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A note on willingness to pay and willingness to accept

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Abstract. In a recent paper Loehman (1991) derives some relationships between measures of benefit for nonmarket goods. This note presents some remarks on Loehman's methodology, several results, if the good considered is normal, and an extension to the case of pollution, i.e. public 'bads'.

1. Introduction

Measures of benefit for nonmarket goods are important for cost benefit analysis and in environmental economics. In a recent paper Loehman (1991) at first derives some relationships between various indicators for these goods and then shows that these measures can often be determined by the use of empirical data on Marshallian demand functions. In view of the significance of this field for theory and application this note presents some remarks on Loehman's methodology, some corrections of the results and an obvious, but important extension to the case of pollution, i.e. public 'bads'. But it deals only with the first topic, namely the relations between appropriate measures of benefit.

Section 2 briefly repeats the underlying model in order to make the note self-contained. Here the various measures are derived and interpreted in detail. Their connection to the usual Hicksian measures is demonstrated. Loehman does not take into account the ordinal character of utility functions. She uses the sign of the second derivatives, which is *always* indefinite. Therefore these assumptions are not very meaningful. In Sect. 3 alternative assumptions are proposed and interpreted which do not suffer from this deficiency. The implications of these conditions are investigated in Sect. 4. Here we consider the situation with no income effects as specific case. Finally Sect. 5 examines private nonmarket goods which are detrimental to consumers or public 'bads' as well. This type of com-

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modities plays an important role in the assessment of environmental programs and in surveys on the evaluation of external effects. Section 6 concludes the paper.

2. Measures of willingness to pay and willingness to accept

We will focus on a single typical consumer. She can buy a bundle of commodities on markets for a given system of market prices. Moreover she consumes a non-market good Y, a good she cannot influence and she has not to pay for. Since we are mainly interested in the evaluation of (changes of) Y we use only Y and the exogenous income M as parameters. All other variables, i.e. essentially prices, are dropped a priori because they are *not* changed during the analysis. But of course they are used implicitly. We need two tools for a description of the consumer's behavior: the first one is the indirect utility function $U(M, Y)$ giving the maximum level of utility attained by the consumer who faces the market prices, the income M and the quantity Y. The other one is the corresponding expenditure function $E(\bar{U}, Y)$. It reflects the minimum income which is necessary to obtain the utility level[1] $\bar{U}$ if market prices and Y are given. Both functions are assumed to be twice continuously differentiable. It should be stressed that U and E do depend on each other. If we use another indirect utility function V representing the *same* underlying preference ordering, we get another directly related expenditure function $\bar{E}(\bar{V}, Y)$. All following derivations do *not* depend on the choice of the representation U for the consumer's preference ordering! Finally we make two further assumptions: The utility function U is quasi-concave in M and Y. This implies that

$$U_M^2 U_{YY} - 2\, U_Y U_M U_{YM} + U_Y^2 U_{MM} \leq 0 \tag{1}$$

and means the following: If two combinations of (M, Y) lead to the *same* level of utility, every (convex) mixture of these 'bundles' never decreases utility, i.e. is in general better for the consumer. Obviously this property depends on the preference *ordering* and thus inequality (1) is an ordinal condition. It is fullfilled for every utility function representing the preference ordering. Moreover it is supposed that Y is a good desired by the consumer. Therefore its marginal utility $\partial U/\partial Y = U_Y$ is positive. The expenditure function reacts negatively to increases in Y, since less income is necessary to attain a given level $\bar{U}$, i.e. $\partial E/\partial Y = E_Y$ is negative.

Now we consider a status quo in which the consumer gets or consumes the quantity Y_0. Then she attains the utility level $\bar{U}_0 = U(M, Y_0)$. We want to derive the consumer's evaluation of an increase or decrease of Y_0 by (a small quantity) y. In each case there are (at least) two possibilities. We use the Hicksian equivalent and compensating variation.[2] Let us first look at an increase y. The consumer's willingness to accept WTA^e is equal to the minimum amount of income, she must get, if she is to be as well off as in the *new* situation (after the increase in Y), but if Y_0 is not increased really: It is defined implicitly by

$$U(M + WTA^e, Y_0) = U(M, Y_0 + y) =: \bar{U}_2 \; . \tag{2}$$

[1] Utility levels are denoted by $\bar{U}$. The utility function is given by U.

[2] The interpretation provided differs slightly from that given by Loehman. Here we start from a defining relation which is directly based on the utility function. Cf. Ebert (1984) for a characterization of the Hicksian measures.

By using duality relations we know that

$$E(\bar{U}_2, Y_0) = M + WTA^e \ , \tag{3}$$

i.e. the minimum amount of income implying the level $\bar{U}_2$ is equal to the income M and the willingness to accept WTA^e. Therefore this measure can be expressed as

$$WTA^e = E(\bar{U}_2, Y_0) - E(\bar{U}_0, Y_0) \tag{4a}$$

$$= E(\bar{U}_2, Y_0) - E(\bar{U}_2, Y_0 + y) \tag{4b}$$

since $M = E(\bar{U}_0, Y_0) = E(\bar{U}_2, Y_0 + y)$. WTA^e is the Hicksian equivalent variation for this case.

Similarly we can derive the willingness to pay WTP^c. It is the maximal amount of income the consumer can do without if good Y is really provided in the quantity $Y_0 + y$ and if she is to be as well off as in the old situation (status quo). It must satisfy the condition

$$U(M - WTP^c, Y_0 + y) = U(M, Y_0) \ . \tag{5}$$

Analogous operations lead to

$$WTP^c = E(\bar{U}_2, Y_0 + y) - E(\bar{U}_0, Y_0 + y) \tag{6a}$$

$$= E(\bar{U}_0, Y_0) - E(\bar{U}_0\, Y_0 + y) \ . \tag{6b}$$

Obviously WTP^c corresponds to the Hicksian compensating variation.

In the same way we can proceed for a decrease in Y. At first we consider the willingness to pay WTP^e (equivalent variation). It equals the maximum amount of income the consumer could give away in the old situation (status quo) if she is to be as well off as with the actual decrease in Y:

$$U(M - WTP^e, Y_0) = U(M, Y_0, -y) =: \bar{U}_1 \ . \tag{7}$$

She could pay WTP^e as an equivalent to the welfare loss. This loss is given (in absolute terms) by

$$WTP^e = E(\bar{U}_0, Y_0) - E(\bar{U}_1, Y_0) \tag{8a}$$

$$= E(\bar{U}_1, Y_0 - y) - E(\bar{U}_1, Y_0) \ . \tag{8b}$$

Finally we define the willingness to accept WTA^c (compensating variation). It describes the amount of income the consumer needs in the new situation (with $Y_0 - y$) in order to attain the old level of utility

$$U(M + WTA^c, Y_0 - y) = U(M, Y_0) \ . \tag{9}$$

She would be willing to accept the amount WTA^c since it guarantees the utility level $\bar{U}_0$. WTA^c can be represented as

$$WTA^c = E(\bar{U}_0, Y_0 - y) - E(\bar{U}_1, Y_0 - y) \tag{10a}$$

$$= E(\bar{U}_0, Y_0 - y) - E(\bar{U}_0, Y_0) \ . \tag{10b}$$

Summing up we see that we obtain four different measures. Each can be expressed implicitly by means of the utility function. This definition is the basis of an interpretation. But they can be computed directly by using the expenditure function as a money metric ((4a), (6a), (8a), (10a)). We recognize that the measures defined above use three different measuring sticks: Prices are always assumed to be constant, *but* the quantity of the nonmarket good Y is Y_0, Y_0+y, and Y_0-y, respectively. This good is exogeneous to the consumer. Obviously her welfare depends on it and the welfare change, i.e. the willingness to pay or to accept, as well. Therefore only WTA^e and WTP^e are directly comparable with one another. On the other hand the indicators can be represented by the difference in expenditures for the change in Y if the level of utility is hold constant ((4b), (6b), (8b), (10b)). This representation proves to be useful in a comparison of magnitudes examined below.

3. Normality of Y

Up to this point we assumed only quasi-concavity of the indirect utility function with respect to M and Y. Now we consider income effects in addition. The *marginal* willingness to pay or to accept is given by the marginal rate of substitution between M and Y

$$MRS=\frac{dM}{dY}=\frac{U_Y(M,Y)}{U_M(M,Y)} . \tag{11}$$

It seems reasonable to postulate that Y is a normal good, i.e. the demand for Y is increasing in income M. This is equivalent to an increasing MRS for a given quantity Y:

(Normality)

$$\frac{d}{dM}\,MRS(M,Y)=\frac{U_{YM}\cdot U_M-U_Y\cdot U_{MM}}{U_M^2}\geq 0 . \tag{12}$$

This condition is ordinal, i.e. independent of the particular choice of the indirect utility function U. If it is fulfilled for one function U, it is satisfied for *all* indirect utility functions representing the given preference ordering. Furthermore in principle it can be tested by observing the demand function. Inequality (12) replaces Loehman's postulates $U_{YM}\geq 0$ and $U_{MM}\leq 0$ which do not make sense since the sign of these derivatives does depend on the utility function chosen. Under certainty there is no possibility to check whether these conditions are fulfilled.

Below we will examine one specific case, namely the situation where there are no income effects. It is given by

$$\frac{U_{YM}\cdot U_M-U_Y\cdot U_{MM}}{U_M^2}=0 . \tag{12a}$$

It will turn out that on the basis of (12a) we get some unambiguous results.

Next we come to the implication of quasi-concavity and normality. The marginal bid for the nonmarket good Y, defined by $-\partial E/\partial Y$, plays the central role in the analysis. We get (analogously to Loehman's Lemma 1):

Lemma 1. *Let the conditions* (1) *and* (12) *be satisfied. Then the marginal bid* $-\partial E(\bar{U}, Y)/\partial Y$ *is nonincreasing in* Y *and nondecreasing in* $\bar{U}$. *If there are no income effects* (*defined by* (12a)) $-\partial E/\partial Y$ *does not depend on* $\bar{U}$.[3]

Proof. From duality we have $E(U(M, Y), Y) = M$ and therefore

$$\frac{\partial E}{\partial U} \cdot \frac{\partial U}{\partial Y} + \frac{\partial E}{\partial Y} = 0 \quad \text{(cf. Loehman)} .$$

It implies

$$-\frac{\partial E}{\partial Y}(\bar{U}, Y) = \frac{U_Y(E(\bar{U}, Y), Y)}{U_M(E(\bar{U}, Y), Y)} . \tag{13}$$

Differentiating this expression with respect to Y and $\bar{U}$ and using quasi-concavity (1) and normality (12), respectively, yield the above results. Q.E.D.

This Lemma has been formulated and proved in a completely ordinal framework.

4. Comparisons of benefit measures reconsidered

As discussed above the measures are based on different measuring sticks. Nevertheless we consider all possible six pairwise combinations of measures in order to make this contribution and its purely ordinal methodology comparable with Loehman's paper. Furthermore it is of interest to confront WTA^e with WTP^c for an increase from Y_0 to $Y_0 + y$ and WTP^e with WTA^c for a corresponding decrease in y since these measures reflect the answer of interviewees in surveys. E.g. in case of an increase of Y you can ask two different questions: what are you willing to pay for this change, if it is made (WTP^c) or what are you willing to accept *instead* of this change (WTA^e)? Typically one finds that $WTP^c < WTA^e$ (cf. e.g. Coursey, Hovis, Schulze (1987), Knetsch (1990) and the references therein). So the question arises whether this relationship can be founded on theoretical grounds. For similar reasons it is interesting to confront WTP^e and WTA^c. We obtain

Theorem 2. *Quasi-concavity of* U *(condition* (1)*) and normality of* Y (*condition* (12)) *yield:*

$$WTA^e \geq WTP^c \tag{14a}$$

$$WTA^c \geq WTP^e \tag{14b}$$

and

$$WTA^c \geq WTP^c . \tag{14c}$$

The size relationship of (WTA^e *and* WTP^e), (WTA^e *and* WTA^c), *and* (WTP^e *and* WTP^c) *can be arbitrary.*

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[3] The diminishing marginal utility of income $\partial^2 U/\partial M^2$, as used by Loehman, is not well defined and will not be employed here.

Proof. By (4b) we have

$$WTA^e = E(\bar{U}_2, Y_0) - E(\bar{U}_2, Y_0 + y)$$
$$= \int_{Y_0}^{Y_0+y} -\frac{\partial E}{\partial Y}(\bar{U}_2, x)\,dx \ ,$$

similarly by (6b)

$$WTP^c = E(\bar{U}_0, Y_0) - E(\bar{U}_0, Y_0 + y)$$
$$= \int_{Y_0}^{Y_0+y} -\frac{\partial E}{\partial Y}(\bar{U}_0, x)\,dx \ .$$

Since $-\partial E/\partial Y$ is nondecreasing in $\bar{U}$ we get $WTA^e \geq WTP^c$. (14b) is proved analogously. The definition of WTA^c implies

$$WTA^c = \int_{Y_0-y}^{Y_0} -\frac{\partial E}{\partial Y}(\bar{U}_0, x)\,dx \ .$$

Therefore we obtain $WTA^c \geq WTP^c$, since $-\partial E/\partial Y$ is nonincreasing in Y.

In all other cases the relative sizes cannot be determined in general as is easily seen by an investigation of analogous integral representation or more simply by comparing the respective areas under marginal bid curves (cf. Fig. 1b in Loehman (1991)). We always get opposing effects in general: $-\partial E/\partial U$ is increased by a change in Y and decreased by a change in $\bar{U}$ or conversely. Q.E.D.

Obviously the inequalities (14a) and (14b) present an explanation of the findings in surveys. These relations have to be expected if consumers behave like maximizers of utility. But there remain some ambiguities. They disappear if we exclude income effects. Then the marginal $-\partial E/\partial Y$ is independent of the utility level and only changes in Y play a role. This assumption yields

Theorem 3. *Assume quasi-concavity of U (condition* (1)*) and no income effects (condition* (12a)*). Then all benefit measures can be compared. The ordering is given by*

$$WTA^c = WTP^e \geq WTA^e = WTP^c \ .$$

Proof. Analogous to the proof of Theorem 1. Q.E.D.

Here two points are interesting. First, for an increase and decrease in Y the respective equivalent and compensating measures are identical: $WTP^e = WTA^c$ and $WTA^e = WTP^c$. That seems to be reasonable since only the difference in the nonmarket good Y is important. Second, we obtain $WTA^e \leq WTP^e$: The willingness to accept for an increase in Y is smaller than or equal to the willingness to pay if Y is decreased by the same number of units. This result is implied by the quasi-concavity of U with respect to M and Y and can intuitively be interpreted by a decreasing marginal rate of substitution.

The results presented here in Theorem 2 and 3 are the same as in Loehman's Theorem 2. But we use different assumptions and the proof is based on (4b) and (6b). In Theorem 2 (page 286) Loehman presents a further condition implying a certain size relationship of welfare measures. The condition is an ordinal one, i.e. its does not depend on the choice of the representation U. If the marginal rate of substitution between M and Y: $MRS = dM/dY$ is *in*creasing (!) in Y this condition is implied. Thus this case seems to be of no interest.

5. Detrimental goods

A nonmarket good Y, e.g. pollution, can harm a consumer or can yield a negative (marginal) utility. The above framework can be easily changed in order to reflect this type of situation. Then we have to take into account that

$$\partial U/\partial Y < 0 \quad \text{and} \quad \partial E/\partial Y > 0 \ .$$

Moreover we have to change the definition of measures giving the willingness to pay or to accept. As an example let us consider an increase in pollution ($Y_0 \to Y_0 + y$) and the equivalent variation. It should be defined as $\overline{WTP}^e$ satisfying

$$U(M - \overline{WTP}^e, Y_0) = U(M, Y_0 + y) \ .$$

$\overline{WTP}^e$ is the maximum amount the consumer is willing to pay if she is to be as well off as in a situation with an increase of Y to $Y_0 + y$. $\overline{WTP}^e$ is defined in analogy to WTA^e, but the sign is changed in order to make it positive. Furthermore it is the willingness to *pay* now, since we examine an *in*crease in pollution and the consumer is worse off. Proceeding in this way we obtain four measures for this case:

$$\overline{WTP}^e = -WTA^e, \overline{WTA}^c = -WTP^c \ ,$$

$$\overline{WTA}^e = -WTP^e, \quad \text{and} \quad \overline{WTP}^c = -WTA^c \ .$$

If we assume quasi-concavity of U in M and Y, and inferiority of Y (the marginal rate of substitution between M and Y defined by $-U_Y/U_M$ is increasing in M, i.e. it means that a high income consumer prefers *less* of the nonmarket good Y), we obtain

Lemma $\bar{1}$. *Let condition* (1) *and the condition*

$$\frac{d}{dM} MRS(M, Y) = \frac{d}{dM}\left(-\frac{U_Y(M, Y)}{U_M(M, Y)}\right)$$

$$= -\frac{U_{YM} U_M - U_Y U_{MM}}{U_M^2} \geq 0 \qquad (\overline{12})$$

be satisfied. Then the marginal compensation (needed to make the consumer indifferent) $\partial E(\bar{U}, Y)/\partial Y$ is nondecreasing in Y and $\bar{U}$. If there are no income effects $\partial E/\partial Y$ is independent of $\bar{U}$.

Using this Lemma we can compare the benefit measures again. Confining ourselves to the interesting comparisons we note that

$$\overline{WTP}^c \leq \overline{WTA}^e \quad \text{and} \quad \overline{WTP}^e \leq \overline{WTA}^c$$

under the assumptions given above, and that

$$\overline{WTP}^e = \overline{WTA}^e, \overline{WTP}^c = \overline{WTA}^e \ , \quad \text{and} \quad \overline{WTA}^e \leq \overline{WTP}^e$$

if there are no income effects. Thus the main relationships are unchanged.

6. Summary

This note draws attention to the assumptions which allow to compare various willingness to pay and willingness to accept measures. It corrects a shortcoming of Loehman (1991) and shows that some comparisons are generally possible. The results support empirical findings from surveys, which can be found in the literature. Furthermore it is demonstrated that similar relations hold for non-market goods which are detrimental.

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