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# Adult-equivalence scales and the economic implementation of interpersonal comparisons of well-being*

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**Abstract.** *Equivalence Scale Exactness* (ESE) or *Independence of Base* (IB), a condition on household preferences and interpersonal comparisons, makes adult-equivalence scales independent of utility levels. ESE is characterized by *Income-Ratio Comparability* (IRC) which assumes that utility equality is preserved by income scaling. If ESE/IRC is a maintained hypothesis, equivalence scales can be estimated from behaviour alone if preferences are not piglog. This condition is not met by a family of translog expenditure functions or by the Almost Ideal Demand System. A translog expenditure function can be used for the 'reference' household, however, together with an independent specification of the equivalence scale.

## 1. Introduction

In economic environments, all practical social evaluations (applications of welfare economics) must deal with two important problems: (1) in different states of affairs prices are likely to be different, and (2) most 'welfarist' social ethics require that the well-being (utility) of individual members of different households be compared.

The first problem means that households must be treated separately unless all of them have identical incomes and characteristics (number of adults and children, location, needs, states of health, and preferences). If they do, then this problem

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reduces to a standard-of-living-index exercise. If not, then some procedure must be found that allows for the different price responses of different households.

The second problem is complicated by the fact that individual people enjoy utilities while households generate demands and allocate goods and services to individual household members and to household public and semi-public consumption. A similar problem arises when investigators employ adult-equivalence scales to characterize demand systems.

To deal with these issues, we assume that the demands of the household are rationalizable by utility-maximizing behaviour which results from maximization of a household social-evaluation function. Given this, a procedure must be found to compare utilities across households.

A simple and appealing way of doing this is to use adult-equivalence scales. If, for example, a household of two adults and two children with an income of $ 30,000 has an equivalence value of three, then the household members can be regarded as enjoying the same level of well-being as a 'reference' single adult with $ 10,000, facing the same prices the household does. For social-evaluation purposes the household is equivalent to four reference single adults with incomes of $ 10,000 each. Thus, adult-equivalence scales permit the conversion of a complex problem in social evaluation into one where each household consists of a single person, and all households have the same preferences. This procedure is used to define equivalence scales formally (for a traditional treatment see Deaton and Muellbauer [1980][1]) and it requires comparisons of *levels* of utility between individuals in different households.

In this paper, we provide a theoretical rationale for household equivalence scales and their use in normative economics. In addition, we investigate the possibility of using household demand behaviour to make statements about the levels of well-being of people in different households.

We introduce the model and attendant technical assumptions in Sect. 2 and the theory of adult-equivalence scales in Sect. 3. Although interhousehold comparisons of well-being are required, utilities may be ordinally measurable – cardinal measurement of utilities is not necessary. We then present a simple condition on preferences that is necessary and sufficient for adult-equivalence scales to be independent of the utility level of the household members which we call *Equivalence-Scale Exactness* (ESE). It was discovered and characterized independently by Lewbel (1989) who calls it *Independence of Base.* It is satisfied by most practical adult-equivalence scales.

Equivalence scales that satisfy ESE are a special case of our general equivalence scales. We show in Sect. 4 that ESE is completely characterized by a joint restriction on preferences and interhousehold comparisons which we call Income-Ratio Comparability (IRC). It requires that, if there exist incomes such that the members of two households facing the same prices are equally well-off, then any scaling of household incomes preserves equality of well-being. ESE/IRC allows preferences to differ from household to household and to be nonhomothetic.

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[1] We restrict our attention to commodity-independent equivalence scales. See also Blackorby and Donaldson (1991a, 1993), Blundell and Lewbel (1991), Browning (1988, 1992), Grunau (1988), Lewbel (1989, 1991), Muellbauer (1974, 1975, 1976), Pollak and Wales (1979, 1981). For discussions of commodity-specific scales see Barten (1964), Blackorby and Donaldson (1991a), Browning (1988, 1992), Gorman (1976), Nelson (1988), and Ray (1992). Jorgenson and Slesnick (1983, 1984a, b, 1987) combine the two types of scales.

We make and prove, in Sect. 5, a claim which may surprise the reader. If ESE/IRC is a maintained hypothesis and preferences are globally regular and nonhomothetic the equivalence scales can be estimated from behaviour alone. Alternatively, if preferences are only locally regular, ESE/IRC allows the scales to be estimated from behaviour as long as preferences are not piglog.

In Sect. 6 we provide necessary and sufficient conditions for a family of translog expenditure functions – one for each household type – to satisfy ESE, and compare this to the exact aggregation requirements which are frequently imposed on the translog.[2] We then note that the resulting family cannot be used to identify equivalence scales using ESE/IRC as a maintained hypothesis. A procedure that uses the translog is available, however, and it has been implemented by Phipps (1990). A translog utility or expenditure function is assigned to the reference household only, and a functional form is chosen for the equivalence scale. This results, in general, in expenditure functions for other household types that are *not* translog, and the equivalence scale can be identified from demand behaviour.

In Sect. 7 we show that the Almost Ideal Demand System also fails to identify equivalence scales using ESE/IRC. Concluding remarks and an Appendix containing proofs complete the paper.

Although we find the equivalence-scale methodology appealing, we do not offer this paper in its defence. Rather, we attempt to discover the restrictions that its imposition requires – both on household preferences and on interpersonal comparisons of well-being. Any index number approach has its costs in terms of modelling flexibility, and equivalence scales are no exception. At least one study (Dickens, Fry, and Pashardes 1992) has indicated that the ESE restriction may not be very serious, given a good specification for household preferences (see Sects 7 and 8), but more work is needed before a conclusive judgement can be made.

## 2. The model

We consider a simple model of utilities and preferences of household members in an economic environment with $m$ private goods. The population consists of $n$ people, $n \geq 2$, and it is grouped into $H$ households, $2 \leq H \leq n$. Households are indexed by $h$ and are described by vectors of characteristics. $\mathbf{A}$ is the set of all possible vectors of characteristics ($|\mathbf{A}| \geq 2$) and $\alpha_h$ describes household $h$. $N(\alpha_h)$ is the number of people in household $h$, and we therefore have

$$\sum_{h=1}^{H} N(\alpha_h) = n \ . \tag{2.1}$$

$\alpha$ may contain the names of household members, their ages, sexes, locations, states of health, and so on.

Households are assumed to be income-sharing groups of people which make joint consumption decisions. Households, therefore, have preferences that rationalize their demand functions while individuals experience utilities. Following Samuelson (1956), each household is assumed to maximize a continuous, increas-

[2] See Jorgenson, Lau, and Stoker (1980, 1982), and Jorgenson and Slesnick (1983, 1984a, b, 1987).

ing, quasi-concave social-evaluation function of its members' utilities,[3] and individual utility levels, in turn, depend on individual consumption of private, public and semi-public household goods.

The household social-evaluation function may be normalized to measure the equally-distributed-equivalent utility of the household's members: it is the utility level which, if enjoyed by all, would be equivalent, according to the household's social-evaluation function, to the utility vector actually experienced. If a household with characteristics $\alpha \in \mathbf{A}$ contains $l$ members with utilities $(u^1,\ldots,u^l)$, then the equally-distributed-equivalent utility function $\tilde{W}(\cdot,\alpha)$ corresponding to the household social-evaluation function $W(\cdot,\alpha)$ is given by

$$\tilde{W}(u^1,\ldots,u^l,\alpha)=\tilde{u}\ , \tag{2.2}$$

where $\tilde{u}$ is defined by

$$W(\tilde{u},\ldots,\tilde{u},\alpha)=W(u^1,\ldots,u^l,\alpha)\ . \tag{2.3}$$

$\tilde{W}(\cdot,\alpha)$ and $W(\cdot,\alpha)$ are ordinally equivalent.

If the household consumes $x=(w,z)\in E^m_+$ where $w$ is a vector of private goods and $z$ is a vector of pure public goods from the household's point of view,[4] and if the household can make lump-sum transfers to its members, then the household utility function $U(\cdot,\alpha)\colon E^m_+ \mapsto E$ is given by

$$\begin{aligned} U(x,\alpha)&=U(w,z,\alpha)\\ &=\max_{(w^1,\ldots,w^l)}\left\{\tilde{W}(U^1(w^1,z),\ldots,U^l(w^l,z),\alpha)\,\middle|\,\sum_{i=1}^{l} w^i\le w\right\}\ , \end{aligned} \tag{2.4}$$

where $w^i$ is person $i$'s consumption of private goods and $U^i(w^i,z)$ is his or her utility level, $i=1,\ldots,l$. $U(\cdot,\alpha)$ measures a utility level that can be assigned to each household member and the vector of equal utilities $(\tilde{u},\ldots,\tilde{u})$ is ethically indifferent, according to the household's own ethical preferences, to the actual utility vector that results from the maximization in (2.4). We assume each $U^i$ is continuous, monotonic (nondecreasing and locally nonsatiated) in $w$ and $z$, and concave, and (given that $W(\cdot,\alpha)$ is continuous, increasing, and quasi-concave) it follows that $U(\cdot,\alpha)$ is continuous, monotonic, and quasi-concave.[5] Household preferences and equally-distributed utility levels are therefore given by a single utility function $U\colon E^m_+\times\mathbf{A}\mapsto E$.

If the household social-evaluation function is *maximin*, with

$$\tilde{W}(u^1,\ldots,u^l,\alpha)=\min\{u^1,\ldots,u^l\}\ , \tag{2.5}$$

$U(x,\alpha)$ is the utility level of *each* household member. In that case, individuals actually experience the utility level $U(x,\alpha)=U(w,z,\alpha)$. This paper is consistent with the more general model, but we write as if the model were rationalized by a maximin social-evaluation function for each household.

Because $\alpha$ can, in principle, name household members, (2.4) is not a serious theoretical restriction. In practice, however, each $\alpha$ describes groups, and (2.4)

[3] Interpersonal comparisons of utilities are needed for this.

[4] All goods are private goods in the economy. We ignore semi-public household goods for mathematical convenience.

[5] See Negishi (1963) for a proof in the private-goods case; it can be extended easily to cover pure public goods.

implies that households with the same characteristics have the same preferences and that, if utility levels are comparable across households, individuals in different households with the same characteristics and consumption vectors are equally well off.

Welfare analyses should be conducted by using a social-evaluation function which depends on the utilities of all $n$ individuals in the economy. In the maximin case, the utility level of (2.5) may be assigned to each person in the household without ethical difficulty. Thus, all people, including children, count in social comparisons. In the general case, however, equally-distributed-equivalent utilities must be used, and this forces investigators to accept the ethics of the household as appropriate for intra-household social decisions, an ethically unattractive compromise that is absent in the equal-utilities case.

The indirect utility function $V$ and the expenditure function $C$ corresponding to $U$ are given by

$$u = V(p, y, \alpha) = \max_x \{U(x, \alpha) \mid p \cdot x \leq y\} \tag{2.6}$$

and

$$C(u, p, \alpha) = \min_x \{p \cdot x \mid U(x, \alpha) \geq u\} \ . \tag{2.7}$$

For each $\alpha \in \mathbf{A}$, $V$ is continuous and homogeneous of degree zero in $p$ and $y$, increasing in $y$, and quasi-convex, nonincreasing and locally nonsatiated in $p$. $C$, on the other hand, is continuous in $(u, p)$, increasing in $u$, and homogeneous of degree one, concave, and nondecreasing in $p$. In addition $C$ and $V$ are related by the identity

$$y = C(u, p, \alpha) \leftrightarrow u = V(p, y, \alpha) \ . \tag{2.8}$$

Some household models use a utility function that represents the parents' preferences, which are assumed to include some concern for the well-being of children, if any are present (see Browning 1992). Although our model may be interpreted in this way, for welfare economics applications we prefer a formulation which permits each individual to count directly.[6]

## 3. Interpersonal comparisons and adult equivalence scales

The simplest and probably the most common form of interpersonal comparison is made through the use of adult-equivalence scales. These scales deal with two phenomena. The first concerns the fact that different households contain different numbers and types (adults, children, disabled people, etc.), and therefore have different preferences and needs. The second concerns the fact that there are economies of scale in household consumption (due to public and semi-public consumption within the household). As an example, suppose that a household consists of two adults with an income of \$ 60,000. If we say that the number of adult equivalents in the household is 1.5, then we mean that the household is equivalent, for utility purposes, to two single reference adults with incomes of

[6] See Blackorby and Donaldson (1991a) for other results on equivalence scales which employ this model. In addition, characterizations of the cost-of-children rules proposed by Rothbarth and others are contained in Blackorby and Donaldson (1991b).

\$ 40,000 each (\$ 60,000 divided by 1.5). The farther the number of adult equivalents for a two-adult household is below 2, the greater is the economy of scale.

Let $d$ be the number of adult equivalents in a household with characteristics $\alpha$ and income $y$ facing prices $p$; $d$ is defined implicitly by

$$u = V(p, y, \alpha) = V\left(p, \frac{y}{d}, \alpha^r\right) . \tag{3.1}$$

$\alpha^r$ is the characteristics vector of a reference household with $N(\alpha^r) = 1$ (a single adult). We assume that for all $\alpha \in \mathbf{A}$ there exists $d$ such that (3.1) has a solution. because $V$ is increasing in $y$, the solution is unique.

The definition of $d$ in (3.1) is not meaningful unless the information structure allows comparisons of levels of utilities[7] between members of different households. If $V$ is replaced by $\tilde{V}$ where

$$\tilde{V}(p, y, \alpha) = \phi\,(V(p, y, \alpha), \alpha)) \tag{3.2}$$

and $\phi$ is increasing in its first argument, then the number of adult equivalents can change. For each $\alpha$, $\tilde{V}(\cdot, \cdot, \alpha)$ is ordinally equivalent to $V(\cdot, \cdot, \alpha)$ and each household's preferences are unchanged. If, on the other hand, $V$ is replaced by

$$\hat{V}(p, y, \alpha) = \psi\,(V(p, y, \alpha)) \tag{3.3}$$

($\psi$ increasing), then the number of adult equivalents (defined by (3.1)) remains unchanged. $\hat{V}$ and $V$ make the same interhousehold comparisons of utility levels.

We require, therefore, that the information structure support interhousehold comparisons of levels of utility (at least). We call this condition Ordinal Full Comparability Plus (OFC + ), and require that any two utility functions regarded as informationally equivalent be related by (3.3).

(3.1) implicitly defines a function $d = D(u, p, \alpha)$.[8] If the reference adult has an income $y/D(u, p, \alpha)$ and faces prices $p$, then he or she enjoys a utility level exactly equal to the one enjoyed by each member of a household with characteristics $\alpha$ and income $y$, facing prices $p$. Using (2.8), $d$ is given by

$$d = D(u, p, \alpha) = \frac{C(u, p, \alpha)}{C(u, p, \alpha^r)} =: \frac{C(u, p, \alpha)}{C^r(u, p)} . \tag{3.4}$$

It is the expenditure needed to bring each member of a household with characteristics $\alpha$ to utility level $u$ divided by the expenditure needed to bring the reference person to the same level of utility. $D$ is homogeneous of degree zero in $p$, and $D(u, p, \alpha^r) = 1$ for all u, $p$.

The function $D$ depends on the utility level of the members of the household, a number that is normally unobservable. A practical solution to this problem is to use a single reference level of utility, $u^r$, and to define an index

$$\bar{d} = \bar{D}(p, \alpha) = \frac{C(u^r, p, \alpha)}{C(u^r, p, \alpha^r)} . \tag{3.5}$$

If $u^r$ is the poverty utility level, then $\bar{d}$ is the ratio of the poverty line for the household in question to the poverty line for the reference household (at the same prices).

$\bar{D}$ uses much less information than $D$. All that is necessary is that a single

[7] See Blackorby, Donaldson and Weymark (1984) or Sen (1977) for a discussion.
[8] Lewbel (1989, 1991) calls this the *cost of characteristics* function.

indifference surface (corresponding to $u^r$) be identified for each $\alpha$. This means that interpersonal comparisons need only be made for a single level of utility. A method for doing this is to find reference consumption bundles $X(\alpha)$, $\alpha \in \mathbf{A}$, such that, for all $\alpha$

$$U(X(\alpha),\alpha)=u^r\ . \tag{3.6}$$

If $u^r$ is the poverty utility level, then $X(\alpha)$ is a poverty consumption bundle for a household with characteristics $\alpha$.

Social evaluations performed with the above indexes of adult equivalence are approximate because the index $\bar{D}(p,\alpha)$ is not always equal to the adult-equivalence measure $D(u,p,\alpha)$. Social rankings will be correct if and only if the index is exact, that is, if and only if

$$\bar{D}(p,\alpha)=D(u,p,\alpha) \tag{3.7}$$

for all $(u,p,\alpha)$. Exactness therefore requires that $D$ be independent of $u$. Lemma 1 of Lewbel (1989) demonstrates that the necessary and sufficient condition for this, and therefore for exact social evaluation, is that the expenditure function can be written as

$$C(u,p,\alpha)=\bar{C}(u,p)\,\hat{C}(p,\alpha) \tag{3.8}$$

for all $(u,p,\alpha)$.[9]

When (3.8) holds, we say that utilities satisfy *equivalence-scale exactness* (ESE). Lewbel calls this *independence of base* (IB). (3.8) is a restriction on $U$, $V$, and $C$; more specifically, it restricts them both *interpersonally* and *intrapersonally.* We can rewrite (3.8) as a relationship between $C^r$, the reference cost function (derived using (2.7) and (3.4)), the exact (independent of $u$) equivalence scale, and $C$, the expenditure function. The exact equivalence scale $D$ is given by

$$\Delta(p,\alpha):=D(u,p,\alpha)=\frac{\hat{C}(p,\alpha)}{\hat{C}(p,\alpha^r)} \tag{3.9}$$

and in this case (3.7) holds. It is homogeneous of degree zero in $p$ because $C$ is homogeneous of degree one in $p$, and, of course, $\Delta(p,\alpha^r)=1$ for all $p$. Given (3.8) and (3.9), the expenditure function can be rewritten as

$$C(u,p,\alpha)=C^r(u,p)\Delta(p,\alpha)\ . \tag{3.10}$$

This equation shows that one household's preferences may be chosen arbitrarily (corresponding to $C^r(u,p)$). The equivalence scales are allowed to depend on $p$, a reasonable condition because economies of scale in consumption are likely to be different for different goods and services. Given $C^r$, choice of an equivalence scale, $\Delta$, completely determines household preferences for all $\alpha$, resulting in a significant restriction on preferences. Using (2.8) and (3.10), the indirect utility function $V$ corresponding to $C$ can be written as

$$V(p,y,\alpha)=V^r\left(p,\frac{y}{\Delta(p,\alpha)}\right) \tag{3.11}$$

[9] There is a certain amount of arbitrariness here as $\hat{C}$ can be homogeneous of any degree as long as $\bar{C}$ is homogeneous of one minus that degree. In any case, however, $\Delta$ is homogeneous of degree zero in prices. Either $\bar{C}$ or $\hat{C}$ can be chosen to be homogeneous of degree one; see Blackorby and Donaldson (1991a).

where $V^r(p,y):=V(p,y,\alpha^r)$.

(3.11) conditions both preferences and interhousehold comparisons of utility. Preferences must be related by

$$V(p,y,\alpha)=\phi\left(V^r\left(p,\frac{y}{\Delta(p,\alpha)}\right),\alpha\right). \tag{3.12}$$

In addition, (3.11) requires $\phi(u,\alpha)=u$ for all $(u,\alpha)$, a condition on interhousehold comparisons.

It is possible to find a direct representation of preferences which satisfies (3.8) and (3.11) under certain additional restrictions. It is always possible to rewrite (3.8) as

$$C(u,p,\alpha)=[C^b(u,p)]^\gamma[C^s(p,\alpha)]^{1-\gamma} \tag{3.13}$$

where $C^b$ and $C^s$ are homogeneous of degree one in prices.

**Theorem 3.1.** *If there exists a* $\gamma$, $0<\gamma<1$, *such that* $C^b$ *and* $C^s$ *are concave in prices, the direct utility function satisfying* ESE *can be written as*

$$U(x,\alpha)=\max_{\{x^b,x^s\}}\{U^b(x^b[U^s(x^s,\alpha)]^\beta)\,|\,x^b+x^s=x\} \tag{3.14}$$

*where* $U^s$ *is homogeneous of degree one in* $x^s$ *and* $\beta=\gamma/(1-\gamma)$.

*Proof.* See the Appendix.

(3.10) (or (3.11)) admits two special cases: (i) the function $\hat{C}$ is independent of $p$, and, therefore, the equivalence scale $\Delta$ is independent of $p$; (ii) $\bar{C}$ is independent of $p$, and each household's preferences are homothetic. We call the former *Engel equivalence extactness* and the latter *full homotheticity.*

$U$ satisfies Engel equivalence exactness if, for all $(u,p,\alpha)$,

$$C(u,p,\alpha)=C^r(u,p)\mathring{\Delta}(\alpha) \tag{3.15}$$

and, therefore, for all $(p,y,\alpha)$

$$V(p,y,\alpha)=V^r\left(p,\frac{y}{\mathring{\Delta}(\alpha)}\right); \tag{3.16}$$

it satisfies full homotheticity if, for all $(u,p,\alpha)$,

$$C(u,p,\alpha)=\tilde{C}(u)\check{C}(p,\alpha) \tag{3.17}$$

and, for all $(p,y,\alpha)$,

$$V(p,y,\alpha)=\tilde{V}\left(\frac{y}{\check{C}(p,\alpha)}\right) \tag{3.18}$$

where $\check{C}$ is homogeneous of degree one in $p$. Notice that full homotheticity is an interhousehold condition which is stronger than the homotheticity of each household's preferences.

Lewbel (1989) refers to $y/\hat{C}(p,\alpha)$ (see (3.8)) as *scaled income* and uses it in welfare analysis. It is clear that, unless $\hat{C}$ is homogeneous of degree one in $p$, scaled income cannot be homogeneous of degree zero in $(p,y)$, and, therefore, cannot be ordinally equivalent to any indirect utility function. $y/\hat{C}(p,\alpha)$ is ordinally equivalent to $V(p, y, \alpha)$ if preferences satisfy full homotheticity, (3.18); full homotheticity can be shown to be necessary as well.[10]

To foreshadow our main result, note that, given full homotheticity, the equivalence scale may be modified by multiplying by an arbitrary function of $\alpha$ without changing preferences. Full homotheticity is preserved by this change and preferences remain unchanged. Thus many equivalence scales are consistent with a given set of preferences in this special case.

ESE has the consequence that the budget-share equation for any commodity decomposes additively into a function of $(u, p)$ and a function of $(p, \alpha)$. The first of these is the share equation for the reference household, and this means that the income elasticity of demand for any pure children's good such as day-care is *one.* This is probably not satisfied by real household preferences.

## 4. Income-ratio comparability

Equivalence-scale exactness imposes no restriction on the preferences of a single household, but, given that household's preferences, all the others are linked to it by (3.10). This requirement has implications for interpersonal comparisons of well-being as well. It is these additional implications which we explore in this section.

Suppose that two households face the same prices (with possibly different incomes) and their members enjoy the same level of well-being. We say that *income-ratio comparability* (IRC) is satisfied when equality of well-being is preserved by arbitrary common scalings of the household's incomes.

**Income-ratio comparability (IRC).** *Utilities satisfy income-ratio comparability if and only if, for all* $p \in E^m_{++}$, $\bar{y}, \tilde{y} \in E_+$, $\bar{\alpha}, \tilde{\alpha} \in \mathbf{A}$,

$$V(p, \bar{y}, \bar{\alpha}) = V(p, \tilde{y}, \tilde{\alpha}) \leftrightarrow V(p, \lambda \bar{y}, \bar{\alpha}) = V(p, \lambda \tilde{y}, \tilde{\alpha}) \tag{4.1}$$

*for all* $\lambda > 0$.

It is tempting to conclude, because income-ratio comparability involves the scaling of incomes, that some kind of interhousehold homotheticity is involved. That is not the case in general (although it is when $U(\cdot, \alpha^r)$ is homothetic) because prices are the same on both sides of (4.1).[11] It is simply a normalization along the income-consumption curves, but these are not required to be straight lines.

Although OFC+ is necessary for income-ratio comparability to be meaningful, IRC imposes structure on the comparisons themselves. Further, it provides a characterization of ESE.

**Theorem 4.1.** *Utilities satisfy equivalence-scale exactness if and only if they satisfy income-ratio comparability.*

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[10] See Blackorby and Donaldson (1993).

[11] See the discussion of *Comparision Homotheticity* in Blackorby and Donaldson (1987).

It is easy to show that ESE implies IRC. Because the proof is instructive, we do so here and leave the converse to the Appendix. ESE and (3.11) imply

$$\begin{aligned} V(p,\bar{y},\bar{\alpha}) &= V(p,\tilde{y},\tilde{\alpha}) \\ &\leftrightarrow V^r\left(p,\frac{\bar{y}}{\Delta(p,\bar{\alpha})}\right)=V^r\left(p,\frac{\tilde{y}}{\Delta(p,\tilde{\alpha})}\right) \\ &\leftrightarrow \frac{\bar{y}}{\Delta(p,\bar{\alpha})}=\frac{\tilde{y}}{\Delta(p,\tilde{\alpha})} \\ &\leftrightarrow \frac{\lambda\bar{y}}{\Delta(p,\bar{\alpha})}=\frac{\lambda\tilde{y}}{\Delta(p,\tilde{\alpha})} \\ &\leftrightarrow V^r\left(p,\frac{\lambda\bar{y}}{\Delta(p,\bar{\alpha})}\right)=V^r\left(p,\frac{\lambda\tilde{y}}{\Delta(p,\tilde{\alpha})}\right) \\ &\leftrightarrow V(p,\lambda\bar{y},\bar{\alpha})=V(p,\lambda\tilde{y},\tilde{\alpha})\ . \end{aligned} \tag{4.2}$$

The first and last lines of (4.2) follow from ESE, given by (3.11), and the second and fourth lines from increasingness of $V^r$ in its second argument.

Theorem 4.1 means that, given level comparability of utilities, an axiomatic justification of ESE (or IB) can be provided by income-ratio comparability. We believe that IRC is a plausible a priori condition on preferences and interpersonal comparisons. That ESE and IRC are equivalent is a pleasant coincidence.

## 5. Behaviour and interpersonal comparisons

We remarked in Sect. 3 that the general model we employ can make no inferences about interpersonal comparisons from behaviour alone. Specifically, if the utility function $U$ is replaced with $\tilde{U}$, where, as in (3.2),

$$\tilde{U}(x,\alpha)=\phi\,(U(x,\alpha),\alpha)\ , \tag{5.1}$$

behaviour is the same for all households but interpersonal comparisons between people from households with different characteristics can be changed arbitrarily.

Once ESE is imposed, however, this arbitrariness of interpersonal comparisons does not hold. The reason is that ESE imposes structure on interpersonal comparisons. In this section, we investigate the consequences of ESE for behaviour both globally and locally.

First, we demonstrate that if preferences are locally regular and the expenditure function is not log-linear in utility (not piglog), if equivalence-scale exactness/income-ratio comparability is maintained, and if an additional technical condition holds, then the equivalence scales can be determined from behaviour alone. Then we show that, if preferences are globally regular and nonhomothetic and if one of two technical conditions is satisfied, the same result obtains: the equivalence scales can be determined from behaviour alone.

In order to demonstrate that this is true we proceed by examining the conditions under which two different equivalence scales could be consistent with the same set of preferences, given ESE/IRC. We first show that this can be true if and only if

$$C(u,p,\alpha)=a(p)[\phi(u)]^{r(p)}\Delta(p,\alpha) \tag{5.2}$$

or, equivalently, that

$$\ln C(u,p,\alpha)=r(p)\ln\phi(u)+\ln[a(p)\Delta(p,\alpha)]\ . \tag{5.3}$$

This requires a local argument only. Using global regularity conditions, we then show that monotonicity of the expenditure function in prices and utility requires the function $r$ to be independent of prices, which implies full homotheticity.

Suppose that there are two different equivalence scales, $\Delta(p,\alpha)$ and $\Delta'(p,\alpha)$ that are consistent with the same household behaviour; then there exist indirect utility functions $V^r$ and $V^{r'}$, and a function $\phi$, increasing in its first argument, such that

$$V^{r'}\left(p,\frac{y}{\Delta'(p,\alpha)}\right)=\phi\left[V^r\left(p,\frac{y}{\Delta(p,\alpha)}\right),\alpha\right] \tag{5.4}$$

for all $(p,y,\alpha)$. Theorems 5.1 and 5.2 give necessary and sufficient conditions for (5.4) to hold locally and globally respectively, given a range condition on the ratio $\Delta'(p,\alpha)/\Delta(p,\alpha)$. Define the function $g$ by

$$g(p,\alpha):=\frac{\Delta'(p,\alpha)}{\Delta(p,\alpha)}\,. \tag{5.5}$$

$g$ must be sensitive to $\alpha$ because $g(p,\alpha^r)=1$ for all $p$ and there exists $\bar{\alpha}\in\mathbf{A}$ such that $g(p,\bar{\alpha})\neq 1$ (since $\Delta'$ and $\Delta$ are assumed to be different). We assume, for Theorems 5.1 and 5.2, that there exists a $\bar{p}\in E^m_{++}$ such that the range of $g(\bar{p},\cdot)$ contains an interval. This in turn requires some component of $\alpha$, such as age, to be a continuous variable.

**Theorem 5.1.** *Given income-ratio comparability (or equivalence-scale exactness), two different equivalence scales, $\Delta'(p,\alpha)$ and $\Delta(p,\alpha)$, satisfying the range condition (above), are consistent with the same locally regular (utility-maximizing) household behaviour,* (5.4), *if and only* (5.2) *holds for all* $(u,p,\alpha)$, *and in that case*

$$\Delta'(p,\alpha)=[S(\alpha)]^{r(p)}\Delta(p,\alpha) \tag{5.6}$$

*where $S(\alpha)$ is positive for all $\alpha$ and $S(\alpha^r)=1$.*

*Proof.* See the Appendix.

Theorem 5.1 means that (given the range condition) behaviour and the assumption that ESE holds are sufficient to find the equivalence scale uniquely as long as preferences are not log-linear in some transform of utility. Hence the requisite interpersonal comparisons are actually revealed by the data.

If one is willing to assume that the expenditure function is everywhere increasing in utility and prices, then, a stronger result is available.

**Theorem 5.2.** *Given income-ratio comparability (or equivalence-scale exactness), two different equivalence scales, $\Delta'(p,\alpha)$ and $\Delta(p,\alpha)$, satisfying the range condition (above), are consistent with the same globally regular (utility-maximizing) house-*

*hold behaviour,* (5.4), *if and only if full homotheticity holds for all* $(u, p, \alpha)$, *and in that case*

$$\Delta'(p,\alpha) = S(\alpha)\Delta(p,\alpha) \tag{5.7}$$

*where* $S(\alpha)$ *is positive for all* $\alpha$ *and* $S(\alpha^r) = 1$.

*Proof.* See the Appendix.

An objection to the claim that, given ESE and nonhomotheticity, behaviour is sufficient to determine $\Delta(p,\alpha)$ might be that the range assumption in Theorems 5.1 and 5.2 is too strong. It is possible, however, to specify sufficient conditions that allow behaviour to determine $\Delta(p,\alpha)$ when **A** contains only two (or more) elements. The money metric representation of $V^r$, employing the reference price vector $\tilde{p}$, is a normalization[12] (an increasing transform) of $V^r$ with

$$\tilde{V}^r(\tilde{p}, y) = y\ . \tag{5.8}$$

If $\tilde{V}^r$ is assumed to be continuously differentible in $y$ for all $p$, and, in addition, $\partial\tilde{V}(p,0)/\partial y$ is greater than zero and finite, then the result of Theorem 5.2 is true without the range condition.[13]

We assume that, for some $\mathring{\alpha} \in \mathbf{A}$,

$$\Delta'(p,\mathring{\alpha}) \neq \Delta(p,\mathring{\alpha}) \tag{5.9}$$

for all $p$. This means that at $\mathring{\alpha}$ the two equivalence scales are different at every price vector.

**Theorem 5.3.** *Given income-ratio comparability (or equivalence-scale exactness), if (i) there are two equivalence scales satisfying* (5.9), *and (ii) if the money metric representation of* $V^r$, $\tilde{V}^r$, *is continuously differentiable in* $y$ *with the derivative at* $y = 0$ *positive and finite for every* $p$, *then* $\Delta'$ *and* $\Delta$ *are consistent with the same (utility maximizing) behaviour,* (5.4), *if and only if full homotheticity holds.*

*Proof.* See the Appendix.

These three theorems provide surprising (to us at least) relationships between behaviour and interpersonal comparisons.[14] Given the maintained hypothesis that preferences satisfy ESE/IRC, then behaviour can identify the equivalence scales uniquely. However, this unique identification of the equivalence scales generates, in addition, the interpersonal comparisons of utility.

## 6. ESE/IRC and the translog indirect utility function

Jorgenson and Slesnick (1983, 1984a, b) in a series of articles (later followed by Kwong (1985)) employed transcendental logarithmic (translog) indirect utility functions to implement a sophisticated methodology for applied welfare analysis

[12] See Weymark (1985) and Blackorby and Donaldson (1988).
[13] Clearly, this condition holds when $p = \tilde{p}$ because $\partial\tilde{V}(\tilde{p},0)/\partial y = 1$. The condition therefore prevents great variation in the derivative as $p$ moves away from $\tilde{p}$.
[14] In a two-good (consumption and leisure), two-person model of the household, Chiappori (1992) shows that, with a maintained hypothesis of Pareto efficiency, observations of individual labour supplies and aggregate consumption are sufficient to identify individual preferences.

that allows preferences to differ across households. Later Jorgenson and Slesnick (1987) used this methodology to construct household equivalence scales.

In this section, we find necessary and sufficient conditions on a family of translog utility functions for equivalence-scale exactness to be satisfied. We then compare these conditions with those imposed on the translog family of functions by exact aggregation. We show that exact aggregation implies ESE/IRC and not conversely. Furthermore we show that the translog cannot identify uniquely the family of equivalence scales given ESE/IRC. Using this we reinterpret some of the results of Jorgenson and Slesnick (1987).

The translog indirect utility function $V$ is

$$V(p,y,\alpha)=\mathring{V}\left(\sum_{i=1}^{m} a_i(\alpha)\ln\left(\frac{p_i}{y}\right)\right.$$
$$\left.+\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij}(\alpha)\ln\left(\frac{p_i}{y}\right)\ln\left(\frac{p_j}{y}\right),\alpha\right), \tag{6.1}$$

where

$$\sum_{i=1}^{m} a_i(\alpha)=-1 \tag{6.2}$$

and

$$b_{ij}(\alpha)=b_{ji}(\alpha) \tag{6.3}$$

for all $i,j=1,\ldots,m$ and for all $\alpha\in\mathbf{A}$.

The expenditure function is found by setting $V(p,y,\alpha)$ in (6.1) equal to $u$ and solving for $y$ (that is, using (2.8)). ESE/IRC requires $C$ to be multiplicatively separable or, equivalently, the logarithm of $C$ to be additively separable, so that

$$\ln C(u,p,\alpha)=\ln\bar{C}(u,p)+\ln\hat{C}(p,\alpha)\ . \tag{6.4}$$

This condition on $C$ is used in Theorem 6.1 to find necessary and sufficient conditions for the translog family to satisfy equivalence-scale exactness.[15] We require some variation in $\Delta$ in order to be able to prove the following result.

**Theorem 6.1.** *Given that there exist at least two values of $\alpha\in\mathbf{A}$ such that $\Delta(p,\alpha)\neq 1$ for all $p$, the translog family,* (6.1), *satisfies equivalence-scale exactness if and only if*

$$B_i(\alpha):=\sum_{j=1}^{m} b_{ij}(\alpha)=\bar{B}_i\ , \tag{6.5}$$

*for all $\alpha\in\mathbf{A}$ and $i=1,\ldots,m$;*

$$B(\alpha):=\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij}(\alpha)=\sum_{i=1}^{m} B_i(\alpha)=\sum_{i=1}^{m}\bar{B}_i=0 \tag{6.6}$$

[15] Lewbel (1991) investigates the translog family with Barten commodity-specific equivalence scales when ESE (IB) is imposed. Blackorby and Donaldson (1991a) contains a general result on ESE in conjunction with Barten scales.

*for all $\alpha \in \mathbf{A}$;*

$$\ln C(u, p, \alpha) = \frac{\phi(u) + \sum_{i=1}^{m} a_i(\alpha) \ln p_i + \sum_{i=1}^{m} \sum_{j=1}^{m} b_{ij}(\alpha) \ln p_i \ln p_j + \psi(\alpha)}{-1 + \sum_{i=1}^{m} \bar{B}_i \ln p_i} \tag{6.7}$$

*where $\phi$ is increasing, and*

$$V(p, y, \alpha) = \overset{*}{V}\left[\sum_{i=1}^{m} a_i(\alpha) \ln\left(\frac{p_i}{y}\right) + \sum_{i=1}^{m} \sum_{j=1}^{m} b_{ij}(\alpha) \ln\left(\frac{p_i}{y}\right) \ln\left(\frac{p_j}{y}\right) - \psi(\alpha)\right] \tag{6.8}$$

*where $\overset{*}{V}$ is increasing.*

*Proof.* See the Appendix.

Given the translog form which satisfies ESE, the equivalence scale can be easily computed and is given by

$$\ln \Delta(p, \alpha) = \ln C(u, p, \alpha) - \ln C(u, p, \alpha^r)$$
$$= \frac{\sum_{i=1}^{m} [a_i(\alpha) - a_i(\alpha^r)] \ln p_i + \sum_{i=1}^{m} \sum_{j=1}^{m} [b_{ij}(\alpha) - b_{ij}(\alpha^r)] \ln p_i \ln p_j + [\psi(\alpha) - \psi(\alpha^r)]}{-1 + \sum_{i=1}^{m} \bar{B}_j \ln p_j}. \tag{6.9}$$

These scales depend on prices through the coefficients and on $\alpha$ through $\psi$.

Inspection of (6.7) and (6.9) makes it clear that the translog cost function satisfies ESE/IRC if and only if it is log-linear, satisfying (5.3). Hence the translog cannot be used to identify equivalence scales without making some other assumptions about them. Theorem 5.1 demonstrates that this functional form cannot identify the equivalence scales uniquely. More specifically, (6.9) shows that the equivalence scale contains an arbitrary function of household characteristics $\psi(\alpha)$ which must be determined by some other means.

If $\bar{B}_j = 0$ for all $j$, then the translog utility functions are homothetic. Homoeticity is therefore not required by ESE in the translog case.

Jorgenson and Slesnick (1987) impose exact aggregation restrictions – in the sense of Lau (1982) – on the translog family. In our notation, this is defined as follows:

**Exact aggregation restrictions:**

$$b_{ij}(\alpha) = \bar{b}_{ij} \tag{6.10}$$

*for all j, k, $\alpha$,*

$$\sum_{i=1}^{m} \sum_{j=1}^{m} b_{ij}(\alpha) = 0 \tag{6.11}$$

*for all* $\alpha$, *and* $\mathring{V}$ *in* (6.1) *satisfies*

$$\mathring{V}(t,\alpha)=\overset{*}{V}(t-\psi(\alpha)) \tag{6.12}$$

*for all* $t$ *in the domain of* $\mathring{V}(\cdot,\alpha)$ *and for all* $\alpha \in \mathbf{A}$.

A glance at these restrictions yields an obvious result.

**Theorem 6.2.** *If the translog family satisfies the exact aggregation condition, then it satisfies equivalence scale exactness.*

Because ESE does not imply (6.10) (it only requires $\sum_j b_{ij}(\alpha)$ to be independent of $\alpha$) ESE is a weaker condition than exact aggregation.

The equivalence scale used by Jorgenson and Slesnick is based on the translog family that satisfies exact aggregation (although they used the additional restriction that $b_{ij}$ is independent of $\alpha$). Hence, interpretation of their results requires interpersonal comparisons that (implicitly) satisfy income-ratio comparability.

It is interesting that a translog family which satisfies ESE and hence exact aggregation cannot be used to identify the equivalence scales from behaviour. This is what one should expect from Theorem 5.1, of course, because the translog family which satisfies ESE is log-linear.[16]

Although Theorem 6.1 indicates that ESE cannot be imposed on a family of translog utility functions (one for each household type) without losing the main advantage of ESE – the identification of the equivalence scale from demand behaviour alone – all is not lost. The utility function of the reference household $V^r$ (or expenditure function $C^r$) may be specified to be an unrestricted translog. Then, a functional form such as

$$\Delta(p,\alpha)=\delta(\alpha)\,p_1^{\gamma_1(\alpha)}\dots p_m^{\gamma_m(\alpha)}\ , \tag{6.13}$$

where $\delta(\alpha)>0$ and $\sum_j \gamma_j(\alpha)=0$ may be chosen for the equivalence scale. The utility function $V$ (or the expenditure function $C$) is found by using (3.11) (or (3.10)). The utility function $V(\cdot,\cdot,\alpha)$ will not be a translog for $\alpha \neq \alpha^r$. This procedure has been used successfully by Phipps (1990).

## 7. ESE/IRC and the almost ideal demand system

The almost ideal demand system introduced by Deaton and Muellbauer (1980) has been used by Blundell and Lewbel (1991) and Browning (1988) to estimate and to interpret adult equivalence scales. In this section we show that if the almost ideal demand system satisfies ESE/IRC, then it satisfies (5.3) and hence cannot identify equivalence scales exactly.

[16] Jorgenson and Slesnick (1987) also employ Barten commodity-specific equivalence scales for the translog family which, in conjunction with the above, identifies the scales. This imposes a condition on interpersonal comparisons that is stronger than IRC.

The almost ideal demand system is given by

$$\ln C(u,p,\alpha)=\ln a(p,\alpha)+b(p,\alpha)\ln\phi(u) \tag{7.1}$$

where $a$ is a translog and $b$ is a Cobb-Douglas.

It is clear that the AIDS satisfies ESE/IRC if and only if $b$ is independent of $\alpha$. However, in that case, the AIDS satisfies (5.3) and hence cannot uniquely identify the household equivalence scales. To be more precise about this problem, note that if (7.1) satisfies ESE/IRC then the expenditure function of the reference household can be written as

$$C^r(u,p)=a(p,\alpha^r)[\phi(u)]^{b(p)} . \tag{7.2}$$

The equivalence scale is given by

$$\begin{aligned}\Delta(p,\alpha)&=\frac{C(u,p,\alpha)}{C(u,p,\alpha^r)}\\ &=\frac{a(p,\alpha)}{a(p,\alpha^r)} ,\end{aligned} \tag{7.3}$$

and the expenditure function of a household with characteristics $\alpha$ is

$$C(u,p,\alpha)=C^r(u,p)\Delta(p,\alpha) . \tag{7.4}$$

Now define another equivalence scale

$$\bar{\Delta}(p,\alpha)=[S(\alpha)]^{b(p)}\Delta(p,\alpha) \tag{7.5}$$

with a corresponding expenditure function

$$\bar{C}(u,p,\alpha)=C^r(u,p)\bar{\Delta}(p,\alpha) . \tag{7.6}$$

Using (7.2) and (7.5) $\bar{C}$ can be written as

$$\bar{C}(u,p,\alpha)=a(p)\Delta(p,\alpha)[S(\alpha)\phi(u)]^{b(p)} \tag{7.7}$$

which generates the same demand system (the same economic behaviour) as (7.4). Hence, ESE/IRC equivalence scales cannot be identified uniquely by the almost ideal demand system.

Blundell and Lewbel (1991) employed the AIDS with ESE but, in order to identify the equivalence scales, added the hypothesis that alcohol is an 'adult good'.[17] In their tests of ESE/IRC, Blundell and Lewbel (1991) and Browning (1988), who used the AIDS as well, rejected ESE. Subsequently, Dickens, Fry and Pashardes (1992) have shown that ESE may be falsely rejected when overly simple preference specifications such as the AIDS are used. These investigators rejected ESE as well, however, but found that the imposition of ESE does not have a large impact on the equivalence scale. Blundell and Lewbel (1991) reached a similar conclusion.

[17] Adult goods are a feature of the Rothbarth model which is discussed in Blackorby and Donaldson (1991b). The presence of the additional assumption in Blundell-Lewbel means that the test for ESE in fact tested the joint hypothesis.

## 8. Conclusion

Adult-equivalence scales are a theoretically attractive way to structure comparisons of the well-being of individuals in different households. In their general form however, they contain an unobservable – the utility of the members of the household in question. Equivalence-scale exactness (ESE) (or independence of base) removes this difficulty by restricting attention to those scales that are independent of the utility level.

Income-ratio comparability (IRC) provides a complete characterization of equivalence scale exactness (Theorem 4.1). It requires that common scaling of incomes preserves utility equality for households facing the same prices. If this condition is found to be appealing a priori, it can be used as an axiom for ESE.

ESE/IRC cannot be tested completely by standard economic techniques, but, because it restricts the way preferences can differ across household types, (3.11), a partial test can be performed. Although ESE is rejected in such tests, Blundell and Lewbel (1991) and Dickens, Fry and Pashardes have shown that its imposition has only a small effect on estimated equivalence scales.

If ESE/IRC is accepted as a maintained hypothesis, then Theorems 5.1–5.3 show that the equivalence scale $\Delta$ can be estimated from behaviour alone, provided that reference preferences do not satisfy homotheticity (for globally regular preferences) or piglog (for locally regular preferences).

Section 6 argues that ESE cannot be imposed on a family of translog indirect utility functions – one for each type – without losing identification of equivalence scales. Section 7 makes the same claim for the AIDS family. It is possible, however, to specify an unrestricted translog for the reference household and a functional form for the equivalence scale without losing identification.

Price-sensitive equivalence scales have been estimated using translog reference preferences (Phipps 1990). The scales are reasonable: they lie in the right range, are sensitive to prices in the expected ways, and agree with one set of 'poverty relatives' (the ratios of poverty lines for different households types) for Canada.

If ESE/IRC is able to pass these tests, it will permit empirical social evaluation to be undertaken with flexibility about the facts of household differences, interpersonal comparisons that reflect the different needs and preferences of different households, and social values that pay due attention to inequality in the distribution of individual well-being.

## Appendix

■ *Proof of Theorem 3.1.*

*(i)* First extend $C$ to the set of nonnegative prices to obtain a function $\tilde{C}$ by continuity from above. The distance (or transformation) function dual to $C$ is given by

$$D(u,x,\alpha) := \min_p \{p \cdot x \mid \tilde{C}(u,p,\alpha) \geq 1,\ x \in E^m_{++}\}\ . \tag{A.1}$$

If (3.13) holds with $C^b$ and $C^s$ concave in $p$, then extend $C^b$ and $C^s$ to the boundary by continuity from above as well, obtaining $\tilde{C}^b$ and $\tilde{C}^s$; define their duals by

$$D^b(u,x) := \min_p \{p \cdot x \mid \tilde{C}^b(u,p) \geq 1,\ x \in E^m_{++}\} \tag{A.2}$$

and

$$D^s(x,\alpha) := \min_p \{p \cdot x \mid \tilde{C}^s(p,\alpha) \geq 1,\ x \in E^m_{++}\} \ . \tag{A.3}$$

Then $D$ itself can be written (using (3.13)) as

$$D(u,x,\alpha) = \max_{\{x^b,x^s\}} \{[D^b(u,x^b)]^\gamma [D^s(x^s,\alpha)]^{1-\gamma} \mid x^b + x^s = x,\ x \in E^m_{++}\} \tag{A.4}$$

by application of Theorem 9 on page 57 of McFadden (1978).[18] Extending $D$, $D^b$, and $D^s$ to the boundary to obtain $\bar{D}$, $\bar{D}^b$, and $\bar{D}^s$ we find the utility function $U$ and define the utility functions $U^b$ and $U^s$ by

$$\bar{D}(u,x,\alpha) = 1 \leftrightarrow U(x,\alpha) = u \ , \tag{A.5}$$

$$\bar{D}^b(u,x) = 1 \leftrightarrow U^b(x) = u \ , \tag{A.6}$$

and

$$\bar{D}^s(x,\alpha) =: U^s(x,\alpha) \ . \tag{A.7}$$

Using (A.4) through (A.7) yields the direct utility function which is required, namely

$$U(x,\alpha) = \max_{\{x^b,x^s\}} \{U^b(x^b[U^s(x^s,\alpha)]^\beta) \mid x^b + x^s = x\} \ . \tag{A.8}$$

*(ii)* Now suppose that (3.14) is satisfied with $U^b$ and $U^s$ quasi-concave in $x^b$ and $x^s$ respectively, and $U^s$ homogeneous of degree one in $x^s$. Define $C^s$ by

$$C^s(p,\alpha) := \frac{\min\{p \cdot x \mid U^s(x^s,\alpha) \geq \xi\}}{\xi} \tag{A.9}$$

for any $\xi > 0$. Because $U^s$ is homogeneous of degree one in $x$, the numerator on the right side is linear in $\xi$ and therefore the right side of (A.9) is independent of $\xi$. Then defining

$$C^b(u,p) := a\bar{C}^b(u,p) \tag{A.10}$$

where

$$\bar{C}^b(u,p) := \min_x \{p \cdot x \mid U^b(x) \geq u\} \tag{A.11}$$

[18] The reader should note that the intersection and summation operations for this case are opposite to that stated in the text.

and $a$ is some positive number, we find that

$$
\begin{aligned}
C(u,p,\alpha) &= \min_{x}\{p\cdot x \mid \max_{\{x^b,x^s\}}\{U^b(x^b[U^s(x^s,\alpha)]^\beta) \mid x^b+x^s=x\}\geq u\} \\
&= \min_{\{x^b,x^s\}}\{p\cdot x^b+p\cdot x^s \mid U^b(x^b[U^s(x^s,\alpha)]^\beta)\geq u\} \\
&= \min_{\{x^b,\xi\}}\{p\cdot x^b+\xi C^s(p,\alpha) \mid U^b(x^b\cdot\xi^\beta)\geq u\} \\
&= \min_{\{z,\xi\}}\left\{p\cdot\frac{z}{\xi^\beta}+\xi C^s(p,\alpha) \mid U^b(z)\geq u\right\} \\
&= \min_{\xi}\left\{\frac{\bar{C}^b(u,p)}{\xi^\beta}+\xi C^s(p,\alpha)\right\} \\
&= \frac{\bar{C}^b(u,p)}{\overset{*}{\xi}{}^\beta}+\overset{*}{\xi}C^s(p,\alpha)\ .
\end{aligned}
\tag{A.12}
$$

The optimizer in (A.12), $\overset{*}{\xi}$, is given by

$$
\overset{*}{\xi}=\left[\frac{\beta\cdot\bar{C}^b(u,p)}{C^s(p,\alpha)}\right]^{\frac{1}{1+\beta}}; \tag{A.13}
$$

letting $\dfrac{1}{1+\beta}=\gamma$ we obtain

$$
\begin{aligned}
C(u,p,\alpha) &= [\beta^{\gamma-1}+\beta^\gamma][\bar{C}^b(u,p)]^\gamma[C^s(p,\alpha)]^{1-\gamma} \\
&= [C^b(u,p)]^\gamma[C^s(p,\alpha)]^{1-\gamma}
\end{aligned}
\tag{A.14}
$$

by suitable choice of $a$. □

■ *Proof of Theorem 4.1.*

(4.2) shows that *equivalence-scale exactness* implies *income-ratio comparability*. Suppose that $U$ satisfies *income-ratio comparability*; then, given $(p,y,\alpha)$, $y>0$, find $\bar{y}$ such that

$$
u=V(p,y,\alpha)=V(p,\bar{y},\alpha^r)\ . \tag{A.15}
$$

Then,

$$
y=C(u,p,\alpha) \tag{A.16}
$$

and

$$
\bar{y}=C(u,p,\alpha^r)=:C^r(u,p) \tag{A.17}
$$

where $\bar{y}>0$ because $y>0$. By *income-ratio comparability*,

$$
u=V\left(p,\frac{y}{\bar{y}},\alpha\right)=V(p,1,\alpha^r)\ . \tag{A.18}
$$

Because $V$ is increasing in income, (A.18) can be solved for $y/\bar{y}$, and it is given by

$$\frac{y}{\bar{y}} = \psi(p, \alpha) \ . \tag{A.19}$$

Hence, using (A.17) and (A.19) we obtain

$$C(u, p, \alpha) = C^r(u, p)\, \psi(p, \alpha) \ , \tag{A.20}$$

and *equivalence-scale exactness* is satisfied. □

■ *Proof of Theorem 5.1.*

(5.4) can be rewritten as

$$V^r\left(p, \frac{y}{\Delta'(p, \alpha)}\right) = \phi\left[V^r\left(p, \frac{y}{\Delta(p, \alpha)}\right), \alpha\right] \tag{A.21}$$

without loss of generality because setting $\alpha = \alpha^r$ makes $\Delta'(p, \alpha^r) = \Delta(p, \alpha^r) = 1$, so that $V^{r'}$ is an increasing transform of $V^r$. (A.21) is equivalent to

$$C^r(u, p)\, \Delta'(p, \alpha) = C^r(\psi(u, \alpha), p)\, \Delta(p, \alpha) \tag{A.22}$$

where $\psi(\cdot, \alpha)$ is the inverse of $\phi(\cdot, \alpha)$ and is, therefore, increasing. Define

$$g(p, \alpha) := \frac{\Delta'(p, \alpha)}{\Delta(p, \alpha)} \tag{A.23}$$

so that (A.22) becomes

$$C^r(\psi(u, \alpha), p) = C^r(u, p)\, g(p, \alpha) \ . \tag{A.24}$$

Setting $p = \bar{p}$ (see the text),

$$\psi(u, \alpha) = f^{-1}(f(u)\, h(\alpha)) \tag{A.25}$$

where $f := C^r(\cdot, \bar{p})$ and $h := g(\bar{p}, \cdot)$. Therefore,

$$C^r(f^{-1}(f(u)\, h(\alpha)), p) = C^r(f^{-1}(f(u)), p)\, g(p, \alpha) \tag{A.26}$$

and, defining $\tilde{C}$ by

$$\tilde{C}(t, p) := C^r(f^{-1}(t), p) \ , \tag{A.27}$$

(A.26) becomes

$$\tilde{C}(f(u)\, h(\alpha), p) = \tilde{C}(f(u), p)\, g(p, \alpha) \ . \tag{A.28}$$

Now define $\dot{g}$ by

$$\dot{g}(p, t) := g(p, h^{-1}(t)) \tag{A.29}$$

so that (A.28) becomes

$$\tilde{C}(f(u)\, h(\alpha), p) = \tilde{C}(f(u), p)\, \dot{g}(p, h(\alpha)) \ . \tag{A.30}$$

Finally, defining $w := f(u)$, and $z := h(\alpha)$, we obtain

$$\tilde{C}(wz, p) = \tilde{C}(w, p)\, \dot{g}(p, z) \ . \tag{A.31}$$

The *range condition* makes it possible to move $z = h(\alpha)$ through an interval of (positive) numbers. Since $f$ must be increasing in $u$ and positive, the same applies to $w$. The solution to (A.31) (see Eichhorn 1978, Theorem 3.6.4) is given by

$$\tilde{C}(w,p) = a(p)\, w^{r(p)} \tag{A.32}$$

and

$$\dot{g}(p,z) = z^{r(p)} \tag{A.33}$$

so that

$$C^r(u,p) = \tilde{C}(f(u),p) = a(p)[f(u)]^{r(p)} \ . \quad \square \tag{A.34}$$

■ *Proof of Theorem 5.2.*

From the definition following (A.25) we know that $f(u) = C^r(u,p)$ so that $f(u) > 0$. Hence, $a(p) > 0$ for all $p$. In addition, $w = f(u)$ can take on any positive value. If $r$ is not independent of $p$, then there exist $p$ and $p' \in E^m_{++}$ with $p'_k > p_k$ for some $k$ and $p'_j = p_j$ for all $j \neq k$ such that $r(p) \neq r(p')$. Rewriting (A.34) as

$$\ln C^r(u,p) = \ln a(p) + r(p) \ln w \ , \tag{A.35}$$

we obtain

$$\ln C^r(u,p') - \ln C^r(u,p)$$
$$= [r(p') - r(p)] \ln w + [\ln a(p') - \ln a(p)] \geq 0 \tag{A.36}$$

because $C^r(u,p') \geq C^r(u,p)$. If $r(p') > r(p)$, then let $w \to 0$. This makes (A.36) negative, a contradiction; if $r(p') < r(p)$, then, letting $w \to \infty$ violates (A.36) as well. This means that $r(\cdot)$ must be independent of $p$, and hence that

$$C^r(u,p) = a(p)[f(u)]^r \tag{A.37}$$

so that preferences are homothetic. From (5.5) and (A.29)

$$g(p,\alpha) = \dot{g}(p,h(\alpha)) = [h(\alpha)]^r =: S(\alpha) \tag{A.38}$$

so that (5.7) is satisfied. Sufficiency is discussed in the text. □

■ *Proof of Theorem 5.3.*

(5.3) implies (A.35) as in the previous proof, and, because $\tilde{V}^r$ is ordinally equivalent to $V^r$, (A.21) holds with $\tilde{V}^r$ replacing $V^r$.

Defining $z := y/\Delta(p,\alpha)$, (A.35) becomes

$$\tilde{V}^r\left(p, \frac{z}{g(p,\alpha)}\right) = \phi(\tilde{V}^r(p,z),\alpha) \ . \tag{A.39}$$

Setting $p = \tilde{p}$ in (A.39) and (5.8), yields

$$\phi(z,\alpha) = \frac{z}{g(\tilde{p},\alpha)} \ . \tag{A.40}$$

Set $\alpha = \mathring{\alpha}$, and define $\gamma = 1/g(\tilde{p},\alpha)$ and $h(p) := 1/g(p,\mathring{\alpha})$; using (A.40), (A.39) becomes

$$\tilde{V}^r(p,h(p)z) = \gamma \tilde{V}^r(p,z) \tag{A.41}$$

for all $p$ and $z$. Setting $z=0$ in (A.41) requires that either $\gamma=1$ or that $\tilde{V}^r(p,0)=0$. The former is impossible because it violates (5.9), so that the latter is true. Differentiating (A.41) with respect to $z$ yields

$$\tilde{V}_y^r(p,h(p)z)h(p)=\gamma\tilde{V}_y^r(p,z) \ . \tag{A.42}$$

Setting $z=0$ in (A.42) yields $h(p)=\gamma$. (A.42) becomes

$$\tilde{V}_y^r(p,z)=\tilde{V}_y^r(p,\gamma z) \tag{A.43}$$

and applying (A.43) $t$ times yields

$$\tilde{V}_y^r(p,z)=\tilde{V}_y^r(p,\gamma^t z) \tag{A.44}$$

for all positive integers $t$. Because $\gamma=\Delta'(p,\mathring{\alpha})/\Delta(p,\mathring{\alpha})$, we may assume without loss of generality that $\gamma<1$. Hence,

$$\lim_{t\to\infty}\tilde{V}_y^r(p,\gamma^t z)=\tilde{V}_{yx}^r(p,0) \ . \tag{A.45}$$

However, from (A.44), the sequence on the left side of (A.45) is constant, which implies that

$$\tilde{V}_y^r(p,z)=\tilde{V}_y^r(p,0) \ . \tag{A.46}$$

Integrating (A.46) yields

$$\tilde{V}^r(p,z)=\tilde{V}^r(p,0)z+\kappa(p) \ . \tag{A.47}$$

However, from the argument following (A.41) we know that $V^r(p,0)=0$ for all $p$, and hence that $\kappa(p)=0$. Therefore, reference preferences are homothetic. Because $h(p)=\gamma$, $g(p,\alpha)$ must be independent of $p$, which in turn implies (5.7) and full homotheticity. □

■ *Proof of Theorem 6.1.*

Suppose that the translog family (6.1) satisfies ESE, so that

$$V(p,y,\alpha)=V^r\left(p,\frac{y}{\Delta(p,\alpha)}\right) \tag{A.48}$$

for all $p$, $y$, $\alpha$. Set $p=1_m$ in (A.48) so that, defining $\delta(\alpha):=\Delta(1_m,\alpha)$, we obtain

$$\begin{aligned}&\mathring{V}(\ln y+B(\alpha)[\ln y]^2,\alpha)\\&=\mathring{V}([\ln y-\ln\delta(\alpha)]+B(\alpha^r)[\ln y-\ln\delta(\alpha)]^2,\alpha^r) \ .\end{aligned} \tag{A.49}$$

Defining $z:=\ln y$, and $w:=\ln\delta(\alpha)$, and $\bar{B}(w):=B(\alpha)$ and substituting these into (A.49) we obtain (remember that $\delta(\alpha^r)=1$)

$$g(z+\bar{B}(w)[z-w]^2,w)=h([z-w]+\bar{B}(0)[z-w]^2) \ , \tag{A.50}$$

where $g$ is increasing in its first argument and $h$ is increasing. Rewriting (A.50) yields

$$z+\bar{B}(w)z^2=f([z-w]+\bar{B}(0)[z-w]^2,w) \ . \tag{A.51}$$

$z=\ln y$ may take on any real values and $w$ has at least two values, $w=\ln\delta(\alpha^r)=\ln 1=0$, and $\bar{w}=\ln\delta(\bar{\alpha})\neq 0$, since, by assumption, there exists $\bar{\alpha}\in\mathbf{A}$ with $\delta(\bar{\alpha})=\Delta(1_m,\bar{\alpha})\neq 1$.

The next step in the proof is to show that $B(\alpha^r)=0$. To do this we proceed by contradiction. Assume that $\bar{B}(0)=B(\alpha^r)\neq 0$ and define $x:=[z-w]$ to obtain (from (A.51))

$$[x+w]+\bar{B}(w)[x+w]^2=f(x+\bar{B}(0)x^2, w) \ . \tag{A.52}$$

First, set $x=0$ and $w=\bar{w}$ so that

$$f(0,\bar{w})=\bar{w}+\bar{B}(\bar{w})\bar{w}^2 \ . \tag{A.53}$$

Next set $x=-1/\bar{B}(0)$ to obtain

$$\begin{aligned} f(0,\bar{w})=&\left[-\frac{1}{\bar{B}(0)}+\bar{w}\right]\\ &+\bar{B}(\bar{w})\left[\frac{1}{\bar{B}(0)^2}-\frac{2\bar{w}}{\bar{B}(0)}+\bar{w}^2\right]=\bar{w}+\bar{B}(\bar{w})\bar{w}^2 \ , \end{aligned} \tag{A.54}$$

using (A.53). Solving (A.54) for $\bar{B}(\bar{w})$, if $2\bar{w}\bar{B}(0)\neq 1$, then

$$\bar{B}(\bar{w})=\frac{\bar{B}(0)}{1-2\bar{w}\bar{B}(0)} \ . \tag{A.55}$$

If $2\bar{w}\bar{B}(0)=1$, then we have an impossibility and $\bar{B}(0)=0$. If not, (A.52) holds, and from (A.52) and (A.55)

$$f(x+\bar{B}(0)x^2,\bar{w})=[x+\bar{w}]+\frac{\bar{B}(0)[x+\bar{w}]^2}{1-2\bar{w}\bar{B}(0)} \ . \tag{A.56}$$

Define $t:=x+\bar{B}(0)x^2$ so that

$$x=\frac{-1\pm[1+4\bar{B}(0)t]^{\frac{1}{2}}}{2\bar{B}(0)} \tag{A.57}$$

for all $t$ such that $(1+4\bar{B}(0)t)$ is positive. (A.56) and (A.57) imply that

$$\begin{aligned} f(t,\bar{w})=&\left[-\frac{1}{2\bar{B}(0)}+\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{2\bar{B}(0)}+\bar{w}\right]\\ &+\left[\frac{\bar{B}(0)}{1-2\bar{w}\bar{B}(0)}\right]\left[-\frac{1}{2\bar{B}(0)}+\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{2\bar{B}(0)}+\bar{w}\right]^2\\ =&\left[-\frac{1}{2\bar{B}(0)}-\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{2\bar{B}(0)}+\bar{w}\right]\\ &+\left[\frac{\bar{B}(0)}{1-2\bar{w}\bar{B}(0)}\right]\left[-\frac{1}{2\bar{B}(0)}+\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{2\bar{B}(0)}+\bar{w}\right]^2 \ . \end{aligned} \tag{A.58}$$

Solving (A.58) yields

$$\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{\bar{B}(0)} = \left[\frac{\bar{B}(0)}{1-2\bar{w}\bar{B}(0)}\right]\left[\frac{[1+4\bar{B}(0)t]^{\frac{1}{2}}}{\bar{B}(0)^2} - \frac{2\bar{w}[1+4\bar{B}(0)t]^{\frac{1}{2}}}{\bar{B}(0)}\right], \tag{A.59}$$

or, upon rearranging,

$$1 = \left[\frac{\bar{B}(0)}{1-2\bar{w}\bar{B}(0)}\right][1-2\bar{w}\bar{B}(0)], \tag{A.60}$$

which implies that

$$\bar{B}(0)=1 \tag{A.61}$$

or an impossibility if $\bar{B}(0)\neq 1$ (which means that $\bar{B}(0)=\bar{B}(\alpha^r)=0$). If $\bar{B}(0)=1$, then, returning to (A.56), we obtain

$$f(x+x^2,\bar{w}) = [x+\bar{w}] + \frac{[x+\bar{w}]^2}{1-2\bar{w}} = \frac{x+x^2+\bar{w}-\bar{w}^2}{1-2\bar{w}}, \tag{A.62}$$

or, setting $t=x+x^2$

$$f(t,\bar{w}) = \frac{t+\bar{w}-\bar{w}^2}{1-2\bar{w}}. \tag{A.63}$$

Because $w=\ln\delta(\alpha)$, and using the definition of $f$ in (A.51), we know that we can rewrite (6.1) as

$$\frac{\sum_j a_j(\alpha)\ln\frac{p_j}{y} + \sum_j\sum_k b_{jk}(\alpha)\ln\frac{p_j}{y}\ln\frac{p_k}{y} + \phi(\alpha)}{\psi(\alpha)} = \sum_j a_j(\alpha^r)\left[\ln\frac{p_j}{y} + \ln\Delta(p,\alpha)\right] + \sum_j\sum_k b_{jk}(\alpha)\left[\ln\frac{p_j}{y} + \ln\Delta(p,\alpha)\right]\left[\ln\frac{p_j}{y} + \ln\Delta(p,\alpha)\right], \tag{A.64}$$

where $\phi(\alpha):=\ln\delta(\alpha)-[\ln\delta(\alpha)]^2$ and $\psi(\alpha):=1-2\ln\delta(\alpha)$. (Note that $\phi(\alpha^r)=0$ and that $\psi(\alpha^r)=1$.) Hence,

$$\sum_j a_j(\alpha)\ln\frac{p_j}{y} + \sum_j\sum_k b_{jk}(\alpha)\ln\frac{p_j}{y}\ln\frac{p_k}{y} = \sum_j \psi(\alpha)\left[a_j(\alpha^r) - 2\sum_k b_{jk}(\alpha^r)\ln\Delta(p,\alpha)\right]\ln\frac{p_j}{y} + \sum_j\sum_k \psi(\alpha) b_{jk}(\alpha^r)\ln\frac{p_j}{y}\ln\frac{p_k}{y} - [\psi(\alpha)+1]\ln\Delta(p,\alpha) + [\psi(\alpha)+1][\ln\Delta(p,\alpha)]^2. \tag{A.65}$$

Moving everything to the left side of the equation except the last line, we observe that prices on the right side appear only in $\Delta$ (or not at all); moving prices through a level set of $\Delta$ leaves both sides of the equation constant for all $y$. Hence, we obtain

$$\ln \Delta(p,\alpha) = [\ln \Delta(p,\alpha)]^2 \ . \tag{A.66}$$

This means that either $\Delta(p,\alpha)=1$ or $\Delta(p,\alpha)=e$ (and hence that $\Delta(p,\alpha^r)=1$ and $\Delta(p,\bar{\alpha})=e$). If there are *two* values of $\alpha \in \mathbf{A}$ with distinct $\Delta(p,\alpha)\neq 1$, then this is an impossibility and so

$$\sum_j \sum_k b_{jk}(\alpha^r) = B(\alpha^r) = 0 \ , \tag{A.67}$$

which completes this part of the proof.

Next substitute (A.67) into (A.49) to obtain

$$\mathring{V}(\ln y + B(\alpha)[\ln y]^2, \alpha) = \mathring{V}([\ln y - \ln \delta(\alpha)], \alpha^r) \ . \tag{A.68}$$

or, upon inverting,

$$K(\ln y + B(\alpha)[\ln y]^2, \alpha) = \ln y - \ln \delta(\alpha) \ . \tag{A.69}$$

This holds true for all values of $y$, so that

$$B(\alpha) = 0 \ . \tag{A.70}$$

Using (A.70) in (6.1) implies that there exists a function $\Gamma$ which is increasing in its first argument so that

$$\ln C(u,p,\alpha) = \frac{\Gamma(u,\alpha) + \sum_j a_j(\alpha)\ln p_j + \sum_j \sum_k b_{jk}(\alpha)\ln p_j \ln p_k}{1 - \sum_j B_j(\alpha)\ln p_j} \ . \tag{A.71}$$

Because ESE requires $C$ to be multiplicatively separable, $\ln C$ must be additively separable and so

$$\frac{\Gamma(u,\alpha)}{1 - \sum_j B_j(\alpha)\ln p_j} \tag{A.72}$$

must be additively separable. Setting $p = 1_m$ makes

$$\Gamma(u,\alpha) = \psi(u) + \phi(\alpha) \ , \tag{A.73}$$

for some increasing $\psi$ and some $\phi$, and given this, $B_j(\alpha)$ must be independent of $\alpha$, yielding (6.6) and (6.7). Sufficiency is immediate. □

## References


Barten A (1964) Family composition, prices, and expenditure patterns. In: Hart P, Mills G, Whittaker J (eds.) Econometric Analysis for National Economic Planning: 16th Symposium of the Calston Society. Butterworth, London, pp 277–292

Blackorby C, Donalsdson D (1987) Welfare ratios and distributionally sensitive cost-benefit analysis. J Publ Econ 34: 265–290

Blackorby C, Donaldson D (1988) Money metric utility: a harmless normalization? J Publ Econ 46: 120–129

Blackorby C, Donaldson D (1991a) Adult-equivalence scales, interpersonal comparisons of well-being, and applied welfare economics. In: Elster J, Roemer J (eds.) Interpersonal Comparisons and Distributive Justice. Cambridge University Press, Cambridge, pp 164–199

Blackorby C, Donaldson D (1991b) Measuring the cost of children: a theoretical framework. Discussion Paper no. 91-40, University of British Columbia (forthcoming In: Blundell R (ed.) The Measurement of Household Welfare. Oxford University Press, Oxford)

Blackorby C, Donaldson D (1993) Household equivalence scales and welfare comparisons: a comment. J Publ Econ 50: 143–146

Blackorby C, Donaldson D, Weymark J (1984) Social choice with interpersonal utility comparisons: a diagrammatic introduction. Int Econ Rev 25: 327–356

Blundell R, Lewbel A (1991) The information content of equivalence scales. J Econometrics 50: 49–68

Browning M (1988) The effects of household characteristics on behaviour and welfare, mimeo, Department of Economics, McMaster University

Browning M (1992) Children and household economic behaviour. J Econ Literat 30: 1434–1475

Chiappori P (1992) Collective labour supply and welfare. J Pol Econ 100: 437–468

Deaton A, Muellbauer J (1989) Economics and Consumer behaviour. Cambridge University Press, Cambridge

Dickens R, Fry V, Pashardes P (1992) Non-linearities, aggregation and equivalence scales, mimeo, Institute for Fiscal Policy, London

Eichhorn W (1978) Functional Equations in Economics. Addison-Wesley, Reading Mass

Gorman WM (1976) Tricks with utility functions. In: Artis M, Nobay R (eds.) Essays in Economic Analysis. Cambridge University Press, Cambridge, pp 211–243

Grunau R (1988) Consumption technology and the intrafamily distribution of resources: adult equivalence scales reexamined. J Pol Econ 96: 1183–1205

Jorgenson D (1990) Aggregate consumer behaviour and the measurement of social welfare. Econometrica 58: 1007–1040

Jorgenson D, Lau L, Stoker T (1980) Welfare comparison under exact aggregation. Am Econ Rev 80: 268–272

Jorgenson D, Lau L, Stoker T (1992) The transcendental logarithmic model of aggregate consumer behaviour. In: Basmann R, Rhodes G, Jr (eds.) Advances in Econometrics, Vol. 1. JAI Press, Greenwich, pp 161–163

Jorgenson D, Slesnick D (1983) Individual and social cost of living indexes. In: Diewert WE, Montmarquette C (eds.) Price Level Measurement. Statistics Canada, Ottawa, pp 241–323

Jorgenson D, Slesnick D (1984a) Aggregate consumer behaviour and the measurement of inequality. Rev Econ Stud 51: 369–392

Jorgenson D, Slesnick D (1984b) Inequality in the distribution of individual welfare. In: Basmann R, Rhodes G, Jr (eds.) Advances in Econometrics, Vol. 3. JAI Press, Greewich, pp 67–130

Jorgenson D, Slesnick D (1987) Aggregate consumer behaviour and household equivalence scales. J Business Econ Stat 5: 219–232

Kwong S (1985) Price-Sensitive Inequality Measurement. Ph.D. Dissertation, University of British Columbia

Lau L (1982) A note on the fundamental theorem of exact aggregation. Econ Lett 9: 119–126

Lewbel A (1989) Household equivalence scales and welfare comparisons. J Publ Econ 39: 377–391

Lewbel A (1991) Cost of characteristics indices and household equivalence scales. Eur Econ Rev 35: 1277–1293

McFadden D (1978) Duality of production, cost, and profit functions. In: Fuss M, McFadden D (eds.) Production Economics: A Dual Approach to Theory and Applications. North Holland, Amsterdam, pp 1–109

Muellbauer J (1974) Household composition, Engel curves and welfare comparisons between households: a duality approach. Eur Econ Rev 5: 103–122

Muellbauer J (1975) Aggregation, income distribution, and consumer demand. Rev Econ Stud 62: 269–273

Muellbauer J (1976) Community preferences and the representative consumer. Econometrica 44: 979–999

Negishi T (1963) On social welfare function. Quart J Econ 77: 156–158

Nelson J (1988) Household economies of scale in consumption: theory and evidence. Econometrica 56: 1301–1314

Phipps S (1990) Price-sensitive adult-equivalence scales for Canada, mimeo, Dalhousie University, revised, 1992

Pollak RA, Wales TJ (1979) Welfare comparisons and equivalence scales. Am Econ Rev 69: 216–221

Pollak RA, Wales TJ (1981) Demographic variables in demand analysis. Econometrica 49: 1533–1551

Ray R (1992) Modelling the impact of children on household expenditure and welfare, mimeo, University of British Columbia, 1992

Samuelson P (1956) Social indifference curves. Quart J Econ 70: 1–22

Sen A (1977) On weights and measures: informational constraints in social welfare analysis. Econometrica 45: 1539–1572

Weymark J (1985) Money-metric utility functions. Int Econ Rev 26: 219–232