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FRACTURE PROPAGATION IN ORGANIC GLASSES

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ABSTRACT

A quantitative assessment of the density of hyperbolic markings in the fracture surfaces of polymethyl methacrylate has been made showing that the density is proportional to the fracture toughness. In general the fracture toughness at any instance is a random function, but there is a definite tendency for it to increase with velocity of fracture propagation.

It is suggested that the craze material ahead of the true crack tip may be assumed to have constant strength and Dugdale's model for elasto–plastic materials applied to organic glasses as well.

INTRODUCTION

The fracture surfaces of organic glasses (in particular that of polymethyl methacrylate) have an aesthetic appeal. Around the fracture origin, in a typical sharply notched tensile specimen, the surface is very smooth and is usually referred to as the 'mirror area'. This region exhibits a brilliant color effect, which Kambour[1,2] has shown to be produced by a thin layer of craze material of low refractive index which causes optical interference. Further away from the fracture origin, hyperbolic markings[3] are present in the fracture surface. These markings increase in density with distance from the origin until the surface becomes extremely rough with some cracks visible beneath the main fracture surface. If the specimen is large enough the fracture finally branches into two or more separate surfaces[4]. The hyperbolic markings represent the confluence of a secondary fracture, radiating outwards, with the main fracture front. Although such markings have been observed for many years, there has been no quantitative study of their density. It is intuitive to believe that the denser the markings on the fracture surface, the greater the fracture toughness will be.

The effective surface energies of organic, unlike inorganic, glasses are many orders of magnitude greater than the true surface energies[5]. The craze observed on the fracture surface precedes the tip of the true crack and absorbs a considerable portion of the energy expended on fracture[6]. Some energy will also be dissipated in viscous flow around the craze material. At low velocities of fracture propagation the fracture toughness may decrease with increase in velocity because the viscous work decreases when the strain rate at the tip of the crack increases. This effect could cause the instability of fracture propagation associated with wedging[7] and be responsible for the rib markings observed in such fractures. In tensile fracture this phase is very quickly passed and the fracture toughness increases with crack growth. That the fracture toughness increases, is evident in the increasing roughness of the fracture surface. Although the fracture toughness does increase with velocity of fracture it will be shown that it is not a function of velocity.

Craze material has been shown to be highly expanded polymer containing many voids[8]. Under the action of the stress ahead of the tip of the crack these voids will open and coalesce, but they will only grow appreciably when they are extremely near the tip of the crack, so that it is unlikely that the hyperbolic markings are caused by voids opening by viscous flow. However, some of the voids in the craze will propagate as cracks. When the fracture propagates at low velocities the length of the craze extending ahead of the crack is small and consequently the secondary fractures do not develop to an appreciable size before the fracture front arrives. However, some of these nascent fractures have been observed by Kambour[6]. If it is assumed that the origins of the secondary fractures which cause the hyperbolic markings exist in the craze material one must assume that the extent of the craze is much larger in those regions. However, a larger region of craze necessarily means a larger stress field, but in the first stages of fracture, the stress field does not grow larger, since the fracture toughness decreases. It is not, therefore, immediately apparent why the craze should ever grow in extent and give rise to large numbers of hyperbolic markings. The growth of the craze must be a self generating process. If the crack is obstructed by any means the stress field ahead of the crack will increase and the craze will grow to a larger extent than it would otherwise. More energy will have been expended which will slow the propagation of the fracture and still further increase the stress field. Thus a small disturbance can produce an avalanche.

EXPERIMENTS ON POLYMETHYL METHACRYLATE

The fracture specimens were rectangular and made from 1/8 in. thick commercial polymethyl methacrylate sheet. Two large specimens 65in. long by 48 in. wide were tested with central slits of 0.96 in. and 1.38 in. respectively. The central slits were cut from a 1/4 in. diameter hole with a 1/16 in. saw – the ends of the slits were finished with a 0.006 in. saw and natural cracks produced at

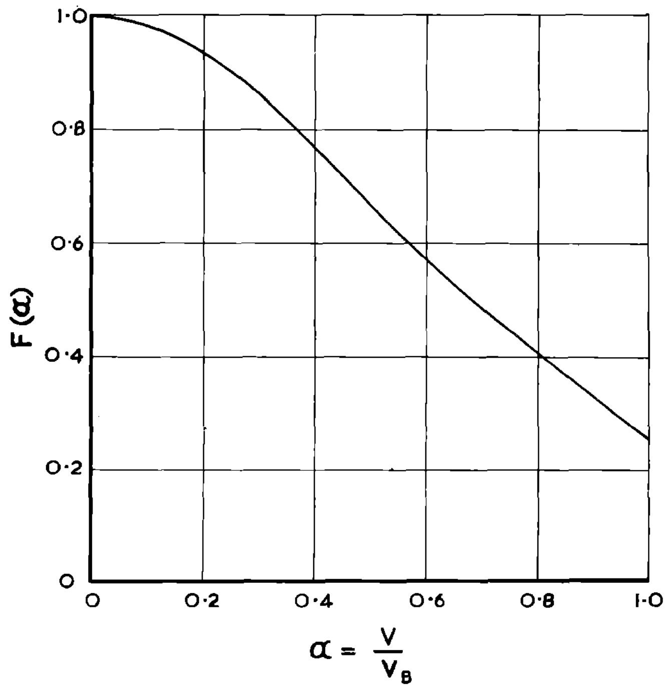

Fig. 1. Velocity modification to crack extension force.

their tips with a wedge. A number of smaller specimens 20 in. long by 12 in. wide were also used. These specimens all had edge slits. The specimens were all fractured in simple tension. In the large specimens strain gauges were used to measure the stress since it was not constant across the section dropping by about 10% from the center to the quarter point. Since results were taken only from the center section of the large specimens, where the fracture was not affected by waves reflected from the ends of the specimens, the stress at the center has been used in the calculations for the crack extension force.

The velocity of propagation of the fractures was measured by placing lines of a conducting paint (a suspension of fine silver particles in a solution of polymethyl methacrylate in chloroform) normal to the fracture path and photographing the voltage trace displayed on a cathode ray oscilloscope when these ruptured.

The crack extension force can be calculated from Broberg's analysis[9] of an infinite plate loaded in simple tension (σ) with a crack propagating at a constant velocity (V) and is expressed by

$$G = \frac{\sigma^2 \pi a}{E} F(\alpha) \tag{1}$$

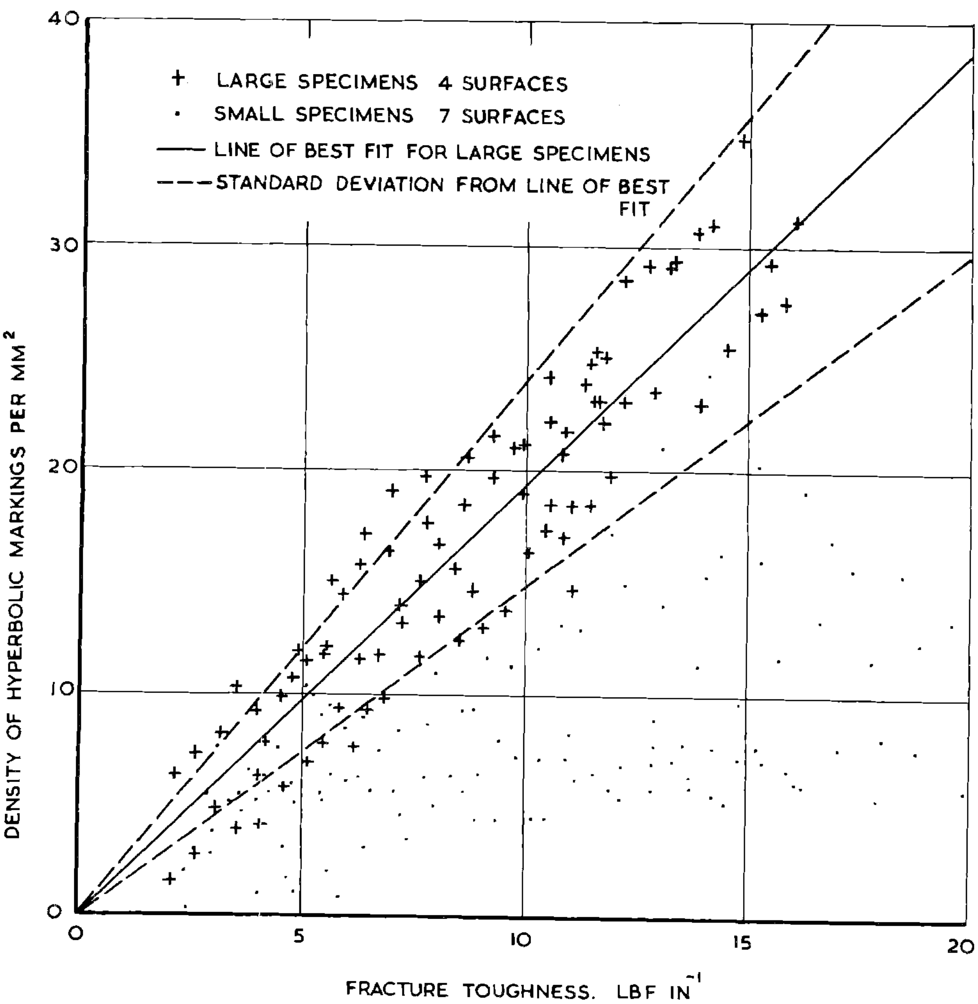

Fig. 2. Density of hyperbolic markings compared with fracture toughness.

where a is the half crack length, E, Young's modulus and $F(\alpha)$, ($\alpha = V/V_B$) is a function of velocity and is shown in figure 1. The velocity V_B at which the fracture branches has been chosen for a dimensional basis, since it can be easily measured[4]. It is assumed that branching occurs when the principal stress trajectory deviates to either side of the prolongation of the crack[10]. The crack extension force will necessarily be identical to the fracture toughness for a propagating fracture.

Successive photographs of the fracture surface were made with a Zeiss microscope with 10 x enlargement using transmitted light. These photographs were used to measure the density of the hyperbolic markings. To avoid any edge effect the markings were counted only over the central half of the surface in each photograph which has a field length of approximately 8 mm.

The fracture toughness is compared with the density of hyperbolic markings in figure 2. Those results obtained from the large specimens indicate that the density of hyperbolic markings is proportional to the fracture toughness – the results from the small specimens fall below the line of proportionality because unloading waves have reduced the stress level and equation (1) is not valid. The validity of equation (1) is in someways justified if one looks at figure 3 where the

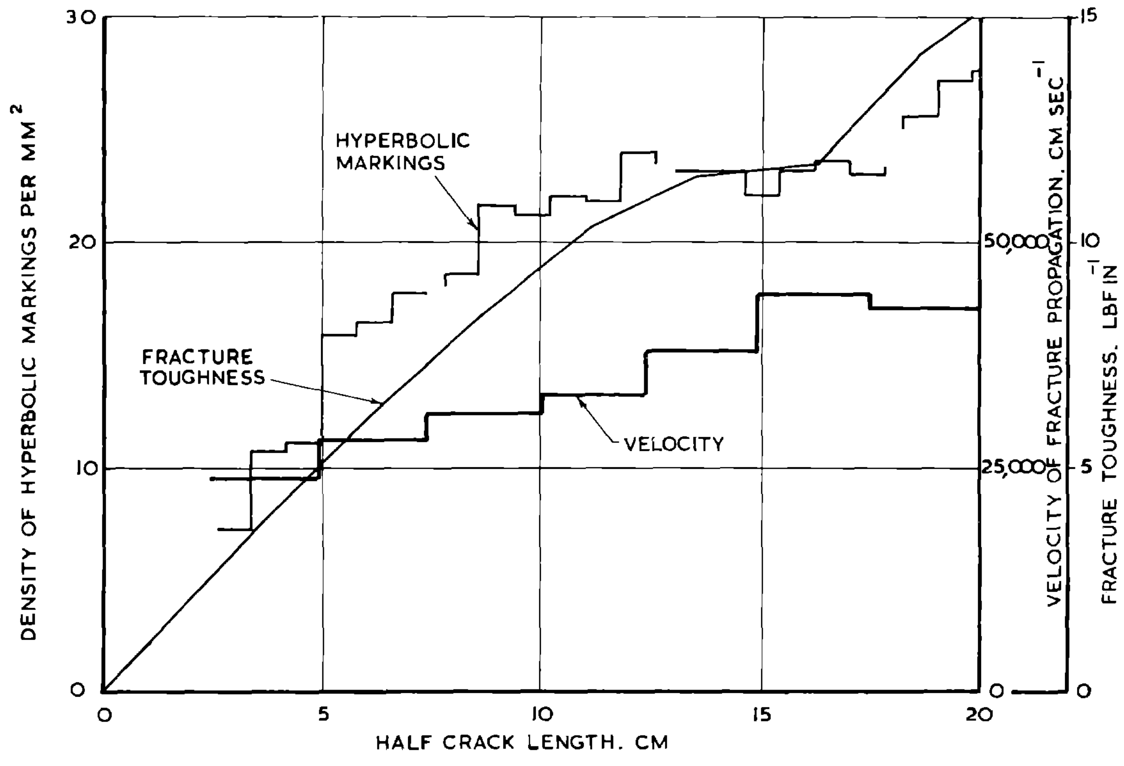

Fig. 3. Development of density of hyperbolic markings, velocity of fracture propagation and fracture toughness in a large specimen.

fracture toughness, velocity and density of markings is plotted against half crack length for one of the large specimens. The dip in the density of markings is mirrored in the dip in fracture toughness caused by the increase in velocity.

The author has suggested[4] previously that fracture toughness is a function of velocity, but if it is agreed on the above evidence that the density of the hyperbolic markings forms a measure of the fracture toughness, then figure 4 (a plot of density of hyperbolic markings against velocity) would invalidate such a suggestion. Thus it would appear that fracture toughness is not a function

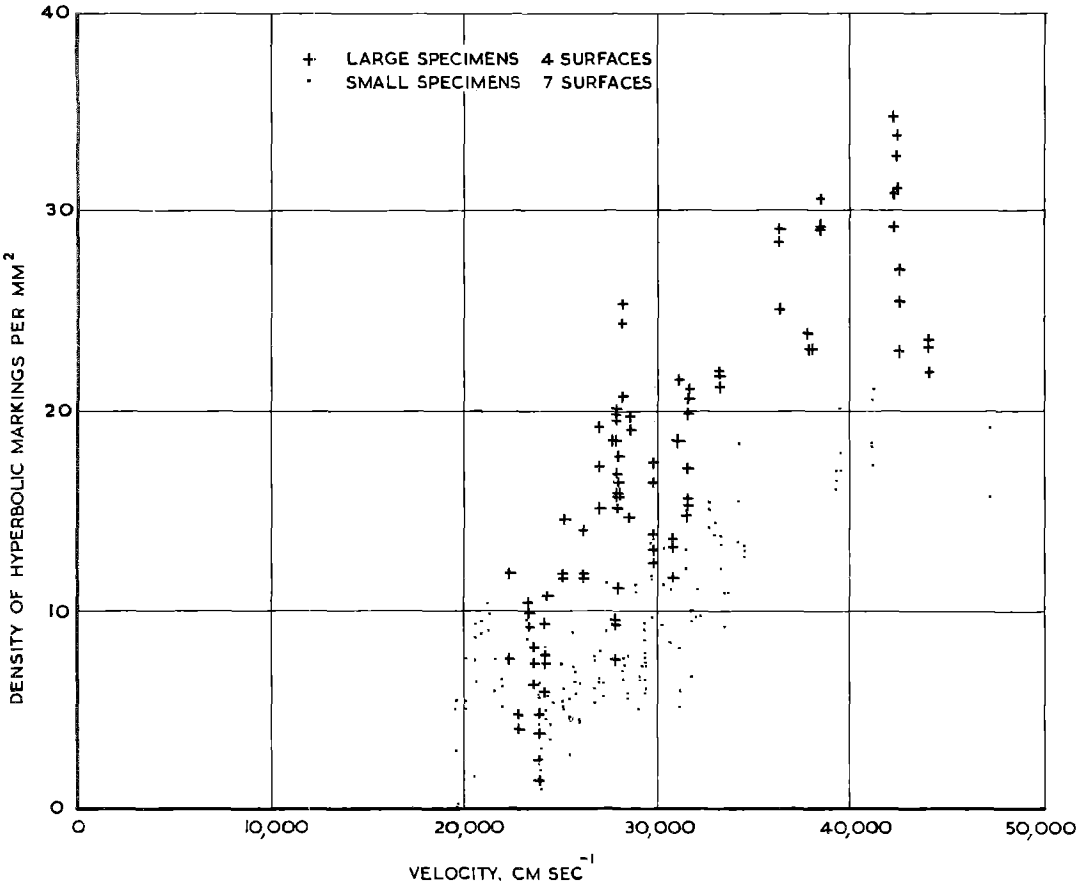

Fig. 4. Density of hyperbolic markings compared with velocity of propagation.

of the velocity, and that its value at any instance is, to some extent, random. Naturally since velocity is linked to the crack extension force through equation (1) there is loose dependence of fracture toughness on velocity, but since the crack length and velocity are independent variables the relationship is not unique.

Leeuwerik[11] suggests that the secondary fractures causing the hyperbolic markings have their origins in flaws that are randomly distributed throughout the material. Such an assumption does lead to the prediction that the density of the hyperbolic markings would be proportional to the fracture toughness, but the assumption is nevertheless wrong. If it were true one would expect that flaws would be activated at distances of the same order either to the side or ahead of the main fracture. However, compare figure 5 showing the surface roughness of the fracture with figure 6 showing the distance from the focus of a hyperbolic marking to its tip. In the next section it will be proposed that the hyperbolic markings have their origin in the craze material ahead of the main fracture front. Since the distance from the focus to the tip of the hyperbolic markings is, on the whole, greater than 25μ, which is the extent of the craze material at zero velocity[6], the extent of craze must grow with fracture propagation.

THE MECHANICS OF THE CRAZE AHEAD OF THE FRACTURE

Kambour[6], using optical interference techniques, has shown that the craze preceding the tip of the true crack in organic glasses has the shape shown in figure 7. The tip of the craze forms a

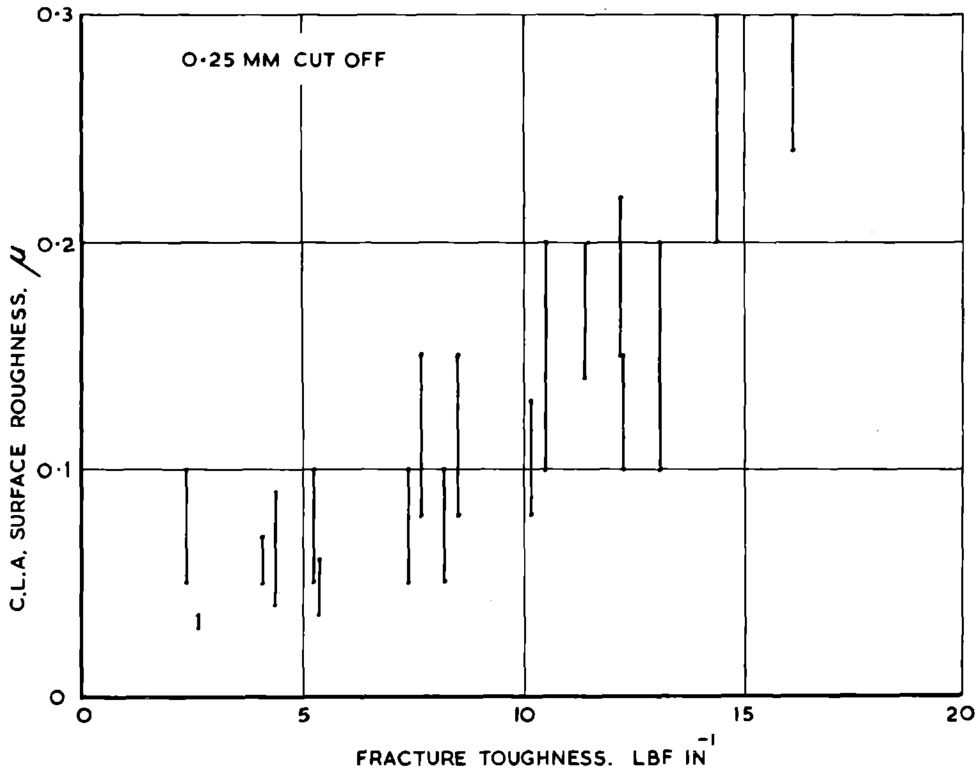

Fig. 5. Surface roughness compared with fracture toughness.

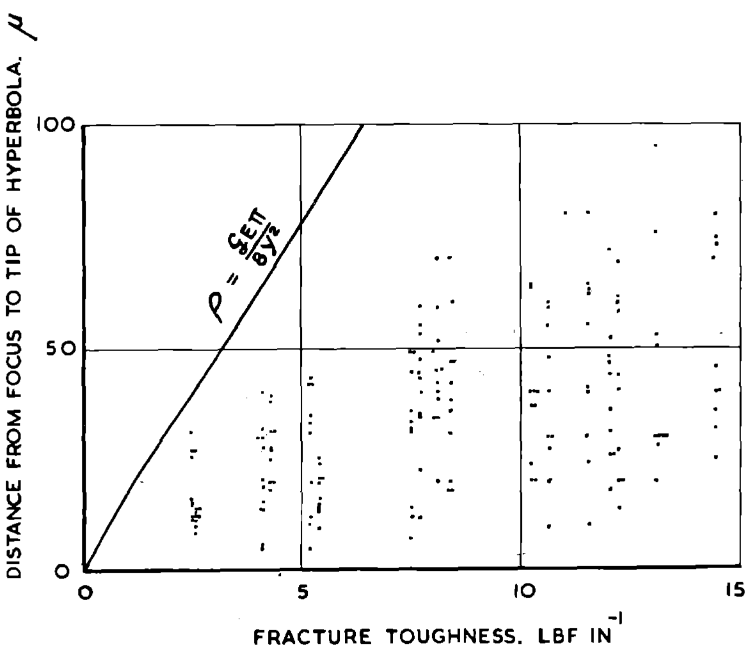

Fig. 6. Size of hyperbolic marking compared with fracture toughness.

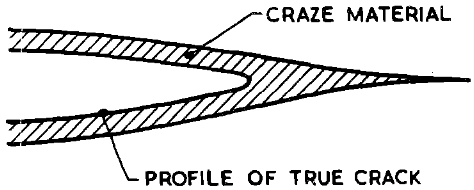

Fig. 7. Development of craze around a crack tip.

cusp and its boundary has a shape very similar to that suggested by Barenblatt[12] for a perfectly brittle material if the stresses are to be finite. Although the craze will carry a considerable stress, it has a very low modulus[8]. Thus it is tempting to assume that the stress in the craze is constant and to make use of the model Dugdale[13] used for plastic zones. The length of the craze and the width at the tip of the true crack is given by[14].

$$\rho/a = \sec\beta - 1 \tag{2}$$

$$v = (4Ya/\pi E)\log\sec\beta \tag{3}$$

where $\beta = (\pi\sigma/2Y)$, σ is the applied stress and Y is the strength of the craze material.
When β is small these expressions become

$$\rho = (GE\pi)/(8Y^2) \tag{4}$$

$$v = G/(2Y) \tag{5}$$

where $G = \sigma^2\pi a/E$ the crack extension force.
Eliminating Y between these two equations we have

$$v^2/\rho = (2G)/(\pi E) \tag{6}$$

The values of v^2/ρ measured by Kambour[6] are compared in table 1 with those calculated from the equation (6) using the fracture data of Berry[5].

	Polymethyl Methacrylate	Polystyrene
$2\gamma = G$	2.8×10^5 ergs/cm^2	14.2×10^5 ergs/cm^2
E	2.4×10^{10} dynes/cm^2	2.0×10^{10} dynes/cm^2
v measured	0.9 μ	6.0 μ
ρ measured	25.0 μ	550.0 μ
v^2/ρ measured	0.03 μ	0.07 μ
$v^2/\rho = 2G/\pi E$	0.07 μ	0.45 μ
Y	1.6×10^9 dynes/cm^2	1.2×10^9 dynes/cm^2

Table 1. Comparison of v^2/ρ measured and calculated.

If it is postulated that voids in the craze material propagate as cracks to form the hyperbolic markings in the fracture surface and that the density of the voids in the craze material is constant, then, if the strength of the craze material is not very sensitive to strain rate, the number of hyperbolic markings per unit area of fracture will be proportional to the thickness of the craze. Since the above statement contains many unsubstantiated assumptions it seems fruitless to refine equation (5) to take into account velocity effects other than to use the dynamic value of the crack extension force, since these refinements will not alter the form of the equation, but only make the constant of proportionality velocity dependent.

Thus from these simple assumptions it appears that the density of the hyperbolic markings should be proportional to the fracture toughness, which is what is observed in figure 5. One would expect if the markings have their origin in voids that are randomly distributed with a constant density, that the density measured over a given area would form a Poisson distribution. Thus the normalized standard deviation would be expected to be unity, but in figure 2 it is only 0.230. This difference is significant, but no explanation can be given for it.

CONCLUSIONS

As far as the mechanics of fracture is concerned it appears that the craze material ahead of the true crack can be treated as if it has constant strength. Thus the Dugdale model[13] for elasto–plastic materials can equally well be applied to the fracture of organic glasses. With such a model the whole of the elastic energy released is used in deforming the craze material[14]. Kambour's[6] estimates of the energy absorbed in the craze material indicate that this may not be too inaccurate.

The density of the hyperbolic markings is shown to be proportional to the fracture toughness which to a large extent is a random function of the velocity of crack propagation, although it is more probable to observe large values of fracture toughness at high than low velocities.

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RÉSUMÉ – Une mesure quantitative de la densité de stries hyperboliques sur les surfaces de rupture de methacrylate de polymethyle a permis de démontrer que la densité des stries est proportionnelle a la ténacité à la rupture du materiau. En règle générale, la ténacité est à tout moment une fonction statistique; il semble toutefois qu'elle tende a s'accroître avec la vitesse de propagation de fissure.

On suggère que la zone de matière qui se trouve en avant de l'extrémité de la fissure réelle puisse être considérée comme ayant une résistance constante, et que le modèle de Dugdale pour l'étude des matériaux élasto-plastiques est applicable a l'étude des verres organiques.

ZUSAMMENFASSUNG – Es wurde eine quantitative Einschätzung der Festigkeit von Hyperbelmarkierungen in den Bruchoberflachen von Polymethyl–Methacrylaten vorgenommen welche zeigte, dass die Festigkeit proportional zur Bruchfestigkeit ist. Die Bruchfestigkeit ist gewöhnlich eine Zufallsfunktion, aber sie hat definitive Tendenz sich mit der Bruchverbreitungsgeschwindigkeit zu vergrössern.

Es wurde angenommen, dass das Rissmaterial vor der wahren Rissspitze konstante Kraft hat und das Dugdale's Modell für elastisch–plastisches Material auch für organisches Glass angewandt werden kann.