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A Circumferential Crack in a Cylindrical Shell under Torsion*

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ABSTRACT
The fully anti-symmetric problem for a cylindrical shell with a circumferential crack is considered. The solution of the problem is reduced to that of a system of singular integral equations of the first kind. As an example the torsion of the cylinder is discussed and membrane and bending components of the stress intensity factor ratio are given.

1. Introduction

In recent years the elastostatics of cylindrical and spherical shells containing a straight through crack has attracted certain amount of attention [1–8]. References [1–3] give the asymptotic solutions for internally pressurized cracked spherical shell, cylindrical shell with an axial crack and cylindrical shell with a circumferential crack. These solutions are valid only for small values of the shell parameter (i.e., for $\lambda \ll 1$) for which (comparatively speaking) the deviation from a flat plate solution is not very significant. More complete solutions of the problem (i.e., for λ up to 5 to 10) are given in [4], [6] and [7]. In [5] the effect of a circumferential stiffener in a cylindrical shell with an axial crack is studied. Reference [8] gives the results for plastic deformations and the crack opening displacement in spherical and cylindrical shells. In all these studies it is assumed that the shell is "symmetrically loaded". More precisely, the solutions are based on the assumption that the unknown functions F, the stress function, and w, the out-of-plane displacement are even functions of the independent variables X and Y measured along the (projected) orthogonal axes**.

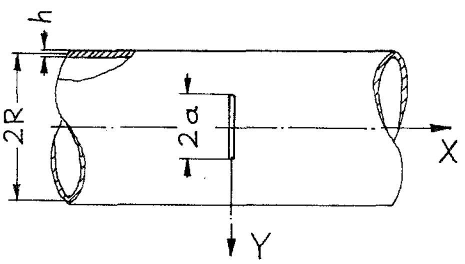

Figure 1. The geometry and loading.

In this paper we will consider the "fully anti-symmetric" problem for a cylindrical shell with a circumferential crack. That is, we will assume that F and w are both odd functions of the coordinates X and Y (Figure 1). The most practical application of the problem would be the cylindrical shell under torsion. As in all the previous studies, in this paper too we will be concerned with the perturbed stress state around the crack only. We will assume that the over-all stress state M_{ij}^0, N_{ij}^0 in the imperfection-free shell is known. Hence, the solution for the cracked shell can be obtained by adding to M_{ij}^0, N_{ij}^0 the perturbed stresses M_{ij}, N_{ij}, which are obtained from the tractions $-M_{ij}^0$, $-N_{ij}^0$ applied on the crack surfaces. In this sense, the usefulness of the present solution obviously goes beyond its application to torsion problem only.

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** With the exception of [5], in which there is only one plane of symmetry.

2. Formulation of the problem

The problem will be solved within the confines of an 8th order linearized shallow shell theory, i.e., under the Kirchhoff-type assumption regarding the boundary conditions. Thus the following equations due to Marguerre [9] will be used to formulate the problem:

$$\frac{Eha^2}{R}\frac{\partial^2 w}{\partial x^2}+\nabla^4 F=0$$

$$\nabla^4 w-\frac{a^2}{RD}\frac{\partial^2 F}{\partial x^2}=\frac{qa^4}{D} \tag{1}$$

$$x=X/a,\ y=Y/a,\ D=Eh^3/12(1-\nu^2)$$

where F is the stress function, w is the normal displacement, q is the normal traction, E and ν are the elastic constants, and the coordinates X, Y and the dimensions a, h, R are shown in Figure 1. The stress resultants N_{ij}, the moment resultants M_{ij} and the components of transverse shear Q_i are given in terms of F and w in the usual manner. The relations relevant to this study are:

$$N_{xx}=\frac{1}{a^2}\frac{\partial^2 F}{\partial y^2},\qquad N_{xy}=-\frac{1}{a^2}\frac{\partial^2 F}{\partial x\partial y}$$

$$M_{xx}=-\frac{D}{a^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

$$V_x=-\frac{D}{a^3}\left[\frac{\partial^3 w}{\partial x^3}+(2-\nu)\frac{\partial^3 w}{\partial x\partial y^2}\right]$$

In the fully anti-symmetric problem we will assume that the only external loads acting on the shell are the (statically self-equilibrating) tractions applied on the crack surface which are given by

$$N_{xx}=0=M_{xx},\quad x=0,\quad |y|\geqq 0 \tag{3}$$

$$N_{xy}=-N_{xy}^0(y),\quad V_x=-V_x^0(y),\quad x=0,\quad |y|<1 \tag{4}$$

In (3) we have $N_{xx}=0=M_{xx}$ for $|y|\geqq 1$ as well as for $|y|<1$ because of assumed symmetry.

Using Fourier transforms, the solution of the system of differential equations (1) satisfying the proper symmetry conditions and the conditions at infinity may be expressed as follows:

$$\begin{aligned}w(x,y)=\operatorname{sgn}(x)\int_0^\infty &\{P_1\exp[(\alpha_1-s_1)|x|]\\&+P_2\exp[-(\alpha_1+s_1)|x|]+P_3\exp[-(\alpha_2+s_2)|x|]\\&+P_4\exp[(\alpha_2-s_2)|x|]\}\sin ys\,ds\end{aligned}$$

$$\begin{aligned}F(x,y)=\frac{iEha^2}{\lambda^2 R}\operatorname{sgn}(x)\int_0^\infty &\{P_1\exp[(\alpha_1-s_1)|x|]\\&+P_2\exp[-(\alpha_1+s_1)|x|]-P_3\exp[-(\alpha_2+s_2)|x|]\\&-P_4\exp[(\alpha_2-s_2)|x|]\}\ \sin ys\,ds\end{aligned} \tag{5}$$

where

$$s_1=(s^2+\alpha_1^2)^{\frac{1}{2}},\quad s_2=(s^2+\alpha_2^2)^{\frac{1}{2}},$$

$$\alpha_1=\frac{\lambda}{2}e^{\pi i/4},\quad \alpha_2=\frac{\lambda}{2}e^{-\pi i/4},\quad \lambda=\left(\frac{Eha^4}{R^2D}\right)^{\frac{1}{4}} \tag{6}$$

and $P_i(s)$ are unknown. The homogeneous conditions at $x=0$ give the following two relations for P_i:

$$P_3+P_4 = P_1+P_2$$
$$P_3-P_4 = \frac{is_1}{s_2}(P_1-P_2) - \frac{(1-\nu)s^2}{\alpha_2 s_2}(P_1+P_2) \tag{7}$$

Two more relations for P_1 are obtained in the form of a system of dual integral equations by using the boundary conditions at $x=0$. Thus, for $|y|<1$ substituting from (5) and (2) into (4) and using (7) we obtain

$$\lim_{|x|\to 0}\int_0^\infty \Big\{\alpha_1(P_1-P_2)\left(e^{-s_1|x|}+\frac{s_1}{s_2}e^{-s_2|x|}\right) + (P_1+P_2)\Big[-s_1 e^{-s_1|x|}+s_2 e^{-s_2|x|} - (1-\nu)\frac{s^2}{s_2}e^{-s_2|x|}\Big]\Big\} s\cos ys\,ds = \frac{\lambda^2 R}{iEh}N^0_{xy}(y),\quad |y|<1$$

$$\lim_{|x|\to 0}\int_0^\infty \Big\{(P_1-P_2)\Big[e^{-s_1|x|}(4\alpha_1^3+(1+\nu)\alpha_1 s^2)-e^{-s_2|x|}\cdot\left(4\alpha_2^2\alpha_1\frac{s_1}{s_2}+(1+\nu)\alpha_1\frac{s_1}{s_2}s^2\right)\Big]+(P_1-P_2)\Big[e^{-s_1|x|}\cdot(-4\alpha_1^2 s_1+(1-\nu)s^2 s_1)+e^{-s_2|x|}\cdot\left(-4\alpha_2^2 s_2+(1-\nu)s^2\left(s_2+\frac{4\alpha_2^2}{s_2}\right)+(1-\nu^2)\frac{s^4}{s_2}\right)\Big]\Big\}\cdot(1-\cos ys)\frac{ds}{s} = \frac{a^3}{D}\int_0^y V^0_x(y)dy,\quad |y|<1 \tag{8}$$

where, for dimensional consistency, the second equation has been integrated in y. Also, in order to separate the divergent parts of the kernels in the analysis that follows, $e^{-s_i|x|}$ terms have been retained under the integral sign.

Outside the cut, all the physical quantities and their first derivatives must be continuous. To fulfill these conditions, it is sufficient to have

$$\frac{\partial^n}{\partial x^n}\binom{F}{w}^+ = \frac{\partial^n}{\partial x^n}\binom{F}{w}^-,\qquad (n=0,1,2,3),\quad |y|>1,\quad x=0 \tag{9}$$

where the superscripts + and − refer to the values of the function as x approaches zero from + and − sides, respectively. Analytically this simply requires that the functions which are odd in x must vanish for $x=0$, $|y|>1$. After some manipulations it can be shown that (9) will be satisfied if the functions $P_i(s)$ satisfy the following conditions:

$$\int_0^\infty (P_1+P_2)\sin ys\,ds = 0,\qquad |y|>1 \tag{10}$$

$$\int_0^\infty (P_1+P_2)s^2\sin ys\,ds = 0,\qquad |y|>1 \tag{11}$$

$$\int_0^\infty \alpha_1 s_1(P_1-P_2)\sin ys\,ds = 0,\qquad |y|>1 \tag{12}$$

Here (10) is the condition $w=0$ at $x=0$, $|y|>1$. Since (11) follows from (10), (10–12) are equivalent to only two conditions, which, together with (8) give the dual integral equations to determine P_1 and P_2.

To solve the system of dual integral equations (8), (10–12) we will first define the following auxiliary functions:

$$\int_0^\infty \alpha_1 s_1 (P_1 - P_2) \sin ts\, ds = u_1(t)$$
$$\int_0^\infty s^2 (P_1 + P_2) \sin ts\, ds = u_2(t) \tag{13}$$

where, by (11) and (12), $u_1(t) = 0 = u_2(t)$ for $|t| > 1$. It should be noted that u_1 and u_2 are defined in terms of (11) and (12) rather than (10) and (12) for again dimensional consistency*. u_1 and u_2 are related to the second derivatives of F and w and have the same type of singularity at $y = \mp 1$, $x = 0$.

Taking the inversions of (13) and substituting into (8) we obtain

$$\frac{1}{\pi}\int_{-1}^{1} \sum_1^2 h_{ij}(t, y) u_j(t) dt = f_i(y), \qquad |y| < 1, \quad (i = 1, 2)$$
$$f_1(y) = \frac{\lambda^2 R}{iEh} N_{xy}^0(y), \quad f_2(y) = \frac{a^3}{D}\int_0^y V_x^0(y) dy \tag{14}$$

where the kernels $h_{ij}(t, y)$ $(i, j = 1, 2)$ are given in the Appendix. It can be shown that h_{21} is bounded for all values of t and and the remaining kernels have a Cauchy type singularity. For example, adding and subtracting the asymptotic value of the integrand, h_{11} may be expressed as

$$h_{11}(t, y) = \lim_{|x| \to 0} \int_0^\infty \left(\frac{e^{-s_1 |x|}}{s_1} + \frac{e^{-s_2 |x|}}{s_2} - \frac{2e^{-s|x|}}{s} \right) \cdot$$
$$\cdot s \sin s(t-y) ds + \lim_{|x| \to 0} \int_0^\infty 2e^{-s|x|} \sin s(t-y) ds$$
$$= \int_0^\infty \left(\frac{s}{s_1} + \frac{s}{s_2} - 2 \right) \sin s(t-y) ds + \frac{2}{t-y}$$

where because of uniform convergence in the first integral limit has been put under the integral sign. After similar operations, (14) may be written as

$$\int_{-1}^{1} \sum_1^2 a_{ij} u_j(t) \frac{dt}{t-y} + \int_{-1}^{1} \sum_1^2 k_{ij}(t, y) u_j(t) dt = \pi f_i(y)$$
$$a_{11} = 2, \quad a_{12} = -1 + \nu, \quad a_{21} = 0, \quad a_{22} = -(1-\nu)(3+\nu)$$
$$(i = 1, 2), \quad |y| < 1 \tag{15}$$

where the kernels k_{ij} are also given in the Appendix.

The singular integral equations of the type (15) has been extensively studied (see e.g., [10]). From (15) and the definition of u_i it is easy to show that the fundamental function of the system is $(1 - t^2)^{-\frac{1}{2}}$ and the index is $+1$. Hence the solution of (15) will be determinate within a pair of arbitrary constants. To determine these constants we use the condition of continuity of the displacement w for $|y| > 1$, $x = 0$. As remarked earlier this condition is given by (10) and has not yet been satisfied. Referring to (10) and (11), it is seen that (11), which is satisfied by the choice of u_i as in (13) may be regarded as the second derivative of (10) or w in y. Thus, in order to have w vanish for $|y| > 0$, u_2 must satisfy the following two conditions:

* Note that the condition on w, (10) has not yet been satisfied. This condition will be used to determine the arbitrary constants arising from the solution of the integral equations.

$$\int_{-1}^{1} u_2(t)\mathrm{d}t = 0, \qquad \int_{-1}^{1} \mathrm{d}y \int_{-1}^{y} u_2(t)\mathrm{d}t = 0 \tag{16}$$

3. Solution of the integral equations and the stress intensity factors

Noting that the fundamental function, $(1-t^2)^{-\frac{1}{2}}$ of the system of singular integral equations (15) is the weight of the Chebyshev polynomials $T_n(t)$, we will express the solution of (15) in the following form

$$u_1(t) = (1-t^2)^{-\frac{1}{2}} \sum_1^\infty A_n T_{2n-1}(t)$$

$$u_2(t) = (1-t^2)^{-\frac{1}{2}} \sum_1^\infty B_n T_{2n-1}(t) \tag{17}$$

The first condition in (16) is satisfied by the choice of u_2 as an odd function. It is not difficult to show that the second condition in (16) gives the same result as obtained below by directly writing $w=0$ at $y=\mp 1$, $x=0$:

$$0 = w(0, 1) = -w(0, -1) = 2\int_0^\infty (P_1+P_2)\sin s\mathrm{d}s$$

$$= 2\int_0^\infty \sin s\mathrm{d}s \frac{2}{\pi s^2}\int_0^1 \sum_1^\infty B_n(1-t^2)^{-\frac{1}{2}} T_{2n-1}(t)\sin ts\mathrm{d}t$$

$$= \frac{4}{\pi}\sum_1^\infty B_n \int_0^1 (1-t^2)^{-\frac{1}{2}} t T_{2n-1}(t)\mathrm{d}t = B_1$$

To determine the remaining constants A_n and B_n the technique described in [11] is used. Once u_1 and u_2 are obtained, all the field quantities in the shell may be expressed as infinite integrals. For this we first obtain P_1 and P_2 by substituting from (17) into (13) as follows

$$\alpha_1 s_1 (P_1 - P_2) = \sum_1^\infty (-1)^{n-1} A_n J_{2n-1}(s)$$

$$s^2 (P_1 + P_2) = \sum_1^\infty (-1)^{n-1} B_n J_{2n-1}(s)$$

which, with (7), (5) and the relations of the type (2) give the complete solution.

As an example we consider the torsion problem in which we have

$$N_{xy}^0(y) = N_0, \; V_x^0(y) = 0 \tag{19}$$

Defining the following normalized auxiliary functions

$$\bar{u}_k(t) = \frac{\bar{u}_k(t)}{u_0}, \qquad u_0 = \frac{N_0 \lambda^2 R}{Eh}, \qquad (k=1, 2)$$

$$\bar{u}_1(t) = (1-t^2)^{-\frac{1}{2}} \sum_1^\infty a_n T_{2n-1}(t) \tag{20}$$

$$\bar{u}_2(t) = (1-t^2)^{-\frac{1}{2}} \sum_1^\infty b_n T_{2n-1}(t)$$

and noting that $A_n = u_0 a_n$, $B_n = u_0 b_n$, through (18), (7), (5) and equations of the form (2), the leading terms of the membrane and bending components of the stresses around the crack tip $y=1$, $x=0$ may be obtained as follows:

$$\sigma_{xy}^m = \frac{N_{xy}}{h} = -\frac{iEu_0}{\lambda^2 R}\sum_1^\infty\left(\frac{a_n}{2} - \frac{(1-\nu)b_n}{4}\right)\frac{1}{\sqrt{2r/a}}\left(3\cos\frac{\theta}{2} + \cos\frac{5\theta}{2}\right)$$

$$+ 0(1) = \frac{N_0\sqrt{a}}{h\sqrt{2r}}\left[-\frac{i}{4}\sum_1^\infty(2a_n-(1-\nu)b_n)\left(3\cos\frac{\theta}{2} + \cos\frac{5\theta}{2}\right)\right]$$

$$+0(1)$$

or, if we define the corresponding flat plate stress intensity factor k_p and a shell curvature correction factor C_m by

$$k_p = \frac{N_0\sqrt{a}}{h}, \qquad \frac{k_s^m}{k_p} = C_m = -i\sum_1^\infty(2a_n-(1-\nu)b_n) \tag{21}$$

σ_{xy}^m and the remaining membrane stresses become

$$\sigma_{xy}^m = \frac{k_pC_m}{\sqrt{2r}}\frac{1}{4}\left(3\cos\frac{\theta}{2} + \cos\frac{5\theta}{2}\right) + O(1)$$

$$\sigma_{xx}^m = \frac{k_pC_m}{\sqrt{2r}}\frac{1}{4}\left(\sin\frac{5\theta}{2} - \sin\frac{\theta}{2}\right) + O(1) \tag{22}$$

$$\sigma_{yy}^m = \frac{k_pC_m}{\sqrt{2r}}\frac{1}{4}\left(7\sin\frac{\theta}{2} + \sin\frac{5\theta}{2}\right) + O(1)$$

where k_s^m is the membrane component of the shell stress intensity factor and r and θ are the polar coordinates around the crack tip $y=1$, $x=0$, where θ is measured from y axis.

Similarly

$$\sigma_{xy}^b = \frac{12ZM_{xy}}{h^3} = -\frac{EZ}{1+\nu}\frac{u_0}{a^2}\sum_1^\infty\frac{b_n}{4}\frac{1}{\sqrt{2r/a}}\left[(5+3\nu)\cos\frac{\theta}{2} - (1-\nu)\cos\frac{5\theta}{2}\right]+O(1)$$

$$= \frac{2ZN_0\sqrt{a}}{h^2\sqrt{2r}}\left[-\frac{[3(1-\nu^2)]^{\frac{1}{2}}}{4(1+\nu)}\sum_1^\infty b_n\right]\left[(5+3\nu)\cos\frac{\theta}{2} - (1-\nu)\cos\frac{5\theta}{2}\right]+O(1)$$

or substituting $Z=h/2$, $\theta=0$ and defining the bending component of the stress intensity ratio by

$$C_b = \frac{1}{k_p}\lim\sqrt{2r}\,\sigma_{xy}^b = -[3(1-\nu^2)]^{\frac{1}{2}}\sum_1^\infty b_n\,, \qquad Z=\frac{h}{2}, \quad \theta=0 \tag{23}$$

σ_{xy}^b and the remaining bending stresses may be expressed as

$$\sigma_{xy}^b = \frac{k_pC_b}{\sqrt{2r}}\frac{2Z}{h}\frac{1}{4(1+\nu)}\left[(5+3\nu)\cos\frac{\theta}{2} - (1-\nu)\cos\frac{5\theta}{2}\right]+O(1)$$

$$\sigma_{xx}^b = -\frac{k_pC_b}{\sqrt{2r}}\frac{2Z}{h}\frac{1-\nu}{4(1+\nu)}\left(\sin\frac{\theta}{2} - \sin\frac{5\theta}{2}\right)+O(1) \tag{24}$$

$$\sigma_{yy}^b = \frac{k_pC_b}{\sqrt{2r}}\frac{2Z}{h}\frac{1}{4(1+\nu)}\left[(9+7\nu)\sin\frac{\theta}{2} - (1-\nu)\sin\frac{5\theta}{2}\right]+O(1)$$

(22) and (24) are, respectively, identical to the solution of a plane problem and that of a plate bending problem utilizing a fourth order theory.

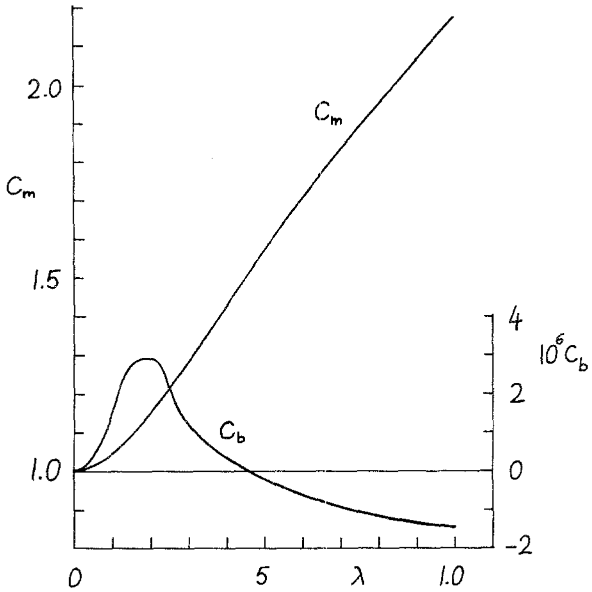

Figure 2. The stress intensity factor ratios $C_m = k_s^m/k_p$, $C_b = k_b^m/k_p$.

For the example under consideration, i.e., a cylindrical shell containing a circumferential crack which is subjected to uniform torsion away from the location of the crack, the membrane and bending components of the stress intensity factor ratio, C_m and C_b are given in Figure 2*. The membrane component C_m is practically identical to the corresponding value of $(k_s^m/k_p) = B_m$ for the cylindrical shell with a circumferential crack under uniform axial tension which is considered in [7]. This can better be seen from Table I in which the stress intensity factor ratios for the shell under axial tension and that under torsion are tabulated. However, the table also shows that the bending stress intensity factor ratios B_b and C_b corresponding to the symmetric and the anti-symmetric cases are considerably different, even though they are both small compared to the membrane components.

TABLE 1
Stress intensity factor ratios for tension and torsion

λ	Tension		Torsion	
	B_m	$10^3 \times B_b$	C_m	$10^6 \times C_b$
0	1.0	0	1.0	0
1	1.0439	1.9851	1.0440	1.5930
2	1.1496	2.6240	1.4960	2.9339
3	1.2847	2.6633	1.2847	1.1360
4	1.4290	1.6377	1.4291	0.3703
5	1.5715	−1.1083	1.5714	−0.1830
6	1.7069	−6.0770	1.7068	−0.4778
7	1.8339	−13.411	1.8338	−1.0644
8	1.9530	−22.870	1.9529	−1.1555
9	2.0657	−33.952		
10	2.1712	−46.062	2.1714	−1.4616

It should be noted that, as in References [1]–[8], the solution given in this paper is based on an eight order shell theory in which Kirchhoff assumption is made regarding the transverse shear and the twisting moment on the boundaries, including the crack surface. In this sense, the solution is approximate. If a tenth order theory is used to account for all the boundary

* As in the case of the previous solutions, the numerical results in this paper are obtained for $\nu = \frac{1}{3}$.

conditions separately, one would expect a shell thickness effect on the bending component, and a slight change in the membrane component of the stress intensity factor (See [12] for the thickness effect in cracked plates under bending). However, in the present problem, as seen from Table I, the bending component is negligible. Therefore, the error in the membrane stress intensity factor would be expected to be very insignificant.

Appendix

The kernels h_{ij} and k_{ij}:

$$h_{11}(t,y) = \lim_{|x|\to 0}\int_0^\infty \left(\frac{e^{-s_1|x|}}{s_1} + \frac{e^{-s_2|x|}}{s_2}\right) s \sin s(t-y)\,ds$$

$$h_{12}(t,y) = \lim_{|x|\to 0}\int_0^\infty \left[-\left(\frac{s}{s_1} + \frac{\alpha_2^1}{ss_1}\right)e^{-s_1|x|} + \left(\frac{\nu s}{s_2} + \frac{\alpha_2^2}{ss_2}\right)e^{-s_2|x|}\right] \sin s(t-y)\,ds$$

$$h_{21}(t,y) = \lim_{|x|\to 0}\int_0^\infty \left[\left(\frac{4\alpha_1^2}{s_1} + (1+\nu)\frac{s^2}{s_1}\right)e^{-s_1|x|} - \left(\frac{4\alpha_2^2}{s_2} + (1+\nu)\frac{s^2}{s_2}\right)e^{-s_2|x|}\right] [\sin st - \sin s(t-y)]\,\frac{ds}{s}$$

$$h_{22}(t,y) = \lim_{|x|\to 0}\int_0^\infty \left[\left(-\frac{4\alpha_1^2 s_1}{s_2} + (1-\nu)s_1\right)e^{-s_1|x|} + \left(-\frac{4\alpha_2^2 s_2}{s^2} + (1-\nu)\left(s_2 + \frac{4\alpha_2^2}{s_2}\right) + (1-\nu^2)\frac{s^2}{s_2}\; e^{-s_2|x|}\right]\cdot$$
$$\cdot [\sin st - \sin s(t-y)]\,\frac{ds}{s}$$

$$k_{11}(t,y) = \int_0^\infty \left(\frac{s}{s_1} + \frac{s}{s_2} - 2\right)\sin s(t-y)\,ds$$

$$k_{12}(t,y) = \int_0^\infty \left(-\frac{s}{s_1} + 1 + \frac{\nu s}{s_2} - \nu - \frac{\alpha_1^2}{ss_1} + \frac{\alpha_2^2}{ss_2}\right)\sin s(t-y)\,ds$$

$$k_{21}(t,y) = \int_0^\infty \left[\frac{4\alpha_1^2}{ss_1} - \frac{4\alpha_2^2}{ss_2} + (1-\nu)\left(\frac{s}{s_1} - \frac{s}{s_2}\right)\right]\cdot$$
$$\cdot [\sin st - \sin s(t-y)]\,ds$$

$$k_{22}(t,y) = \int_0^\infty \left[(1-\nu)\left(\frac{s}{s_1} + \frac{s}{s_2}\right) + (1-\nu^2)\frac{s}{s_2} - (1-\nu)(3+\nu)\right]\cdot$$
$$\cdot [\sin st - \sin s(t-y)]\,ds$$
$$+\int_0^\infty \left[-\frac{(3+\nu)\alpha_1^2}{ss_1} - \frac{4\alpha_1^4}{s_1 s^3} + \frac{(1-5\nu)\alpha_2^2}{ss_2} - \frac{4\alpha_2^4}{s_2 s^3}\right]\cdot$$
$$\cdot (1-\cos ys)\sin ts\,ds + (1-\nu)(3+\nu)\frac{1}{t}$$

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RÉSUMÉ

On considère, sous l'angle d'un problème complètement antisymétrique, le cas d'une enveloppe cylindrique comportant une fissure circonférentielle. La solution du problème est ramenée à celle d'un système d'équations intégrales singulières du premier ordre. A titre d'exemple, on applique ces considérations théoriques au cas d'un cylindre soumis à torsion uniforme, et l'on fournit les composantes de flexion du quotient du facteur d'intensité des contraintes de l'enveloppe par le facteur d'intensité des contraintes correspondant à une tôle plane.

ZUSAMMENFASSUNG

Es wird das komplette anti-symetrische Problem einer zylindrischen Hülle mit einem Umfangsriß untersucht. Die Lösung wird auf diejenige eines Systems von singularen Integralgleichungen erster Ordnung zurückgeführt. Als Beispiel wird die Torsion eines Zylinders erörtert und es werden die Membran- und Biegekomponenten des Spannungsintensitätsfaktors gegeben.