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Research and Development in the Growth Process

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This paper introduces into Schumpeterian growth theory an important element of heterogeneity in the structure of innovative activity—namely, the distinction between research and development. We construct a simple model of growth to investigate how the (steady-state) rate of growth affects and is affected by the relative mix between research and development. Although we assume for simplicity that the total supply of innovative activity is given it turns out that, with one important exception, the growth rate responds to most parameter changes in the same way as in previous models where growth was determined by the total amount of innovative activity. In particular, the level of research tends to covary positively with the rate of growth, even in the extreme case where the general knowledge that underlies long-run growth is created only by secondary innovations arising from the development process. The exception concerns the effects of competition on growth. Although simpler Schumpeterian growth models implied that increased competition would reduce growth by reducing the incentive to innovate, introducing the distinction between research and development implies that this effect is likely to be reversed.

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1. Introduction

One advantage that Schumpeterian growth models have over human-capital based models is their greater specificity concerning how knowledge is used, how it is generated, and how it creates losses as well as gains. But specificity often comes at a high cost in generality. The typical Schumpeterian model postulates just one kind of innovative activity, generating a very particular kind of knowledge. In fact, there are many kinds of innovative activity, generating many different kinds of knowledge. An aggregate theory that fails to distinguish between these different activities is potentially misleading if the distinctions matter. Whether growth will be enhanced by a subsidy to innovation, for example, might depend crucially on whether product or process innovations are subsidized or on whether basic or applied research is encouraged by the subsidy.

The purpose of this paper is to introduce into Schumpeterian growth theory an important element of heterogeneity in the structure of innovative activity—namely, the distinction between research and development. We construct a simple model of growth to investigate how the (steady-state) rate of growth affects and is affected by the relative *mix* between research and development, how that equilibrium mix varies with the parameters of the economy, how it compares with the efficient mix that maximizes the representative agent's lifetime utility, and how the results of simpler growth models are affected by taking this element of heterogeneity into account.

In our view, the main distinction between research and development is that they are aimed at generating different kinds of knowledge. Research produces *fundamental* knowledge, which by itself may not be useful but which opens up windows of opportunity, whereas the purpose of development is to generate *secondary* knowledge, which will allow those opportunities to be realized. In this respect the distinction is much the same as that between basic and applied research, between invention and innovation, or between innovation and diffusion. Thus research and development are complementary activities; in order to profit from the fundamental knowledge generated by research a firm must spend resources developing applications, while development by itself would be of no use if there were no fundamental ideas to be developed. We capture this distinction by supposing that each innovation resulting from research consists of a potential line of new products and that each innovation resulting from development consists of a workable plan for producing one of those products.

Now a case (see Rosenberg and Birdzell, 1986) can be made that most of the fundamental discoveries that led to what we now recognize as basic science were not the intended outcome of basic research but rather the (often serendipitous) outcome of narrowly focused problem-solving activities. This suggests that fundamental knowledge can be generated by development as well as by research. To accommodate this phenomenon we need to recognize a third kind of knowledge, which we call *general* knowledge—the common scientific, technological, and cultural heritage potentially available to everyone. In our model, both general and fundamental knowledge open up opportunities for future breakthroughs. But general knowledge can be used by everyone in the economy and cannot be appropriated, whereas fundamental knowledge can be used only to develop a particular line and is appropriable (otherwise research would not be freely undertaken). Although fundamental knowledge is created only be research, we assume that the growth of general knowledge is enhanced by both research and development.

Two relationships will jointly determine the steady-state rate of economic growth and the amount of research relative to development. The first is a *growth equation*, which governs the evolution of general knowledge over time, and thereby determines the steady-state growth rate as a function of the mix between research and development. The second is an *arbitrage* equation that results from the attempt by skilled workers to engage in the most profitable type of innovative activity, either research or development, depending on the growth rate.

The first main result of the paper is that when the sum of resources available for both research and development is exogenously fixed, the rate of growth tends to be positively correlated with the level of research and hence negatively correlated with the level of development. Almost all parameter changes affect growth and research in the same direction because a steady state can occur only at a point where they are positively related in both the growth equation and the arbitrage equation. This is true even in the extreme case where general knowledge can be produced only by *secondary* innovations arising from the *development* process.[1] For the more forward-looking nature of research (research is aimed at capturing rents from future lines, whereas development is concerned only with present lines) and the positive net rate of interest[2] imply that even the flow of secondary innovations will be affected more, on the margin, by research than by development.

Moreover, we find that the level of research is less than optimal. Thus a (small) subsidy that switched resources out of development and into research would increase not only the rate of growth but also the expected present value of output. An aggregate approach that ignored the distinction between research and development could easily be misleading, for although an increase in the overall level of innovation will raise growth and welfare, the opposite effect could result from an innovation subsidy that happened to be targeted at development and induced substitution away from research.

Another striking result is that any parameter change that raises the productivity of the R&D process will shift human capital out of development and into research. This is even true of a parameter change whose only direct effect is to raise the productivity of *development*. This result reflects to some extent the more forward-looking nature of research, which means that researchers are better able to capitalize the benefits of increased growth than are developers. It also reflects the complementarity between research and development, and the way rents are shared between the two types of innovators; an increase in the productivity of development will generate increased rents for both.

As a first step in modeling the rent-sharing arrangements between researchers and developers we assume that there is perfect competition in the market for developers, and no problems of enforcing contracts. However, our framework could be used to introduce more complex contractual considerations, such as those studied by Aghion and Tirole (1994), as we point out in Section 3.1 below. In this way we see our paper as opening the door for studying organizational aspects of R&D in endogenous growth theory.

The second main result of the paper is that the level of research and the rate of growth can be increased if developers become more *adaptable*—that is, if the rate at which they are able to switch from developing old lines to developing new ones increases. This result supports Lucas's (1993) claim to the effect that the key to success of some newly industrialized countries is their ability to move skilled workers quickly between sectors. When we endogenize this adaptability parameter, we also find that the same result implies a *positive effect of competition on growth.* That is, an increase in the substitutability between new and old product lines, which implies an increase in competitiveness between them, will induce developers to leave old lines more rapidly, with the possible effect of inducing a higher level of research. *Contrary to previous Schumpeterian models this implies that increased competition may lead to faster growth.*

The paper is organized as follows: Section 2 lays out a simple growth model based on the division between research and development and derives the growth equation. Section 3 determines the equilibrium payoffs to research and development and derives the arbitrage equation and the main comparative-statics results. Section 4 lays out the efficiency analysis. Section 5 endogenizes the adaptability parameter and examines the effects of competition on growth. Section 6 suggests some applications of our analysis to patent policy and to Schumpeterian waves and briefly reviews recent growth literature touching on the distinction between fundamental and secondary knowledge.

2. The Basic Model

2.1. *Basic Assumptions*

We consider an infinite-horizon continuous time model with constant masses H of skilled workers and L of laborers, each of whom lives forever. Each skilled worker can choose whether to engage in research or development. There is a single final output, which can be used only for consumption, and a continuum of intermediate goods, which constitute the only inputs into producing the final good. All individuals have intertemporally additive riskneutral preferences over consumption, with a constant rate of time preference ρ. There is no disutility of work.

The production (or "input-output") matrix can be described as follows:

1. Final output is produced with a continuum of intermediate goods of different vintages. Intermediate goods of more recent vintage are better than older ones because they embody a higher level of *general knowledge*. More specifically, let A_r denote the state of general knowledge at date τ, let S_{tr} denote the number of different intermediate goods of vintage τ for which plans have been developed by time $t > \tau$, and let $\ell_{t,\tau}$ denote the labor input used in the production of each such good. Each intermediate good is produced by labor alone under constant returns to scale, so by an appropriate normalization $\ell_{t,\tau}$ also equals the output of each intermediate good of vintage τ at date t. Then aggregate final output is just:

$$Y_t = \int_{-\infty}^{t} S_{t,\tau} A_\tau F(\ell_{t,\tau}) d\tau = \int_{-\infty}^{t} Y_{t,\tau} d\tau, \tag{1}$$

   where F is increasing and concave (for example, $F(\ell) = \ell^\alpha, 0 < \alpha < 1$) and $Y_{t,\tau} = S_{t,\tau} A_\tau F(\ell_{t,\tau})$ denotes all the aggregate final output produced using intermediate goods of vintage τ.

2. The vintage of an intermediate good is not the date at which its plan was developed but the date of invention of the product line from which the plan was developed. New intermediate goods (or "plans") of vintage τ are invented by skilled workers who have chosen to develop a product line of that vintage, with the assistance of the researcher who discovered the line. *Secondary* innovations (plans for new intermediate goods) arrive to each developer at a Poisson arrival rate λ_τ^d.[3]

3. *Fundamental* innovations (new product lines) are made by skilled workers doing research, making use of general knowledge. Let H_τ^r denote the mass of researchers at date τ. The flow of new lines at that date is taken to be equal to $\lambda^r \cdot H_\tau^r$, where λ^r is each researcher's Poisson arrival rate, an exogenous parameter.

4. Finally, new general knowledge is created by research and development throughout the entire economy, using the existing stock of general knowledge.[4]

Before filling in all the details of the model we can use Figure 1 to give an idea of the structure of output in the economy at any given date t. Because nothing can be produced

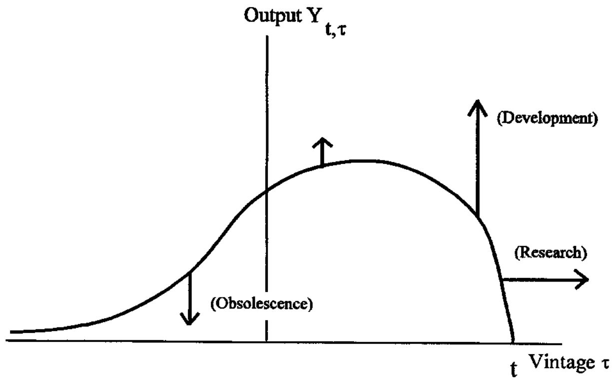

Figure 1. The profile of output across lines of different vintages, at date t

on lines that have not yet been invented, $Y_{t,\tau} = 0$ for all $\tau \geq t$. Because older vintages are less efficient, output of very old lines will be very low. Hence the profile will tend to have the wave form depicted by Figure 1. Over time, the profile will shift to the right, as research opens up new product lines. Near the leading edge the profile will be shifting up, as development creates new goods. But far back from the leading edge the profile will be shifting down, as the rise in real wages associated with growth draws labor from old product lines[5], and the reallocation of old developers into new lines reduces the rate at which new goods are being introduced on old lines. However, there will always be some development taking place no matter how old the line.[6]

2.2. *Determination of Aggregate Output Flows Within and Across Lines*

In order to simplify the analysis we assume provisionally that the rate at which developers can move from developing old lines to new lines is fixed exogenously. That is, once a skilled worker chooses to develop a line, he cannot do anything else until he is exogenously upgraded. Upgrading arrives to each developer at the fixed Poisson rate σ, which is our measure of adaptability. When he is allowed to upgrade, we assume he always chooses to go either into research or into developing a line of the most recent vintage. (In Section 5 below we relax this restriction and allow skilled workers to move instanteously and costlessly between research and development of each line.) Let h_τ^d denote the flow of skilled workers going into development on lines of the most recent vintage at date τ. By time t the number of developers still working on lines of vintage τ will be $h_\tau^d e^{-\sigma(t-\tau)}$. As assumed above, new plans are discovered by each developer with a Poisson arrival rate λ_τ^d. Thus the flow of new plans on lines of vintage τ at date t will be $\lambda_\tau^d h_\tau^d e^{-\sigma(t-\tau)}$, and the number of different goods actually being produced on these lines will be

$$S_{t,\tau} = \int_\tau^t \lambda_\tau^d h_\tau^d e^{-\sigma(s-\tau)} ds = \lambda_\tau^d h_\tau^d \varphi(t-\tau), \tag{2}$$

where $\varphi(t-\tau) \equiv (1 - e^{-\sigma(t-\tau)})/\sigma$.

Let w_t denote labor's real wage rate at date t. The producer of any intermediate good has a monopoly in that good and sells to a competitive final output sector in which the price of the good is its marginal product $A_\tau F'(\ell_{t,\tau})$. Thus the firm's employment $\ell_{t,\tau}$ and profit $\pi_{t,\tau}$ will be

$$\begin{cases} \ell_{t,\tau} = \arg\max\{A_\tau F'(\ell)\ell - w_t\ell\} = \bar{\ell}(w_t/A_\tau), & \bar{\ell}' < 0 \\ \pi_{t,\tau} = \max\{A_\tau F'(\ell)\ell - w_t\ell = A_\tau\bar{\pi}(w_t/A_\tau), & \bar{\pi}' < 0. \end{cases} \tag{M}$$

The corresponding flow of final output will be: $A_\tau\bar{y}(w_t/A_\tau)$, $\bar{y}' < 0$; the flow of aggregate output in the economy will be the sum within and then across all vintages $\tau \leq t$:

$$Y_t = \int_{-\infty}^{t} S_{t,\tau} A_\tau \bar{y}(w_t/A_\tau) d\tau; \tag{3}$$

and the real wage w_t will be determined by the market-clearing condition for labor:

$$L = \int_{-\infty}^{t} S_{t,\tau} \bar{\ell}(w_t/A_\tau) d\tau. \tag{4}$$

2.3. *The Growth Equation*

In accordance with the discussion of the previous section, we assume that the growth of general knowledge is a function of the current flow of innovations of both types, and also of the accumulated *stock* of general knowledge, which embodies all previous innovations.

2.3.1. A Benchmark Case A limiting example of the above occurs when the log of general knowledge is equal to the existing stock of *fundamental* innovations—that is, to the number of existing product lines. Then, the growth of general knowledge is governed by the equation (see also Figure 2)

$$\dot{A}_t = \lambda^r H_t^r \cdot A_t.$$

This special case would arise, for example, if there were an endless list of potential lines, ordered by their productivity parameters Π, such that the log of Π on each line was proportional to the number of that line, and research consisted of moving monotonically through the list, discovering each one in turn. In this special case the state of general knowledge would be represented by the Π of the most recent line, and its rate of growth would be proportional to the speed $\lambda^r H_t^r$ with which research was moving through the list.

Thus, in steady-state where $H_t^r \equiv H^r$, the growth of general knowledge in the benchmark case will simply be governed by the equation

$$\dot{A}_t = \lambda^r \cdot H^r \cdot A_t, \tag{G1}$$

which is quite attractive both because of its simplicity and also because of its affinity with more conventional growth models.

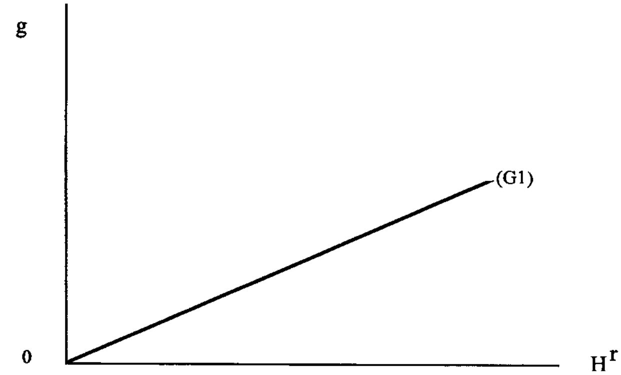

Figure 2. Growth curve in the benchmark case

2.3.2. The General Case More generally, we can suppose that both fundamental and secondary innovations produce general knowledge and that they both generate ideas that allow future researchers to create better lines. Specifically, assume that the log of A_t is proportional to a weighted average of the stock of *fundamental* innovations and the stock of *secondary* innovations[7], with respective weights β and $(1-\beta)$. The benchmark case is just the special case where $\beta = 1$. Assume that the arrival rate of each developer's innovations on a line is given by the function $\lambda_\tau^d = \lambda^d \cdot (\eta_\tau)^{-\nu}$, where $0 < \nu < 1$, $\eta_\tau = \frac{h_\tau^d}{\lambda^r H_\tau^r}$ is the number of developers initially hired on the line[8], and λ^d is an exogenous parameter of the development technology. Appendix B shows that the steady-state growth of general knowledge is governed by the function

$$\begin{aligned} \dot{A}_t/A_t &= G(H^r; \beta, \lambda^r, \lambda^d, \nu, \sigma, H) \\ &\equiv \beta\lambda^r H^r + (1-\beta)\lambda^d(\lambda^r)^\nu (H^r)^\nu (H - H^r)^{1-\nu}\sigma^{-\nu}, \end{aligned} \tag{G2}$$

which satisfies

1. $G = 0$ when $H^r = 0$ and $G = \beta\lambda^r H$ where $H^r = H$,
2. G is strictly concave in H^r if $\beta < 1$,
3. G is increasing in λ^r, λ^d and H, and
4. G achieves a maximum g^* at an intermediate level of research $H^* \in (\nu H, H)$ if $0 < \beta < 1$.

In order to rule out infinite wealth, assume that $g^* < p$ (see Figure 3).

2.3.3. Steady-State Growth Since the stock of developers changes over time according to the relocation equation $\dot{H}_t^d = h_t^d - \sigma H_t^d = h_t^d - \sigma(H - H_t^r)$, in steady state the flow

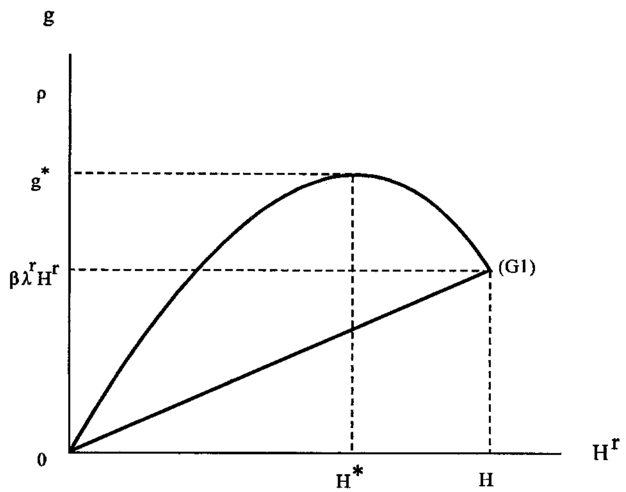

Figure 3. Growth curve in the general case

of skilled workers into developing new lines will be constant and equal to

$$h^d = \sigma(H - H^r). \tag{R}$$

Equations (1) through (4), (G1) or (G2), and (R) imply that the growth rates of output and the wage rate will equal that of *general* knowledge—that is,

$$g = \lambda^r H^r \tag{G1}$$

in the *benchmark* case and

$$g = G(H^r; \beta, \lambda^r, \lambda^d, \nu, \sigma, H) \tag{G2}$$

in the *general* case.

3. Arbitrage and Comparative Statics

3.1. Rent-Sharing Between Researchers and Developers

For both kinds of innovative activity to coexist in a steady state, skilled workers who have just been upgraded must be indifferent between research and development on a new line. To specify the arbitrage equation that reflects this indifference, we must describe how each kind of innovative activity is compensated. Each plan (to produce a new intermediate good) on a line is implemented by a company formed by the researcher who discovered the line and the developer who found the plan. When the developer first begins work on the line, it is agreed that a certain fraction of κ of each company's profits will go to the researcher, with $1 - \kappa$ going to the developer. At each date t there will be $\lambda^r H^r$ researchers with new lines of vintage t competing for developers, using κ as their strategic variable. As we shall see, this competition will define a unique equilibrium value of κ.

Let W_t denote the capitalized value of rents generated on each product line opened up at date t—that is, the present discounted sum of profits generated by all the new intermediate goods that will be produced on the line. Since there are $\lambda^r H^r$ new lines per unit of time, the steady-state value of W_t is $W_t = e^{gt} W$, where, under the normalization $A_0 = 1$,

$$W = \int_0^\infty e^{-\rho s} S_{s,0} \bar{\pi}(w_0 e^{gs}) ds / \lambda^r H^r. \tag{5}$$

As in the previous section, $S_{s,0}$ denotes the number of different intermediate goods being produced on all lines of vintage 0 at date $s \geq 0$.

Now, assume as in Section 2.3 above that the arrival rate of each developer's innovations on a line is given by $\lambda_\tau^d = \lambda^d \cdot (\eta_\tau)^{-\nu}$, $0 < \nu < 1$, where $\eta_\tau = h_\tau^d / \lambda^r H_\tau^r$ is the initial number of developers on each line of vintage τ. From equation (2) and the steady-state condition $\eta_\tau = \eta$ for all $\tau \geq 0$, we have

$$S_{s,0}/\lambda^r H^r = (\lambda^d \cdot \eta^{-\nu}) \cdot h_0^d \cdot \varphi(s)/\lambda^r H^r = \lambda^d \cdot \eta^{1-\nu} \cdot \varphi(s).$$

Substituting this into the above equation (5), one can reexpress W as

$$\begin{aligned} W &= \int_0^\infty e^{-\rho s} \bar{\pi}(w_0 e^{gs}) \lambda^d \eta^{1-\nu} \varphi(s) ds \\ &= \eta^{1-\nu} \times \text{ constant.} \end{aligned} \tag{6}$$

The equilibrium share κ is determined by the condition that under perfect competition each developer is paid his marginal contribution to the private value of the line W_t. Let $V_t^d = V^d e^{gt}$ denote the steady-state expected present value of the income a developer will receive from developing a product line of vintage t. From equation (6),

$$V^d = \partial W / \partial \eta = (1 - \nu) W / \eta. \tag{7}$$

On the other hand, since $(1 - \kappa)$ is the fraction of W to be shared between η developers,

$$\eta V^d = (1 - \kappa) W. \tag{8}$$

From (7) and (8),

$$\kappa = \nu. \tag{9}$$

Remarks.

1. In particular if $\nu = 0$—that is, if there are constant returns to development on a product line—Bertrand competition for developers among the researchers who discovered the newest lines will drive the equilibrium share κ of the researchers down to zero. In other words, this is a case where no research will ever take place in a steady-state equilibrium, and thus where the growth process, if any, will be driven entirely by horizontal product development on the initial lines. This is why in order for both research and development to coexist in a steady state, we must assume $\nu > 0$.

2. The above remark would cease to hold if researchers and developers were involved in specific contractual relationships whereby a developer's private investment combined with a (nondescribable) training effort by the researcher jointly generate new intermediate plans on the line discovered by the researchers. The share of total rents W accruing to the researcher would now depend upon the allocation of *property rights* over intermediate innovations between the researcher and the developer, and unless all the ownership rights are being allocated to the developer, we will have $\kappa \neq 0$ even when the development technology λ_τ^d exhibits constant returns to scale ($\nu = 0$) (see Aghion and Tirole, 1994).[9]

3.2. *The Arbitrage Equation*

Now that we have characterized the equilibrium rent sharing between researchers and developers, we can derive the indifference condition between research and development. Let $V_t^r = V^r e^{gt}$ denote the expected present value of the income that a researcher will receive until his alternative choice as a developer is upgraded to a new line. That is, V_t^r is the value of a claim to all the researchers' rents from fundamental innovations made over the time period (of stochastic length) during which he could have been developing a line of vintage t. Since a newly upgraded skilled worker can freely choose either activity, a steady state with both research and development requires

$$V^r = V^d. \tag{10}$$

In a steady state the value V_t^r grows at the rate g and capitalizes flow payoffs (per unit of time) equal to the flow probability of discovering a new line λ^r times the researcher's share of a new product line κW_t. Since upgrading occurs at Poisson rate σ, the Bellman equation defining the steady-state value of V^r is

$$\rho \cdot V^r = \lambda^r \cdot \kappa \cdot W - \sigma \cdot V^r + g \cdot V^r. \tag{11}$$

Equations (8) through (11) and the steady-state condition (R) yield the arbitrage equation

$$\rho + \sigma - g = \frac{\nu}{1-\nu}\sigma\frac{(H - H^r)}{H^r}. \tag{A}$$

Note that the details of the total present value W of a product line, and in particular the way W depends upon the time sequencing of the development process, do not enter the arbitrage equation (A). This reflects the complementarity of research and development. Anything that raises the *total* payoff W to a line will generate extra revenues to *both* researchers and developers, without affecting the relative profitability of either activity. This means that the model could easily be generalized to include a wide variety of development processes, involving learning, product improvement, and spillovers among developers on a line, or to apply to a much broader dichotomous distinction than research and development—for example, basic and applied research, or innovation and diffusion.

According to (A) an increase in the growth rate will result in a larger equilibrium level or research. This positive effect of growth on research is clearly a reflection of the more forward-looking nature of research as compared with development.

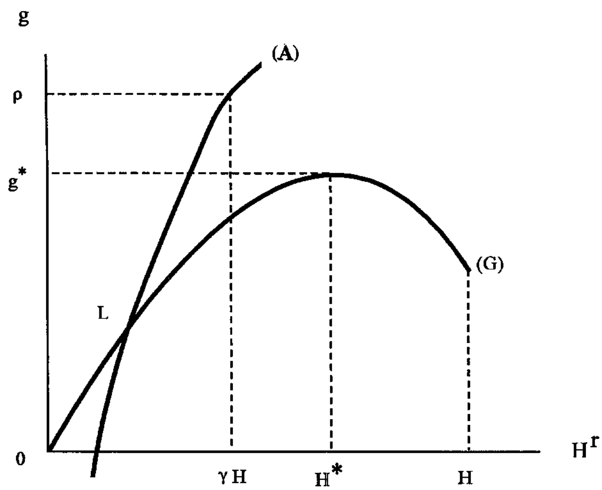

Figure 4. The equilibrium L always occurs on the rising part of the growth curve (G), where it is cut by (A) from below.

3.3. Comparative Statics of Steady-State Growth

We now examine comparative statics in the *general case* where both fundamental and secondary innovations create general knowledge. The steady-state values of g and H^r are jointly determined by the arbitrage equation

$$\rho + \sigma - g = \frac{\nu}{1-\nu}\sigma\frac{(H - H^r)}{H^r} \tag{A}$$

and the growth equation

$$g \equiv G(H^r; \beta, \lambda^r, \lambda^d, \nu, \sigma, H). \tag{G2}$$

The two curves in Figure 4 depict these two equations.

There might exist multiple steady-state equilibria, with (G) and (A) intersecting more than once over the range where the growth curve is increasing. We will restrict attention, however, to cases in which there is a unique steady-state equilibrium. Then it is clear from examination of Figure 4 that this must be an equilibrium in which the arbitrage curve cuts the growth curve from below.[10] Furthermore, our assumption that $g^* < \rho$ ensures that the intersection point (L in Figure 4) lies to the left of the maximal point[11] (H^*, g^*).

Proposition 1: *In the general case defined by (G2), the rate of growth g and the level of research H^r are both (1) increasing in λ^r, λ^d, and H and decreasing in ρ and (2) increasing in σ and ν for β close enough to 1.*

Proof: All of these can be established graphically, using Figure 4. Increasing λ^r or λ^d shifts the (G) curve up without affecting (A). Increasing H shifts the (G) curve up and the

(A) curve to the right. Increase ρ shifts the (A) curve to the left without affecting (G). To establish part (2) note that increasing σ or ν shifts the (A) curve to the right, whereas in the limiting case where $\beta = 1$ (the benchmark case), neither σ nor ν affects (G); thus the results of part (2) hold for $\beta = 1$, and by continuity they hold for β close enough to 1. ∎

Thus the steady-state growth rate responds to parameters changes much as in conventional Schumpeterian models, even though the overall level of innovative activity, which drives growth in conventional models, has been assumed constant. So, for example, an increase in the rate of discount ρ will reduce both research and the growth rate, as in conventional models.

A remarkable feature of these results is that, even in the case where general knowledge is assumed to be created only by secondary innovations arising from the *development* process, the growth rate almost always covaries positively with the level of *research*. The reason for the positive covariance between growth and research is first, the more forward-looking nature of research compared with development (research is aimed at capturing rents from future lines whereas development is concerned only with present lines); and second the *positive* net rate of interest $(\rho - g)$, which implies that research must have a larger marginal product than development in generating either kind of innovation.[12] Therefore, a marginal increase in research at the expense of development will raise the growth rate.

The fact that an increase in λ^d should affect research positively is perhaps surprising. It occurs because λ^d does not impact directly on the arbitrage equation. This property reflects the *complementarity* of research and development; an increase in λ^d enhances the total value W to be shared by researchers and developers, without affecting the relative profitability of either kind of innovation activity. Instead, the impact of λ^d is entirely on the growth equation, where it tends to raise the growth rate. Since, as we have seen, research is more forward looking than development, the prospective increase in growth draws resources out of development and into research.

Also remarkable is the positive effect (when β is close enough to 1) of the upgrading rate σ on research and growth. This effect can be explained as follows: Holding the total supply of skilled workers H constant, an increase in σ implies an increase in the initial flow of developers into newly discovered lines. Although it also increases the speed at which current lines are being depleted of their developers, time discounting implies that the former effect dominates the latter; that is, a higher σ increases the value of being a researcher relative to that of being a developer. Hence the positive effect of the upgrading rate σ on research through the arbitrage equation. Since the growth curve is upward sloping at the steady-state point this will raise growth. As long as β is close enough to unity (we are near the benchmark case), the growth curve will not shift enough to reverse this effect.

This is the "Lucas effect" referred to in the introduction above, whereby adaptability increases growth. However, contrary to Lucas (1993), a higher mobility of developers across lines enhances growth not because it increases aggregate learning by doing but rather because it increases the steady-state mass of researchers. As we show in Section 5 below, the same effect translates into a positive effect of competition on growth when we endogenize σ.

4. Efficiency

To address the efficiency question, we ask how the steady-state values of research and growth under laissez faire compare with those in an efficient steady state; that is, in the steady state of the social problem of maximizing the present value of output, $\int_0^\infty e^{-\rho t} Y_t dt$. To make the problem tractable, we limit attention to the special case in which the production function F is Cobb-Douglas: $F(\ell) \equiv \ell^\alpha, 0 < \alpha < 1$. Equation (A4) of Appendix A and equation (1) above imply that in this case aggregate output is

$$Y_t = L^\alpha \left[\int_{-\infty}^{t} \lambda_\tau^d h_\tau^d \varphi(t-\tau) A_\tau^{1/(1-\alpha)} d\tau \right]^{1-\alpha}.$$

From this, the fact that $\lambda_\tau^d \equiv \lambda^d (h_\tau^d)^{-\nu} (\lambda^r H_\tau^r)^\nu$, and the expression derived in Appendix B for the flow of secondary innovations, the social problem is to choose a path $\{h_t^d, H_t^r, A_t\}_0^\infty$ so as to

$$\begin{cases} \text{Max } \int_0^\infty e^{-\rho t} L^\alpha \left[\int_{-\infty}^t \lambda^d (h_\tau^d)^{1-\nu} (\lambda^r H_\tau^R)^\nu \varphi(t-\tau) A_\tau^{1/(1-\alpha)} d\tau \right]^{1-\alpha} dt \\ \text{subject to: } \dot{A}_t = A_t \cdot \left(\beta \lambda^r H_t^r + (1-\beta) \int_{-\infty}^t \lambda^d \cdot (\lambda^r H_t^r)^\nu \cdot (h_t^d)^{1-\nu} e^{-\sigma(t-r)} d\tau\right) \\ \text{and: } \dot{H}_t^r = \sigma(H - H_t^r) - h_t^d \end{cases} \tag{12}$$

with initial conditions $\{h_t^d, H_t^r, A_t\}_{-\infty}^0$. We show in Appendix C that a steady state to the solution of this problem must satisfy the social arbitrage equation

$$\rho + \sigma - g = \frac{\nu}{1-\nu} \sigma \frac{(H - H^r)}{H^r} + \beta \cdot SP, \tag{S}$$

where SP represents a spillover term. The efficient steady state occurs where (S) and (G) are both satisfied, as shown by the point E in Figure 5.

The social arbitrage equation (S) and the private arbitrage equation (A) are identical except for the spillover term $SP > 0$. It follows immediately that the graph of (S) in Figure 5 lies everywhere to the right of the private arbitrage curve (A), except when $\beta = 0$. By inspection of Figure 5 we have

Proposition 2: *The steady-state level of research under laissez faire is less than efficient, except in the limiting case ($\beta = 0$) where only secondary innovations contribute to the creation of general knowledge.*

Propositions 1 and 2 together show that a marginal subsidy that induced a movement of skilled workers out of development and into research would raise not only growth but also the present value of consumption.

To understand why the equilibrium always involves too little research, not first that the production of general knowledge is a purely external benefit of research and development. Everyone takes the time path of general knowledge as given, even though their collective innovative activities are what make it grow. Furthermore, in the Cobb-Douglas case of this section, this externality is the only reason for the equilibrium to be nonoptimal. (The

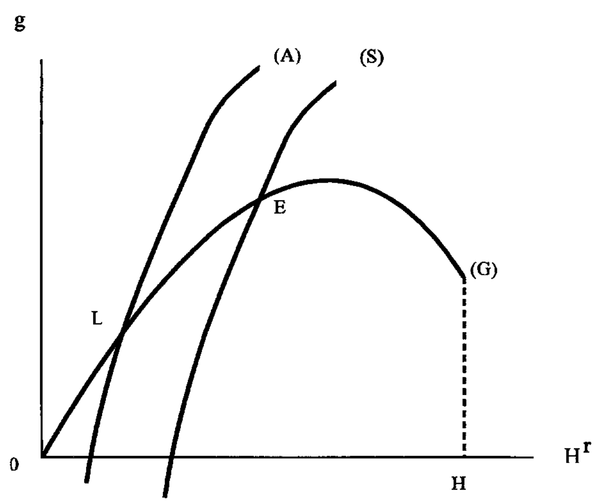

Figure 5. Welfare Comparison

noncompetitive behavior of intermediate-goods producers does not distort because there is an exogenously fixed labor supply, whose allocation across monopolistically competitive sectors is efficient when the elasticity of demand for output is the same in all sectors.)

In general, both research and development create this external benefit because, as noted above in Section 3.3, they both combine to produce secondary innovations. Furthermore, the way research and development combine to produce secondary innovations is the same as the way they combine to produce private rents; that is, according to a Cobb-Douglas function[13] with coefficients ν and $1 - \nu$. Thus in the extreme case ($\beta = 0$) where *only* secondary innovations create general knowledge, the intertemporal maximization of private rents involves the same allocation between research and development as does the intertemporal maximization of external benefits, and the equilibrium fortuitously generates just the right amount of research. In the opposite extreme case, however, of $\beta = 1$ (the benchmark case), only research produces the external benefit, so the equilibrium generates too little research. The general case is thus an average of one extreme where there is just enough and another where there is too little.

5. Perfect Adaptability and Competition

In this section we relax the assumption that developers must wait for an exogenous upgrading before being able to move to research or to developing a line of the most recent vintage. Suppose instead that they are free at each instant of time to engage in research or to do development on any line of their choosing. To keep the model tractable we restrict attention to the Cobb-Douglas case of the preceding section. It turns out that an analogous arbitrage equation results because developers will again relocate at a constant rate in the steady-state equilibrium, except that the rate of upgrading is now endogenously chosen instead of being imposed. The positive results are identical to those obtained under an exogenous σ, except that the Cobb-Douglas parameter α now matters for positive results, because it affects the endogenous upgrading rate.

Let $W_{t,\tau}$ denote the value at date t of a plan of vintage τ: $\int_t^\infty e^{-\rho(s-t)}\pi_{s,\tau}ds$, where $\pi_{s,\tau}$ is the flow of profits according to the decision problem (M) above. In the Cobb-Douglas case it is easily shown that $\pi_{s,\tau} = \delta A_\tau^{1/(1-\alpha)} w_s^{\alpha/(\alpha-1)}$, where δ is a positive constant. Since A_τ grows at the constant rate g in a steady state, therefore

$$W_{t,\tau} = W_{t,t}e^{-g(t-\tau)/(1-\alpha)}.$$

Since all skilled workers are mobile across all innovative activities, they must all earn the same expected income x_t at date t. In particular, a researcher who has a line of vintage τ at date t will have to pay x_t to each developer he employs at that date. Thus he will choose $\eta_{t,r}$ so as to maximize his flow of new development royalties:

$$Max\left\{\lambda^d \eta_{t,\tau}^{1-\nu} W_{t,t}e^{-g(t-\tau)/(1-\alpha)} - x_t\eta_{t,\tau}\right\}. \tag{13}$$

(Recall that each developer's arrival rate is $\lambda^d\eta_{t,\tau}^{-\nu}$). The solution to this maximization problem is

$$\eta_{t,\tau} = \left[x_t/(1-\nu)\lambda^d W_{t,t}e^{-g(t-\tau)/(1-\alpha)}\right]^{-1/\nu} = \eta_{t,t}e^{-g(t-\tau)/\nu(1-\alpha)}. \tag{14}$$

This shows that the unique candidate for an endogenous steady-state relocation rate is $\sigma = g/[\nu(1-\alpha)]$.

It turns out that this endogenous rate satisfies the same relocation equation (R) as before:

$$\eta_{t,t} = \eta - \sigma \cdot \frac{H-H^r}{\lambda^r H^r} = \frac{g}{\nu(1-\alpha)} \cdot \frac{H-H^r}{\lambda^r H^r}. \tag{R$'$}$$

This follows straightforwardly from the equilibrium price x_t equating the supply and demand for developers. The supply is $H - H^r$, and since the number of lines of each vintage is $\lambda^r H^r$, the demand is

$$\int_{-\infty}^t \lambda^r H^r \eta_{t,\tau}d\tau = \lambda^r H^r \int_{-\infty}^t \eta_{t,t}e^{-g(t-\tau)/\nu(1-\alpha)}d\tau.$$

Equating these two yields the above equation (R′).

Using (13), (14), and (R′) we obtain the same arbitrage equation as before, but with σ being replaced by the endogenous relocation rate $\frac{g}{\nu(1-\alpha)}$—that is[14],

$$\rho + [g/\nu(1-\alpha)] - g = \frac{\nu}{1-\nu}[g/\nu(1-\alpha)]\frac{(H-H^r)}{H^r}. \tag{A$'$}$$

Note that the modified arbitrage equation is identical to the original one (A) except that the upgrading rate σ in (A) has been replaced by the term $[g/\nu(1-\alpha)]$ in (A′). The interpretation of this replacement is simply that, as equation (14) shows, when skilled workers are perfectly mobile they will choose to behave as if they faced an exogenous upgrading rate of $g/\nu(1-\alpha)$.

A steady-state equilibrium occurs when the growth equation (G) and the modified arbitrage equation (A′) are both satisfied. It is straightforward to verify that the curve representing (A′) in Figure 4 would be upward sloping and would be affected by parameter changes in exactly the same direction as is the curve representing (A), except that now the Cobb-Douglas parameter α will shift it to the right because it has a direct effect on the endogenous upgrading rate $g/\nu(a-\alpha)$, whereas neither α nor any parameter of the general production function F had an effect when the upgrading rate was exogenous.

Thus the only effect that endogenizing the upgrading rate has on the comparative-statics results of the model is to add an effect of α. In particular, since an increase in α has the effect of increasing the upgrading rate $[g/\nu(1-\alpha)]$, it will work through the "Lucas effect" described above to shift the modified arbitrage curve to the right, resulting in more research, and more growth, at least when β is close enough to 1.

What is remarkable about this result is that α can be constructed as a direct measure of the degree of competition facing a potential innovator trying to take advantage of his innovation. That is, as explained in Aghion and Howitt (1992), it is an inverse measure of the degree of market power enjoyed by each of the monopolistic producers of intermediate goods. In particular, as α increases monotonically to unity, the equilibrium flow of profits enjoyed by each producer goes monotonically to zero. In other Schumpeterian models this increase in competition would have the effect of discouraging research and lowering the growth rate.[15] In this model, however, it stimulates research by drawing innovators away from the alternative activity of development. That is, the effect on the overall level of development of innovators choosing to remain for a shorter time on old lines outweighs the effect created by more of them going into lines of the most recent vintage, with the result that there will be fewer people in development, and hence more in research, in the new steady state. Of course, if there were a variable sum of research and development, then we would expect to find both our effect and the traditional Schumpeterian effect at work.

6. Concluding Remarks

In this paper we have tried to analyze the interaction between growth and the mix between research and development, taking into account what we see as the most salient aspects of that distinction. We believe that this analysis opens up a number of interesting avenues for future research on endogenous growth and its microeconomic foundations. Let us just mention two potential applications of our analysis, before concluding with a brief review of related literature.

6.1. Intellectual Property Rights

Compared with previous endogenous growth models, the framework provides a new rationale for the use of patents, in addition to their role in protecting innovators' rents against potential imitators: namely, the fact that patents can help achieve a better coordination between research and development through imposing more efficient rent sharing. For example, going back to a previous remark in Section 3 above, if the development technology $\lambda^d(\eta)$

exhibits constant returns to scale ($\nu = 0$) and both researchers and developers contribute a specific investment to the development of new intermediate plans, then it is obviously an efficient policy for a social planner to grant a minimum property right to the researchers. The allocation of property rights on intermediate innovations will effect the equilibrium allocation of innovative activity between research and development and help adjust this equilibrium toward the efficient mix.[16] The scope for such patent provisions remains when $\nu > 0$ because the intertemporal spillover effect pointed out in the above efficiency section is positive, making the laissez-faire level of research too small.

A natural issue raised by the above discussion is the fact that, in practice, governments, or any third party in general, cannot easily distinguish between research and development. It often takes years before figuring out the true nature of a discovery—that is, how much of a breakthrough this discovery represents. This, in turn, limits the extent to which the patent legislation can itself discriminate between research and development. In other words, it may be quite inefficient for a government to rely entirely on patent policy as the instrument for intervention in the R&D sector for the double objective of inducing an efficient total amount of innovative activity H and of efficiently allocating these activities between research and development. A first way around this problem is to make the patent policy sufficiently flexible (such as through being contingent on the breadth of innovations or on the organization of R&D, for example distinguishing between innovations generated by independent laboratories and innovations generated by integrated research units). A second, somehow more natural approach is for the government to combine patent policy with direct subsidies to institutions like universities or research laboratories that are most likely to perform fundamental research activities.

An interesting avenue for future research would be to open the black box of contracting process between research and development (for example, following the recent work of Green and Schotchmer, 1995) and then investigate the implications for growth of various patenting and/or subsidy devices.

6.2. Toward More Radical Fundamental Innovations

There is one important aspect of the fundamental/secondary distinction, however, which the above analysis does not capture; that is, the radical nature of fundamental innovations. In our analysis there is a steady stream of knowledge of all types coming on line, which pushes out the frontier of knowledge in a gradual and continuous fashion. Thus the model seems to come down on the side of Kuznets (1940), who criticized Schumpeter for his emphasis on large economywide innovations, and against the recent Schumpeterian models (including our own, 1992) that emphasize such radical innovations.

The only reason for assuming that innovations coming from research are gradual in our model is simplicity. It allows us to avoid the changes that would take place during the intervals between the discovery of new lines in the incentive to engage in research versus development. The next task will be to analyze this dynamic decision, in the hopes of deriving endogenous Schumpeterian waves.[17] In ongoing research on this subject, we are investigating the case in which each fundamental innovation consists of a process innovation, and each secondary innovation discovers how to apply the new process to a different sector of the economy.[18] We are examining the aggregate consequences of structural changes that take place as a new process innovation filters through the economy. The first sectors to apply it produce more but with fewer workers. As is the case with other sectoral shocks, the impact effect will be to raise unemployment. But subsequent development in other sectors that raises aggregate output, and hence aggregate demand, will eventually reverse the initial decline in unemployment. This analysis allows us to examine the relationship between the charactistics of waves and the average growth rate.[19] These and other aspects of the fundamental/secondary distinction emphasized in this paper are the subject of our current research.[20].

6.3. Relation to Earlier Literature

The analysis in this paper builds on several recent contributions to growth theory that have modeled more than one endogenous source of knowledge.

6.3.1. Learning by doing Stokey (1988) was first to emphasize the distinction between new products that come from innovations and cost reductions that come from *learning by doing*. However, the innovations are produced as an automatic by-product of the learning by doing and are not influenced independently by purposive research and development.

Young (1993a) assumes that deliberate research produces new goods but automatic learning reduces the cost of producing them. There is still only one kind of deliberate research in this scheme. In our view, even learning from experience requires real resources to be intentionally spent in reflection, experimentation, eliciting information from customers and from the shop floor, and so forth.

There is, however, a simple way to reinterpret the above model as one of innovations and *learning-by-doing*: Suppose as before that there are two kinds of labor, *skilled* and *unskilled*. While unskilled labor (l) can be used only in manufacturing, skilled labor can be employed in both research (H^r) and manufacturing (x).

The output flow of an intermediate producer employing l unskilled workers and x skilled workers is Cobb-Douglas, equal to $A_\tau \cdot l^\alpha \cdot x^{1-\alpha}$.

And while the discovery of new product lines still results from purposive research activites, the arrival of new intermediate plans on a line is entirely driven by learning-by-doing, which in turn is an increased (concave) function of the total amount of skilled workers currently employed on the line. By contrast with Young (1993a) learning-by-doing is *not bounded* on each line, even though the arrival rate of new intermediate plans will asymptotically converge to zero as the line's age increases and skilled workers are being continuously reallocated to newer lines.

Another noticeable difference with Young's model is the fact that the contribution of learning-by-doing to the generation of new intermediate plans on a given line is *internalized* by skilled workers (both in research and manufacturing) and in proportions that again depend on the *competitive* or *contractual* environment. On the other hand, as in Young (1993a), the contribution of learning-by-doing to the growth of general knowledge (such as according to the growth equation $\dot{A}/A = G(H^r, Y)$, Y being the aggregate output flow) will not be internalized by researchers and manufacturers within existing lines. Overall, this reinterpretation of the above model produces a richer framework than Stokey or Young to analyze the *multiple* dimensions of learning-by-doing in relation to innovations and growth.

6.3.2. Other Models with Radical Versus Incremental Innovations The two-dimensional nature of the research decision had already been explicitly taken into account in the partial equilibrium analyses of Jovanovic and MacDonald (1994), who consider the choice of whether to innovate or imitate; Jovanovic and Rob (1990), whose distinction between intensive and extensive search corresponds closely to our distinction between fundamental and secondary research; and Jovanovic and Nyarko (1994). Andolfatto and MacDonald (1993) extend the Jovanovic-MacDonald analysis to a general equilibrium setting. In that analysis there are two kinds of research but only one kind of knowledge, which can be discovered either by inventing or by mimicking another firm.

Segerstrom (1991), Zeng (1993), and others consider the choice between innovation and imitation, taking the resource cost of imitation into account, but assuming that there is no complementarity between these alternative activities. Likewise, Grossman and Helpman (1991) model innovation and imitation, by assuming that the same quality (or technology) can be produced at lower labor cost in the South, thereby making the Northern inventions obsolete in the absence of trade barriers. We are interested in cases where development enhances previous researchers' rents rather than stealing them.

Closer to the present paper is Amable (1993), who also assumes that innovators can choose which kind of activity to engage in and derives comparative statics results on steady-state growth. However, this paper differs from ours in several respects, in particular in the way it models the asymmetry between radical and incremental innovations.[21] Barro and Sala-i-Martin (1994) study a model in which incumbents make incremental quality improvements while outsiders make more radical quality innovations by inventing completely new products.

Young (1993b) analyzes the relationship between innovations that complement earlier ones and those that substitute for earlier ones, a difference that characterizes our research and development distinction. He does not, however, allow people to choose which kind of innovation to attempt. Instead, he assumes that each innovation is the same and goes through the same life cycle, evolving from being complementary with other innovations to being a substitute.

Finally, the analysis of Bresnahan and Trajtenberg (1992) of "general purpose technologies" is closely related to our analysis. It does not, however, allow for the rent sharing and the lifetime career choice at the heart of our arbitrage equation. Instead, it is concerned with the strategic interaction between two separate sectors, one in which the technologies are invented and one in which they are applied.

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Notes

1. This extreme case makes it clear that what we call "development" is quite different from the manufacturing labor that can be substituted for research in conventional Schumpeterian models.
2. Net of the growth rate.
3. In Section 3.1 we shall be more precise about the development technology—that is, about λ_τ^d.
4. See the following Section 2.3 for more details on the dynamics of the general knowledge parameter A_τ.
5. A parallel can be drawn between our model and Young's (1993a), by reinterpreting our product lines as corresponding to Young's "innovations" and our development as his "learning-by-doing." The main difference is that in our model both research and development are deliberate, resource-consuming activities, whereas there is only one kind of deliberate research in Young's model. In addition, Young does not make our distinction between fundamental and general knowledge.
6. Hence the model exhibits at least part of what economic historians sometimes call the "sailing-ship effect," whereby development continues even on obsolescent lines.
7. The results of the paper go through if we follow Caballero and Jaffe (1993) and allow for obsolescence and slow diffusion of innovations.
8. This assumption of *diminishing returns to development* ($0 < \nu$) will be used repeatedly in the analysis. A natural justification for it is in terms of the limited ability of the fundamental innovator of a line to help developers solve problems that arise in the development process. For example, the quality of each developer on a given line could depend on the amount of training initially provided to him by the fundamental innovator (or researcher). Such training is typically aimed at better acquainting the developer with new technology brought about by the product line. Whenever (1) the researcher's training efforts are nonobservable (noncontractible *ex ante*) and (2) the overall training cost incurred by the researcher is a convex function of the sum of training efforts devoted to all the developers on the line, then the equilibrium training effort per developer and therefore the productivity of each developer will both decrease with η_τ.
9. The optimal allocation of property rights will essentially depend on the relative marginal efficiencies of the researcher's and developer's investments.
10. Uniqueness is guaranteed in the benchmark case because (G) is linear, (A) is concave, and, as we show in the next footnote, (A) lies above (G) at $H^r = H^* = H$.
11. Substituting $H^r = \nu H$ into the arbitrage equation (A) shows that the height of (A) at νH in Figure 4 is ρ. Since (A) is upward sloping and (G) never reaches the height ρ, therefore (A) lies above (G) everywhere to the right of νH. From this and property (d) of G2 above, the intersection point must lie to the left of H^*.
12. The difference in marginal products of research and development in the prodution function for secondary innovations is proportional to

$$\frac{\nu}{H^r} - \frac{1-\nu}{H^r},$$

whereas the arbitrage equation (A) can be rewritten as

$$\rho - g = \sigma \cdot \left[\frac{H - H^r}{1-\nu}\right] \cdot \left[\frac{\nu}{H^r} - \frac{1-\nu}{H - H^r}\right].$$

Thus research has a larger marginal product than development in generating secondary innovations. Research also has a larger marginal product than development in producing fundamental innovations since the marginal product of development is simply equal to zero.

13. Compare (6) above with the expression for $S(t)$ in Appendix B.

14. More precisely, it follows from (14) that the flow of new development royalties that will accrue to a researcher at date t from a line discovered at τ is $\frac{\nu}{1-\nu}x_t\eta_{t,\tau}$. Since the researcher aiming to discover a new line at date t must also earn an expected flow of rents equal in expected value to x_t, we have

$$\begin{aligned} x_t &= \lambda^r \int_t^\infty e^{-\rho(s-t)} \frac{\nu}{1-\nu} x_s \eta_{s,t} ds \\ &= \lambda^r \int_t^\infty e^{-\rho(s-t)} \frac{\nu}{1-\nu} x_t e^{g(s-t)} \eta_{t,t} e^{-g(s-t)/\nu(1-\alpha)} ds. \end{aligned}$$

Cancelling x_t on each side of the equation, calculating the integral, eliminating h^d using (R′), and rearranging yields the modified arbitrage equation (A′).

15. See however Aghion, Dewatripont, and Rey (1995) and Aghion, Harris, and Vickers (1995) for alternative attempts to obtain a positive correlation between product market competition and growth in a Schumpeterian context.
16. The socially efficient amount of research will generally remain strictly positive. In the benchmark case of (G1) it is defined by

$$\rho + \sigma - g^* = \sigma(H - H^{r*})\lambda^r/(1-\alpha)(\rho - g^*), \quad \text{with } g^* = \lambda^r H^{r*}.$$

17. See Jovanovic and Rob (1990) and Cheng and Dinopoulos (1993) for earlier attempts to generate Schumpeterian waves, also based on the dichotomy between fundamental and incremental innovations.
18. Helpman and Trajtenberg (1994) and Cohen and St. Paul (1994) have examined exogenous waves of this sort.
19. If there were strong enough learning spillovers in development, for example, there would be multiple equilibria. If everyone believed that there would be no developers of a particular fundamental innovation, then no one would have an incentive to develop it, whereas the contrary expectation would cause a bandwagon effect.
20. For example, the fundamental/secondary paradigm may be used to analyze the implications of intraline (or interline) developers mobility for the relationship between growth and demand uncertainties. Another potential application of our framework is to the so-called vintage models of physical capital. These might indeed be extended into endogenous growth models using the above paradigm, by reinterpreting the accumulation of physical capital of a given vintage as the development of a fundamental line. The direct cost of fundamental research activities may have offsetting effects on the average growth rate of the economy, in particular through the average age of a given capital vintage and the corresponding value of research activities
21. Amable assumes the former to be random, the latter being deterministic; the former displacing all existing intermediate goods instantaneously and replacing them with new intermediate goods produced under perfect competition.

Appendix A

In the special case where $F(\ell) \equiv \ell^\alpha, 0 < \alpha, < 1$, the solution to the intermediate good producer's problem (M) is

$$\ell_{t\tau} = \bar{\ell}(w_t/A_\tau) \equiv (w_t/\alpha^2 A_\tau)^{\frac{1}{\alpha-1}}, \tag{A.1}$$

which implies

$$\bar{y}(w_t/A_\tau) \equiv (w_t/\alpha^2 A_\tau)^{\frac{\alpha}{\alpha-1}}. \tag{A.2}$$

Substituting from (A.1) into (4) yields the equilibrium wage:

$$w_t = L^{\alpha-1}\left[\int_{-\infty}^t S_{t\tau}(\alpha^2 A_\tau)^{\frac{1}{1-\alpha}} d\tau\right]^{1-\alpha}. \tag{A.3}$$

Substituting from (A.2) and (A.3) into (3) yields the equilibrium value of aggregate output:

$$Y_t = L^\alpha \left[\int_{-\infty}^{t} S_{t\tau} A_\tau^{\frac{1}{1-\alpha}} d\tau \right]^{1-\alpha}. \tag{A.4}$$

Appendix B Deriving the Growth Equation (G2)

The description of general knowledge given just before (G2) in the text implies that $\dot{A}_t/A_t = \beta F(t) + (1-\beta)S(t)$, where $F(t)$ and $S(t)$ are the flow of fundamental and secondary innovations, respectively, at date t. Clearly, $F(t) = \lambda^r H_t^r$. The flow of secondary innovations is the integral of all the flows across all existing lines. There are $\lambda^r H_\tau^r$ lines per vintage τ, each with $\eta_\tau e^{-\sigma(t-\tau)}$ remaining developers, and with an arrival rate of secondary innovations equal to $\lambda^d \cdot \eta_\tau^{-r}$ per developer. Thus,

$$S(t) = \int_{-\infty}^{t} \lambda^r \cdot H_\tau^r \cdot \lambda^d \eta_\tau^{1-\nu} e^{-\sigma(t-\tau)} d\tau = \int_{-\infty}^{t} \lambda^d \cdot (\lambda^r H_\tau^r)^\nu (h_\tau^d)^{1-\nu} e^{-\sigma(t-\tau)} d\tau.$$

In steady state with $H_\tau^r \equiv H^r$ and $h_\tau^d \equiv \sigma \cdot (H - H^r)$, we have $F(t) \equiv \lambda^r H^r$, $S(t) \equiv \lambda^d (\lambda^r)^\nu (H^r)^\nu (H - H^r)^{1-\nu} \sigma^{-\nu}$, and hence

$$\dot{A}_t/A_t \equiv \beta \lambda^r H^r + (1-\beta)\lambda^d (\lambda^r)^\nu (H^r)^\nu (H - H^r)^\nu \sigma^{-\nu},$$

which confirms (G2).

Properties 1 through 4 follow immediately from the construction of G(·). To establish 4 note that, by direct calculation, $\partial G/\partial H^r = \beta\lambda^r > 0$ when $H^r = \nu H$, and $\partial G/\partial H^r \to -\infty$ as $H^r \to H$. Thus the maximum of G, where $\partial G/\partial H^r = 0$, occurs in the interval $(\nu H, H)$.

■

Appendix C

Define:

$$Z_t \equiv \int_{-\infty}^{t} \lambda^d (h_\tau^d)^{1-\nu} (\lambda^r H_\tau^r)^\nu A_\tau^{\frac{1}{1-\alpha}} \frac{1 - e^{-\sigma(t-\tau)}}{\sigma} d\tau, \tag{C.1}$$

$$K_t \equiv \int_{-\infty}^{t} \lambda^d (h_\tau^d)^{1-\nu} (\lambda^r H^r)^\nu A_\tau^{\frac{1}{1-\alpha}} e^{-\sigma(t-\tau)} d\tau, \text{ and} \tag{C.2}$$

$$B_t \equiv \int_{-\infty}^{t} \lambda^d (\lambda^r)^\nu (H_\tau^r)^\nu (h_\tau^d)^{1-\nu} e^{-\sigma(t-\tau)} d\tau. \tag{C.3}$$

Then clearly $\dot{Z}_t = K_t$. Also, from (A.4), (2) and the definition of φ, $Y_t = Z_t^{1-\alpha}$ in the Cobb-Douglas case. Thus the social problem (12) can be rewritten as

$$\begin{array}{lll} \max \int_0^\infty e^{\rho t} Z_t^{1-\alpha} dt & & \\ \text{subject to:} & \dot{Z}_t = K_t & \\ & \dot{K}_t = \lambda^d (h_t^d)^{1-\nu} (\lambda^r H_t^r)^\nu A_t^{\frac{1}{1-\alpha}} - \sigma K_t & \quad (\text{C.4}) \\ & \dot{A}_t = A_t \cdot [\beta\lambda^r H_t^r + (1-\beta) B_t] & \\ & \dot{B}_t = \lambda^d \cdot (\lambda^r)^\nu (H_t^r)^\nu (h_t^d)^{1-\nu} - \sigma B_t & \\ & \dot{H}_t^r = \sigma(H - H_t^r) - h_t^d, & \end{array}$$

with initial conditions: $(Z_0, K_0, A_0, B_0, H_0^r)$. this is a standard optimal control problem with a single control variable h^d and five state variables (Z, K, A, B, H^r). To solve it, define the Hamiltonian:

$$\begin{aligned} H \equiv\ & Z^{1-\alpha} + \mu^Z K + \mu^K [\lambda^d (h^d)^{1-\nu} (\lambda^r H^r)^\nu A^{1/(1-\alpha)} - \sigma K] \\ & + \mu^A [\beta\lambda^r H^r + (1-\beta) B] A + \mu^B [\lambda^d (\lambda^r H^r)^\nu (h^d)^{1-\nu} - \sigma B] \\ & + \mu^H [\sigma(H - H^r) - h^d]. \end{aligned}$$

By Pontryagin's maximum principle, the solution must obey

$$\begin{aligned} \dot{\mu}^Z &= \rho\mu^Z - (1-\alpha) Z^{-\alpha} && (\text{C.5}) \\ \dot{\mu}^K &= \rho\mu^K - \mu^Z + \sigma\mu^K && (\text{C.6}) \\ \dot{\mu}^A &= \rho\mu^A - [1/(1-\alpha)]\mu^K \lambda^d (h^d)^{1-\nu} (\lambda^r H^r)^\nu A^{\alpha/(1-\alpha)} && (\text{C.7}) \\ &- \mu^A [\beta\lambda^r H^r + (1-\beta) B] && \\ \dot{\eta}^B &= \rho\mu^B - \mu^A \cdot A \cdot (1-\beta) + \sigma\mu^B && (\text{C.8}) \\ \dot{\mu}^H &= \rho\mu^H - \nu\lambda^d (h^d)^{1-\nu} (\lambda^r)^\nu (H^r)^{\nu-1} A^{1/(1-\alpha)} \mu^K && (\text{C.9}) \\ & -\mu^B \lambda^d (\lambda^r)^\nu (h^d/H^r)^{1-\nu} \cdot \nu - \mu^A \beta\lambda^r A + \sigma\mu^H && \\ 0 &= (1-\nu)\mu^K \lambda^d (h^d)^{-\nu} (\lambda^r H^r)^\nu A^{1/(1-\alpha)} && (\text{C.10}) \\ & +\mu^B \lambda^d (\lambda^r H^r)^\nu (h^d)^{-\nu} (1-\nu) - \mu^H && \end{aligned}$$

We are interested in the steady state of this dynamical system, in which $\dot{A}/A = g \equiv \beta\lambda^r H^r + (1-\beta)B$ and $\dot{h}^d = \dot{H}^r = 0$, and all the μ's grow at constant rates. From (C.1) and (C.2) we thus have $\dot{Z}/Z = \dot{K}/K = g/(1-\alpha)$. From (C.5): $\dot{\mu}^Z/\mu^Z = -g\alpha/(1-\alpha)$; from this and (C.6): $\dot{\mu}^K/\mu^K = -g\alpha/(1-\alpha)$; from this and (C.7): $\dot{\mu}^A/\mu^A = 0$; from this, (C.8) and (C.9): $\dot{\mu}^B/\mu^B = \dot{\mu}^H/\mu^H = g$. Combining these results with (C.9) and (C.10) yields

$$g\mu^H = \rho\mu^H - \left[\lambda^d (\lambda^r H^r)^\nu (h^d)^{1-\nu}\right] \left[A^{\frac{1}{1-\alpha}} \mu^K + \mu^B\right] \left(\frac{\nu}{H^r}\right) - \beta\mu^A A \lambda^r + \sigma\mu^H \quad (\text{C.9}')$$

$$\mu^H = \left[\lambda^d (\lambda^r H^r)^\nu (h^d)^{1-\nu}\right] \left[A^{\frac{1}{1-\alpha}} \mu^K + \mu^B\right] \left(\frac{1-\nu}{h^d}\right) \quad (\text{C.10}')$$

Using (C.10′) to substitute for the factors in square brackets in (C.9′) yields

$$g\mu^H = \rho\mu^H - \frac{\nu}{1-\nu} \cdot \frac{h^d}{H^r}\mu^H - \beta\mu^A A\lambda^r + \sigma\mu^H.$$

Dividing both sides by μ^H and using (R) to replace h^d yields

$$\rho + \sigma - g = \frac{\nu}{1-\nu} \cdot \sigma \cdot \frac{H - H^r}{H^r} + \beta\left(\frac{\mu^A A\lambda^r}{\mu^H}\right),$$

which is the social arbitrage equation (S) of the text, with $SP \equiv \frac{\mu^A A\lambda^r}{\mu^H} > 0$. ∎

References

Aghion, Philippe, Mathias Dewatripont, and Patrick Rey. (1995). "Competition, Financial Discipline and Growth." Unpublished paper.

Aghion, Philippe, Christopher Harris, and John Vickers. (1995). "Competition and Growth with Step-by-Step Technological Progress." Unpublished paper.

Aghion, Philippe, and Peter Howitt. (1992). "A Model of Growth Through Creative Destruction." *Econometrica* 60, 323–351.

Aghion, Philippe, and Jean Tirole. (1994). "On the Management of Innovation." *Quarterly Journal of Economics* 109, 1185–1209.

Amable, Bruno. (1993). "Radical and Incremental Innovation: A Model of Endogenous and Unsteady Growth." Unpublished paper, CEPREMAP.

Andolfatto, David, and Glenn M. MacDonald. (1993). "Endogenous Technological Change, Growth, and Aggregate Fluctuations." Unpublished paper, University of Waterloo.

Barro, Robert J., and Xavier Sala-i-Martin. (1994). "Quality Improvements in Models of Growth." Working paper 4610, National Bureau of Economic Research.

Bresnahan, Timothy F., and Manuel Trajtenberg. (1992). "General Purpose Technologies: 'Engines of Growth?' " Working paper 4148, National Bureau of Economic Research.

Caballero, Ricardo J., and Adam B. Jaffé. (1993). "How High Are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth." In *NBER Macroeconomics Annual.* Cambridge, MA: MIT Press.

Cheng, Leonard K., and Elias Dinopoulos. (1993). "Schumpeterian Growth and Stochastic Economic Fluctuations." Unpublished paper, University of Florida.

Cohen, Daniel, and Gilles St.-Paul. (1994). "Uneven Technical Process and Job Destructions." Working paper 9412, CEPREMAP.

Green, Jerry, and Suzanne Scotchmer. (1995). "On the Division of Profit in Sequential Innovation." *Rand Journal of Economics* 26, 20–33.

Grossman, G. M., and E. Helpman. (1991). *Innovation and Growth in the Global Economy.* Cambridge, MA: MIT Press.

Helpman, E., and M. Trajtenberg. (1994). "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies." Working paper 32, CIAR.

Jovanovic, Boyan, and Glenn M. MacDonald. (1994). "Competitive Diffusion." *Journal of Political Economy* 102, 24–52.

Jovanovic, Boyan, and Yaw Nyarko. (1994). "The Bayesian Foundations of Learning by Doing." Working paper, New York University.

Jovanovic, Boyan, and Rafael Rob. (1990). "Long Waves and Short Waves: Growth Through Intensive and Extensive Search." *Econometrica* 58, 1391–1409.

Kuznets, Simon. (1940). "Schumpeter's Business Cycles." *American Economic Review* 30, 257–271.

Lucas, Robert E., Jr. (1993). "Making a Miracle." *Econometrica* 60, 251–272.

Rosenberg, N., and L. E. Birdzell, Jr. (1986). *How the West Grew Rich.* New York: Basic Books.
Segerstrom, P. S. (1991). "Innovation, Imitation, and Economic Growth." *Journal of Political Economy* 99, 807–827.
Stokey, N. L. (1988). "Learning by Doing and the Introduction of New Goods." *Journal of Political Economy* 96, 701–717.
Young, Alwyn. (1993a). "Invention and Bounded Learning by Doing." *Journal of Political Economy* 101, 443–472.
Young, Alwyn. (1993b). "Substitution and Complementarity in Endogenous Innovation." *Quarterly Journal of Economics* 108, 775–807.
Zeng, Jinli. (1993). "Innovative Vs. Imitative R&D and Economic Growth." Unpublished paper, University of Western Ontario.