DUCTILE FRACTURE BY HOLE GROWTH IN SHEAR BANDS

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ABSTRACT

The motion of a hole in an infinite viscous body is given for simple shear with superimposed hydrostatic tension. An estimate is made for a plastic body. The holes close under pressure and shear, but reach a steady-state eccentricity and orientation with tension and shear. A criterion for fracture in shear bands, based on coalescence of neighboring holes, indicates an exponential reduction in fracture strain with tension for plasticity. A simple equation approximating the fracture criterion is presented for problems of practical interest.

INTRODUCTION

The growth and eventual merging of holes from microscopic inclusions within materials results in fracture. This phenomenon has been observed in ductile metals by Tipper(1), Puttick(2) and Rogers(3), and more dramatically by Pelloux(4) and Beachem(5) using electron micrographs of replicas. When these inclusions are large enough to be observed with an optical microscope, fracture due to the growth and merging of holes may be described by continuum mechanics.

The motion of a cylindrical void in an infinite plane viscous body was studied by Berg(6), who found that the free surfaces of initially circular holes deform as rotating ellipses.

Recent investigations(7) have extended Berg's work to describe fracture by the growth of cylindrical holes in infinite viscous solids in triaxial states of stress and strain, under various loading conditions; by comparison with circular holes, approximate results were obtained for plastic materials. These studies dealt only with loadings which did not rotate relative to the material at infinity. However, fracture often occurs in shear bands, in which case there is rotation of the material at large distances from the hole. The irrotational and rotational modes are illustrated by the shear and normal fractures in a necked copper specimen shown in Fig. 1. In this figure the irrotational mode predominates, giving a normal mode of fracture, but the shear band is also in a state of incipient fracture, due to growth, rotation, and overlapping of the holes. Intense bands of shear also form in many metalworking operations, in some of which there is also a transverse compressive stress component(8). Conceivably, the fracture which occurs in such cases arises from rotation in the shear band, sliding across the faces of closed slits, and growth and coalescence of the slits.

PHYSICAL MODEL

Large numbers of holes are assumed to be scattered uniformly throughout a material. The solid may be divided into a number of elements, each containing a single centrally-located hole. Since the spacings between holes turn out to be considerably greater than the hole diameters over most of the strain to fracture, the motion of each hole is calculated as if the hole lay in an infinite body. Likewise, the motion of the boundary of each element is calculated as if it did not contain a hole.

These calculations predict that the voids eventually touch the boundaries of their elements, as indicated in Fig. 1. This will not necessarily cause fracture if the hole spacing is regular, since adjacent holes might touch the boundaries of their elements at different points. However, in a large

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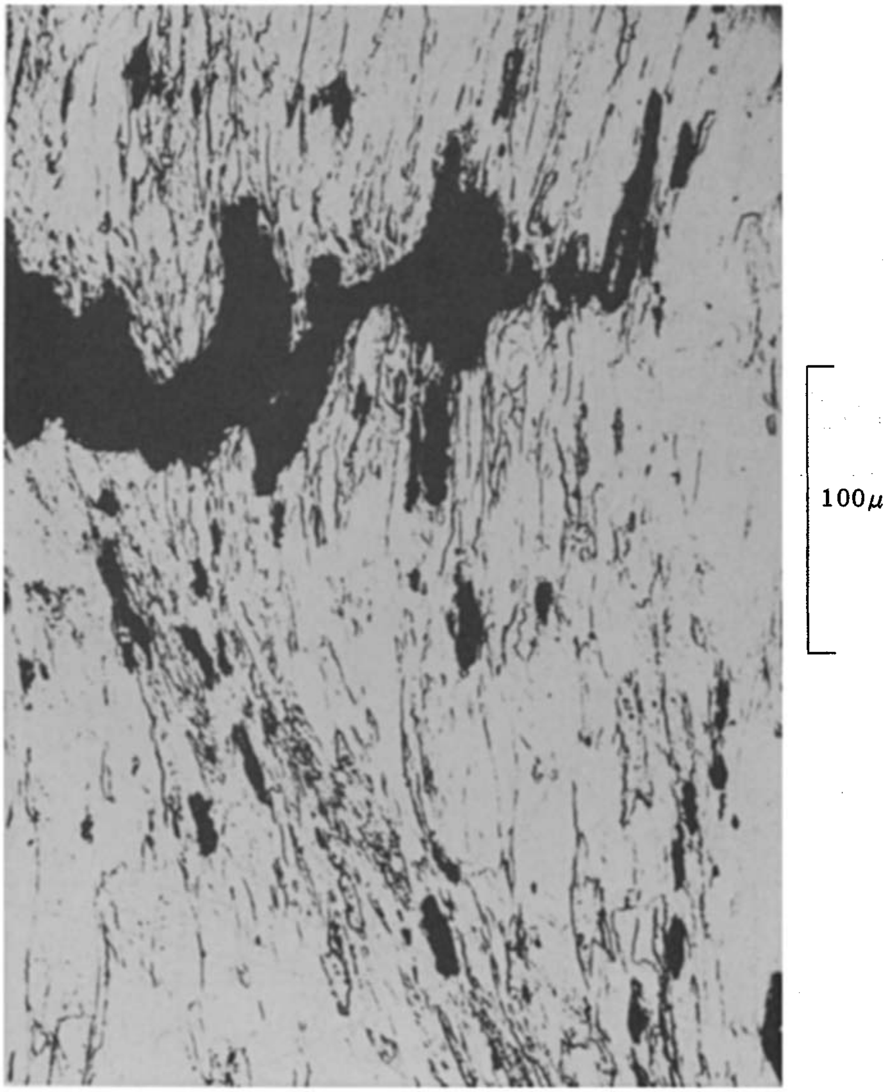

Fig. 1. Coalescence of holes. Bluhm and Morrisey (1965) Courtesy U.S. Army Materials Research Agency.

volume of material, the chances are that there will be some elements so unfavorably located that when holes from adjacent elements touch their boundaries, they at the same time touch each other. Fracture, therefore, will be assumed to occur when holes touch the boundaries of their elements.

In order to obtain expressions for the change in shape of the holes with time, they will be assumed to be cylindrical and to be situated in a viscous material, so that the analysis of Berg[6] can be used. As in previous work[7], the results will be extended to plastic materials by analogy to the deformation of circular holes under biaxial tension.

DEFORMATION WITHOUT HOLE CLOSURE

The plane deformation of an elliptical hole for irrotational motion at infinity was given by Berg[6]. Berg's equations must be modified to describe the motion of the hole with respect to the coordinates of the shear band, relative to which the material at infinity is rotating (Fig. 2). Para-

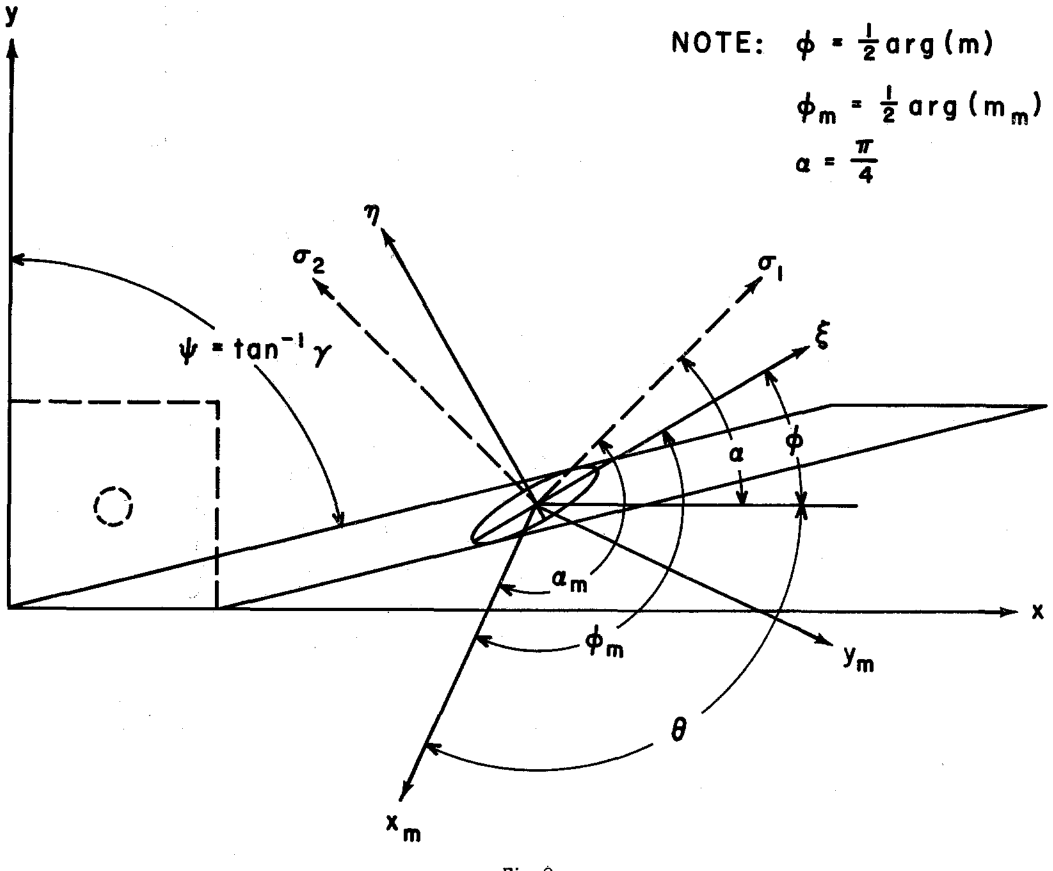

Fig. 2.

meters referred to a coordinate system not rotating with respect to the material will be denoted by the subscript $(\ )_m$. No subscript will be used for parameters referred to the shear band (x, y) coordinate system. The major semi-axis a, the minor semi-axis b, and the orientation $\emptyset$ of an ellipse can be given in terms of a mean radius R and complex eccentricity m by

$$R_m = R = (a + b)/2 \ , \tag{1}$$

$$|m_m| = |m| = \frac{(a - b)}{(a + b)} , \tag{2}$$

and

$$\phi_m = \frac{1}{2} \arg m_m , \quad \phi = \frac{1}{2} \arg m . \tag{3}$$

The two orientations are related by

$$\phi_m = \phi + \theta, \quad \text{or } m = m_m e^{-2i\theta} \tag{4}$$

Berg's equations for the rates of change of the radius and eccentricity of an elliptical hole in material with viscosity μ are expressed in terms of the principal shear stress, τ, the mean normal shear stress, σ, and the angle α_m from the material $(\ )_m$ to the principal stress axes:

$$\frac{dm_m}{dt} = \frac{\tau}{\mu} e^{2i\alpha_m} - \frac{\sigma m_m}{\mu} \tag{5}$$

and

$$\frac{dR}{dt} = \frac{R\sigma}{2\mu} \tag{6}$$

While the material axes x_m, y_m may be oriented so that θ is momentarily zero, $d\theta/dt$ is different from zero. Differentiation of Eq. 4 and evaluation at $\theta = 0$ gives

$$\frac{dm}{dt} = \frac{dm_m}{dt} e^{-2i\theta} - 2 i m_m e^{-2i\theta} \frac{d\theta}{dt} = \frac{dm_m}{dt} - 2 i m_m \frac{d\theta}{dt} \tag{7}$$

For $\theta = 0$, the angle from the material to the principal stress coordinates is

$$\alpha_m = \alpha = \pi/4 \tag{8}$$

The rate of change of angle between the coordinate systems is given in terms of the shear strain rate at infinity $(\frac{d\gamma}{dt})$ by

$$\frac{d\theta}{dt} = \frac{1}{2} \frac{d\gamma}{dt} \tag{9}$$

For convenience in studying different stress histories, the viscosity and differential time, dt, can be expressed in terms of the applied shear strain:

$$dt/\mu = d\gamma/\tau \tag{10}$$

The equations for the hole eccentricity and mean radius relative to the shear band coordinates are found by substituting Eqs. 7-10 into Eqs. 5 and 6:

$$\frac{dm}{d\gamma} + (\frac{\sigma}{\tau} + i)\, m = e^{i\pi/2} = i \tag{11}$$

and

$$\frac{dR}{d\gamma} = \frac{\sigma}{\tau} \cdot \frac{R}{2} \tag{12}$$

For plastic materials, it is reasonable to approximate the *shape* of the hole by the viscous equations[7], but the mean radius is a stronger function of the stress. In terms of the principal stress (σ_a, σ_b), strain (ϵ_a, ϵ_b), equivalent stress and strain ($\bar{\sigma}$, $\bar{\epsilon}$), and the strain hardening coefficient n in $\bar{\sigma} = \bar{\sigma}_1 \bar{\epsilon}^n$ Rhee and McClintock[9] found

$$\frac{dR}{d\bar{\epsilon}} = \frac{\sqrt{3}R}{2(1-n)} \sinh\left[\frac{\sqrt{3}}{2}(1-n)\frac{(\sigma_a + \sigma_b)}{\bar{\sigma}}\right] + \frac{d(\epsilon_a + \epsilon_b)}{2\, d\bar{\epsilon}} \tag{15*}$$

For a strain increment in the shear band

$$\begin{aligned} d\bar{\epsilon} &= d\gamma/\sqrt{3}\,, \\ (\sigma_a + \sigma_b)/2 &= \sigma\,, \\ \bar{\sigma} &= \sqrt{3}\,\tau\,, \\ d\,(\epsilon_a + \epsilon_b) &= 0\,. \end{aligned} \tag{16}$$

Substituting Eqs. 16 into Eq. 15,

$$\frac{dR}{d\gamma} = \frac{R}{2(1-n)} \sinh \frac{(1-n)\sigma}{\tau} \tag{17}$$

In the limit as $n \rightarrow 1$, Eq. 17 approaches the viscous solution, Eq. 12, as expected. Therefore, pending an exact plastic solution, Eq. 17 will be used for the growth of the mean radius of holes in a plastic material deforming in a shear band. For a constant stress ratio σ/τ, the integral of Eq. 17 is

$$\ln(R/R_1) = \frac{\gamma}{2(1-n)} \sinh \frac{(1-n)\sigma}{\tau} \tag{18}$$

The eccentricity is found by integrating Eq. 11 with σ/τ constant:

$$m = \frac{i\,\left(1 - e^{-[(\sigma/\tau)+i]\gamma}\right)}{\left(\frac{\sigma}{\tau} + i\right)} + m_1\, e^{-[(\sigma/\tau)+i]\gamma}\,, \tag{19}$$

For convenience in numerical calculations, the complex eccentricity may be expressed as:

$$m = |m|\, e^{2i\phi} \tag{20}$$

where

$$|m| = \frac{\left[\begin{aligned} &1 - 2e^{-\frac{\sigma\gamma}{\tau}}\cos\gamma + e^{-\frac{2\sigma\gamma}{\tau}} + |m_1|^2\left[1 + \left(\frac{\sigma}{\tau}\right)^2\right]e^{-\frac{2\sigma\gamma}{\tau}} \\ &+ 2|m_1|e^{-\frac{\sigma\gamma}{\tau}}\left\{\cos(\gamma - 2\phi_1) - e^{-\frac{\sigma\gamma}{\tau}}\cos 2\phi_1 - \frac{\sigma}{\tau}\left[\sin(\gamma - 2\phi_1) + e^{-\frac{\sigma\gamma}{\tau}}\sin 2\phi_1\right]\right\} \end{aligned}\right]^{\frac{1}{2}}}{\left[1 + \left(\frac{\sigma}{\tau}\right)^2\right]^{\frac{1}{2}}}$$

* Due to an error in numbering, there are no Eqs. 13 and 14.

$$\phi = \frac{1}{2}\tan^{-1}\left[\frac{\frac{\sigma}{\tau}\left(1-e^{-\frac{\sigma\gamma}{\tau}}\cos\gamma\right)+e^{-\frac{\sigma\gamma}{\tau}}\sin\gamma-|m_1|\left[1+\left(\frac{\sigma}{\tau}\right)^2\right]e^{-\frac{\sigma\gamma}{\tau}}\sin(\gamma-2\phi_1)}{\left(1-e^{-\frac{\sigma\gamma}{\tau}}\cos\gamma\right)-\frac{\sigma}{\tau}e^{-\frac{\sigma\gamma}{\tau}}\sin\gamma+|m_1|\left[1+\left(\frac{\sigma}{\tau}\right)^2\right]e^{-\frac{\sigma\gamma}{\tau}}\cos(\gamma-2\phi_1)}\right] \tag{20b}$$

The changes in eccentricity and orientation are shown in Figs. 3 and 4.

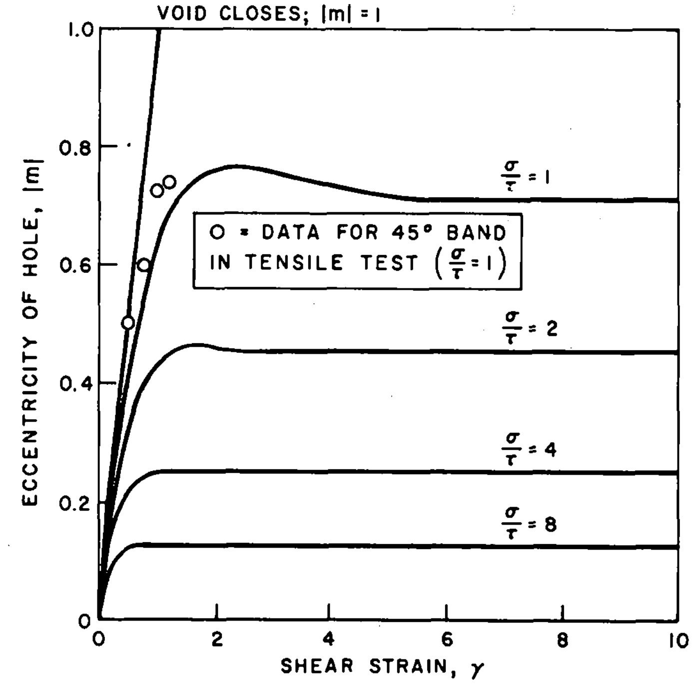

Fig. 3. Eccentricity of Initially Circular Holes.

First the hole elongates in the direction of principal stress ($\phi = 45^\circ$). The deformation of the shear band rotates the hole and tends to close it. As the orientation sweeps past the principal stress direction ($\phi < 45^\circ$) the principal tensile stress tends to round out the hole and rotate it backwards. From this point of view, the steady state which is reached seems natural, although the authors were at first surprised to find it. Once the steady state is reached, the hole grows at constant orientation and eccentricity.

PRELIMINARY EXPERIMENTS

Plasticine, which is convenient, does not strain harden excessively, and has high local ductility, was used to test Eqs. 17 and 20-20b for hole growth in plastic shear bands. For pure shear ($\sigma/\tau = 0$), a Plasticine cylinder with a small, transverse circular hole was twisted. The measured nominal strain and orientation of the hole at closure were in excellent agreement with the theory (Fig. 4).

For a loading at infinity of $\sigma/\tau = 1$, in which case no closure was predicted, a grooved Plasticine bar was pulled (Fig. 5). The groove was sufficiently deep in the material to provide plane strain, (10). Photographs taken during deformation gave the eccentricity, orientation, and mean radius of the hole as a function of shear strain. Figs. 3, 4 and 6 show that the experimental results agree well with the predictions except for

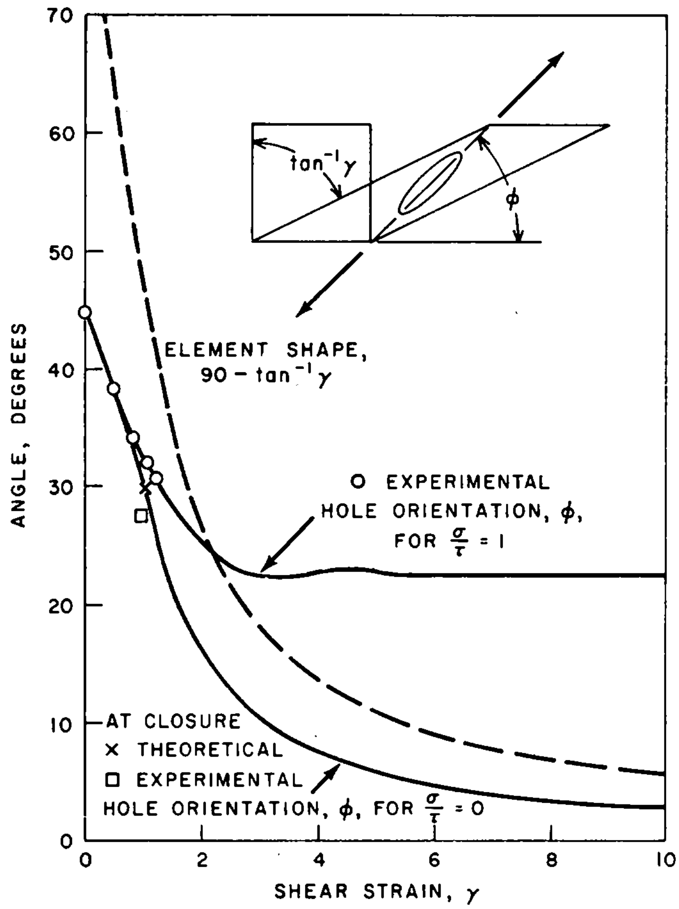

Fig.4. Rotations of Hole and Element.

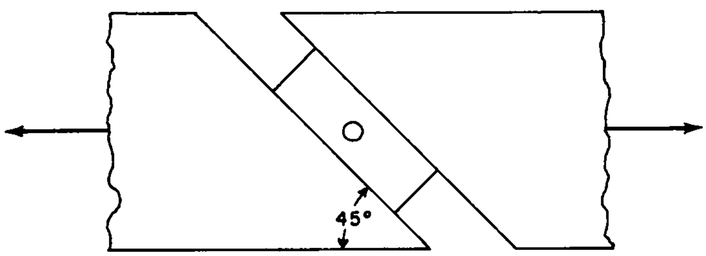

Fig.5. Plasticine Specimen for 45° Band in Tension. $\sigma/\tau = 1$.

very large holes where an accelerated growth occurs.

The measured change in mean radius ratio with strain generally agreed with the experimental data, although the viscous and plastic predictions did not differ enough to make the experiment critical.

These preliminary tests indicate that the extrapolated solution for hole growth in plastic shear bands may be applied with reasonable accuracy to actual situations. Further experimental confirmation would be valuable, particularly with a higher ratio of tension to shear.

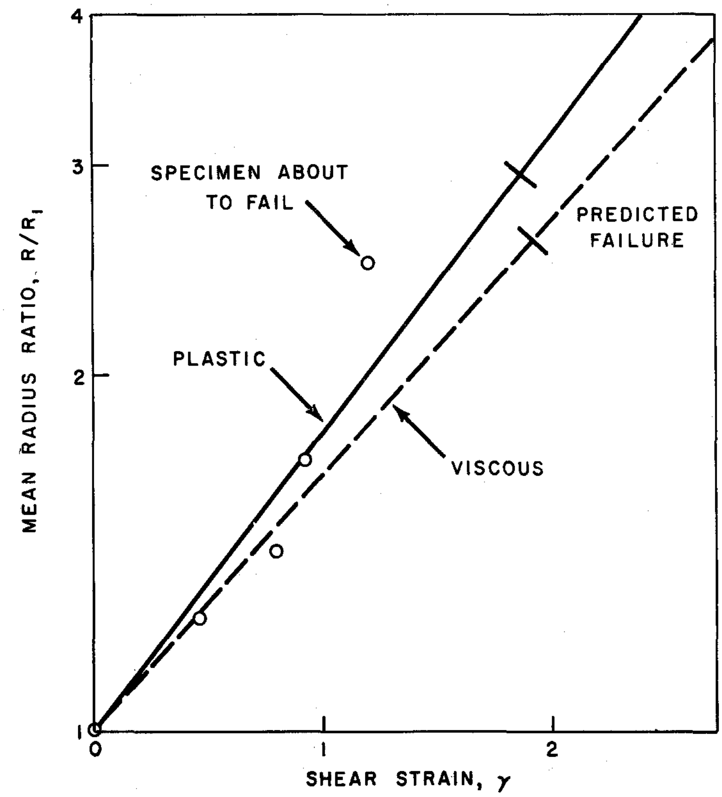

Fig. 6. Comparison of Theoretical and Experimental Data: $\sigma/\tau = 1$.

DEFORMATION AFTER CLOSURE

After a hole has closed and become a slit, it will still tend to elongate and rotate as the faces of the slit slide over each other. The relative sliding of the crack faces is opposed by friction which is assumed to obey Coulomb's law, so that the local frictional traction is proportional to the local normal pressure. To investigate the possibility that the closed crack may elongate enough to cause fracture, the motion of the closed crack in a viscous material is first determined and the results are then extended by analogy to describe the motion of the crack in the strain hardening plastic material (as in Eqs. 11 and 15 above).

Since the material is incompressible, a uniform pressure p applied to the interior of the closed crack can be replaced by the same hydrostatic tension applied at infinity. Thus, an elliptical crack loaded by internal pressure and stress at infinity remains elliptical. One can obtain the motion of a closed (elliptical) slit which slides without friction by superposing on the free surface motion (described by Eqs. 11 and 12) the motion due to a uniform internal pressure which is just sufficient to maintain the slit as a slit (i.e., to maintain the eccentricity $|m|$ at unity). Now, where friction is present, a tangential traction proportional to the (uniform) pressure acts on the face of the crack.

To find the motion caused by the uniform tangential tractions, recall that according to Rayleigh's analogy[14] between elastostatic deformation and quasi-static viscous deformation, the velocities in a viscous incompressible body of a given configuration subject to given loadings are proportional to the displacements in an incompressible elastic body having the same configuration and loading. Berg[11] shows that on the surfaces of a slit in an elastic incompressible material, subject to uniform tangential tractions, the displacements are tangent to the slit. Consequently uniform tangential tractions acting on the closed viscous slit just circulate particles without changing the crack configuration in any way. Thus the sliding closed viscous crack is loaded by uniform normal and tangential tractions.

One may now use Eqs. 11 and 15 to determine the motion of the crack, with σ replaced by σ-p where p is the pressure required to maintain the crack as a slit. These equations become

$$\frac{dm}{d\gamma} + \left((\frac{\sigma - p}{\tau}) + i \right) m = i \;, \tag{21}$$

and

$$\frac{dR}{d\gamma} = \frac{R}{2(1-n)} \sinh \frac{(1-n)\ (\sigma - p)}{\tau} \tag{22}$$

where n is the strain hardening exponent, and as $n \longrightarrow 1$ the viscous behavior is obtained. Since the slit remains a slit, m is given by

$$m = \cos 2\,\phi + i \sin 2\,\phi \;. \tag{23}$$

Inserting (23) into (21) and separating real and imaginary parts, one finds

$$\sigma - p = \tau \sin 2\,\phi \tag{24}$$

which shows that the pressure on the crack face equals the compressive stress applied transverse to the crack at infinity; thus the slit will just remain shut when the compression across the crack at infinity is zero. In addition

$$\frac{d(\cot\,\phi)}{d\gamma} = 1, \text{ or } \cot\phi - \cot\phi_c = \gamma - \gamma_c \;, \tag{25}$$

where the subscript $(\;)_c$ indicates conditions at closure. Equation 25 shows that the slit rotates as a material line.

Introducing Eq. 24 into Eq. 22, calculating $d\gamma$ from Eq. 25 and taking the limit of Eq. 22 as $n \longrightarrow 1$ for the viscous case, one finds

$$\frac{dR}{R} = \frac{\sin 2\,\phi}{2} d(\cot\,\phi) = -\frac{d(\sin\,\phi)}{\sin\,\phi} \;;$$

$$\text{i.e. } R \sin\phi = R_c \sin\phi_c \tag{26}$$

which shows that the closed crack elongates as if it were a material line.

It is natural that the viscous closed crack should rotate and elongate as a material line because the configuration of the crack is not influenced by the uniform frictional traction distribution. If at each instant the tangential traction were equal to the value of the shear stress on the plane of the crack, the crack would be "stuck" and would behave exactly as a material line with no circulation of particles. Since the closed viscous crack elongates and rotates as a material line, it cannot cause fracture by the mechanism shown in Fig. 2.

The rotations of a closed crack in plastic strain hardening and viscous materials are the same; however, Eq. 24 shows that in the plastic strain hardening material the crack does elongate somewhat more rapidly than in the viscous case (because $|\sinh \alpha| \geq |\alpha|$). Inserting Eq. 24 into Eq. 22, using Eq. 26 to obtain $\sin\phi$ and $\cos\phi$ after setting γ_c equal zero, one obtains after some rearranging

$$\frac{dR}{R} = -\frac{1}{2(1-n)} \sinh \left\{ \frac{2(1-n)\ (\gamma + \cot \phi_c)}{1 + (\gamma + \cot \phi_c)^2} \right\} d(\gamma + \cot \phi_c) \qquad (27)$$

Now, γ starts at zero and increases, and the cracks for which $\cot \phi_c < 0$ shrink initially. The argument of the hyperbolic sine has its maximum at $\gamma + \cot \phi_c = 1$, and as γ increases to large values, the argument becomes very small, the hyperbolic sine approaches its argument and the growth of the plastic crack approaches that of a viscous crack. The plastic crack which will undergo the maximum growth is an initially vertical crack ($\cot \phi_c = 0$) in a non-hardening material ($n = 0$). The total growth of this crack is given by

$$\log (R/R_c) = \frac{1}{2} \int_0^{\gamma} \sinh \left(\frac{2\gamma'}{1 + \gamma'^2}\right) d\gamma' \qquad (28)$$

The integration need be carried out only up to $\gamma = 8$; at that point the argument is 0.245 (and decreasing), and the sinh is 0.247. Beyond $\gamma = 8$ the crack elongates as a material line. Numerical integration of (28) up to $\gamma = 8$ gives

$$\frac{R(\gamma = 8)}{R(\gamma = 0)} = 9.52 \qquad (29)$$

The elongation ratio of an initially vertical material line in a shear band subjected to a strain of 8 is 8.06. Thus, the crack which has the greatest total plastic growth elongates only 18% more than would a material line in its place. Such a hole could grow to intersect the boundary of its cell (Fig. 1) *only* if the length of the hole at closure were greater than 82% of the width of the cell at the time of closure. If the holes were this large relative to their typical spacing (i.e., the cell size of Fig. 1) ductile fracture would be imminent. Thus within the accuracy contemplated in this paper one finds that if ductile fracture has not occurred by the time the voids are closed, it will not occur after closure. Some other mechanism must be contributing to fracture in compression.

FRACTURE CRITERION

As discussed earlier, fracture is assumed to occur when (according to the equations for an infinite solid) the hole just touches the boundary of the deforming element in which it lies. The point of contact may be either at the side of the element, as shown in Fig. 7, or at the top, to be considered later. The initial ratio of spacing along the band to mean diameter, $L_L/2R_1$, that leads to contact at this instant will be termed the hole growth factor F_L. With respect to the axes ξ, η of the hole, the coordinates of the point of contact P must satisfy

$$\left(\frac{\xi_P}{a}\right)^2 + \left(\frac{\eta_P}{b}\right)^2 = 1 \qquad (30)$$

The slope at P is:

$$\frac{d\eta_P}{d\xi_P} = -\tan \beta = \cot (\phi + \tan^{-1} \gamma) \qquad (31)$$

This is found by expressing the normal to the surface in terms of both

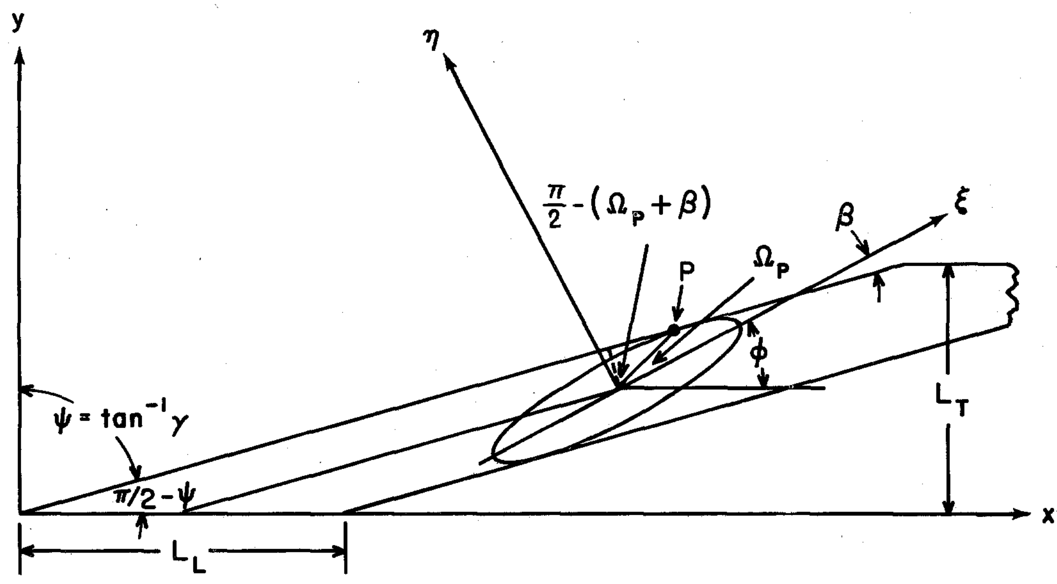

Fig. 7. Geometry of Failure Due to Hole Growth.

the dimensions of the element and the dimensions of the ellipse:

$$\sqrt{\xi_p^2 + \eta_p^2}\,\sin(\Omega_p + \beta) = (L_L/2)\cos\psi = L_L/2\,\sqrt{1+\gamma^2}. \qquad (32)$$

The coordinates of the point of contact are found by differentiating the equation for the ellipse,

$$(\xi_p/a)^2 + (\eta_p/b)^2 = 1, \qquad (33)$$

expressing the slope in terms of the angle β,

$$d\eta_p/d\xi_p = -\tan\beta, \qquad (34)$$

and solving:

$$\xi_p = \frac{a(a/b)\tan\beta}{\sqrt{1+(a/b)^2\tan^2\beta}},\quad \eta_p = \frac{b}{\sqrt{1+(a/b)^2\tan^2\beta}}. \qquad (35)$$

The semi-axes a and b can be expressed in terms of the mean radius and eccentricity by Eqs. 1 and 2. The angle β is expressed in terms of the orientation of the ellipse and the shear strain γ by

$$\beta = \phi + \tan^{-1}\gamma - \pi/2. \qquad (36)$$

The angle Ω_P is found from the coordinates in terms of the eccentricity $|m|$ and the angle β:

$$\Omega_p = \tan^{-1}\frac{\eta_p}{\xi_p} = \tan^{-1}\left\{\left(\frac{1-|m|}{1+|m|}\right)^2\cot\beta\right\} \qquad (37)$$

The fracture condition, Eq. 32 is now that the growth factor F_L becomes

$$F_L = \frac{L_L}{2R_1} = \sqrt{1+\gamma^2}\left(\frac{R}{R_1}\right)\sqrt{\frac{(1-|m|)^4 + (1+|m|)^4\tan^2\beta}{(1-|m|)^2 + (1+|m|)^2\tan^2\beta}}\,\sin(\Omega_p+\beta). \qquad (38)$$

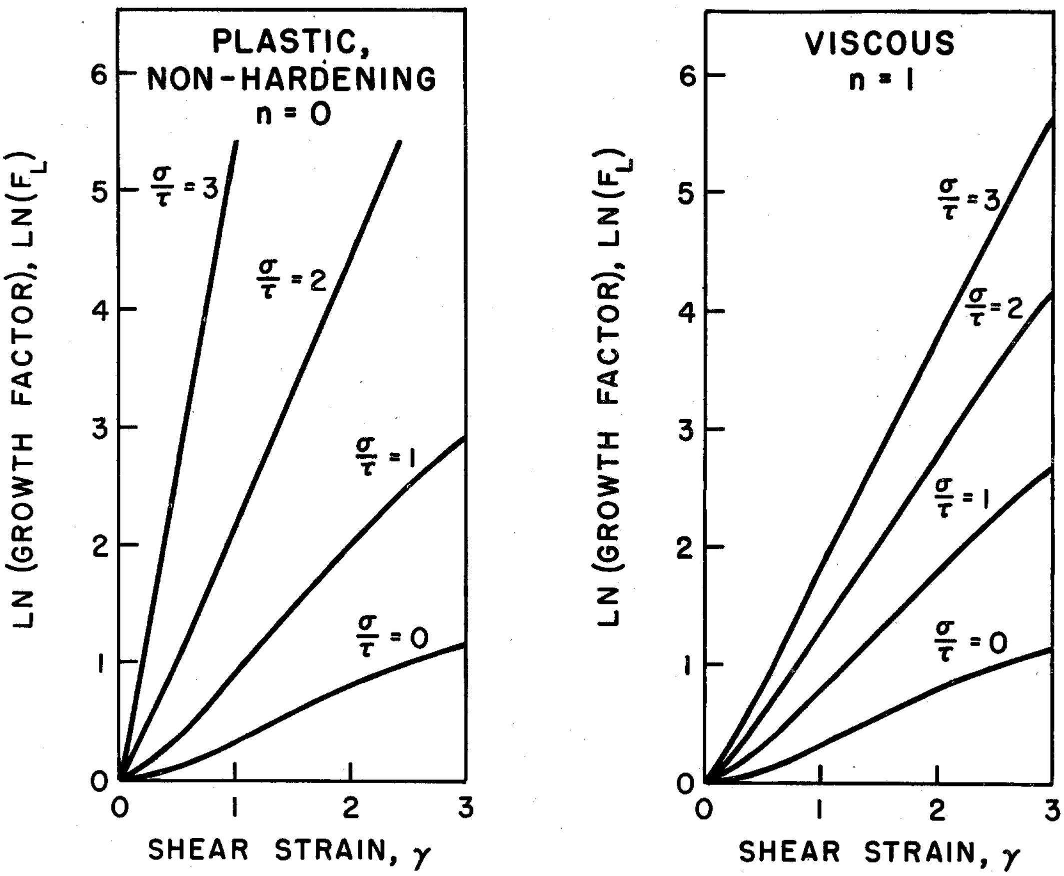

Fig. 8.

If the point of contact is at the top, the critical ratio is that of the spacing of holes across the band to the initial mean radius $L_T/2R_1$. The condition for contact, Eq. 32, becomes

$$\sqrt{\xi_p^2 + \eta_p^2}\,\sin\,(\Omega_p + \phi) = L_T/2. \tag{39}$$

Now the point of tangency is determined by

$$\beta = \phi \tag{40}$$

Eq. 37 still gives Ω_p, and the fracture condition becomes

$$F_T = \frac{L_T}{2R_1} = \left(\frac{R}{R_1}\right)\sqrt{\frac{(1-|m|)^4 + (1+|m|)^4 \tan^2\phi}{(1-|m|)^2 + (1+|m|)^2 \tan^2\phi}}\,\sin\,(\Omega_p + \phi). \tag{41}$$

These equations are also valid when the holes have just closed (m = 1).

APPROXIMATE FRACTURE CRITERION

The complexity of Eqs. 38 and 41, as well as of Eqs. 11, 15, and 37 for the angles ϕ and Ω_p, makes it desirable to have approximate forms for rough calculation and interpretation of the effects of major variables. In

Eqs. 38 and 41, the radical and the sine factors may be taken to be unity, to a first approximation. These approximations are summarized in Table 1, and plotted in Fig. 8a and Fig. 8b, for plastic and viscous, (or linearly hardening) materials, respectively. Note that the fracture conditions for the length and thickness directions of the shear band differ only by a factor of $\sqrt{1 + \gamma^2}$. Therefore in plotting the relations in Fig. 8, only the longitudinal criterion was plotted. The thickness criterion can be obtained, if desired, by subtracting the lowest curve from the others, (which then become straight lines).

TABLE 1. Summary of Approximate Equations

Fracture condition

for hole contact in the longitudinal direction of the shear band,

$$\ln F_L = \ln L_L/2R_1 = \ln \sqrt{1 + \gamma^2} + \ln R/R_1 \qquad (38a)$$

for hole contact in the thickness direction of the shear band,

$$\ln F_T = L_T/2R_1 = \ln R/R_1 \; . \qquad (41a)$$

Radius ratio

for holes up until the instant of closure

$$\ln R/R_1 = \frac{\gamma}{2(1-n)} \sinh \frac{(1-n)\sigma}{\tau} \qquad (18)$$

For a material with a given initial ratio of hole spacing to diameter, fracture occurs when the curve corresponding to the given state of stress reaches the ordinate corresponding to that value of $L/2R_1$. The most striking result of these curves is the very large reduction in strain to fracture for plastic materials under hydrostatic tension, seen by comparing Fig. 8a with 8b. The fracture criteria given above apply to voids which have not closed, since from Section 5, once a hole has closed it can no longer grow enough to cause fracture.

CONCLUSIONS

1. Approximate equations have been derived for the deformation of holes in shear bands in both plastic and viscous (or linearly hardening) materials. With sufficient normal stress, the holes rotate beyond the direction of maximum principal stress, after which the tension tends to prevent further rotation and a steady state is attained. Under moderate amounts of tension, $\sigma/\tau < 0.7$, the holes may close temporarily, rotate further, and then reopen. With compression across the band, the holes remain closed and do not cause fracture, regardless of the friction present.

2. If the exponent n in $\tau = \tau_1 \gamma^n$ is constant, as is the ratio of normal to shear stress on the shear band, the applied shear strain γ for holes to run together in the longitudinal direction of the shear band is

$$\ln L_L/2R_1 = \ln \sqrt{1 + \gamma^2} + \frac{\gamma}{2(1-n)} \sinh \frac{(1-n)(\sigma)}{\tau} \qquad (18 \text{ and } 34a)$$

3. Exact plasticity solutions should be obtained for the deformation of the holes in place of those assumed here. The effects of fracture in the

high strain region at the tips of cracks should be further investigated, especially since the present criterion indicates no fracture once the holes have closed, and yet ductile fracture is known to occur under such conditions.

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RÉSUMÉ - Le mouvement d'un trou dans un corps visqueux et infini est donné pour le cas du tondage simple avec la tension hydrostatique superposée. Les trous se ferment sous la pression et le tondage, mais avec la tension et le tondage ils attaignent une excentricité et une orientation d'état fixe. Un critère pour la fracture dans les bandes tondues ce qui est fondé sur la coalescence des trous voisins indique pour la plasticité qu'il y a une réduction exponentielle dans la déformation nécessaire à fracturer avec tension. On présent une équation simple qui approxime le critère de fracture pour les problèmes d'intérêt pratique.

ZUSAMMENFASSUNG - Unter Voraussetzung eines unbegrenzt grossen viskosen körpers wird die Veränderung und Bewegung eines Loches unter einfacher Schubbeanspruchung mit überlagerter hydrostatische Spannung dargestellt.
Für einen plastischen Körper wird eine Abschätzung durchgeführt.
Die Löcher schliessen sich unter Druck und Schub, erreichen aber bei Spannung und Schub eine stationäre Exzentrizität und Orientierung. Ein Kriterium für den Bruch in Schubzonen, herrührend vom Zusammenschluss benachbarter Löcher zeigt bei plastischen Körpern mit wachsender Spannung eine exponentielle Verringerung der erforderlichen Bruchbeanspruchung.
Für Probleme von praktischem Interesse wird eine einfache Gleichung, die das Bruchkriterium annähernd wiedergibt vorgestellt.