigma_f^*/\sigma_Y - 1] - 1 \qquad (4)$$

$$(R_F < R_\beta)$$

On a microscopic scale, the fracture moment M_F is that value of M which is required to spread the plastic zone out to a distance R_F ahead of the notch(2).

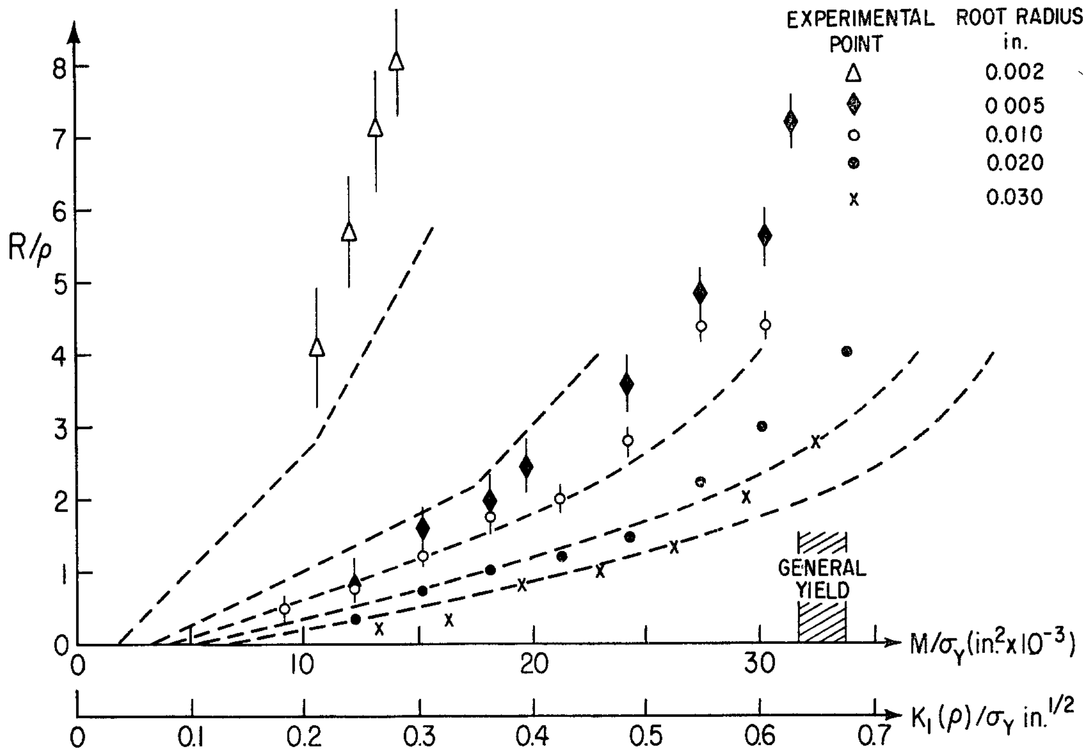

Figure 2. Comparison between experimental and theoretical dependence of R/ρ with M/σ_Y, for Charpy type specimens of varying root radii(2).

The importance of this approach is that it allows the determination of the cleavage strength σ_f^* in materials that are normally ductile in simple tension tests, even at cryogenic temperatures, but which fracture by brittle cleavage when they are in the form of thick structures containing sharp notches. This permits a quantitative assessment of the effect of alloying additions or heat treatment (e.g., a temper embrittlement treatment) on the cleavage strength σ_f^*(8). All that is required is 1) a measurement of the fracture moment M_F at a given temperature and 2) a measurement of the tensile yield strength σ_Y at that temperature, when σ_Y is evaluated at a strain rate corresponding to that which exists in the plastic zone ahead of the notch.* Then, from critical curves such as those shown in Figure 2 for a *particular* notch geometry, it is possible to evaluate R_F/ρ from the measured M_F/σ_Y and, from equation (4), determine the value of σ_f^*/σ_Y. σ_f^* is then determined, knowing the value of σ_Y from the tensile test. It should be noted that these tests must be performed at a sufficiently low temperature or high strain rate such that fracture occurs before general yielding and also that $\sigma_Y > \dfrac{\sigma_f^*}{1 + \pi/2 - \omega/2}$, since equation (4) only applies in the region ($R < R_\beta$) before maximum triaxiality is reached.

THE PHYSICAL SIGNIFICANCE OF σ_f^*

In our previous work(2) on hot rolled steel we observed that at –196, –180 and –170°C,

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* For example, for standard Charpy V notch geometry loaded with a cross head velocity of 0.1″/min. the strain rate in the plastic zone is about 4.8×10^{-1}/min.

R_F/ρ, and hence σ_f^*, appeared (within the experimental scatter) to be independent of ρ for ($0.010'' < \rho < 0.050''$). We also observed that $\sigma_f^* = 194 \pm 3$ ksi was independent of temperature in this range. However, we also noted that for very sharp notches $\rho < 0.010''$, R_F/ρ (and hence σ_f^*) increased sharply as ρ decreased and that this increase was responsible for the existence of a 'limiting effective radius', ρ_o(9–11), below which M_F and $K_{Ic}(\rho)$ remained constant as ρ decreased. In addition, we, as well as other workers(4,5), have observed that σ_f^* was about 60% greater than the cleavage strength σ_f of a plane tensile specimen. Since the latter can be thought of as a shallow notched specimen with $\rho = 1.0''$ (the gauge length) or greater, this observation indicates that σ_f^* also decreases with increasing ρ at large values of ρ. The present investigation was undertaken to investigate the effect of ρ on σ_f^* in more detail.

Notched bars of Charpy geometry of a hot rolled, high nitrogen steel, similar to that used previously, were fractured in slow bending at temperatures between –196°C and –150°C. The average grain diameter in the present investigation was 0.00125 inches and the root radius of the notched bars was varied from 0.002 inches to 0.050 inches. Values of R_F/ρ were obtained by the method outlined above and these are plotted in Figure 3 as a function of $\ln(1/\rho)$. One notes that

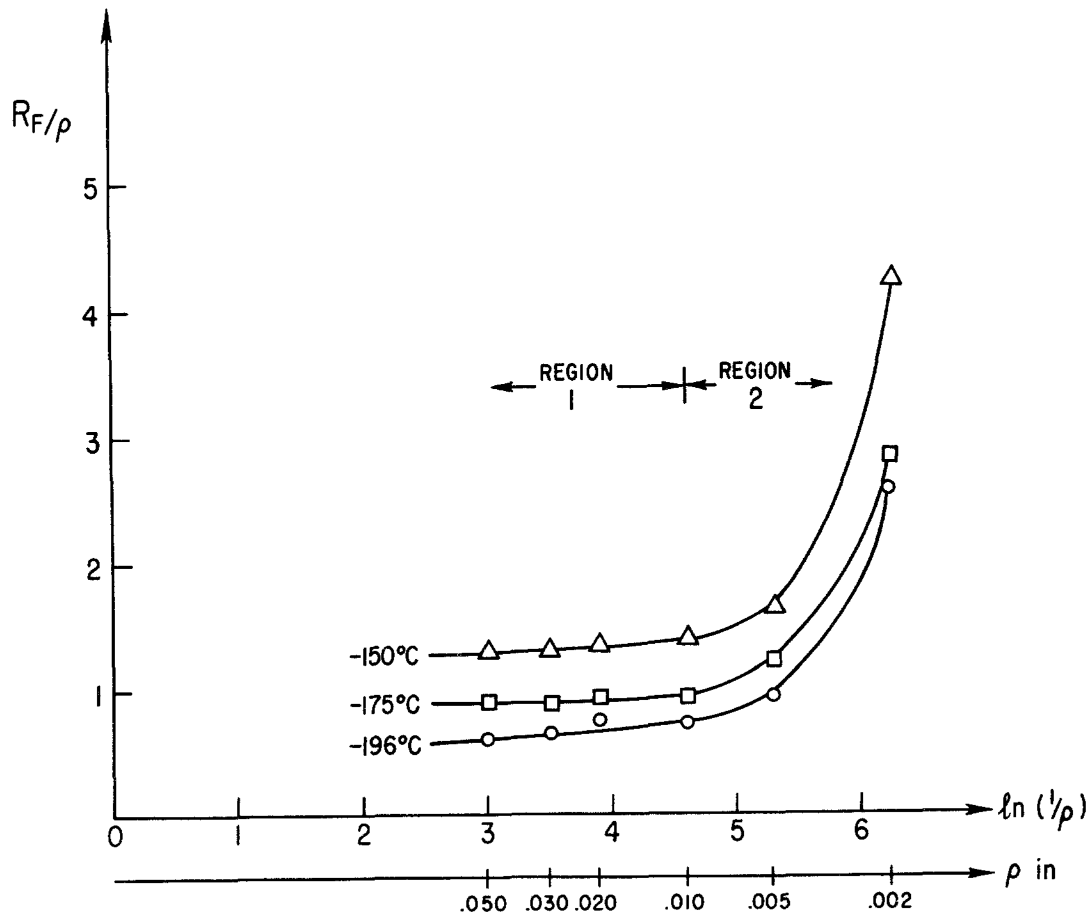

Figure 3. Experimentally determined values of R_F/ρ as a function of $\ln(1/\rho)$, at –196, –175 and –150°C.

there are two distinct regions. First, for $\rho > 0.010''$, there is a small but definite increase in R_F/ρ, and hence in σ_f^*, with decreasing ρ. Second, for $\rho < 0.010''$, (Region 2), R_F/ρ, and hence σ_f^*, increase sharply as ρ decreases. Values of σ_f^* were computed for each of the R_F/ρ data points according to equation (4) and within experimental scatter σ_f^* again appeared to be temperature independent between −196 and −150°C. Figure 4 shows the measured values of σ_f^* and the two

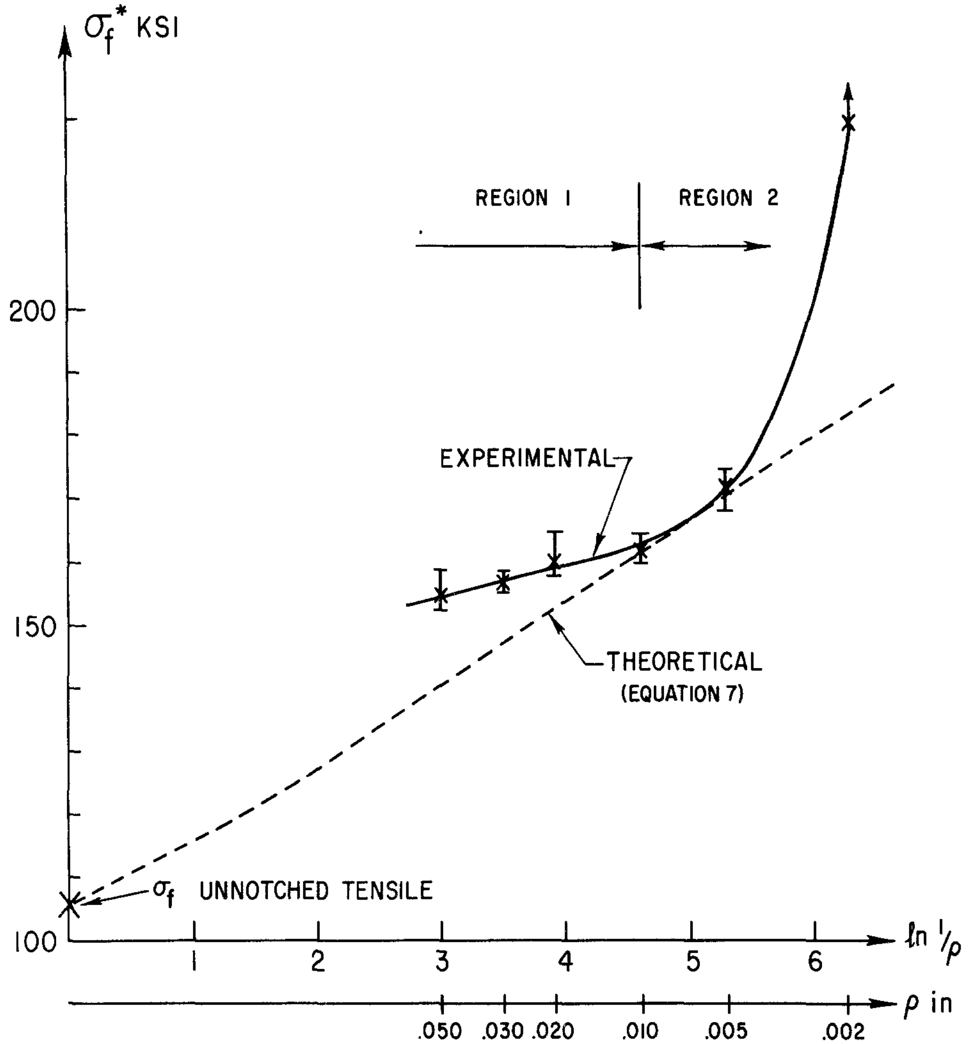

Figure 4. Experimentally determined and calculated (equation 7) variation of σ_f^* with $\ell n\,(1/\rho)$.

regions where σ_f^* varies slightly and sharply with ρ. We shall now discuss each of these regions in turn.

Region 1 – σ_f^* increasing slightly with decreasing ρ.

Cleavage fracture in mild steel requires that three critical steps take place[(8)]: a) microcrack nucleation, b) microcrack propagation through the grain in which the crack was nucleated and c) microcrack propagation through the boundaries that surround the nucleating grain. In materials that contain brittle grain boundary carbides (e.g., mild steel) steps (a) and (b) are easier than (c) and consequently non–propagating cracks may be observed in fractured specimens[(12–14)]. Once a crack breaks through the first boundary that it encounters, and spreads to the next boundary in the presence of a constant longitudinal stress field, the microcrack will have a higher stress intensity factor and it will be able to overcome this boundary as well, and successive boundaries, causing unstable fracture. At cryogenic temperatures, where microcrack linking by localized plastic flow does *not* precede fracture, the stress level at which unstable fracture occurs is that stress at which *one* suitably oriented microcrack can continue spreading through the boundaries that surround the nucleating grain. Consequently, a necessary condition for fracture in notched bars is that $\sigma_{yy} = \sigma_f^*$ *over at least one grain diameter in the plastic zone ahead of the notch.*

Since a certain number of cracks will nucleate before *one* crack is formed that can accomplish step (c), and cause unstable fracture, we would therefore expect σ_f^* to be influenced by statistical considerations; the larger the volume of material under stress, the larger will be the number of cracks that form and the greater will be the probability of nucleating one crack that can cause instability. Recent data[14] indicate that the microcrack density in plane tensile specimens of low carbon iron (number of cracks/in^3) varies as the 9th power of the tensile stress, over a wide range of test conditions. At −196°C, for example, the fracture stress in that material coincides with the tensile stress required to produce 1700 cracks. Thus, the fracture criterion

$$\text{(cracks/volume) (volume of specimen)} = C \tag{5}$$

where C is assumed to be a constant independent of stress at a particular temperature, for a particular material, and (cracks/volume) = $A\sigma^n$ where n = 9 for low carbon iron. On this basis, a large fraction of the difference between σ_f^* and σ_f, and the variation of σ_f^* with ρ in region (1), Figure 4, can be attributed to the difference between the volume of the plane tensile specimen V_t and the volume of the stressed region ahead of the notch, V_N. At a given temperature, equation (5) gives the ratio of σ_f^* and σ_f as

$$\sigma_f^* = \sigma_f \left(V_t/V_N\right)^{1/n} \tag{6a}$$

which is the same as Weibull's[15] law.

Suppose that the highly stressed region ahead of the notch has a gauge length ℓ, a gauge width equal to two grain diameters 4d, and a thickness t equal to the thickness of the structure, so that

$$\sigma_f^* = \sigma_f\left(\frac{V_t}{4d \cdot \ell \cdot t}\right)^{1/n} \qquad \begin{pmatrix} \ell = \rho \text{ for } \rho \geqslant 2d \\ \ell = 2d \text{ for } \rho < 2d \end{pmatrix} \tag{6b}$$

The gauge length ℓ is approximately equal to the root radius ρ for ρ greater than the grain size; however, since the minimum critical volume element is one grain, ℓ is equal to 2d for $\rho < 2d$.

In the present work, the fracture stress of the plane tensile specimen was 106 ksi at −196°C. Plastic flow was observed over only 10% of the 1″ gauge length, and since the specimen thickness was 0.070 inch and the specimen width was 0.22 inch, $V_t = 1.54 \times 10^{-3}$ in^3. For the notched bend specimens, ρ was always greater than the grain size 2d = 0.00125″ so that $\ell = \rho$ in eqn. (6b). Then, since t = 0.394″, the calculated value of σ_f^* is

$$\sigma_f^* = 106\left(\frac{1.57}{\rho}\right)^{1/n} \tag{7}$$

The value of n in this material is not known. Figure 4 indicates that in region 1, equation (7) is in fairly good agreement with the experimentally determined value of σ_f^* for n = 12. While further studies of the stress dependence of microcrack formation for this material need to be carried out to determine n exactly, and whether C is stress dependent, there is a strong indication that the difference between σ_f^* and σ_f, and the variation of σ_f^* with ρ for relatively blunt cracks, can be attributed to a statistical size effect.

Region 2 – σ_f^* increasing sharply with decreasing ρ.

For small values of ρ, σ_f^* increases sharply as ρ decreases and statistical considerations alone (Figure 4) cannot account for this increase. As stated previously, this increase is extremely important from a practical viewpoint because it leads to the existence of a lower limiting radius, ρ_o, at which $K_{Ic}(\rho)$ reaches its limiting value, K_{Ic}, and further decreases in ρ below ρ_o produce no further decrease in K_{Ic}. We believe that the sharp increase in σ_f^* is an *apparent effect*, that occurs because σ_{yy}^{max} must equal σ_f^* over at least one whole grain diameter to allow microcrack propaga-

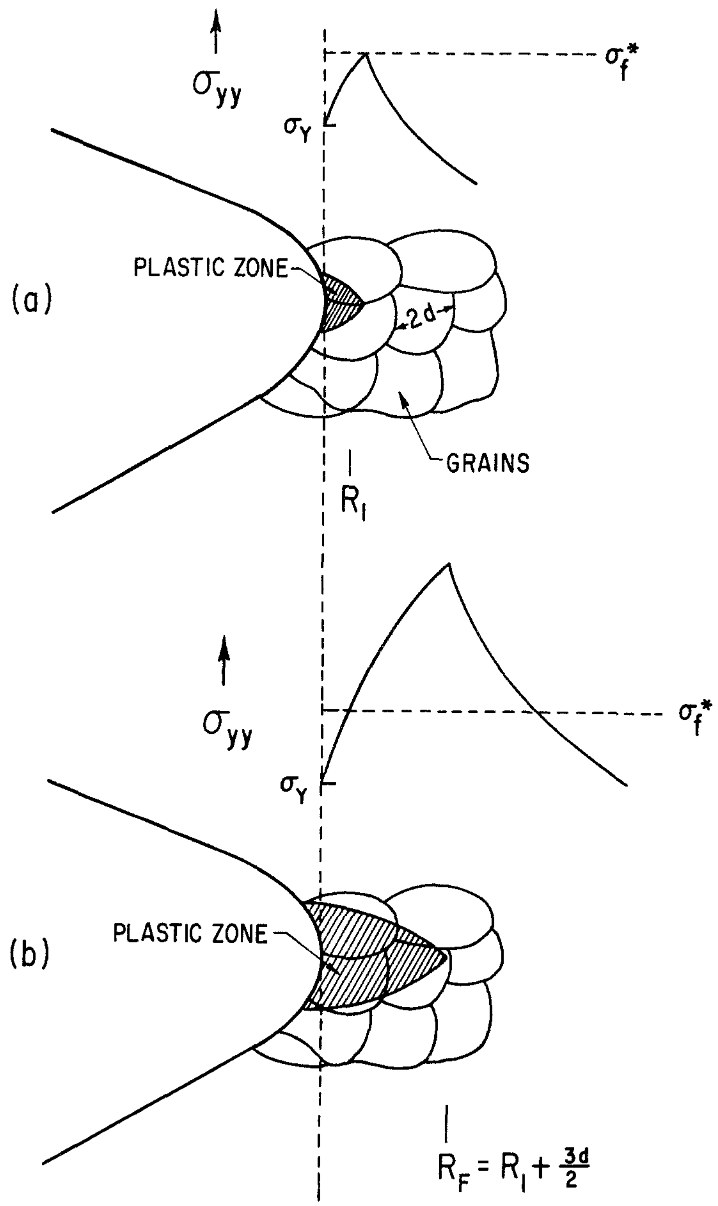

Figure 5. The effect of a steep stress gradient on the criterion for unstable fracture. See text for explanation.

tion through the boundaries that surround the nucleating grain and hence to allow unstable fracture. For example, when $\sigma_{yy}^{max} = \sigma_f$ at $R = R_1$ (Figure 5a), unstable fracture cannot occur because the *sharp stress gradient* causes $\sigma_{yy} < \sigma_f^*$ at the grain boundary through which the microcrack would have to spread. To a first approximation unstable fracture will occur if $\sigma_{yy} \geqslant \sigma_f^*$ over one whole grain in the plastic zone. Thus, according to the diagram shown in Figure 5b, it is assumed that the plastic zones must spread one and one–half grain diameters beyond R_1. The required value of R_F is then

$$R_F = R_1 + \frac{3}{2}(2d). \tag{8}$$

Since $\sigma_{yy}^{max} = \sigma_f^*$ at $R = R_1$, we have, from equation (4),

$$\frac{R_F}{\rho} = \frac{R_1}{\rho} + \frac{3}{2}\frac{(2d)}{\rho} = \exp\left[\sigma_f^*/\sigma_Y - 1\right] - 1 + (3d/\rho) \tag{9}$$

and introducing the statistical dependence of σ_f^* on ρ, (equation 6b) gives

$$\frac{R_F}{\rho} = \exp\left[\frac{\sigma_f}{\sigma_Y}\left(\frac{V_t}{4t \cdot \ell \cdot t}\right)^{1/n} - 1\right] - 1 + \frac{3}{2}\left(\frac{2d}{\rho}\right). \tag{10a}$$

$\ell = \rho$ for $\rho \geqslant 2d$

$\ell = 2d$ for $\rho < 2d$

Substituting the measured values of σ_f, σ_Y, V_t, 2d, t, and n at −196°C gives

$$\frac{R_F}{\rho} = \exp\left[\frac{106}{106}\left(\frac{1.57}{\rho}\right)^{1/12} - 1\right] - 1 + \frac{0.00375}{\rho}. \tag{10b}$$

The measured and calculated values of R_F/ρ at −196°C are compared in Figure 6. An exact analysis of the stress intensity factor of a microcrack in a sharp stress gradient is required to refine the calculation, but the trend is evident. For $\rho > 0.010''$ the term $3d/\rho$ has a relatively small effect on R_F/ρ unless the grain size is extremely coarse. This is convenient, since standard Charpy specimens ($\rho = 0.010''$) can generally be used to obtain a good measure of σ_f^* without introducing complications due to stress gradients. Alternatively, when $3d/\rho$ is large (very sharp cracks, very coarse grained materials), it will be the dominant term in equation (10) and the fracture criterion becomes

$$R_F \approx \frac{3}{2}(2d). \tag{11}$$

Physically, this means that plastic flow must occur over about one and one–half grain diameters ahead of the notch root before unstable fracture can occur. The limiting 'effective' radius, ρ_o, is then the value of ρ at which $3d/\rho$ becomes significantly large that equation (11) describes the fracture criterion.

SUMMARY

An elastic–plastic analysis was used to accurately measure the microscopic cleavage strength σ_f^* of notched bars of high nitrogen steel. It was shown that σ_f^* increases as the root radius of the notch ρ decreases. For $\rho > 0.010''$, the variation of σ_f^* with ρ, and the difference between σ_f^* and the cleavage fracture strength of a plane tensile specimen, σ_f, appears to result from a statistical

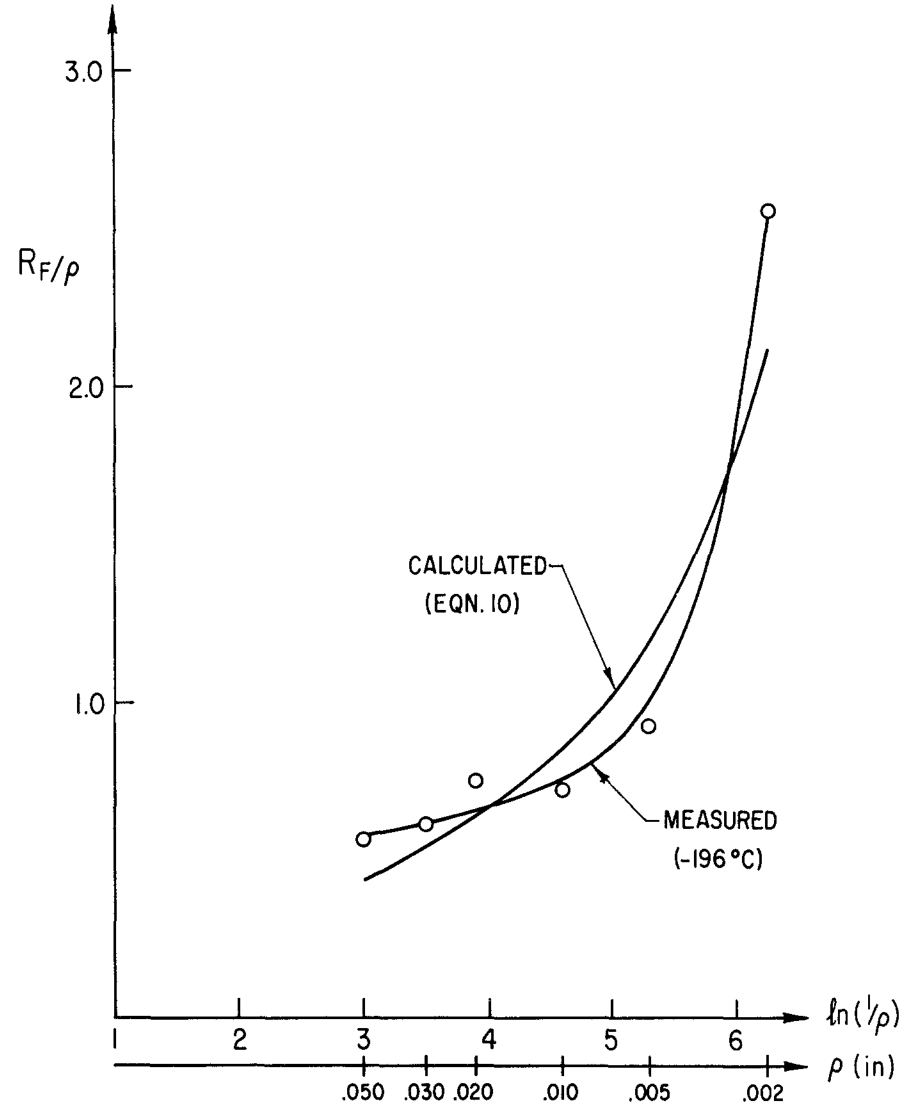

Figure 6. Measured (−196°C) and calculated (equation 10) dependence of R_F/ρ with ℓn $(1/\rho)$.

effect, due to differences in the volume of highly stressed material in the plastic zone. For $\rho < 0.010''$, the primary reason for the apparent increase in σ_f^* with decreasing ρ is the steep stress gradient at the notch tip, which forces the critical plastic zone size to extend further to insure that unstable microcrack propagation can occur. Both the statistical and stress gradient effects have been quantitatively evaluated and found to be in good agreement with the experimental data.

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RÉSUMÉ – Une analyse élastoplastique a été utilisée pour la mesure précise de la résistance microscopique du clivage σ_f^* de barreaux entaillées d'acier à haute teneur en azote et soumis à flexion.

On a trouvé que σ_f^* s'accroît lorsque diminue le rayon d'arrondi ρ à la racine de l'entaille. Lorsque ρ est supérieur à 0,25 mm, la variation de σ_f^* en fonction de ρ est due à un effet statistique dû aux différences de volume de matière soumis, dans la zone de déformation plastique, à des contraintes élevées. Il en est également de même l'écart entre σ_f^* et la contrainte de rupture par clivage d'une éprouvette de traction sans entaille.

Lorsque ρ est inférieur à 0,25 mm, la raison principale de l'accroissement de σ_f^* avec des valeurs de ρ décroissantes réside dans l'existence d'un gradient aigu des contraintes à la pointe de la fissure. Un tel gradient force à s'accroître les dimensions de la zone critique de déformation plastique, pour que puisse se produire une propagation instable d'une microfissure.

On a pu évaluer quantitativement ces effets statistiques d'une part et de gradients de contraintes d'autre part, et l'on s'est trouvé en accord satisfaisant avec les données expérimentales.

ZUSAMMENFASSUNG – Man benutzte eine elastisch–plastische Analyse, um die mikroskopische Spaltungsstärke σ_f^* von eingekerbten Stahlbarren mit hohem Stickstoffgehalt im Krümmen sorgfältig zu messen. Man stellte fest, dass σ_f^* sich vergrösserte während der Wurzelradius der Kerbe ρ sich verringerte. Da $\rho > 0.010''$ ist, ist die Variation von σ_f^* mit der Spaltungsfrakturstärke einer planaren Spannungsprobe, σ_f, durch einen statistischen Effekt und durch Unterschiede in dem Volumen von stark angespanntem Material in der plastischen Zone verursacht. Da $\rho > 0.010''$ ist, ist der ursprüngliche Grund für die augenscheinliche Zunahme in σ_f mit verringertem ρ, der tiefe Anspannungsgradiente an der Kerbenspitze, durch welche sich die kritische, plastische Zonengrösse weiter ausdehnen muss, um die labile Mikrorissausbreitung zu sichern. Die statistischen und Anspannungsgradienteffekte wurden quantitative ausgewertet und man fand gute Übereinstimmung mit den experimentellen Werten.