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Public pension reform, demographics, and inequality*

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Abstract. Starting from a simple, descriptive model of individual income, an explicit link between the age composition of a population and the personal distribution of incomes is established. Demographic effects on income inequality are derived. Next, a pay-as-you-go financed state pension system is introduced. The resulting government budget constraint entails interrelations between fiscal and demographic variables, causing an additional, indirect demographic impact on the distribution. This is shown not only to change, but in some cases even to reverse the distributional incidence of an aging population. Several policy conflicts arise. The point is re-emphasized by an analysis of the German Pension Reform Act of 1992. The study reveals that the design of the pension formula decisively drives the relation between demographics and inequality.

I. Overview

In most industrialized countries the post-war baby boom has been followed by a drastic decline in birth rates; at the same time, mortality rates have continued to fall as the salubrious effects of higher living standards and public health measures have been reinforced by a broad range of medical advances. Since the consequences of these developments only become apparent several decades later, politics and

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science have ignored the relationship between *demographics* and *economics* for a long time. Meanwhile, the effects of demographic change render policy reactions unavoidable. It is up to economic science to bridge the gap between demographic and economic variables by establishing a structural framework in which the issues can be examined in a consistent way. The present paper tries to do this on the subject of old-age pensions and the distribution of income.

Public interest in questions of economic inequality has increased considerably in highly developed economies, particularly in Germany and the United States. In the political arena, distributional policy decisions are typically based on cross-sectional examinations, i.e., on the dispersion of periodic (annual) incomes. However, cross-sectional information used in practical discussions of redistribution measures is influenced by a factor which has nothing to do with economic inequality per se: the *age structure* of the population. In the face of continuous demographic change, the ensuing problem of normative evaluation is of grave importance. Though many empirical studies have been trying to quantify the 'age-effect' on the distribution, the theoretical consequences, particularly in judging distributional equity, have received little attention to date.[1]

In aging populations, caused by declining birth and death rates, not only are immediate distributional distortions observable, but the financing of public transfer systems, which themselves influence the distribution of disposable income, is endangered. The focus of the economic discussion on resource assignment in the face of rising proportions of the elderly are *inter*generational transfer programs, especially old-age pension schemes. Meanwhile, the growing discrepancy between pension contributions and pension benefits has so gravely affected the financial solvency of the state pension system, that reform is inevitable. The *Pension Reform Act of 1992* is the result of scientific and political discussions in Germany of the possibilities of solving this problem. It is taken in this paper as an example and starting point to explore potential interdependencies between age composition, old-age insurance, and income distribution, posing questions like: How does the demographic change affect the dispersion of incomes? How does a pay-as-you-go financed state pension system bear upon the incidence of demographic shifts? Which are the distributional implications of a balanced budget policy reacting to population aging? How does the Pension Reform control burden sharing between workers and retirees, and what are the new consequences for the demographic inequality impact? Until the present time, in spite of their importance for the formulation of financial and distributional policies, these questions have remained largely unanswered. This paper tries to clarify some of these demo-economic interrelations.

II. Pension financing, budget incidence, and intergenerational burden sharing

The fundamental goal of the subsequent analysis is to devise a simple descriptive framework which is capable of capturing certain crucial aspects of the aforementioned interdependencies, thus facilitating the recognition of possible misinterpretations of the empirical picture of inequality.

The population consists of two groups: persons who are gainfully employed, and those who receive old-age pensions. Net earnings of worker j, Y_j, are given by:

$$Y_j = (1-c)A_j \ , \quad 0<c<1 \ , \quad A_j>0 \ , \tag{1}$$

where c denotes the rate of contributions to the state pension fund and A_j marks gross earnings of worker j. Pensioner i's retirement income, P_i, is specified as:

$$P_i = p\mu_A L_i \ , \quad 0<p<1 \ , \quad L_i>0 \ , \tag{2}$$

where p is the retirement benefit rate, μ_A the average gross earnings of the working population, and L_i the pension claim basis for retiree i (which is typically linked to his earnings history and his number of insurance years). Eq. (2) is based on pension formulas currently used in a number of nations. In particular, it reflects the built-in flexibility of state pensions increasing in line with gross earnings per worker.

To move from the micro level characterized by (1) and (2) to the macro level, i.e., to the population as a whole and thus to the *distribution* of individual incomes, we must aggregate across all j's and i's. The present study concentrates on the first two moments. The linear structure of the model would allow a derivation of moments of higher order, but no meaningful economic interpretation can be given to them. Including higher order moments would provide more precise information as to the functional form of the aggregate distribution. Yet, from a policy viewpoint, the functional form as such is of little importance. A major part of the distributional information relevant to political issues of economic inequality is already contained in the first two central moments, indicating per capita income and the variance of income. Moreover, from these moments the squared coefficient of variation can be determined, which for purposes of illustration is taken as a summary measure of the degree of inequality.

If the first two moments are calculated on the basis of Eqs. (1) and (2) for workers and retirees and then aggregated over the corresponding population subgroups, we can determine the squared coefficient of variation for the overall distribution of disposable income, V^2:[2]

$$V^2 = \frac{\sigma^2}{\mu^2} \ , \tag{3}$$

where:

$$\sigma^2 = x\sigma_Y^2 + (1-x)\sigma_P^2 + x(1-x)(\mu_Y - \mu_P)^2 \ ,$$

$$\mu = xm_Y + (1-x)\mu_P \ ,$$

and:

$$\sigma_Y^2 = (1-c)^2\sigma_A^2 \ ,$$

$$\mu_Y = (1-c)\mu_A \ ,$$

$$\sigma_P^2 = p^2\mu_A^2\sigma_L^2 \ ,$$

$$\mu_P = p\mu_A\mu_L \ ;$$

$$x = \frac{E}{E+R} = \frac{1}{1+R/E} \ .$$

This represents a simple closed-form decomposition of the overall level of inequality into *economic* and *demographic* components. It is analytically flexible and easy to interpret, both theoretically and empirically. It is keyed to the *intergenerational* impact of demographic change and forms the basis of the subsequent incidence analysis. σ^2 represents the variance and μ the average disposable income of the total population. σ_Y^2 depicts the variance and μ_Y the average net earnings of the working population, σ_P^2 gives the variance of pensions and μ_P the average pension. The distributional influence of an aging population is captured by x, the fraction of workers in the total population. This is a monotonically decreasing function of the *old-age dependency ratio* R/E, the ratio of the number of retirees R to the number of active workers E.

Birth rates in most developed countries currently are at or below those necessary to sustain population. At the same time, mortality rates have continued to decline. One of the main results of these demographic trends is an increase in the ratio of pensioners to active workers. While this ratio has already been gone up now for many years, sharp additional increases will occur in the next two decades within most developed nations. How does this affect the distribution of disposable income? Which are the direct effects of a higher dependency ratio R/E (i.e., a lower x) on the level of relative income dispersion V^2? Taking the derivative of V^2 in (3) with respect to x, we obtain the condition:

$$\frac{\partial V^2}{\partial x} \gtreqless 0 \Leftrightarrow -(\mu_Y-\mu_P)^2[x\mu_Y-(1-x)\mu_P]-\sigma_P^2[(1-x)(\mu_Y-\mu_P)+\mu_Y]$$

$$-\sigma_Y^2[x(\mu_Y-\mu_P)-\mu_P] \gtreqless 0 \ .$$

If the number of workers exceeds the number of retirees, and provided that average net earnings are distinctly greater than the average pension – more precisely: if $E>R\geq 0$ ($\Rightarrow \frac{1}{2}<x\leq 1$) and $\mu_Y \geq (1+\frac{1}{x})\mu_P$ – then $\partial V^2/\partial x$ is unambiguously negative. These two conditions are sufficient but not necessary. In fact, the weaker, empirically corroborated assumptions[3] $E>R$ and $\mu_Y>\mu_P$ are enough to guarantee that the first line of the sign condition is negative; if the overall sign condition is evaluated by means of the empirically-based benchmark parameter set given in Table 1, then we see that the negative terms in the first line clearly dominate the term in the second line, so that:

$$\frac{\partial V^2}{\partial x}<0 \ ; \tag{4}$$

a higher ratio of retirees to workers $\left(\frac{R}{E}\uparrow, \text{ or } x\downarrow\right)$ thus causes a greater relative variation of income. To go from this intermediate result of specific incidence of demographic change to our actual focus of interest, namely, demographic *budget incidence*, we must include the financial constraints. In the present case, these are restricted to a single national program: to a pay-as-you-go financed state pension scheme. When total contributions are set against total pension payments, the following accounting equation is obtained from the microrelations (1) and (2):

$$\sum_{j=1}^{E} cA_j = \sum_{i=1}^{R} P_i \ , \tag{5}$$

or:

$$Ec\mu_A = Rp\mu_A\mu_L \ .$$

For (5) to hold in light of changing economic and demographic constellations, one degree of freedom is lost in setting policy variables: c and p will no longer be mutually independent instruments. Thus, the policymaker must decide which decision variable is to be fixed and which will be endogenous. If the retirement benefit rate p is politically determined, the corresponding budget balancing ("*BB*") contribution rate is given by:

$$c_{BB} = \frac{R}{E}\frac{\mu_P}{\mu_A} = \frac{1-x}{x} p\mu_L \ . \tag{6}$$

If, on the other hand, the contribution rate c is institutionally fixed, p will be endogenous and we have:

$$p_{BB} = \frac{x}{1-x}\frac{c}{\mu_L} \ . \tag{7}$$

Equations (6) and (7) induce *indirect* demographic inequality effects which distort the cross-sectional picture above and beyond the direct effect captured by (4). This has been largely neglected by the previous literature. The subsequent analysis illustrates that these effects may be critical to the conclusions drawn.

One immediate question is whether the incidence results are robust with respect to the fundamental policy decision (6) versus (7): is the direction of the total demographic effect independent of whether policymakers choose (6) or (7) to adapt to population development? To settle this question, Eq. (6) must be integrated in place of c into the previously developed distributional structure (3), and then, correspondingly, Eq. (7) in place of p.

If the contribution rate c is replaced in (3) by its endogenous form c_{BB} from (6), we have:

$$V^2_{BB} = V^2\langle x, c_{BB}\langle x\rangle\rangle \ ,$$

so that:

$$\frac{dV^2_{BB}}{dx} = \underbrace{\frac{\partial V^2}{\partial x}}_{<0} + \underbrace{\underbrace{\frac{\partial V^2}{\partial c_{BB}}}_{<0}\underbrace{\frac{dc_{BB}}{dx}}_{<0}}_{>0} \ .$$

The first term on the righthand side gives the direct demographic inequality effect, and the second term the indirect distribution effect resulting from the balanced budget Eq. (6). The signs of the partial derivatives originate from (4) and (6), as well as from:

$$\frac{\partial V^2}{\partial c} \gtreqless 0 \Leftrightarrow -\left[\frac{\mu_P}{\mu_Y}\sigma_Y^2 - \sigma_P^2 + (\mu_Y - \mu_P)\mu_P\right] \gtreqless 0 \ ,$$

which implies $\partial V^2/\partial c < 0$, given the empirically corroborated assumptions $\mu_Y > \mu_P$ and $V_Y^2 > V_P^2$.[4] The signs of the partial derivatives do not, however, allow us to make a final judgement as to the direction of the total demographic effect. If the following condition is evaluated using the empirically corroborated benchmark parameter set in Table 1,

$$\begin{aligned}\frac{dV_{BB}^2}{dx} \gtreqless 0 &\Leftrightarrow [(3-x)\mu_P - x\mu_A](1-c_{BB})\frac{\sigma_A^2}{\mu_A} - (2-x)\sigma_P^2 + [(3-2x)\mu_P - x\mu_A] \\ &\quad \times (\mu_{Y,BB} - \mu_P) \gtreqless 0 \\ &\Leftrightarrow [(3-x)p\mu_L - x]\left(1 - \frac{1-x}{x}p\mu_L\right)\sigma_A^2 \\ &\quad + \left[[(3-2x)p\mu_L - x]\left(1 - \frac{p\mu_L}{x}\right) - (2-x)p^2\sigma_L^2\right]\mu_A^2 \gtreqless 0 \ ,\end{aligned}$$

then we obtain for the demographic budget equilibrium effect based on (6):

$$\frac{dV_{BB}^2}{dx} > 0 \ ; \tag{8}$$

an increase in the proportion of retirees ($x\downarrow$) thus induces a *reduction* in relative income disparity. Note that the additional inequality effect results from a purely *fiscal* reaction to disturbances of budget equilibrium, not from any redistributional reaction to changes in the personal distribution of incomes. The restoration of government budget equilibrium brought out of order by demographic shifts entails unintended (or intended . . .) distributional effects which put the incidence of demographic trends in a different light.

Which is the effect of an endogenous retirement benefit rate p_{BB}? If we exchange the so far exogenous parameter p for the equilibrium rate (7) in Eq. (3), then we have:

$$V_{BB}^2 = V^2\langle x, p_{BB}\langle x\rangle\rangle \ .$$

Combining the inequality-decreasing influence of the benefit rate ($\partial V^2/\partial p < 0$), obtained from:[5]

$$\frac{\partial V^2}{\partial p} \gtreqless 0 \Leftrightarrow -\left(\sigma_Y^2 - \frac{\mu_Y}{\mu_P}\sigma_P^2\right) - (\mu_Y - \mu_P)\mu_Y \gtreqless 0 \ ,$$

with (4) and (7), gives:

$$\frac{dV_{BB}^2}{dx} = \underbrace{\frac{\partial V^2}{\partial x}}_{<0} + \underbrace{\underbrace{\frac{\partial V^2}{\partial p_{BB}}}_{<0}\underbrace{\frac{dp_{BB}}{dx}}_{>0}}_{<0} < 0 \ ; \tag{9}$$

i.e., economic inequality *increases* with pensioners' population share ($R/E\uparrow$, or $x\downarrow$) when (7) is used as the budget balancing device.

The extreme cases at the disposal of policymakers within the framework of a pay-as-you-go financed old-age insurance system for making adjustments to population dynamics (either by varying the contribution rate or by varying the pension benefit rate, ceteris paribus) induces *contrary* demographic inequality effects according to (8) and (9). The political need for redistribution, as derived typically from cross-sectional information (adequate longitudinal data are still missing), is thus subject to the whim and will of policymakers, as long as the question of intergenerational burden division has no well-founded basis. Notice that two distinct distribution levels are intertwined. One level relates to the distribution of the burden of financing old-age pensions between the generations, and the other is characterized by distributional policy decisions pertaining to current income disparity. Consequently, decisions made on the latter level are *prejudiced* through decisions made on the former level. The empirical inequality findings can be manipulated in *both* directions through the continuous transition from a pure contribution rate adjustment [⇒ (8)] to a pure benefit rate adjustment [⇒ (9)].[6]

III. Pension reform and demographic distortion

The results from the previous section made plain how central the question of intergenerational burden sharing is for the budget incidence of demographic change and thus for the policy evaluation of cross-sectional data. None of the extreme possibilities at disposal within the framework of a pay-as-you-go financed state pension system seems to be politically acceptable. The central issue in the public discussion of old-age insurance is, rather, the division of the financial burden of an aging population between both population groups concerned, workers *and* retirees. However, what determines such a compromise? What do the actual reform attempts look like in Germany and what implications do they have for the above inequality interactions?

The standard accounting equation for a pay-as-you-go financed state pension scheme is:

$$\sum_{j=1}^{E} cA_j + G = \sum_{i=1}^{R} P_i \ , \tag{10}$$

or:

$$Ec\mu_A + G = R\mu_P \ .$$

G represents a state subsidy to the pension budget, common in Germany as well as in many other countries.[7] All other symbols have already been described in Sect. II. Under the previous German pension law, adjustments for demographic shifts were made by varying the contribution rate such that total contributions plus government subsidy equal total pension payments. If (10) is solved for this policy instrument, we obtain:

$$c_{BB} = \frac{R}{E}\frac{\mu_P}{\mu_A} - \frac{G}{E\mu_A} \ , \tag{11}$$

where R/E represents the dependency ratio, and μ_P/μ_A the average gross pension level (or replacement rate). Whereas in 1986 approximately two workers supported one pensioner, this relationship will be roughly one-to-one by the year 2030.[8] Taking into consideration this age-structure shift has lead to the concurrent conclusion that a contribution rate c_{BB} of 37–42% would be required for a balanced budget if the pension law and the prevailing pension level are maintained.[9] This, however, is an economically and politically unacceptable burden on the workforce and has produced an intensive scientific and political discussion on possible solutions to the financing problem and, subsequently, to associated questions of distribution. The *Pension Reform Act of 1992* is the result of that discussion in Germany. On the one hand, this act is based on existing law, but on the other hand, it contains a number of new structural elements. Among the points of contact are the wage- and contribution-based old-age pension (there will be neither a tax-financed basic pension nor a need-based minimum insurance – two propositions which were widely discussed), a uniform contribution rate (neither a contribution rate based on the number of children nor a value-added based contribution will be introduced), the significance of the pay-as-you-go system for financing benefits (extended capital funds based pension programs and other deviations from pay-as-you-go financing of old-age insurance are no longer topics of discussion), as well as the persons covered by the system.[10] Some important structural novelties of the 1992 law are:[11]

- The pension progression changes from gross- to net-orientation.
- The public grant is founded on a new perpetuation basis: beginning in 1992 the state subsidy is not only coupled to changes in average gross earnings but also to changes in the rate of contributions to the public pension fund.

With these changes the government hopes to counter demographically-caused financial shortages. The central idea of the novelties consists in establishing a self-regulating link between pension revenues and expenditures, i.e., between the contribution rate, state subsidies, and pension benefits, so that the financial burden arising from population aging is commonly carried in a foreseeable way by all parties affected. From the self-regulating interconnection of these three variables, *feedback effects* result which constrain the financial burden and reduce the otherwise necessary increase in the contribution rate: given an impended upward adjustment of the contribution rate, as a consequence of the new dynamization, we can now already take into account a corresponding rise in the federal subsidy and a lower progression of retirement incomes, so that the increase of the contribution rate may turn out less severe. Moreover, the self-regulating relationship causes disposable earnings and retirement incomes to develop synchronized with each other: pensions will no longer increase any more than net wages, the average net pension level (i.e., the net replacement rate) keeps stable.

Given the analytics of demographic incidence developed in Sect. II, the question of demo-economic inequality effects triggered by the Pension Reform arises. To address this issue, the key reform elements must be translated into the language of the above model.[12] The coupling of federal subsidies to the development of both average gross earnings and the contribution rate can be made explicit by:

$$G = \gamma c \mu_A \ , \quad \gamma > 0 \ , \tag{12}$$

where γ represents a federally determined subsidy rate.[13] The transition from gross to *net* progression of pensions can also be easily integrated. The changed dynamics of retirement incomes can be expressed as:

$$P_i = p\mu_Y L_i \ , \quad 0<p<1 \ , \quad L_i>0 \ . \tag{13}$$

Compared with (2), the new quantity is μ_Y, which is known from Eqs. (1) or (3): $\mu_Y = (1-c)\mu_A$ represents average net earnings of the working population.

If the modified Eqs. (12) and (13) are substituted into the pension budget restriction (10), then exactly what is meant in the law by "self-regulatory link" becomes clear:

$$cE\mu_A = (1-c)Rp\mu_A\mu_L - c\gamma\mu_A \ . \tag{14}$$

If, as a result of demographic change, there is an increase in the number of retirees ($R\uparrow$), then according to previous law, the rate of contributions to the state pension fund, c, is raised correspondingly to bring expenditures and revenues back into equilibrium. Under the new law the budget of the public insurance system will also be balanced by a variation of the contribution rate. However, if we include the reform elements, we see from the righthand side of (14) that an upgraded contribution rate causes two feedback effects: first, the amount of per capita pension is diminished due to the present orientation on net adjustment; secondly, the amount of the public grant is marked up. Both of these help restrain the increase of the contribution rate which would otherwise be necessary. Solving (14) with respect to c, we obtain:

$$c_{BB} = \frac{1}{1+\dfrac{E+\gamma}{Rp\mu_L}} \ . \tag{15}$$

What effects do these automatic feedbacks have on average disposable incomes of the younger and older generations? What effects do changes in the age structure now cause on the burden sharing between gainfully employed and retired people? According to the previous pension law, living standards developed one-sidedly to the detriment of the active generation: net earnings per worker decreased as the ratio of retirees to workforce participants increased; the average pension remained unaffected. One of the main goals of the Pension Reform Act of 1992 consisted of the abolition of this unequal burden development. The financial burden resulting from foreseeable shiftings of the age composition should be allocated both to the workers as well as to the retirees. The way in which this goal is achieved by the Reform can be seen from the modified per capita figures, which are obtained by substituting (15) into the average values of (1) and (13):

$$\mu_{Y,BB} = (1-c_{BB})\mu_A \ , \tag{16}$$

$$\mu_{P,BB} = p(1-c_{BB})\mu_A\mu_L \ . \tag{17}$$

Both quantities decrease as the number of retirees goes up ($R\uparrow$). The intergenerational distribution of the demographically-induced finanical burden of the state

pension scheme is oriented on the criterion of a stable average net pension level.[14] Based on (16) and (17), it can be easily seen that percental changes of average net labour income and disposable average pension are equal:

$$\varepsilon_{\mu_{Y,BB},\bullet} = \varepsilon_{\mu_{P,BB},\bullet} \ , \tag{18}$$

where $\bullet$ stands for R and/or E, respectively. This correspondence of demographic elasticities specifies the meaning of the reform announcement of a "balanced" development of disposable earnings and pensions.

The central result of the previous section was that burden-sharing between generations and budget incidence of an aging population are coupled with each other. This immediately suggests the question of how the reform proposals affect the V^2_{BB}-incidence of demographic change: How does a shift in the age structure impact upon the cross-sectional distribution of disposable income, given the solution to intergenerational burden-sharing as addressed in the 1992 Pension Reform Act? In order to see the effects, the above modifications have to be included into the previously-developed framework (3). The difference between the previous structure (3) and the one now under consideration consists of changes in the first two central moments of retirement incomes:

$$\mu_P = p\mu_Y\mu_L = p(1-c)\mu_A\mu_L \ , \tag{19}$$

$$\sigma^2_P = p^2\mu^2_Y\sigma^2_L = p^2(1-c)^2\mu^2_A\sigma^2_L \ ; \tag{20}$$

consequently:

$$\mu = x\mu_Y + (1-x)\mu_P = (1-c)[x+(1-x)p\mu_L]\mu_A \ , \tag{21}$$

and

$$\begin{aligned}\sigma^2 &= x\sigma^2_Y + (1-x)\sigma^2_P + x(1-x)(\mu_Y-\mu_P)^2 \\ &= (1-c)^2[x\sigma^2_A + (1-x)p^2\mu^2_A\sigma^2_L + x(1-x)(1-p\mu_L)^2\mu^2_A] \ .\end{aligned} \tag{22}$$

If the relation $\mu_Y > \mu_P$ (cf. Sect. II) held before the Pension Reform Act, then subsequent to the Reform it holds more than ever, since the new value for μ_P from (19) is lower than the former from (3). Based on the previously mentioned reference parameter set, this means that the specific incidence result (4) is still valid:

$$\frac{\partial V^2}{\partial x} < 0 \ . \tag{23}$$

To ascertain the consequences of the Pension Reform Act for the *budget incidence* of demographic change, the modified financial restrictions must be included as the next step. The equilibrium rate (15), by the way, cannot be represented as a direct function of x (or R/E);[15] therefore, I will concentrate on R as an indicator (equivalent to x) of the aging process. It is just a question of interpretation whether the analysis refers to an increasing number of retirees ($R\uparrow$) or to a decreasing workers' population share ($x\downarrow$); remember that $x := E/(E+R)$ is a

monotonically decreasing function of R. To subsequently enable a direct qualitative comparison with the result of budget incidence *before* the Pension Reform Act, (8) is transformed via $V^2_{BB} = V^2_{BB}[x(R)]$ to:[16]

$$\frac{dV^2_{BB}}{dR} = \underbrace{\frac{dV^2_{BB}}{dx}}_{>0} \underbrace{\frac{dx}{dR}}_{<0} < 0 \; . \tag{8a}$$

If the contribution rate c is replaced in (3) by its endogenized form c_{BB} as given in (15), noting the new relations (19)–(22), then, formally:

$$V^2_{BB} = V^2\langle x\langle R\rangle \; , \; c_{BB}\langle R\rangle\rangle \; .$$

Totally differentiating this with respect to R yields:

$$\frac{dV^2_{BB}}{dR} = \underbrace{\underbrace{\frac{\partial V^2}{\partial x}}_{<0} \underbrace{\frac{dx}{dR}}_{<0}}_{>0} + \underbrace{\underbrace{\frac{\partial V^2}{\partial c_{BB}}}_{=0} \underbrace{\frac{dc_{BB}}{dR}}_{>0}}_{=0} > 0 \; . \tag{24}$$

The sign of the first partial derivative indicates the specific incidence effect from (23). The sign of dx/dR ensues from the definition of x. The partial influence of a variation of the contribution rate on the relative variation of income V^2 is derived from the equations of the first two moments of the overall distribution, (21) and (22). The effect of a higher number of pensioners on the contribution rate c_{BB} originates from the equilibrium relation (15). A dynamization of pensions according to the development of average net earnings, a key characteristic of the Reform, means that the contribution rate c acts as a *proportional tax* on *both* individual gross earnings and retirement incomes [cf. (1) and (13)]; consequently, *relative* income disparity V^2 remains unaffected by variations in c. This, however, implies that all indirect demographic inequality effects cease. The second summand in (24) is equal to zero and one obtains an unambiguous sign for the budget incidence: Given the reform measures, shifts in the age structure manifesting in an increase in the number of retirees ($R\uparrow$), causes economic inequality to *rise*.

For lack of adequate longitudinal information, a political justification of any redistributional activity will employ cross-sectional periodic data. According to the above results, the distorting impact of an aging population on current income inequality *before* [cf. (8a)] and *after* [cf. (24)] the Pension Reform is in *opposite* directions. The goal of the Pension Reform Act of 1992 is to get hold of the financial burden of the state pension program and to distribute this burden as evenly as possible between the active and the retired generations. It is certainly not the goal of the Pension Reform to influence the decision basis of future policy measures in the area of personal income distribution. However, precisely this is the case. Until now, the interdependency between *fiscal measures of old-age insurance reform* and *demographic inequality effects* has been overlooked. The usefulness of presently available distributional information is once more put to question.

This conclusion is underlined by a final investigation of the order of magnitude of the established effects. Since each variable of the theoretical framework

Table 1. Intergenerational burden sharing and demographic budget incidence. (R-Elasticities)

Previous pension law		Pension reform act 1992	
$\varepsilon_{\mu_{Y,BB},R}$	−0.274	$\varepsilon_{\mu_{Y,BB},R}$	−0.177
$\varepsilon_{\mu_P,R}$	0	$\varepsilon_{\mu_{P,BB},R}$	−0.177
$\varepsilon_{V^2_{BB},R}$	−0.093	$\varepsilon_{V^2_{BB},R}$	0.150
$\varepsilon_{V^2_{BB,intra},R}$	−0.044	$\varepsilon_{V^2_{BB,intra},R}$	0.032
$\varepsilon_{V^2_{BB,inter},R}$	−0.509	$\varepsilon_{V^2_{BB,inter},R}$	0.646

Benchmark parameter specification: $x = 0.7$ (or $R/E = 0.43$); $p = 0.0125$; $\gamma = 2.5$; $\mu_A = 36$, $\sigma_A^2 = 374$; $\mu_L = 40$, $\sigma_L^2 = 400$.

has an empirical counterpart, a *quantitative* evaluation of the analytical closed-form relations is a straightforward matter.

The benchmark parameter set shown in Table 1 is based on German data of 1988.[17] The ceteris-paribus-sensitivities calculated on this basis deliver interesting additional insights into the dynamic consequences of the Pension Reform.

- The Reform significantly influences the intergenerational burden division. Whereas under the previous law workers alone have to bear the budgetary burden of a shifting age structure, the Pension Reform Act provides that this will be allocated to both generations involved in the old-age insurance scheme. A one per cent increase in the number of retirees would lead to a decrease in average net earnings by approximately 0.27% *before* the Reform, and by only about 0.18% *after* the Reform. This corresponds to a burden reduction of well over 30%. Retirement incomes which, under previous law, would remain unaffected by an aging population ($\varepsilon_{\mu_P,R} = 0$), adjust in the same way as earned incomes under the new Act [cf. (18)]: per capita pension benefits would fall by approximately 0.18%, given a one per cent increase in R.
- The demographic elasticity of the overall inequality of disposable labour and retirement incomes, $\varepsilon_{V^2_{BB},R}$, changes direction as a consequence of the reform measures [see (8a) and (24)] and increases considerably in absolute terms. Particularly conspicuous is the incidence reversal of the *inter*-group inequality component:[18] whereas before the enactment of the Pension Reform a one per cent increase in the number of retirees led to a *decline* of inter-group inequality by 0.5%, the same increase in retirees induces an *augmentation* in the relative inter-group disparity by almost 0.65% after the enactment of the Pension Reform.

Of course, the quantitative sensitivity results presented in Table 1 do not depict empirical truths; the calculated values reflect an accuracy which a simple model of the present kind cannot possess. Moreover, the magnitudes as such should not be taken too literally; other inequality indicators will report other orders of scale. The values are just to be regarded as *trends*. These trends, however, speak a distinct language; given the otherwise observed stability of the aggregate distribution, they signalize distortions of considerable importance and thus reinforce the significance of demographic change for distributional policy.[19]

IV. Extensions

The main objective of this paper has been to devise a conceptual framework for a consistent positive analysis of an acute policy problem. The approach was aimed at the structurally simplest possible level, in order to isolate fiscal-demographic dependencies which already lead to policy conflicts and potential misinterpretations of the empirical evidence on inequality. It stands to reason that the particular focus on the *demographics* of inequality entails no implication for the relevance or irrelevance of other factors that might affect the distribution of income.

The model can be extended in many ways. Before briefly sketching a few alternatives, let me underline that no generality is claimed beyond the analytics presented above. In particular, the set-up restricts its attention to the first two central moments of the distribution. These moments have a straightforward meaning in the political context of economic inequality. In addition, they can be used to determine the coefficient of variation, a common summary measure of the degree of relative income dispersion.[20] Nevertheless, other measures are also employed in practice. As long as the specific inequality-indicator at hand is a member of the Generalized Entropy family[21] and thus, among other things, additively decomposable by population subgroups, it should be possible, in principle, to derive analytical results similar to the ones of this paper. Numerical calculations based on the same benchmark data as those given in Table 1 led to a reproduction of the above sign reversals for such widely-used inequality indices as the Gini coefficient, the relative mean deviation, the variance of logarithms, the Theil index, and the Atkinson measure (for $\epsilon = 0.5$, 1, and 1.5) – though on differing quantitative scales.

The two starting Eqs. (1) and (2) are of a descriptive nature. While the pension formula (2) leans upon the institutional details of retirement laws in operation in a number of industrialized countries, the earnings equation (1) is to be read as a reduced form of some structural microeconomic model, not explicitly set out here. The full range of earnings theories may be considered to generate A_j.[22] Integrating standard maximizing responses by making labour income an *endogenous* variable entails additional demo-economic inequality interrelations: disincentive reactions involve modifications not only of the distribution of net incomes but also of gross incomes. Hence, demographic shifts indirectly interfere with the process of income formation, opening up another channel of demographic disparity bearings [$c_{BB}(R)$ retroacts on A_j, and thus on μ_A and σ_A^2].[23] Moreover, the present framework could be used to analyze distributional consequences of *retirement decisions*.[24] Incorporating an endogenously determined transition to the group of pensioners triggers enormous entanglements, since the old-age dependency ratio R/E will not only be driven by demographic trends but will then also become a continuous function of economic (and fiscal) variables. Clearly, closed-form solutions will no longer be possible and one would have to resort to numerical simulations.

For consistency reasons, the micro level of the model is restricted to the subgroups of the population and the sources of income which play a direct role in a pay-as-you-go financed state pension scheme. This indeed captures the most important groups and sources for the personal distribution of income.[25] Nevertheless, it is possible to apply the analytical framework to an extended micro level including, e.g., *capital income*. Savings would enter the scene, introducing complex incentive mechanisms and inequality repercussions. It goes without saying

that this addition by far exceeds the descriptive scope of the present paper and constitutes much more than simply an extension.

Calender time is not explicitly entering the above model. In a way, the set-up corresponds to a cut through a two-generations OLG framework in some given period, subsequently conducting a comparative statics analysis of different stationary states. In this sense, the proposed model addresses short-run issues only. Introducing real time and embedding the model in an overlapping generations framework would permit a *dynamization* of the present distributional approach, potentially enabling a simultaneous investigation of both equality and efficiency issues of demographic change.

In which way should the key parameters of a pay-as-you-go financed state pension system be corrected to cope with the solvency problem induced by an aging population? Both groups, pensioners and workers, should play some part in bearing the demographic burden; however, how can society compromise on the distributional dilemma? One answer has been given by the German Pension Reform. Other responses are conceivable. In practice, most proposals are based on a conventional budgetary analysis – a widely used tool in the institutional public finance literature. Yet, this gives an incomplete insight into the trade-off between workers' taxes and retirees' benefits. Indeed, an aging population does not only affect the financial relations of the state pension scheme; it also changes the relative number of *votes* cast by workers and pensioners. This puts conventional conclusions drawn from pure budgetary analyses in a different light. If in a representative democracy the responsible policymakers aim at being reelected then fiscal adjustments of the state pension budget will depend on *political* factors determined by the *age composition* of the population. The political economy of social security is a fairly young area of economic research.[26] In a recent study, I have tried to design an exploratory framework which allows for a demographic impact on the politico-economic trade-off between the level of contribution payments and the level of pensions.[27] The division of the financial cake is shaped by an interplay of population aging, political power distribution, and institutional constraints. It would be interesting and tempting to analyze the intra- and intergenerational inequality effects of these mechanisms by linking this set-up to the distributional framework of the present paper. One could then also try to capture the disparity consequences of a simultaneous determination of c and p in a strategic pension game, thereby explicitly taking into account the possibility of a strike: as the dependency ratio surges, political power will shift from the working population to the older generation, leading to a typical free-rider situation where the old can outvote the young. However, the workers could break away from the generation contract if the old overdo their political pressures. Maybe the Shapley-Value concept of political power could be applied in this context; maybe a sufficiently rich structure of mutual disincentives would do.

Last but not least, the fiscal-demographic inequality relations exposed in this paper motivate a closer examination of the distribution of *lifetime* income. Such an extension of the model would be helpful in two ways: first, public pension reforms could be assessed on a theoretically superior incidence basis, and secondly, longitudinal data could be interpreted more consistently. Panel data scarcely exist today, but this situation could change in the near future.[28] Of course, implementing the lifetime approach requires making some stringent assumptions. Furthermore, for politico-economic reasons, it seems that the lifetime approach is not currently viable as a standard of distributional analysis.[29] Nevertheless,

for a theoretical comparison with the results of this paper it would be an interesting route to follow. In fact, preliminary findings, available from the author upon request, reveal that contrary to the widely held belief that the distribution of lifetime income (as opposed to the distribution of current income) remains largely unaffected by changes in the population age structure, the mechanism of the pension formula leads to demographic distortions also of lifetime inequality.

V. Conclusion

In recent years, public concern at questions of income distribution has increased considerably. In the political arena, such questions are typically (and due to the absence of adequate longitudinal data also by necessity) discussed on the basis of empirical cross-section information. Yet, cross-section data on income disparity are subject to distortions caused by the age composition of the population. Over the last two decades, the population age structure in most industrialized countries has undergone drastic changes: rapidly declining birth and death rates have led to rising proportions of the elderly. This in turn undermines the solvency of pay-as-you-go financed state pension schemes. Such schemes on their part again affect the distribution of income. This interaction between demographic change, old-age insurance, and personal income distribution is vitally influenced by public pension reform. The present study shows that the design of the pension formula decisively drives the relation between demographics and inequality. Until now, the interdependency between fiscal measures of old-age insurance reform and demographic inequality effects has been overlooked. Thus, the empirical cross-section evidence tends to be interpreted in a biased way – the distributional significance of population aging is missed.

Endnotes

[1] The empirical literature originated from Paglin (1975). A useful clarification of a number of controversial points encountered in that literature is provided by Mookherjee and Shorrocks (1982); cf. in addition the revealing paper by Cowell (1984) and the survey by Pestieau (1989). As to a theoretical treatment, see, e.g., v. Weizsäcker (1989).

[2] Note that (3) is based on a moment aggregation over *population subgroups*, which is not to be confounded with a moment calculation of the sum of correlated random variables as met, e.g., in an inequality decomposition by *income components*. See Theil (1967, Chap. 4.A), Shorrocks (1980, 1984), or Lam (1986).

[3] Compare, e.g., BMA (1989); Göseke and Bedau (1983); Koss (1984); BMI/Bundesregierung (1984); Statistisches Bundesamt (1986, 1993); VDR (1993). The demographic supposition $E>R$ does also not contradict recent population projections; see Koss (1984), United Nations (1985), and Hagemann and Nicoletti (1989).

[4] If average net earnings exceed the average pension and if the relative dispersion of net earnings is greater than the relative dispersion of retirement incomes, i.e., if $\mu_Y > \mu_P$ and $V_Y^2 > V_P^2$, then it follows that:

$$\frac{\sigma_Y^2}{\sigma_P^2} > \frac{\mu_Y^2}{\mu_P^2} > \frac{\mu_Y}{\mu_P} (>1) ,$$

and thus: $\mu_P \sigma_Y^2 > \mu_Y \sigma_P^2$.

[5] See endnote 4.

[6] Put another way, by continuity it is possible to choose a combination of a contribution/benefit rate adjustment such that $dV_{BB}^2/dx = 0$, i.e., the demographic inequality effects are balanced and inequality keeps constant.

[7] Questions of financing the public grant are not dealt with here. As it was expounded in the previous sections, this paper concentrates on the demographic incidence effects of the pension budget only. Other redistribution systems are ignored. Tax-based financing of federal subsidies, as seen, e.g., in $Y_j = (1-t)(1-c)A_j$ and $P_i = p\mu_Y L_i$ with $\mu_Y = (1-t)(1-c)\mu_A$, which took into account the deductibility of pension contributions as well as the exemption of retirement income from taxation, could be directly included in the model without affecting the subsequent results; a constant marginal tax rate t would have no influence on relative income disparity. Other tax structures could, of course, lead to additional inequality effects resulting from demographic change. Such aspects, however, are not part of the Pension Reform Act of 1992 and thus are not included in the present analysis of incidence.

[8] See VDR (1987, p. 36).

[9] The range of 5 percentage points arises from differing assumptions with respect to economic growth.

[10] See Dederer and Grintsch (1989).

[11] See BMA (1988, pp. 8–9). The new regulation of retirement age limits will not be explicitly dealt with in the sequel, although this reform element could also directly be integrated into the analysis. The aggregated net effect of raising the age limit is to damp down the demographic-induced increase in the dependency ratio R/E. This has no effect on the subsequent qualitative conclusions. Furthermore, the quantitative results (see Table 1) are as such unaffected; when interpreting the findings, it is only to be minded to proceed from a correspondingly moderated increase in R.

[12] The laws and regulations specific to the *transition period* [BMA (1988, p. 41)] are not specifically addressed in the sequel.

[13] This formulation corresponds in a one-to-one way to the 1992 Act. As will become evident later, it would be more convenient from an analytical perspective to specify the public grant in per worker terms: $G/E = \gamma c \mu_A$. It would then be possible to keep the dependency ratio (or x) as a variable throughout the text.

[14] In the language of the present model: $\mu_{P,BB}/\mu_{Y,BB} = constant$.

[15] This is due to the inclusion of the federal subsidy as specified in (12). See endnote 13.

[16] The reader should note that the incidence result (8) is independent of whether the included balanced budget rate c_{BB} stems from (6) or (11).

[17] See the citings in endnote 2. More recent data on personal income distribution in Germany are heavily restricted by the adoption of a new "Datenschutzgesetz", a law protecting the privacy of personal data.

[18] The squared coefficient of variation, employed in this study for illustrative purposes, is a member of the *Generalized Entropy family* of inequality measures and is thus decomposable in an intra- and an inter-group component of relative income dispersion; cf. Theil (1967, Chap. 4) and Shorrocks (1980). The corresponding decomposition is:

$$V^2 = V^2_{intra} + V^2_{inter} \; ; \quad V^2_{intra} = x\frac{\mu_Y^2}{\mu^2}V_Y^2 + (1-x)\frac{\mu_P^2}{\mu^2}V_P^2 \; , \quad V^2_{inter} = \frac{x(1-x)}{\mu^2}(\mu_Y - \mu_P)^2 \; .$$

[19] An interesting issue in this context is the possible impact of *immigration* on the old-age dependency ratio. Here, it may be worth investigating the quantitative inequality consequences of a short-run increase in E. On this and some other demographic and economic effects of immigration on the distribution of income see v. Weizsäcker (1994b).

[20] German Federal Ministries, e.g., base their empirical grasp of income inequality extensively on the squared coefficient of variation.

[21] See Bourguignon (1979), Cowell (1980), and Shorrocks (1980, 1984). See also Jenkins (1991).

[22] For an extended human capital approach, see, e.g., v. Weizsäcker (1994a).

[23] These interactions may even go one step further when considering the findings of the *endogenous fertility* literature [see Becker (1988) or Becker and Barro (1988)], rendering the age structure itself an economically determined variable.

[24] See, e.g., Stock and Wise (1990) for an instructive study of retirement behaviour.

[25] See Atkinson (1983, Chaps. 5, 6, 11).

[26] For a partial overview see Boadway and Wildasin (1989).

[27] See v. Weizsäcker (1990).

[28] See, e.g., the Michigan Survey of Income Dynamics in the US, or the Socioeconomic Panel in Germany.

[29] See Barthold (1993).

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