ELASTIC FIELD EQUATIONS FOR BLUNT CRACKS WITH REFERENCE TO STRESS CORROSION CRACKING

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ABSTRACT

The elastic stress field equations for blunt cracks are derived and presented in a form equivalent to the usual sharp crack tip stress fields. These stress field equations are employed in analyzing a dissolution model for the arrest of stress corrosion cracking by crack tip blunting, which is often observed with the arrest of stress corrosion cracks.

Stress corrosion, the growth of cracks due to the combined and interrelated action of stress and environment, is a highly complex phenomena in all its aspects. In general, it involves the diffusion of an environment into a crack which, in some way, attacks the highly stressed material in the vicinity of the crack tip causing the crack to grow. There have been recent experimental observations in which the arrest of a stress corrosion crack was accompanied by the apparent blunting of the crack tip. See Figures 1 and 2. These observations suggest that an investigation of the

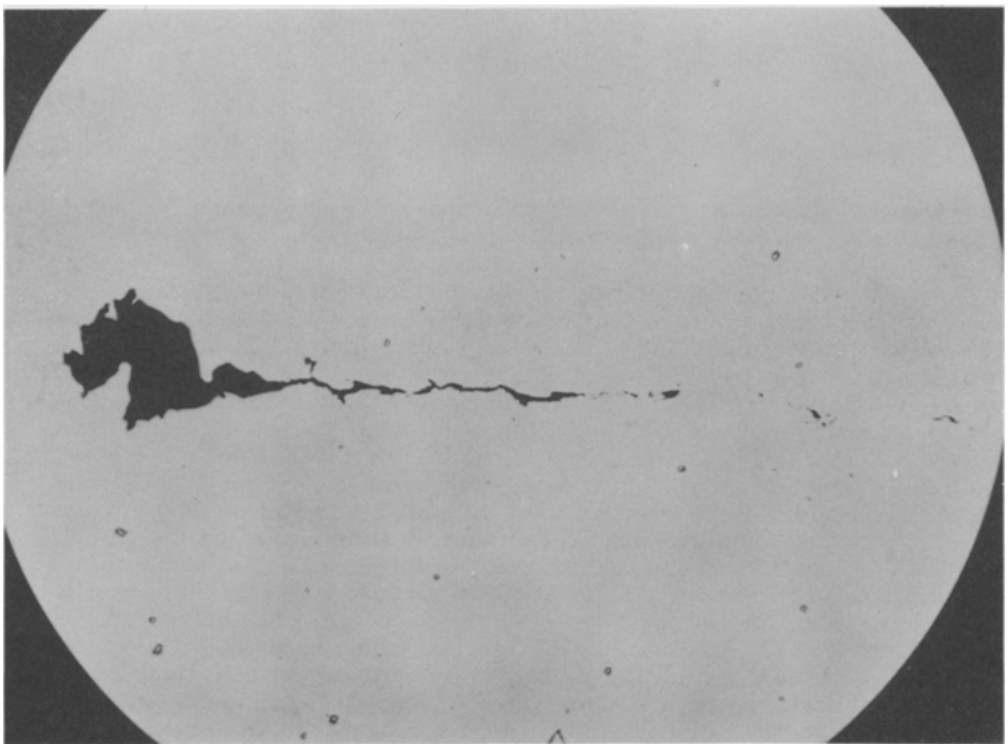

Figure 1: 12% Ni - 5% C_r - 3% Mo - Maraging steel in 3-1/2% NaCl solution at K_i equal to 9500 psi $\sqrt{in}$. exposure time in excess of 300 hours. Courtesy of Floyd Brown, U.S. Naval Research Laboratory.

stability of the shape of the crack tip surface may be of great interest.

In order to attempt to relate the conditions of attack to the stress conditions for blunting, a very simplified mechanical view of the process si-

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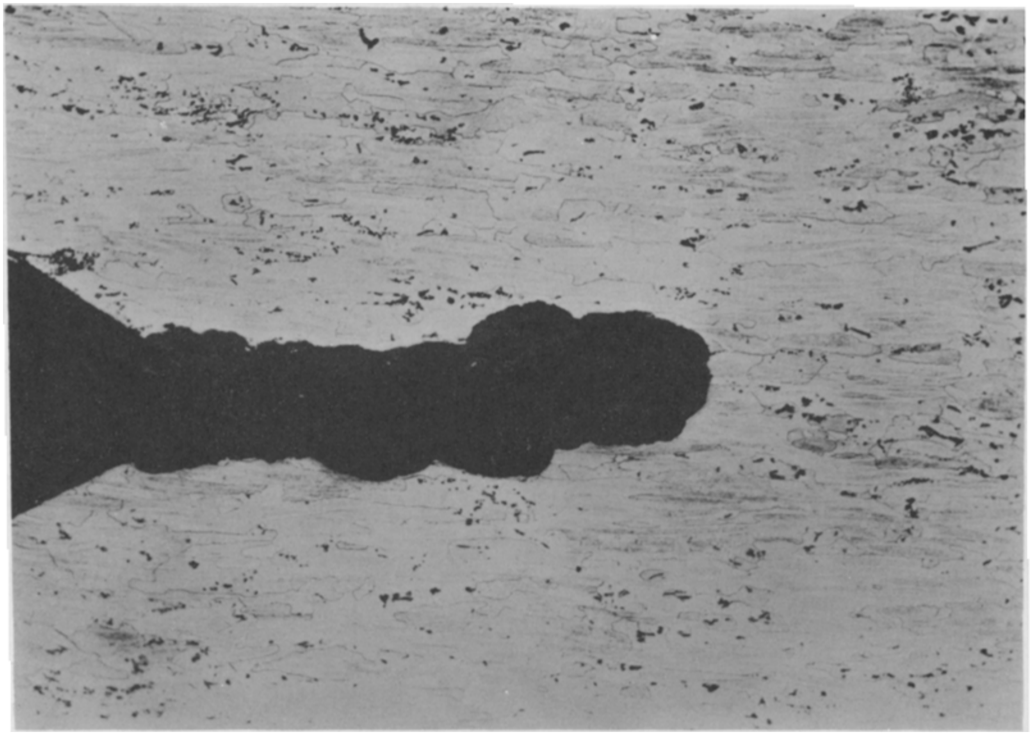

Figure 2: 2024-T351 aluminum alloy in 3-1/2% NaCl solution at K_I equal to 5360 psi $\sqrt{in}$. exposure time in excess of 1000 hours. Courtesy of J. Mulherin, U.S. Army, Frankford Arsenal.

milar to that of Charles and Hillig[1] will be adopted here. The usual continuum model of a crack is a planar void of material, and the corresponding mathematical model is a plane of discontinuity. However, since the chemical attack of a material by the environment at the crack tip is being considered, it is relevant to have as a physical model of the crack, a void that is not the usual plane ending with zero radius of curvature, but a narrow volume with a finite curvature at the tip. This type of blunt crack or notch is conveniently represented mathematically by an elliptical or hyperbolic cylinder, void of material, in which the radius of curvature at the tip is small in comparison to the major dimensions of the void.

It is then of interest to explore the nature of the stress distribution about this blunt crack or notch. As is usual in Fracture Mechanics, an elastic analysis will be attempted which may later be discussed in the light of the effects of nonlinear material behavior near the crack tip. Consequently, the elastic stress distribution in the neighborhood of elliptical holes and hyperbolic notches will be presented. Since regions near the crack tip are of special interest, it is advantageous to expand the expressions for the stresses as a power series in terms of radial distance from some point near the tip and to discard all the second order terms. This is most appropriately done by expanding the expressions for the stresses using the origin chosen in Figure 3, since certain simplifications result. Note that the origin is a distance of $\rho/2$ away from the crack tip, where ρ is the radius of curvature at the crack tip. When ρ/a (a is half the crack length) is small compared to one, the origin is to a very close approximation the focal point of the ellipse or hyperbola that represents the surface of the crack. The results of this expansion for both the elliptic hole and the hy-

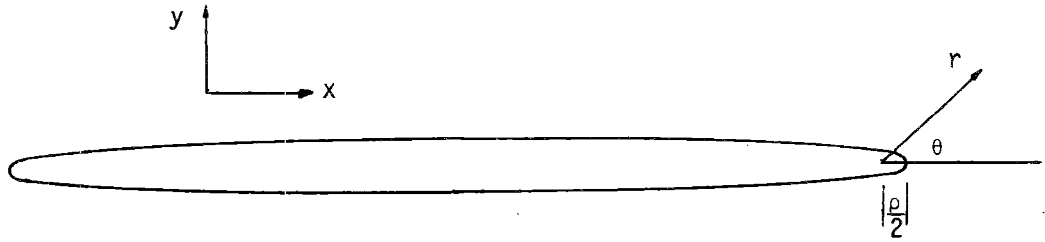

Figure 3: Coordinate system for stress field.

perbolic notch are identical and as follows [2]:

MODE I - OPENING MODE (PLANE-EXTENSION SYMMETRICAL MODE)

$$\sigma_x = \frac{K_I}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3}{2}\theta\right] - \frac{K_I}{(2\pi r)^{1/2}}\frac{\rho}{2r}\cos\frac{3}{2}\theta$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3}{2}\theta\right] + \frac{K_I}{(2\pi r)^{1/2}}\frac{\rho}{2r}\cos\frac{3}{2}\theta \qquad (1)$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3}{2}\theta - \frac{K_I}{(2\pi r)^{1/2}}\frac{\rho}{2r}\sin\frac{3}{2}\theta$$

MODE II - EDGE SLIDING MODE (PLANE-EXTENSION SKEW SYMMETRICAL MODE)

$$\sigma_x = -\frac{K_{II}}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}\left[2 + \cos\frac{\theta}{2}\cos\frac{3}{2}\theta\right] + \frac{K_{II}}{(2\pi r)^{1/2}}\frac{\rho}{2r}\sin\frac{3}{2}\theta$$

$$\sigma_y = \frac{K_{II}}{(2\pi r)^{1/2}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3}{2}\theta - \frac{K_{II}}{(2\pi r)^{1/2}}\frac{\rho}{2r}\sin\frac{3}{2}\theta \qquad (2)$$

$$\tau_{xy} = \frac{K_{II}}{(2\pi r)^{1/2}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] - \frac{K_{II}}{(2\pi r)^{1/2}}\frac{\rho}{2r}\cos\frac{3}{2}\theta$$

MODE III - TEARING MODE (ANTI-PLANE MODE)

$$\tau_{xz} = -\frac{K_{III}}{(2\pi r)^{1/2}}\sin\frac{\theta}{2}$$

$$\tau_{yz} = \frac{K_{III}}{(2\pi r)^{1/2}}\cos\frac{\theta}{2} \qquad (3)$$

These field equations are similar to those for the "mathematically sharp" plane crack. In fact, the relationship describing the third mode stress state is exactly the same as that for the third mode stress state of a plane crack. Moreover, the first and second mode stress state relations differ from their plane crack counterparts by a single additional term dependent upon the radius of curvature at the tip. In these blunt crack stress field equations, the additional term in each may be neglected when ρ/r is negligible compared to one. The remaining terms are the usual sharp crack tip stress field equations which are consequently seen to be applicable to the region $\rho \ll r \ll a$. It is therefore implied that the tips of blunt cracks are imbedded within the usual crack tip stress field and that these usual field equations are disturbed only for mode I and mode II stress states and only in the immediate vicinity of the crack tip. In addition, it is interesting to note that the hydrostatic stress distribution, $\sigma_x + \sigma_y$, is the same for a crack with a finite curvature at the tip and a sharp crack.

For the stress corrosion mechanism under discussion, the crack shape will be considered to be an ellipse and the stress conditions will initially be taken to be entirely elastic. Furthermore, it will be envisioned that the reaction between the solid material and the environment is simply one in which the solid is made to dissolve. Dissolution in this context can be generally viewed as any local degradation in the material's ability to transmit stress. As a consequence, the velocity of dissolution will be considered to be a function of the stress at the material-environment interface.

It is of interest to examine the conditions under which the crack contour could be considered stable. That is, when the shape of the crack at the tip is maintained rather than blunting or sharpening, causing a tendency to arrest or accelerate.

The case where the loading creates a mode I type deformation is of greatest practical interest and therefore will be considered. Referring to Figure 4, $V(\sigma)$ is the dissolution velocity normal to the surface. Note that

Figure 4: Crack extension due to stress corrosion.

for a constant shape of the whole contour near the tip, it is required that the apparent horizontal velocity, V_H, be constant for all points on the contour. This leads to

$$V_H = \frac{V(\sigma)}{\cos\phi} = \text{const} = C_1 \tag{4}$$

From simple geometry, it follows that

$$V_H = \frac{V(\sigma)}{\cos\frac{\theta}{2}} = C_1 \tag{5}$$

The normal stress at the surface is zero so that the stress sum, $\sigma_x + \sigma_y$, equation (1), gives the tangential stress σ_t:

$$\sigma_t = \frac{2K_I}{(2\pi r)^{1/2}} \cos\frac{\theta}{2} \tag{6}$$

Therefore, for the condition of constant shape of the contour, combining equations (5) and (6) gives

$$V(\sigma_t) = C_1 \frac{(\pi \rho)^{1/4}}{K_1^{1/2}} \sigma_t^{1/2} \quad (7)$$

Consequently, if $V(\sigma)$ is more highly dependent on stress than $\sigma^{1/2}$, the crack will tend to sharpen. Alternately, a consideration of the sign of the derivative of $V(\sigma)/\sigma^{1/2}$ will indicate the tendency for accelerated crack growth or crack arrest. Clearly the stress analysis for this model has not been very exacting physically in that effects of plasticity have been ignored. However, since plasticity effects will serve to reduce the gradient of stress along the crack surface, the above analysis represents a limiting case and indicates the minimum stress dependency of the functional relationship for dissolution velocity with contour stability.

As indicated previously by Figures 1 and 2, there have been a number of reported cases where blunting, as might be expected from the above model, has been observed in specimens where crack growth due to stress corrosion cracking has been arrested. At this time, whether this blunting has occurred before or after arrest has not been properly ascertained, but the relationship of this phenomenon and the above model surely warrants further investigation.

It should be pointed out that in addition to their use in the analysis of this particular stress corrosion mode, the stress field equations given by equations (1), (2) and (3) may be used in a number of other applications. For example, they provide a convenient method for determining stress intensity factors using photoelastic methods where the difficulty of working with a plane crack in a brittle photoelastic material can be easily avoided by using a blunt crack.

CONCLUSIONS

1. The elastic stress field equations for blunt cracks are presented in a form equivalent to the usual sharp crack tip stress fields for each of the three modes.
2. These elastic stress field equations are incorporated in showing that a stress corrosion cracking model, based upon material dissolution (or degradation), suggests that the dissolution velocity should be more strongly dependent upon stress than a one-half power law for accelerated stress corrosion cracking to occur.
3. The previous statement is equally correct if there is a plastic zone in the vicinity of the crack tip and therefore may represent a "minimum criteria" for stress corrosion cracking with material dissolution.
4. The stress field equations presented have other applications as well.

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RÉSUMÉ - Les équations de champ de tension élastique pour des fissures épointées sont dérivées et présentées de la même façon que celles qui décrivent le champ de tension des pointes de fissures aigues. Ces équations de champ de tension sont employées dans l'analyse d'un modèle de dissolution décrivant l'arrêt du craquement corrosif sous tension obtenu par l'émoussement des pointes de fissures, qu'on observe souvent dans l'arrêt du craquement corrosif sous tension.

ZUSAMMENFASSUNG - Es wird die Spannungsverteilung vor einem Riss mit einem Kruemmungsradius groesser Null fuer den elastischen Fall abgeleitet und in einer den ueblichen Gleichungen fuer den scharfen Riss aehnlichen Form dargestellt. Diese Spannungsgleichungen werden angewandt, um ein Modell fuer das Anhalten eines Bruchs durch die Abstumpfung der Riss-spitze zu untersuchen. Bei der Spannungskorrosion ist dieses Anhalten eines Bruches haeufig von einer Abstumpfung begleitet.