# Mixed Duopoly, Inefficiency, and Public Ownership\*

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Abstract. If a publicly-owned firm has a *higher* marginal cost than a private firm, partial public ownership may be welfare-improving, if the public firm acts is Stackelberg leader. If the private firm's marginal cost is private information a simple transfer function is truth-eliciting. If the stock market is efficient, the cost of renationalization is "small".

Key words: Mixed oligopoly, public ownership, privatization.

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The aim of this Note is to add to the literature on mixed oligopoly in three respects: (i) by introducing a cost asymmetry between public and private firms; (ii) by relaxing the assumption of complete information; and (iii) by considering the implications of the analysis for policies which contemplate taking part of the industry back into public ownership.

We bias the analysis against partial public ownership by assuming that the public firm is less efficient than the private firm. In spite of this we obtain that for a surprisingly wide range of relevant parameters the cost disadvantage of the public firm is more than offset by the strategic advantage stemming from its position as a Stackelberg leader. We also argue that if the stock market efficiency hypothesis holds and thus shareholders fully discount the future effects of government policy, then the cost of partial public ownership can be lower than commonly assumed, even allowing for inefficient fund-raising by the government.

Although we would warn against the perils of extrapolating the implications of a simple model to the complex phenomenon of (de)privatization policy, we

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<sup>&</sup>lt;sup>1</sup> Cremer et al. (1989) include an additional marginal cost for the public firm, but treat it as a pure transfer.

<sup>&</sup>lt;sup>2</sup> The admirable survey by De Fraja and Delbono (1990) concludes by stating that an "important question which cannot be ignored and can seriously alter the conclusion is the informational requirement by the public authority . . . what should a public firm do when it knows only approximately the features of the market in which it operates?".

believe that our model may make a contribution to the current policy debate on the desirability of some form of restored public ownership.<sup>3</sup>

Section I sketches the simplest mixed duopoly case with cost asymmetries and Section II examines the welfare consequences of privatization and partial public ownership by comparing the welfare of a Stackelberg mixed duopoly and of a private duopoly with two profit-maximizers.

### I. Stackelberg leadership with cost asymmetry

Consider a mixed duopoly with firms 1 and 2 being respectively the "public" and the "private" enterprise. Let demand for the homogeneous good X be linear in output:  $P(x_1+x_2)=\max\{A-(x_1+x_2),0\}, A>2c_1>0$ . Let marginal costs be constant with  $c_1>c_2$ . The assumption that the public firm is less efficient, i.e., has a higher (constant) marginal cost than the private firm, is not meant as an approximation of the true state of affairs, but as a means to bias the analysis against the public firm, thereby strengthening any results in favour of partial public ownership. We wish to avoid the trivial case where the first best is achieved by command and so we restrict the range of instruments deployed by the social-welfare maximizer to indirect controls. In other words, welfare maximization is not pursued by direct instruments like price controls or output quotas for the private firm, but relies on giving the public firm the role of Stackelberg leader with the private firm being free to choose its profit-maximizing output level.

We can justify this assignment of strategic rôles both theoretically and empirically. Unlike the case of private oligopoly where there is no reason why any of otherwise identical firms should be endowed with a first-mover advantage, in a mixed oligopoly the public firm can use its ownership status as a credible commitment. Being state-owned, in fact, means that the whole machinery of government regulation/legislation can be deployed to make any commitment in terms of output (or price) irreversible and hence credible. The actual experience of mixed oligopolies is rich with examples. In fact, given the unambiguous welfare benefits derived from Stackelberg leadership, failure by state-owners to support public firms by means of credible commitments can be taken as evidence of sub-optimal behavior on their part.

We ignore principal-agent problems and simply assume that the private firm maximizes its own profit whereas the public firm maximizes a social welfare function given by the sum of consumers' surplus and industry profits.

<sup>&</sup>lt;sup>3</sup> In the UK, for example, both main opposition parties have expressed interest to a limited reversal of the privatization programme pursued by the Conservative administration.

<sup>&</sup>lt;sup>4</sup> The empirical literature on the relative efficiency of public vs. private firms is ambiguous on this issue; see Yarrow (1986) and Vickers and Yarrow (1988) for a survey.

<sup>&</sup>lt;sup>5</sup> Obvious examples include public service and commercial television and radio (in the UK the BBC is committed to minimum production targets in terms of drama, documentaries, news coverage, etc.); steel (in Italy and France state-controlled firms used to be given output targets).

Consider first the mixed Nash-Cournot case, with the two firms setting  $x_1$  and  $x_2$  simultaneously:

$$\pi_2 = (A - c_2 - x_1 - x_2)x_2 \tag{1}$$

$$\frac{\mathrm{d}\pi_2}{\mathrm{d}x_2} = A - c_2 - x_1 - 2x_2 = 0 \tag{2}$$

$$W_1 = A(x_1 + x_2) - \frac{(x_1 + x_2)^2}{2} - c_1 x_1 - c_2 x_2 \tag{3}$$

$$\frac{\mathrm{d}W_1}{\mathrm{d}x_1} = A - (x_1 + x_2) - c_1 = 0 \tag{4}$$

Solving (2) and (4) yields:

$$x_1^N = A - 2c_1 + c_2 \quad x_2^N = c_1 - c_2 \tag{5}$$

$$W_1^N(x_1^N, x_2^N) = \frac{A^2}{2} + \frac{3c_1^2}{2} + c_2^2 - Ac_1 - 2c_1c_2$$
 (6)

Consider now a scenario in which public firm 1 acts as a Stackelberg leader. In our model Stackelberg leadership takes the very simple form of an output target for the public firm. Thus public firm 1 sets its output so as to maximize (3) subject to (2):

$$W_1^S = \frac{A(A - c_2 + x_1)}{2} - \frac{(A - c_2 + x_1)^2}{8} - c_1 x_1 - c_2 \frac{(A - c_2 - x_1)}{2}$$
 (7)

$$\frac{\mathrm{d}W_1^S}{\mathrm{d}x_1} = \frac{A + c_2}{2} - \frac{(A - c_2 + x_1)}{4} - c_1 = 0 \tag{8}$$

whence we obtain:6

$$x_1^L = A - 4c_1 + 3c_2 \quad x_2^F = 2(c_1 - c_2)$$
 (9)

$$W_1^S(x_1^L, x_2^F) = \frac{A^2}{2} + 2c_1^2 + \frac{3c_2^2}{2} - Ac_1 - 3c_1c_2$$
 (10)

where the superscripts L and F stand respectively for public Leader and private Follower. Comparing (5)–(6) with (9)–(10) we can establish a number of interesting results:

<sup>&</sup>lt;sup>6</sup> A sufficient condition to guarantee that  $x_1^L>0$  is  $c_2>\frac{2}{3}c_1$ . However, as we expect the public authorities to take an interest only in "large" industries (i.e., where  $A\gg c_1$ ), we can safely assume that  $x_1^L>0$  for all relevant cost asymmetries.

- (i) unlike the standard "private" Cournot duopoly case, in which Stackelberg leadership produces an overall increase in output, here leadership by the public firm results in an output *contraction* (by  $c_1 c_2$ ) and hence a corresponding price increase;
- (ii) again unlike the "private" duopoly case in which output produced by the leading firm increases, here inter-firm output allocation changes with the *private* firm now producing twice as much as before;
- (iii) unlike the Nash-Cournot case, under Stackelberg leadership the public firm does *not* apply marginal cost pricing;
- (iv) in spite of the price increase and because of the stronger effect produced by the more efficient inter-firm allocation of output, net welfare *rises* (by  $(c_1 c_2)^2/2$ ).<sup>7</sup>

Figure 1 illustrates the trade-off between the loss of consumer's surplus due to the higher price and the efficiency gain due to the larger market share of the more efficient firm.<sup>8</sup>

#### II. Net Welfare Effects of Privatization and Partial Public Ownership

We can now apply the preceding analysis to the case of either privatization or, in reverse, to the partial restoration of an industry to public ownership. Suppose the public firm's objective were to change from welfare- to profit-maximization

What is required in order to elicit the true marginal cost is a transfer schedule increasing in  $\hat{c}_2$ . This is found by writing the private firm's profit as a function of actual and reported costs:

$$\pi_2(\hat{c}_2) = (2c_1 - c_2 - \hat{c}_2)2(c_1 - \hat{c}_2) + \tau(\hat{c}_2) \tag{11}$$

The FOC for the maximization of (11) is

$$\tau'(\hat{c}_2) = 2(3c_1 - c_2 - 2\hat{c}_2) \tag{12}$$

In order for  $\tau(\hat{c}_2)$  to be truth-eliciting it must be the case that  $\tau'(\hat{c}_2) = 6c_1 - 6\hat{c}_2$  and thus, on integrating, we obtain the following truth-eliciting transfer function:

$$\tau^*(\hat{c}_2) \equiv 3\hat{c}_2(2c_1 - \hat{c}_2). \tag{13}$$

<sup>&</sup>lt;sup>7</sup> Notice that the similar results obtained by De Fraja and Delbono (1989) in a model without cost asymmetries and with increasing marginal costs are due to the altogether different reason of the *higher* output produced by the Stackelberg public firm.

<sup>&</sup>lt;sup>8</sup> A note on truth-revelation. The argument sketched above shares with most of the literature on mixed oligopoly the assumption that the private firm will reveal its true marginal cost to the Stackelberg leader. If more realistically, we assume that only output levels are observable and verifiable and that the true value of the private firm's marginal cost is private information and is not contractible, then the private firm will never report its true cost. Since its profits are monotonically decreasing in its reported marginal cost,  $\hat{c}_2$ , it will in fact report  $\hat{c}_2 = 0$ .

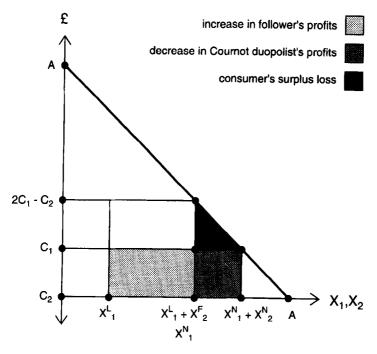


Figure 1. Output and welfare changes when public firm becomes a Stackelberg leader.

because of "privatization",  $^9$  and that as a result  $^{10}$  its marginal cost were to fall to  $c_2$ . Because of the symmetry of the two private firms it is assumed that the newly-privatized firm cannot act as a Stackelberg leader. This is reasonable if it is accepted that whereas the government can enjoy a first-mover advantage simply by setting and announcing an output target for the public firm, a private firm cannot become a leader in the absence of credible commitments. It is simple to confirm that at a Nash-Cournot equilibrium output per firm and associated welfare are respectively

$$x_1^0 = x_2^0 = \frac{A - c_2}{3} \tag{14}$$

$$W^{0}(x_{1}^{0} + x_{2}^{0}) = \frac{4(A - c_{2})^{2}}{9}$$
 (15)

Using (10) and (15) we can compute the change in net welfare brought about by the change in strategic role and technical efficiency involved in either privatization or partial public ownership:

$$\Delta W \equiv W^S(x_1^S, x_2^F) - W^0(x_1^0 + x_2^0)$$

<sup>&</sup>lt;sup>9</sup> A change in the managers' incentive structure could produce the same result.

<sup>&</sup>lt;sup>10</sup> A story in terms of "stock market discipline", reduction in managerial slack, etc. could be told here.

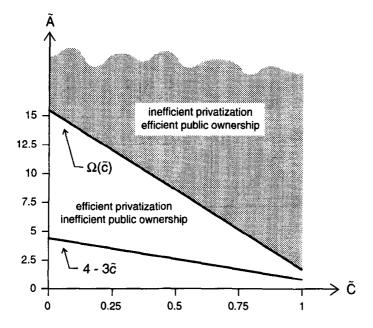


Figure 2. Cost-adjusted market size, cost asymmetry and the (in)efficiency of privatization/partial public ownership.

$$= \frac{1}{18} [A^2 - 2(9c_1 - 8c_2)A + 36c_1^2 + 19c_2^2 - 54c_1c_2]$$
 (16)

Defining  $\tilde{A} = A/c_1$  and  $\tilde{c} = c_2/c_1$  respectively as cost-adjusted market size and cost asymmetry, solving the above quadratic equation in A defines the following straight line in the  $(\tilde{c}, \tilde{A})$  space:

$$\Omega(\tilde{c}) = 9 + 3\sqrt{5} - (8 + 3\sqrt{5})\tilde{c} \tag{17}$$

In Figure 2 at any point above (below)  $\Omega(\tilde{c})$  partial public ownership (privatization) would lead to an increase (decrease) in welfare. Notice that Figure 2 includes the non-negativity constraint on the output of the public firm under Stackelberg leadership (see (9)). Figure 2 shows that the larger the market and the smaller the cost differential, the more likely it is that partial public ownership will lead to a welfare improvement. The policy implications of this conclusion are heightened given that the cost advantage of private ownership may be small<sup>11</sup> and that policy concern in this area is focused on large markets.

Figure 3 illustrates the gains and losses when partial public ownership increases welfare: the gross efficiency loss due to the increase in costs assumed to be brought about by public ownership is more than offset by the gain of consumer surplus and the combined gain in profits of the public leader and private follower.

<sup>11</sup> See Vickers and Yarrow (1988).

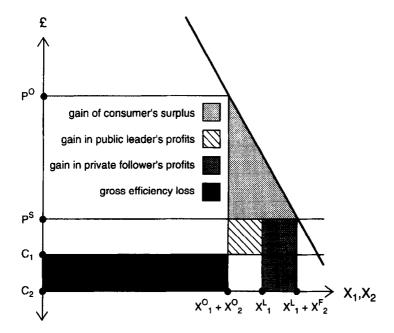


Figure 3. Output and welfare effects of partial public ownership.

The analysis sketched above ignores the outlays for (proceeds from) partial public ownership (privatization), for these are mere transfers. However, if non-distortionary taxes cannot be levied, it is of interest to note that in the case of partial denationalization the cost of buying the target private firm is not given by its profits before being taken into public ownership  $((A-c_2)/3)^2$ , but by the much smaller profits of the firm that remains in the private sector,  $4(c_1-c_2)^2$ . In fact the Government, simply by announcing the output target for the firm taken into public ownership, can, on the presumption that the stock market fully anticipates the effect on profits of a change of regime, drive the market price of both private firms to  $4(c_1-c_2)^2$ . <sup>12</sup>

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<sup>&</sup>lt;sup>12</sup> The share price based on profits earned by the firm that is *not* renationalized is the reservation price for either private firm and the only equilibrium price, assuming that shareholders in the two private firms cannot collude.

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