THE EFFECTS OF SUNSPOTS ON SOLAR IRRADIANCE

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Abstract. Sunspots have an obvious direct effect upon the visible radiant energy falling upon the Earth. We show how to estimate this effect and compare it quantitatively with recent observations of the solar total irradiance (Willson *et al.*, 1981). The sunspots explain about half of the total observed variance of one-day averages. Since the sunspot effect on irradiance produces an asymmetry of the solar radiation, rather than (necessarily) a variation of the total luminosity, we have also estimated the sunspot population on the invisible hemisphere. This extrapolation allows us to estimate the true luminosity deficit produced by sunspots, in a manner that tends toward the correct long-term average value. We find no evidence for instantaneous global re-emission to compensate for the sunspot flux deficit.

1. Introduction

The darkness of a sunspot on the visible hemisphere of the Sun will reduce the solar irradiance on the Earth. This effect has been sought in numerous programs for the observation of the solar constant, most notably the Smithsonian program (e.g., Foukal *et al.*, 1977; Hoyt, 1979a; see Fröhlich, 1977, for a general review). The unambiguous detection of the effects of solar activity on the irradiance had to await continuous high-quality data obtained from space observations on the Solar Maximum Mission (Willson *et al.*, 1981).

Sunspots vary in size, lifetime, and configuration, although some evidence for morphological changes on long time scales does exist (Albregtsen and Maltby, 1978; Hoyt, 1979b). In principle the irradiance deficit for a given spot may be determined by precise photometry (e.g. Bray, 1981). In practice, of course, this would be prohibitively difficult for the whole Sun, and photometric measurements that cover the whole disk have not been carried out effectively. Nevertheless, the counting of spots has continued since their discovery, and routine estimates of spot areas as determined from sketches appear in the *NOAA Solar-Geophysical Data*.

In this paper we show how to obtain a crude estimate of the irradiance deficit produced by sunspots (Section 2) and of the total luminosity reduction for the whole global population of sunspots (Section 3). Although spots obviously influence the solar irradiance and this influence cannot be directly balanced by faculae, since the two phenomena have different patterns of development, it is not valid to conclude that the spots reduce the solar *luminosity* unless both hemispheres are accounted for. As we shall see, the main effect may be instead to make the solar

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radiation pattern anisotropic. Future studies of the variations of solar luminosity on short or long time scales will depend upon our ability to compensate for the known effects of solar active regions, of which spots represent the most obvious components.

2. A Photometric Sunspot Index

In general, the irradiance deficit of a sunspot must be referred to the irradiance of the quiet Sun, given by

$$S_{\odot}=\int_{\Omega_{\odot}} I\,\mathrm{d}\Omega=2\pi I_0\left(\frac{R_{\odot}}{\mathrm{AU}}\right)^2\int_0^1 \mu f(\mu)\,\mathrm{d}\mu\,, \tag{1}$$

where $I=I_0 f(\mu)$ represents the limb variation of intensity with $\mu=\cos\theta$, where θ is the angle between the line of sight and the solar vertical. A sunspot with area A_{spot} will contribute an irradiance

$$S_{\text{spot}}=\mu\frac{A_{\text{spot}}}{(\mathrm{AU})^2}g(\mu)I_{\text{spot}} \tag{2}$$

in terms of the limb-darkening law for the spot $I=I_{\text{spot}}g(\mu)$. Combining these equations, we can express the normalized flux excess as

$$\frac{\Delta S}{S_{\odot}}=-\frac{\mu A_{\text{spot}}[f(\mu)I_0-g(\mu)I_{\text{spot}}]}{2\pi I_0 R_{\odot}^2\int_0^1 \mu f(\mu)\,\mathrm{d}\mu} \tag{3}$$

in terms of the intensities and limb-darkening laws of the spot areas and the quiet Sun. The quiet-Sun irradiance is $S_{\odot}$, the integral over spot area is presumed to allow for both the umbral and penumbral regions. To obtain an approximation to the exact value for the deficit, we may make the following assumptions:

(1) All umbral regions have the same temperature.

(2) All penumbral regions have the same temperature.

(3) The ratio of umbral to penumbral area is constant.

(4) The visibility function for center-to-limb variations of spot numbers is unimportant in this context.

(5) The limb-darkening law is the same in spots as it is in the quiet photosphere, and the Eddington approximation with gray opacity describes the limb-darkening (e.g. Mihalas, 1978) as given by

$$f(\mu)=\frac{3\mu+2}{5}\,. \tag{4}$$

(6) The visibility function for spots near the limb is negligible.

With these assumptions we essentially assume that the spot atmosphere is structurally similar to that of the photosphere, but with a different temperature, and that all spots are similar and have the same temperatures.

The sunspot flux deficit, ψ (irradiance units), is defined by

$$\Psi = -\Delta S = +\mu S_{\odot}\left(\frac{3\mu+2}{2}\right)A_{\text{spot}}\frac{1}{2\pi R_{\odot}^2}\left(1-\frac{I_{\text{spot}}}{I_0}\right) \tag{5}$$

$$= \mu S_{\odot}\frac{3\mu+2}{2}\frac{a_{\text{spot}}}{10^6}\alpha$$

after carrying out the solid-angle integration according to Equation (4). In accordance with assumption (5), we have set $f(\mu) \equiv g(\mu)$. The spot area $a_{\text{spot}} = 10^6(A_{\text{spot}}/2\pi R_{\odot}^2)$ is normalized to millionths of the solar hemisphere, the normal unit of measurement. The temperature-dependent factor α is given by

$$\alpha = \frac{A_u}{A_{\text{spot}}}\left[1-\left(\frac{T_u}{T_0}\right)^4\right] + \frac{A_p}{A_{\text{spot}}}\left[1-\left(\frac{T_p}{T_0}\right)^4\right] \tag{6}$$

and represents an area-weighted average of the blackbody total intensity deficit of the spot normalized to the photosphere. The umbral and penumbral temperatures are (T_u, T_p) and (A_u, A_p). Note that T_0 is the photospheric blackbody temperature, $\sim$6100 K, rather than the solar effective temperature. Using the values tabulated in Allen (1973), we find $\alpha = 0.315$. For comparison Bray (1980) gives values of 0.31 and 0.22 for two well-observed spots. Adopting a constant value for this parameter is a gross approximation, since variations certainly occur, but it is the only possibility at present for dealing with large amounts of data.

The form of the sunspot flux deficit (Equation (5)) shows that both the limb-darkening law and the simple projection of sunspot area contribute to the deficit. The deficit will therefore show a strong maximum when the spot crosses the central meridian of the Sun. Sunspots therefore produce an *anisotropy* of the solar radiation field, and this re-direction of the solar emission must be understood in detail before variations of the solar luminosity may be measured.

3. Global Measure of Spot Flux Deficit

The flux deficit produced by a sunspot must be counterbalanced by an excess emission, on the accepted view that the energy sources in the solar interior may only vary on time scales of millions of years (e.g. Ulrich, 1975; Gough, 1979). This does not mean, however, that the flux must reappear in a 'bright ring' near the spot, since appreciable short-term storage of energy is certainly possible. Observational searches for the bright ring have generally been unsuccessful, at least in terms of the total flux deficit. The faculae or plage around a spot group may represent the missing flux, but the difference in time histories in this case argues very strongly for energy storage.

These considerations make it interesting to attempt to characterize the global distribution of sunspots, including the invisible hemisphere. Of course this should be done with deep-space observatories that can make direct measurements. At present, however, we must deal with observations on one hemisphere and try to interpolate as accurately as possible. In this section we show one way of doing this, although any interpolation scheme must be ambiguous since the spot evolutionary times are comparable to the Sun's rotational period. An earlier estimate of this type (Becker and Kiepenheuer, 1953) in terms of the Zürich index showed that solar-rotation effects could be successfully removed for sunspot *counts* (that is, the Zürich number). The uncertainty in a given scheme for extrapolation to the invisible hemisphere can in principle be restricted to shorter time scales if the scheme imposes the requirement that equal total areas appear on the two hemispheres.

If a spot reappears on successive rotations, one can hope to interpolate its growth curve for the intervening periods of its invisibility (e.g. Dyson, 1925). However for most spots this procedure cannot be carried out easily, especially using routine-quality synoptic data. For such spots, as well as for spots that have short lifetimes and do not recur, an alternative approach is necessary.

We have used the following rules for the interpolation of spot areas on the invisible hemisphere. The rules are intended to apply to synoptic data giving daily measures of the areas of sunspot groups. Figure 1 illustrates the rules listed below, which necessarily involve some arbitrary decisions. We have devised the rules in such a way that the 'invisible' spots tend towards the same total area as the visible ones.

(1) For recurrent spots, we interpolate linearly.

(2) For groups that die on the disk, we assume an equal lifetime on the invisible hemisphere (symmetrical growth curve).

(3) For groups that are born on the disk, we again assume a symmetrical pattern on the invisible hemisphere.

(4) For groups that both start and finish on the visible hemisphere, we assume the existence of a symmetrical identical group on the invisible hemisphere. The area is split between two episodes half a rotation before and after the data.

These rules obviously cannot precisely describe the invisible spots; the estimate will be noisy but it should have the correct long-term behavior.

The excess in solar luminosity due to the total global population of sunspots may be calculated from

$$\Delta L = -L_\odot \frac{\sum A_i}{4\pi R_\odot^2}\alpha\,, \tag{7}$$

where α has the definition given in Equation (6). We can now define the simplest model for the re-emission of the spot deficit, namely that it reappears globally and instantaneously. The excess in *irradiance* can therefore be estimated as

$$\eta = \frac{\Delta L}{L_\odot} S_\odot = +\frac{\alpha S_\odot}{2}\frac{\sum a_i}{10^6} \tag{8}$$

in terms of the 'quiet Sun' irradiance $S_\odot \sim 1370\ \mathrm{W\,m^{-2}}$. The spot areas a_i are expressed in millionths of the hemisphere. Figure 1 shows sketches illustrating the extrapolation rules and the approximate variability to be expected from a single

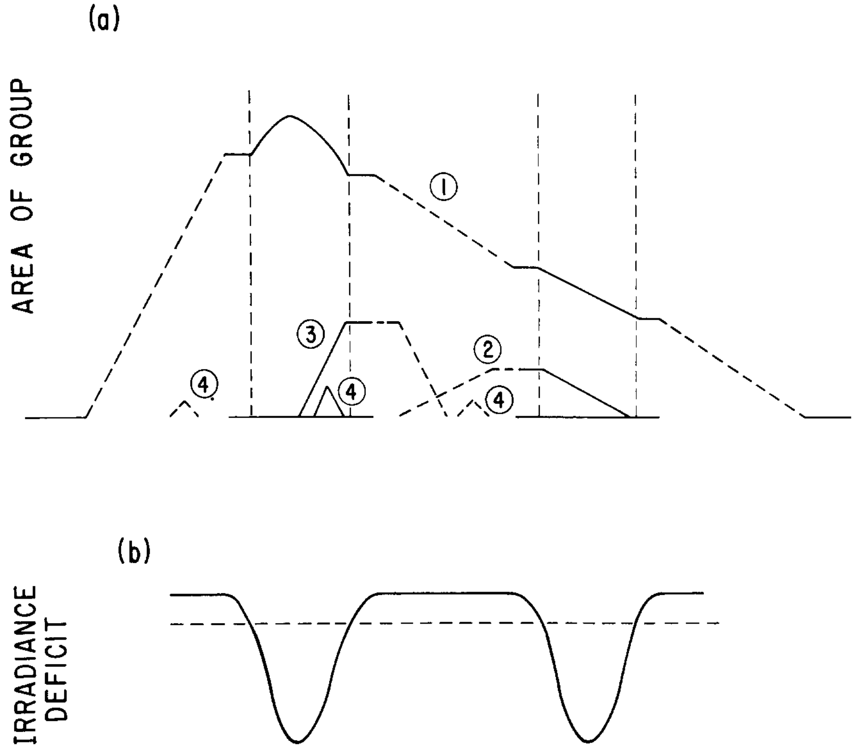

Fig. 1. Sketches (a) illustrating the extrapolation rules for spot areas on the visible hemisphere to obtain an estimate of the area on the invisible hemisphere. Horizontal lines show the visible hemisphere; dashed vertical lines show the ±70° criterion for accepting area measurements. The lower sketch (b) shows the irradiance variability expected from a single permanent spot, based upon the definitions of ψ and η in Equations (5) and (8), according to the approximate limb-darkening law for gray opacity. The steady excess is just $\frac{1}{4}$ of the peak deficit.

large spot, indicating the importance of the anisotropic radiation pattern of the spot deficit.

In order to use only the most reliable data, we have restricted the acceptable data to those active regions whose centers lie within 70° central distance. For central angles between this point and the limb, the last 'good' data point was simply adopted as an estimate of the area. A criterion of 5° was set to define recurrence, based upon the tabulated centroid locations of spot groups. We have used the weekly *Preliminary Reports of Solar-Geophysical Data* produced by the Space Environment Services Center of NOAA. Figure 2 shows the recurrent spot groups based upon the published data, with the 5° criterion for recurrence at central meridian passage, calculated from the differential rotation.

We have carried out this analysis for the first 133 days of data from the ACRIM solar-constant experiment on the Solar Maximum Mission (Willson, 1979; Willson *et al.*, 1981). Figure 3 shows the data in the form of daily averages of irradiance

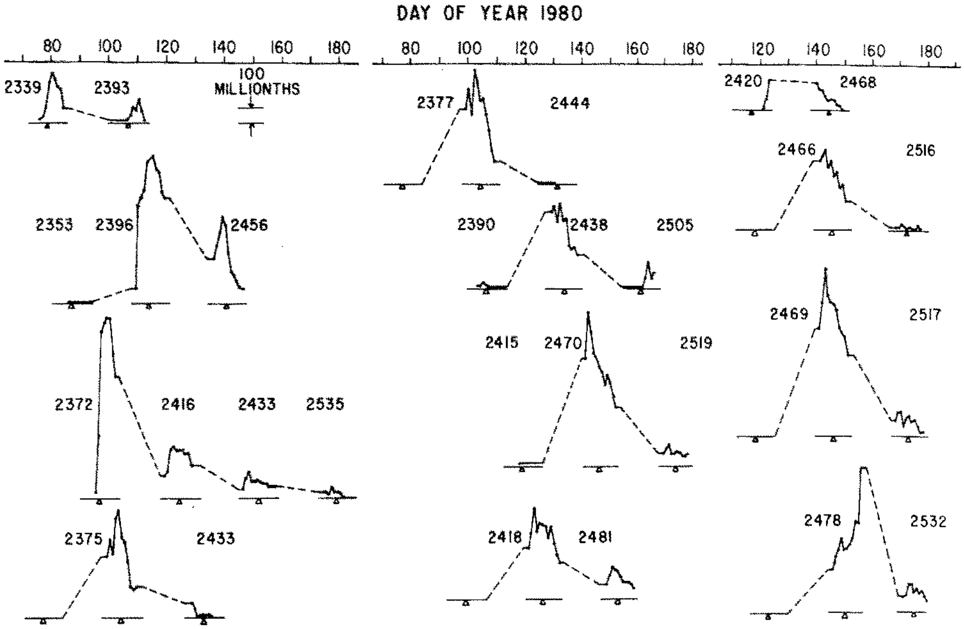

Fig. 2. Some recurrent spot groups found in the *Solar-Geophysical Data, Preliminary Reports of Solar-Geophysical Data,* with a criterion of 5° total displacement at the central-meridian passage computed from differential rotation (Dyson, 1925). The plots show area vs time, with the projection angle removed, for positions within 70° of disc center, for spot groups during February–July, 1980. Horizontal lines show spot passage across the visible hemisphere; central meridian passage is indicated by a small triangle.

observed at the Earth, corrected for distance and satellite motion (Willson and Hudson, 1981b). The sunspot irradiance index, calculated from Equation (5), shows an obvious correlation of the large 'dips' with the the presence of major groups of sunspots, especially in early April and late May, 1980. Figure 3 also shows the total flux blockage computed from the algorithm described above and Equation(7).

4. Models for the Compensating Flux Excess

The flux deficit represented by the sunspots must have a compensating excess, if the internal energy sources of the Sun are time-stationary and isotropic at some deep layer. The excess may appear at some different location or at some different time (e.g. Foukal and Vernazza, 1979); in the latter case we may speak of energy storage. In principle we can gain some knowledge of the mechanisms of the flux blockage and re-emission by comparing the ACRIM irradiance data with data on spots; this is one of the main motivations for the data reduction outlined above.

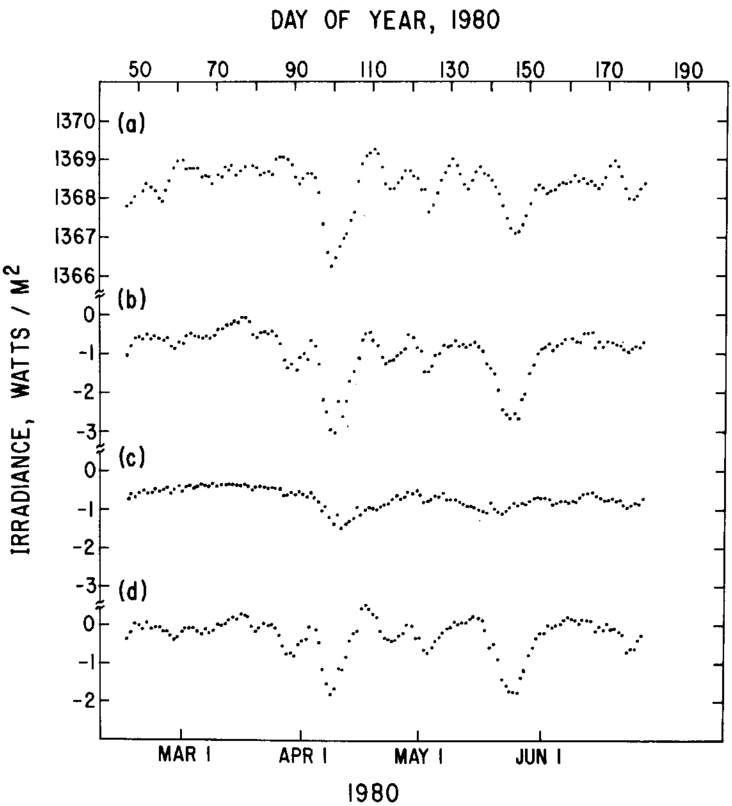

Fig. 3. Daily average values for total solar irradiance (a) compared with (b) the sunspot irradiance contribution $-\psi$ and (c) the global deficit $-\eta$..The simplest model for spot deficit and re-radiation of the missing flux, $-\psi+\eta$, is shown in (d). A value for the quiet-Sun irradiance of 1370 W m^{-2} has been assumed.

As an illustration of this comparison, we consider here a very simple model for the excess radiation: we assume that it appears instantaneously and is spread uniformly across the entire solar area. This model can be calculated directly from Equations (5) and (8), according to

$$\Delta F = -\psi + \eta \tag{9}$$

as shown in Figure 3. We can test such models against the ACRIM data by calculating the linear correlation coefficients shown in Table I. These results show that the dips produced by sunspots have a strong correlation with the ACRIM data, as is obvious in any case from the plots in Figure 3. The addition of the compensating re-emission did not improve the correlation. The variations due to solar activity account for about half of the variance in the ACRIM data. More elaborate models might be expected to yield measurements of parameters that could be related to the solar interior.

TABLE I

Correlation coefficients for simple models ($N = 133$)

(a) ACRIM vs ψ	$r = -0.82$
(b) ACRIM vs η	$r = +0.55$
(c) ACRIM vs $\psi - \eta$	$r = -0.78$
(d) ACRIM$+\psi$ vs η	$r = -0.41$

Sample standard deviations

(a) ACRIM	0.56 W m^{-2}
(b) ACRIM$+\psi$	0.37 W m^{-2}
(c) ACRIM $\psi - \eta$	0.35 W m^{-2}

5. Conclusions

The modeling of the effects of solar-activity on the total irradiance has several purposes: (i) We wish to learn about the energetics of active-region formation; (ii) we wish to use the observed solar variability to probe the solar interior structure; and (iii) we wish to isolate local active-region effects in order to correct the data and show any secular trends more explicitly. We have described methods of using tabulated sunspot data to estimate these effects. This paper illustrates the methods, but we anticipate more sophisticated analyses to follow, perhaps using better sunspot data. The ACRIM data themselves have exceptionally high precision (Willson and Hudson, 1981a).

The data on total solar irradiance obtained with the Active Cavity Radiometer on board the Solar Maximum Mission have been shown to have a good correlation with the darkness of sunspots, based upon a simple model for the spot radiation. This confirms the direct observational evidence that the 'missing flux' does not reappear as a bright ring near the spot, and suggests strongly that (a) energy is stored locally in the convection zone, or (b) that the spot influence reaches far down into the convection zone (Foukal and Vernazza, 1979). We have shown one method for extrapolating visible-hemisphere spot areas to the invisible hemisphere. As an illustration, we have used this extrapolation to calculate a very simple model for the re-radiation necessary to balance the flux deficit: instantaneous balancing on a global scale. This model does not materially improve the correlation. We conclude that either more complicated models of spot development are needed, or that other indicators of solar activity (faculae, plage, or radio emission) could be used more efficiently to remove the effects of solar activity.

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