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CAN THERE BE A REALIST SINGLE-CASE INTERPRETATION OF PROBABILITY?

The purpose of this note is to argue that no realist single-case interpretation of the probability calculus can make sense of conditional probabilities other than in well-defined exceptional circumstances. The argument bears some relation to Paul Humphreys' criticism of propensity theories, reported in Salmon (1979). We begin by sketching Humphreys' line of argument and then show how it may be blocked; in the sequel it is argued that the evasion of Humphreys' criticism affords little comfort to proponents of real single-case probabilities.

1.

Propensity theorists are sometimes guilty of a *non sequitur*: namely, that since probabilities depend on generating conditions, all probabilities are conditional probabilities, the conditioning event being the realisation of the generating conditions. Popper, for example, nearly always formalises propensities as conditional, or, as he calls them, relative probabilities.[1] Against this construal Humphreys' argues effectively, showing that it leads to decidedly odd, if not absurd, interpretations of inverse probabilities. Salmon gives the following example:

> Given suitable "direct" probabilities we can, for example, use Bayes' theorem to compute the probability of a particular cause of death. Suppose we are given a set of probabilities from which we can deduce that the probability that a certain person died as a result of being shot through the head is $\frac{3}{4}$. It would be strange, under these circumstances, to say that this corpse has a propensity (tendency?) of $\frac{3}{4}$ to have had its skull perforated by a bullet.[2]

By recognising the fallacious inference as such propensity theorists can evade the thrust of Humphreys' criticism and retain their theories as interpretations of the orthodox mathematical account of probabilities. According to this account a probability distribution is a function of a single argument from an algebra of possible outcomes – the event space – into the closed interval [0, 1]. Conditional probabilities are introduced, as defined terms, for pairs of elements of the event space. Probabilities of causes, probabilities of hypotheses, inverse probabilities

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do not enter into consideration. The traditional examples for this account are games of chance. That the numerical values of the probabilities attributed to the elements of the event space should depend on the generating conditions which give rise to the possible outcomes is an interpretive matter not reflected in the formal mathematical calculus.

2.

At first sight all is now well with propensity interpretations of probability but as is seen from an analysis of conditional probabilities in realist single-case interpretations this is not the case. Of course, not every propensity theory is a single-case theory, and conversely not every realist single-case theory is a propensity theory. The argument which follows is directed against any realist single-case interpretations of probability.

By a realist single-case interpretation is meant a theory of probability such as Ronald Giere's,[3] in which probabilities are assigned to the outcomes of a particular trial. Such theories are often thought appropriate for the understanding of quantum mechanics, particularly as they allow non-trivial probabilities in indeterministic universes. Determinism entails that all real single-case probabilities are either zero or unity. The probabilities are real in that they are not only objective but also physical, located in the world.

Let us consider an unbiased die in an indeterministic universe in which the real single-case probabilities have their familiar values. If a denotes the outcome '6'-uppermost, and b denotes the event 'even number'-uppermost, then $p(a)=\frac{1}{6}$, $p(b)=\frac{1}{2}$, and, by definition, $p(a \mid b)=\frac{1}{3}$. How is $p(a \mid b)$ to be interpreted? It is certainly not the probability that the outcome a is realised given that the outcome b has been realised, for if b has been realised exactly one of the events '2'-uppermost, '4'-uppermost, or '6'-uppermost has occurred. In the first two cases a's occurrence is impossible, in the third it is certain. The event b is realised by the occurrence of a or of an event incompatible with a. It is the realisation of a or one of these other events which constitutes the occurrence of b. In terms of real single-case probabilities, when b occurs there is no longer any matter of chance, no indeterminacy, about a's occurrence, it is fully determinate.

The problem here is that a realist single-case interpretation of probability is useful only in an indeterministic universe because otherwise the probabilities are all trivial. In such universes the future is "open" with respect to the present and past. Non-trivial conditional probabilities are only possible when the conditioned event occurs later than the conditioning event, a relatively rare occurrence when the event space is generated by the outcomes of a single trial. In illustration of this consider the experiment consisting in the tossing of two coins one after the other. In an obvious notation the basic outcomes which generate the event space are: H_1H_2, H_1T_2, T_1H_2, and T_1T_2. The conditional probability $p(H_2 \mid H_1)$, which is defined as $p(H_1H_2)/(p(H_1T_2)+p(H_1H_2))$, is susceptible to a realist single-case interpretation. The formally symmetric $p(H_1 \mid H_2)$ has no such interpretation. When H_2 is realised the first toss is over and done with, there is no matter of chance, no indeterminacy about the outcome of the first toss, H_1 either has or has not been realised. It's outcome is fixed by the events happening upto and including the realisation of H_2. (If both coins are tossed simultaneously then neither conditional probability has a real single-case meaning.)

In both these examples it is important to note that the occurrence of the conditioning event does not determine the occurrence or otherwise of the conditioned event. What makes the probabilities 0 or 1 is that the occurrence or otherwise of the conditioned event is determinate before or concurrently with the occurrence of the conditioning event.

3.

There is a certain irony in reaching the conclusion that conditional probabilities cannot adequately be treated in realist single-case interpretations of probability, for one aim of propensity theories is to provide such an interpretation. Popper has claimed that the switch from von Mises' theory to his own 'corresponds to the transition from the frequency theory to the measure-theoretical approach',[4] but as Kolmogorov pointed out, what distinguishes probability theory from measure theory is the definition of conditional probability. A consequence of the above argument, of interest to propensity theorists, is that since conditional probabilities pose no problems for a theory which interprets probabilities as propensities to produce long run frequencies, whatever else these propensities may be, they are not real single-case probabilities.

What are the consequences for realist single-case interpretations in general? The argument appeals essentially to only three premises, the rejection of none of which recommends itself. One could ignore conditional probabilities altogether and thus obtain a realist single-case interpretation of normalised measures, not the probability calculus. One could sunder the link between realist single-case interpretations and the indeterminate. Or one could claim that the past and present are as indeterminate as the future. None of these is a particularly happy choice. Far better to surrender real single-case probabilities.

NOTES

[1] Cf. Popper (1957), p. 67.
[2] Salmon (1979), pp. 213–4.
[3] Cf. Giere (1973).
[4] Popper (1957), p. 68.

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