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LINKING FACTUAL AND PROCEDURAL KNOWLEDGE IN SOLVING SCIENCE PROBLEMS: A CASE STUDY IN A THERMODYNAMICS COURSE

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ABSTRACT

Well-specified problems of the type presented boxed in the introduction to this article are extremely common in science courses. Unfortunately, this does not mean that students find them easy to solve, even when a teacher provides model answers to problems which differ only marginally (in the teacher's eyes) from those put before the students. The central difficulty with such courses is that they do not embody instructional principles that reflect students' need for "direction" in problem solving. In this article, we describe how the necessary heuristics and strategic knowledge were built into the remake of a conventional thermodynamics course. In contrast to mainstream American work on learning problem solving we chose to direct our curriculum reconstruction using the Gal'perin theory of stage-by-stage formation of mental actions and Landa's description of the "through" systematization of knowledge. As indicated by both, we first developed an integrated system of instructional objectives: a programme of actions and methods (PAM) to solve problems in thermodynamics. Then the plan of instruction was designed. This plan indicates which instructional procedures and materials should be used to realize the instructional functions, derived from the learning theory. The evaluation design contained two control and three experimental courses. In discussing our main findings, we consider the generalizability of the procedures we followed in constructing the PAM and the instructional plan.

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1. Introduction

As Greeno (1980) has observed, even well-structured problems like the one quoted in the box below require both a factual knowledge base, and a strategic knowledge base for their solution. The complexity of such knowledge-based performance is not always appreciated and phrases such as "merely applying an algorithm" or "just remembering how to do it" express the generally low opinion of performance in which the knowledge used by

The cabin of a mountaineer is situated near a waterfall. The height of the fall is 7 meters. The quantity of the water running through the brook is 0.03 $m^3.s^{-1}$. Its temperature is 8°C during the entire year. The mountaineer wonders if he can use the energy of the water to heat his cabin. To maintain a temperature of 20°C in the room of the cabin 20 kilowatt is needed.
The density of water = 1000 $kg.m^{-3}$; g = 10 $m.s^{-2}$; 1 W = 1 $J.s^{-1}$.
Is it possible, at least in theory, to maintain a temperature of 20°C in the room by means of the water?

the performer is presumed to be understood by the person who is judging the performance (Greeno, 1980, p. 10). Although attitudes are changing, it is rare in our experience for curricula to change in the direction of recognizing the complexities of problem solving. Thus, some teachers in the Department of Chemical Technology at Twente University of Technology, like most people involved in science education, were initially hardly aware of the difficulties students experience when they learn to solve problems in science. It was soon realized that many students use a trial and error method; they have no clear strategy and are not sure which laws or principles to apply, even when the problems are well structured. Further in most courses students are not able to approach new, less well-structured problems in a systematic way.

To find an instructional solution to this problem, the teachers and the authors formed a group that in 1975 started to remake a conventional first-year course in Thermodynamics in the Department of Chemical Technology. In this project we specifically focussed our attention on developing a systematic approach to problem solving, on designing instruction where students learn this approach, and on finding a procedure for remaking and evaluating other courses in problem solving.

This article describes the major activities and results of the project. It is a condensed version of the final project report, which is available from the authors (Mettes and Pilot, 1980). Before describing the main points of the project we must point out, to those familiar with the literature on problem solving, that our concern has been to devise material of use to practising teachers. While many researchers in problem solving have investigated abstract, game-like problems (e.g. De Groot, 1965; Newell and Simon, 1972) we consider it unlikely that such work can immediately inform those interested in the *teaching* or *learning* of problem solving in science.

This is not to say that successful courses in scientific problem solving have not been developed. Indeed Larkin lists several in a recent article (Larkin, 1980, p. 113). However, as she says herself: "with all strong points, . . . these instructional programs . . . remain idiosyncratic products of enthusiastic individuals [and] it is hard to . . . [use such a course elsewhere] because one doesn't know how it works". While her own work is not open to criticism of idiosyncrasy, since she is developing computer-implemented models of how people apply physics principles to solving problems, it is as yet too much laboratory-centred. We hoped to demonstrate that one does not need to invoke sophisticated theories of representing and solving problems, to answer the question of how to teach students to solve problems in a real-life science course. After posing ourselves this question, we split it up into three parts:

1. Which actions and methods should be learned to promote the effectiveness of the problem-solving process?
2. How should students learn these actions and methods? Which instructional procedures and materials should be applied to get an optimal learning process?
3. How should the results of the experimental course be evaluated? What kind of criteria should be applied on what kind of data for judging the worth of the new instructional programme?

Each of these parts is represented in our project and produced an intermediate product in the development and evaluation of the experimental course.

The products of phase 1 were first the principles of instructional learning to be used in course development (section 2), and second the Programme of Actions and Methods (PAM) for solving problems in Thermodynamics that was developed on the basis of these instructional principles and from which a system of heuristics was derived (section 3). The instructional programme consisting of the instructional procedures, materials and teaching activities is described in section 4. Section 5 reviews the evaluation: the data on the processes and the results of teaching and learning, the criteria that were applied and the decisions that were taken. The last section contains some general observations on our approach and on its implementation in other courses at Twente University of Technology.

2. Principles of Instructional Learning

Before developing the new course we looked for a suitable theory of instructional learning. In our opinion such a theory should contain directives which relate instructional objectives to learning processes, and also learning processes to instructional procedures. As stated above, the instructional objectives of the Thermodynamics course involve skills in solving problems found rather difficult by students. Because of this, the only relevant theories of learning of which we knew seemed to be those of Ausubel (1968), Gagné (1977) and Gal'perin (Talyzina, 1973). In this project we eventually chose

Gal'perin's theory of instructional learning supplemented with contributions of Talyzina (1973) and Landa (1975). Our main reasons for choosing this theory are:

1. Gal'perin's theory is the only explicitly instructional one in the sense that Gal'perin gives a definition of an optimal learning result and prescribes the micro behaviour desired of both the teacher and the student.

2. This being a cybernetic theory, the learning result is consistently defined in terms of (mental) operations or actions. Acquisition of knowledge requires the formation of adequate systems of actions, that specify what a student should do to solve problems properly, in terms of particular algorithms and heuristics.

In 2.1 we describe those parts of Gal'perin's theory that are relevant for our course. In 2.2 the elements we took from Landa and Talyzina are presented. Paragraph 2.3 contains the main principles of instructional learning we used in course development.

2.1. GAL'PERIN'S THEORY OF STAGE-BY-STAGE FORMATION OF MENTAL ACTIONS

According to the theory of Gal'perin (Talyzina, 1973) there are four characteristics or parameters in the performance of an action: *form; generalization; completeness of action links;* and *mastery.*

Details of each of these parameters can be found elsewhere (e.g. Talyzina, 1973). Their relevance here is that for Gal'perin learning is the acquisition of new (mental) actions, and instructional learning is a process of planned progressive internalization of external actions. This transformation in the form of the action is accompanied by changes in the other three parameters. So an expert's performance is more transferable, abbreviated and automatic than a student's. At the start of this process the student should perform a complete action in material or materialized form. By observing the completely externalized performance both student and teacher can detect incorrect or incomplete actions and administer feedback. Also they get knowledge of the results on the other parameters of the performance. This knowledge has to be used to ensure that the performance becomes more transferable, abbreviated and automatic. When the action is mastered in material or materialized form the teacher allows the student to proceed to and exercise at the next form and so on, until the student reaches mastery in the mental form.

Gal'perin points out that before starting this stage-by-stage formation of new mental actions, the student must have an orienting basis to be able to perform the action for the first time. He must have information to orientate himself about what to do in what circumstances. This orienting basis should be complete i.e. contain all information necessary for a perfect performance:

such as the goal of the action, the composition of all action links, the conditions in which the action can and cannot be performed. The best orienting basis is both complete and presented to the student in a generalized form i.e. a form that covers a whole class of problems. The quality of the orienting basis is emphasized in the theory of Gal'perin, because it outlines the conditions which are objectively necessary for the student to perform the action successfully i.e. to solve the relevant problems.

2.2. EMPHASIS ON SYSTEMS OF ACTIONS AND KNOWLEDGE

Two other Russian psychologists emphasize the importance of *systems* of actions. In his research on problem solving Landa (1975) pays much attention to forming systems of actions. One way to form such a system is the so-called "through" systematization of knowledge. "Through" systematization of knowledge means: combining in a single system all knowledge relevant for problem solving that is contained in separate sections of a book, a course etc. In this way the subject matter can be reorganized in an operational form (see also Willems, 1981).

Talyzina (1973) developed on the basis of Gal'perin's theory a procedure for the development of instruction. In this procedure systems of actions, subprogrammes in her terminology, occupy an important place. These subprogrammes contain:

1. The bulk of knowledge in a particular subject matter.
2. The rational actions and methods of thinking adequate in learning to apply this knowledge. This subprogramme is divided in two parts:
    a. actions and methods constituting specific types of thinking (specific for this subject matter);
    b. logical actions and methods of thinking (not dependent on a concrete subject).

The rules or suggestions to execute the actions and methods of programme 2a are called algorithms and heuristics. Talyzina remarks that the construction of these programmes is difficult because the actions and methods are not explicitly formulated in the subject matter and also are largely unknown to the teachers.

2.3. SUMMARY OF THE PRINCIPLES OF INSTRUCTIONAL LEARNING

We derived from Gal'perin's theory of instructional learning, supplemented with the findings and reflections of Talyzina and Landa, the following principles of instructional learning:

Presentation of an orienting basis

Orientation on how to act in problem solving is meaningful, because it is rational. Therefore the student should get an orienting basis on how to solve problems in Thermodynamics. Because we deal with heuristic problem solving this basis cannot be complete, but should be as complete and generally applicable as possible. Such an orienting basis consists of: (a) subject matter (knowledge) in operational form, and (b) heuristics and general methods of thinking.

Stage-by-stage formation of mental actions

Actions which are "new", i.e. previously unknown to a student, should first be performed in materialized form (e.g. on paper or on the blackboard) with all action links complete. Once a student has reached mastery of the action in this first stage, he should pass on to the next stage, the verbal form (e.g. talking to the teacher or other students). After mastery of this form he should pass on to the mental form (i.e. solving a problem in his head by thinking of the solution). During this process of passing on to the mental form the action gradually becomes more abbreviated, more generally applicable and is more perfectly performed. The advantage of this training procedure for the student is that in the first stage (the materialized form) he gets acquainted with the coherence of all actions and the consequences of their application. So he has optimal control over his own actions. The advantage for the teacher is that it is easier to give feedback, because the actions are external as far as possible, and hence observable.

Mastery learning

In recent years the principle of mastery learning (Carroll, 1963, 1971; Bloom et al., 1971) has been used in the construction of many types of courses both on a teacher or group paced learning basis, and on an individually paced learning basis. In the Netherlands both types are used (Plomp et al., 1978), but in development and execution of these courses difficulties were met with problem-solving objectives. In this course we used the group paced type as described by Bloom (1976).

3. Analysis of Difficulties and Development of the Programme of Actions and Methods

As stated previously, we had two main objectives:

1. to improve the existing Thermodynamics course;
2. to find a set of procedures for developing and evaluating courses in problem solving in science.

Our project consisted of eight activities, which we compare in Fig. 1 with Provus' model of programme development and evaluation. The first two activities, analysis of difficulties and development of a programme of actions and methods (PAM) are linked as this section shows. Only activities 4 and 5 are not described in this article.

Stages in Provus' model	Activities in the procedure of Mettes et al.
1. Definition of the programme	1. Analysis of the difficulties 2. Development of a PAM 3. Construction of an instructional plan
2. Installation of the programme	4. Experimental try-out 5. Staff training
3. Assessment of initial effects of the programme; making of adjustments	6. Formative evaluation
4. Assessment of achievement of terminal objectives	7. Summative evaluation
5. Assessment of the efficiency of the programme	8. Assessment of the effectiveness of the experimental course

Fig. 1. Stages in Provus' model and the activities in our procedure of development and evaluation.

3.1. THE DEVELOPMENT OF THE PAM

The first two activities were the most essential and difficult ones in the project. The teachers in the course in Thermodynamics and other specialists in this field could not give us an adequate description of problem solving in this subject matter. The literature on Thermodynamics does not contain any adequate system of heuristics. The situation for most subject matter at this moment may, in our opinion, well be similar. Our first attempt at producing a PAM for Thermodynamics was based on the well-known and widely used set of heuristics Polya (1957) developed for problem solving in mathematics. Unfortunately those heuristics for the analysis of problems were too incomplete or gave hints in the wrong direction. No adequate heuristics were found for transforming science problems into recognizable and soluble subproblems, nor was reasoning by analogy successful.

We then decided to do some research on a descriptive model of science undergraduate problem-solving behaviour. The problems in our courses are "specification problems" (Mettes and Pilot, 1980). In typical, well-specified problems of this kind, a situation, certain relations, variables, magnitudes etc. are given; the problem is to find or calculate etc. one or more unknowns, other relations, variables, magnitudes and such-like. If the unknown is found, the situation is better specified. This type of problem is very frequently used in science and technology curricula.

We carried out experiments in which students as well as staff tried to solve problems relevant for the course objectives. They were requested to think aloud, and protocols of their problem-solving behaviour were recorded and transcribed. These protocols were interpreted in terms of a model derived from theories on problem solving of Duncker (1945), De Groot (1965) and Newell and Simon (1972) in an iterative process (details of which are available from the authors). The result of this process was a model (called Transformation to Standard Problem, TSP model). Although we derived this model from studying Thermodynamics protocols, it can be used to *describe* problem-solving behaviour in other subject matter areas in science and technology, with few or no modifications. For example, recently the TSP model was used successfully to describe protocols of problem solving in Electricity and Magnetism (Van Weeren et al., 1980).

In the following phase we tried to develop from this descriptive model a prescriptive one: a Programme of Actions and Methods to be used in the *training* of problem solving in Thermodynamics. When designing this PAM from the TSP model we looked for actions and methods to ensure a systematic and effective problem-solving process, irrespective of whether these actions and methods were found in the protocols or not. We used a number of indications and criteria for desirable actions and methods, such as:

– indications from the protocols, e.g. differences in problem-solving behaviour between students and teachers, such as the frequencies of actions in analyzing the problem, selecting relations etc., the errors made and the ways in which these errors were corrected (Mettes and Pilot, 1980);

– indications from the literature on special heuristics (Marples, 1974);

– indications from the literature on research on PAM's for other subject matter (Talyzina, 1973; Dubovskaja, 1967; Obuchowa, 1973);

– research on frequently made mistakes and difficulties in exercises and exams in this course.

The programme as such contained information that was not suitable for student use. So, the next step was the transformation of this programme into a system of heuristics that students can use to orientate themselves in problem solving. (The teachers can use it also when giving feedback to students.) A summary of this system was condensed to one page, usually referred to as the SAP chart for Thermodynamics, where SAP means Systematic Approach to Problem solving (Fig. 2). As section 4 shows, this summary was adequate for the majority of students' needs, because it formed part of a coherent instructional plan.

The content of the heuristics is essentially similar to the PAM, but there may be considerable functional differences in form and wording of the actions and methods. The SAP chart was drawn up using the following five principles:

1. Only those heuristics were included that refer to actions unknown to the student and strictly necessary for solving the most important problems.

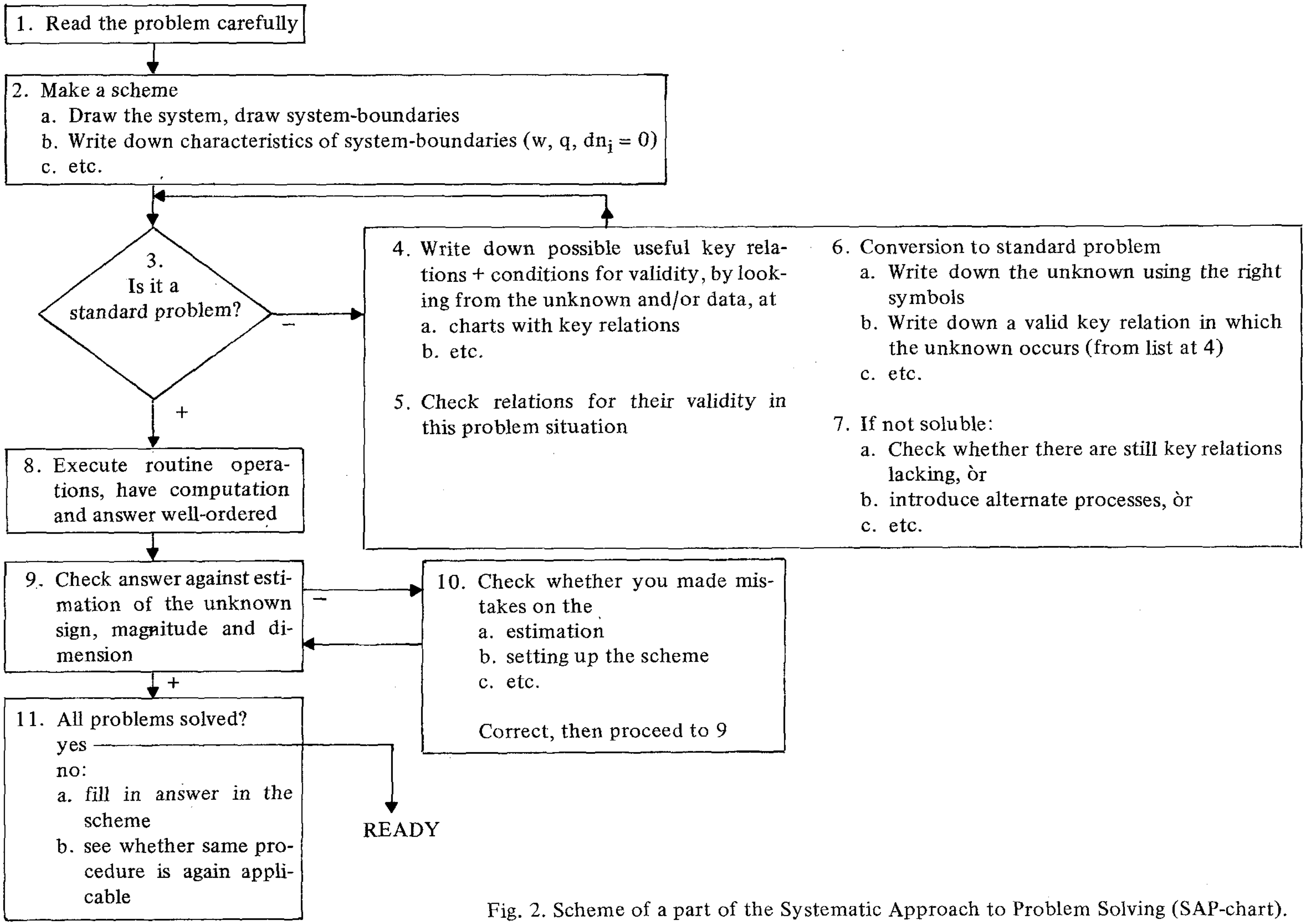


Fig. 2. Scheme of a part of the Systematic Approach to Problem Solving (SAP-chart).

2. The heuristics had to be worded in such a way that the student could readily understand them.

3. The text of the heuristics had to be as complete as possible to enable the student to perform a complete action in materialized form.

4. The heuristics had to be worded in such a way as to ensure their appropriateness throughout the course, even if the subject matter varied. From this general wording, more specific applications – related to specific subject matter – had to be deducible.

5. The imperative mood had to be used to show clearly that the heuristics are directions for desired actions.

3.2. THE PAM

The first design of the SAP chart was checked and corrected in small-scale experiments with students. On the basis of these experiments a more definitive version of the SAP chart – and consequently of the PAM from which it was deduced – was designed and used in two experimental courses. On the basis of the evaluation data of these courses the definitive version of both this PAM and the chart was developed. This version has four principal phases:

Phase 1: Reading the problem thoroughly; careful analysis of the data and the unknown by making a scheme.
Phase 2: Establishing whether or not it is a standard problem, i.e. a problem that can be solved by mere routine operations; if not: Looking for relations between the data and the unknown that can be of use in the transformation of the problem to a standard problem; conversion of the problem to a standard problem.
Phase 3: Execution of routine operations.
Phase 4: Checking the answer, interpretation of the results.

Phase 2 will now be presented in more detail (for information about the other phases see Mettes et al., 1980). We first mention its purpose and then list a number of desired actions. We only list the actions that can be expressed in general terms. For different fields, different specifications of the actions are needed. An example of a problem in Thermodynamics that has been worked out according to the PAM (specified for Thermodynamics) is given in par. 4.2.

Phase 2: Transformation of the problem

Purpose: Conversion of the problem to a standard problem by linking the unknown and the data with given relations between quantities.

Desired actions:

2a. Establishing whether or not the problem is a standard problem

If so, the problem solver can go on to phase 3.

If not, continue with 2b.

2b. Writing down possibly useful relations

2b1. Splitting up the problem (if necessary) into subproblems; choice of the first subproblem to solve (e.g. the easiest or one where results are expected that can be used later on).

2b2. Writing down possibly useful relations from the following sources (taking the unknown and/or the data as the starting point):

- a. Charts with "Key Relations" for this subject. By Key Relations we mean relations that contain the very core of the subject matter in a formulation which makes them a good starting point for solving problems (for more detailed information about Key Relations see par. 4.3).
- b. Charts with relations for other fields (e.g. mathematics, prerequisite subjects).
- c. Relations which follow from the data, directly and indirectly.
- d. Relations which the problem solver at this stage can indicate only in general mathematical terms [e.g. in a rubber band the length (L) is a function of the force (K) and the temperature (T): $F(K,L,T) = 0$].

2b3. Checking the relations found for their validity in this problem situation.

2c. Conversion of the problem to a standard problem

2c1. Trying to interrelate unknown and data by applying the relations to the problem situation and by linking them up. This can be done in many ways, but experience shows that using the unknown as the starting point gives a better chance for a successful solution of the problem (see Fig. 3). When this is done, chances of transformations that are irrelevant or come to a dead end are less than when the data are used as starting point.

2c2. If it is not possible to arrive at a standard problem by the actions in 2c1, the following actions might be tried:

- a. Trying to simplify the problem, e.g. by solving it for an infinitesimally small change, after which integration might be justified.
- b. Trying to restate the problem or to consider it from a different point of view (e.g. larger or smaller scale; setting up the analysis of the problem in a different way).
- c. Trying to solve an analogous problem in a different field; this might generate ideas about how to solve this problem.
- d. Letting the problem rest for some time; difficult problems generally are not solved in one go.

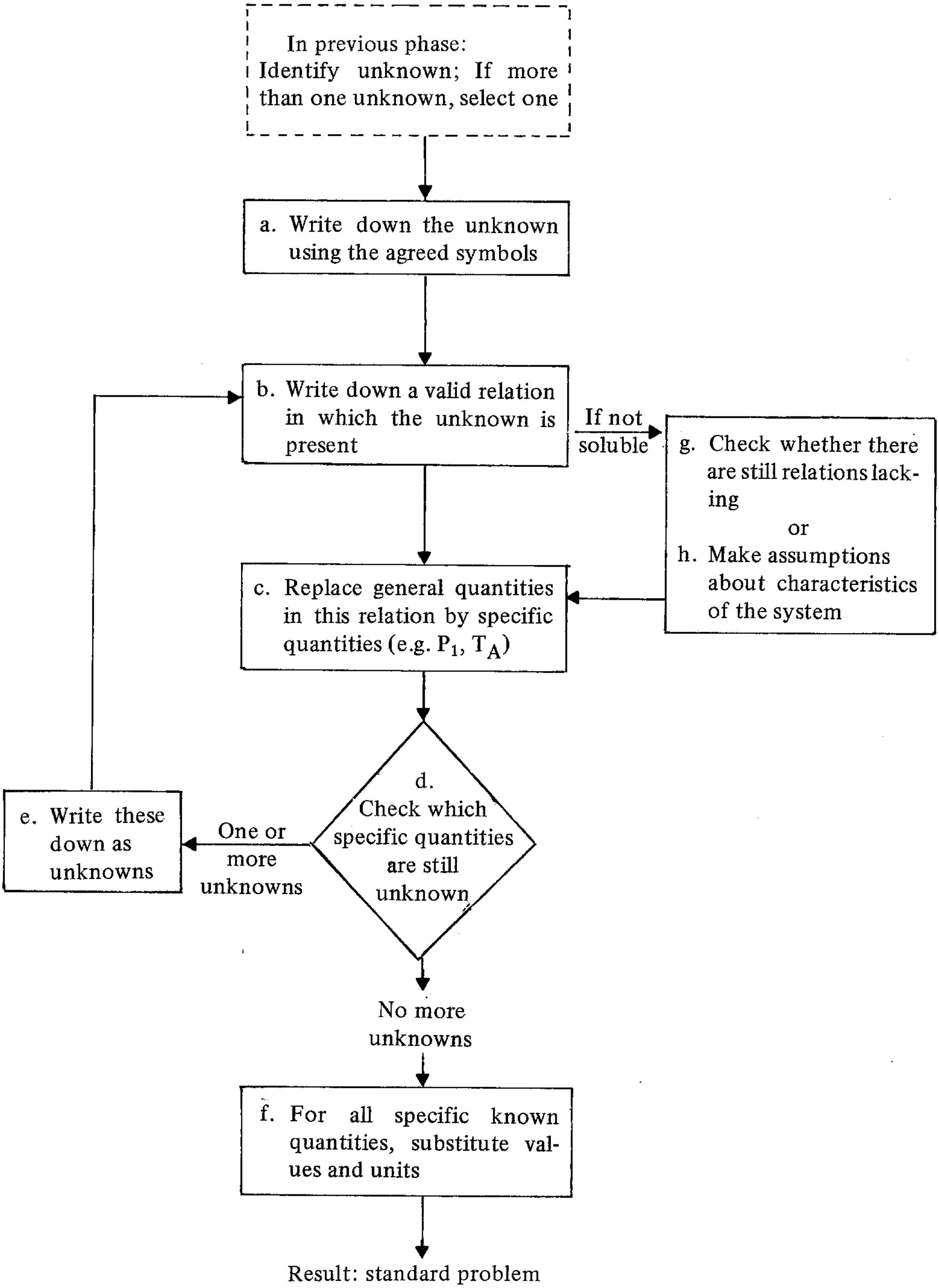


Fig. 3. Diagram of strategy: transformation, using the unknown as starting point.

In a block diagram (Fig. 3) the strategy for transformation to a standard problem, using the unknown as the starting point, is summarized.
In the next section we will discuss the instructional plan for the systematic approach to problem solving.

4. Construction of the Instructional Plan

In section 3 we described our answer to the first question in this project, concerning which actions and methods should be learned (see section 1). The second question was: How should students learn these actions and methods; that is, which instructional procedures should be applied to get an optimal learning process?

Our answer to this question is a plan of instruction that indicates which instructional procedures and materials should be used to stimulate and direct the phases of the learning process. There is a gap between the formulation of instructional objectives and the choice of instructional procedures to realize these objectives. Unfortunately the literature provides little information that can fill this gap. For instance, consider getting the student acquainted with subject matter: many different procedures and materials may be adequate e.g. lectures, lecture notes, literature, films, video tapes, self-study etc. Research does not give guidance on how to choose between them (see Dubin and Taveggia, 1969; Wallen and Travers, 1963).

4.1. INSTRUCTIONAL FUNCTIONS

In our opinion learning theories should bridge the gap between objectives and procedures. We therefore restated the phases of the learning process in terms of *instructional functions.* Instructional functions are defined as general operations or actions that have to be performed in instruction to evoke the necessary phases of the learning process and by doing this to realize the objectives. In other words, the best way to realize an optimal learning process and thus an optimal learning outcome is to guarantee the realization of all instructional functions. Figure 4 gives a survey of the phases of the learning process, the instructional functions we derived from them and the instructional procedures and materials for the realization of each function. The details of the phases of the learning process were described in section 2. (Readers requiring more information about instructional functions in the development and evaluation of instruction should contact the first two authors.)

The best way to realize an instructional function very much depends on the specifics and context of a course. We think that achieving realization of a function is more important than the particular way in which it is realized.

		INSTRUCTIONAL PROCEDURES AND MEANS								
PHASES OF THE LEARNING PROCESS	FUNCTIONS OF THE INSTRUCTION	lecture 1	lecture notes 2	SAP chart*+ 3	KR chart**+ 4	problems for work in groups+ 5	problems for self-study 6	tests and feedback 7	remedial instruction 8	exam 9
1. Learning the conditions of the programme of actions and methods ↔	Orientation									
	1. Presentation of the essential elements of knowledge and actions specific for this subject	●	●							
	2. Making these elements of knowledge and actions operational	●			●		●			
	3. Giving a system of heuristics for problem solving			●						
	4. Realizing the connection with the entering behaviour				●	●				
	5. Giving the student insight in the objectives of instruction					●		●		
2. Learning to perform the programme of actions and methods ↔	Stage by stage exercising									
	6. Exercising the actions and methods of problem solving (PAM)					●	●			
	7. Giving feedback during exercises					●				
3. Getting knowledge of the learning results ↔	Testing PAM									
	8. Checking what learning outcome is reached and establishing whether this satisfies the standard							●		●
4. Improving the execution of PAM ↔	Feedback after a test									
	9. If it is below standard eliminating the cause of the mistakes							●	●	

* SAP = systematic approach to problem solving
** KR = key relations
\+ = class-based exercises
● denotes this procedure or means should make a major contribution to the realization of this function.

Fig. 4. Overview of relations between phases of the learning process, instructional functions and the instructional procedures and means used in the thermodynamics courses.

We therefore selected procedures that differed as little as possible from the procedures teachers are used to in our university. This means that the main procedures are lectures to a large group (± 80 students), supplemented with self-study of lecture notes and classes in which small groups work individually at problems with feedback from a teacher. The main characteristics of the instructional plan are described in this section. As Fig. 4 shows it was constructed by matching procedures and materials with instructional functions and integrating them into a consistent programme. One condition was made beforehand: once devised, the experimental course should not take more time from the teachers and students than the existing course.

In order to achieve a maximal execution of the instructional plan, we organized, before the course started, some training sessions for the three teachers to get used to the new procedures and materials. Once the new course had started we observed all lectures and small group activities to gather data for the evaluation of the instructional process. If there were discrepancies between the planned and the actual procedures the observer consulted the teacher about the causes for this, immediately after the session. Deviations from the plan that endangered the realization of a function were remedied as far as possible and steps were taken to prevent their re-occurrence.

We now discuss the most characteristic elements of the experimental instruction: SAP chart, SAP worksheet and key relations. At the end of this section, some details are given about the organization of the course.

4.2. SAP CHART AND SAP WORKSHEET

The Systematic Approach to Problem solving is presented to the students in several ways. The most important way is via the SAP chart as mentioned in 3.1. On this chart a survey of all heuristics is condensed to one page (see Fig. 2). In the lectures, these heuristics are illustrated by problems used as examples. The teacher uses the heuristics regularly when explaining concepts and laws in the lectures. In the classes after the lectures, the students are encouraged, when solving problems, to proceed in accordance with the heuristics as far as possible. In the first phases of the learning process they practice performing on paper the new actions and methods with completeness of all action links. The paper provided is a special worksheet with a lay-out reflecting SAP. The heuristics are represented on this sheet by key words. Figure 5 shows such a worksheet, with a worked problem (see section 1) on it.

The students in a class work individually or in small subgroups of two or three students. The teacher makes his rounds, checks their work, gives directions and explanation in accordance with the procedure of stage-by-stage exercising. This means, for example, that he avoids showing the students how to do the problem, because the students have to get practice in doing the problems by themselves. Only as a last resort should he actually solve a problem for a student because:

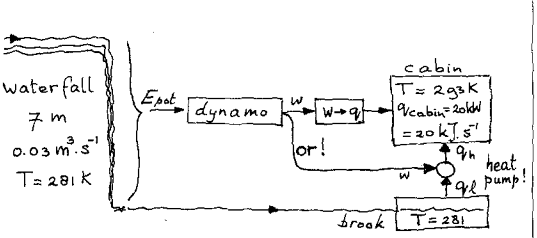
– if the student has made a mistake or does not know what to do, showing how to solve the problem gives too specific information. Probably the student will make the same mistake again in a slightly different problem. Instead the teacher should diagnose the gap in the orienting basis and so equip the student with transferable knowledge.

– if the student has no gaps in his orienting basis but has difficulties in applying his knowledge to a specific problem then the teacher should let him

Worksheet for the systematic approach to solving problems:

ANALYSIS

1. read
2. scheme
a. system
b. boundaries
c. content
d. states
e. processes
f. other data
g. graph
h. unknown
i. estimation



unknown: $q_{in\ cabin} \overset{?}{\geq} 20\ kJ.s^{-1}$

$\rho_{water} = 1000\ kg.m^{-3}$
$g = 10\ m.s^{-2}$
$1 W = 1 J.s^{-1}$

PLAN

4. select relations
a. KR-chart
b. general relations
c. from data

$$E_{pot.} = m.g.h$$

$$\frac{q_1}{q_2} = -\frac{T_1}{T_2} \quad (\text{Carnot-cycle})$$

$$dU = dq + dw$$

$$\oint dU = 0$$

5. check validity

6. Transformation to standard problem
a. unknown
b. relation in which unknown occurs
c. specification
d. new unknowns
e. new start
f. substitution

① direct conversion of $E_{pot.}$ in q with a dynamo
(assumption : no energylosses in the conversion, rather optimistic!)
$q = E_{pot.} = m.g.h. \rightarrow$ standard problem.
② more efficient use of work using a heat pump.
Assume that the heat pump uses a reversible Carnot-cycle:

$q_h \leftarrow$ $$\frac{q_1}{q_2} = -\frac{T_1}{T_2} \longrightarrow \frac{q_h}{q_l} = -\frac{T_{cabin}}{T_{brook}} \longrightarrow \text{only } q_l \text{ yet unknown}$$

$q_l \leftarrow$ $$dU = dq + dw \longrightarrow q_h + q_l + w_{needed} = \oint dU = 0$$

$$\rightarrow w_{needed} = q_h \frac{T_{brook}}{T_{cabin}} - q_h = q_h \left(\frac{T_{brook}}{T_{cabin}} \right) - 1 \longrightarrow \text{standard problem}$$

7. if not soluble
a. other (key) relation
b. alternate processes
c. assumptions

3. standard problem

$$E_{pot.} = 0.03 * 1000 * 10 * 7\ m^3.s^{-1}.kg.m^{-3}.m^2.s^{-2}.m$$

$$= 2100\ kg.m^2.s^{-2} = 2100\ N.m.s^{-1} = 2.100\ kJ.s^{-1}$$

Not enough!

$$w_{needed} = 20 kJ.s^{-1}\left(\frac{281}{293} - 1\right) = 0.81\ kJ.s^{-1}$$

available is $2.1\ kJ.s^{-1}$

EXECUTION OF ROUTINE OPERATIONS

8. calculation
and answer

So, the mountaineer has quite enough energy in the fall to maintain the temperature in the cabin at 20°C, if he uses a heat pump.

9. check
a.
b.
c. ok

if necessary:
10. tracking down mistakes

Fig. 5. Model elaboration of the problem in the box in section 1.

exercise on a lower stage of the learning process (if necessary with help). By showing the student how to solve the problem the learning process of the student is delayed, one should not provide more help than is needed.

The use of the worksheets allows the teacher to closely observe the work of each student. Consequently, the teacher is able to give precise feedback at an early phase. Besides correcting mistakes, the teacher also comments on the learning process of the students, e.g. when a part of the systematic approach is omitted prior to total understanding. In general, students can work reasonably well on their own, because they are guided by the heuristics. As the course proceeds, students continually execute parts of SAP faster and more automatically. This is, in fact, the intention, but every time new subject matter is introduced, the pace is slowed down in order to enable new elements to be carefully integrated, e.g. other aspects in the analysis and new key relations.

4.3. KEY RELATIONS

As indicated in phase 2 of the Programme of Actions and Methods, the core of the problem-solving process is linking up the unknown and data, using relationships between quantities. These relationships in science and technology usually result from laws, formulas, diagrams etc. Such quantitative relationships are referred to as "relations". An important part of all instruction is the derivation and explanation of such relations. In order to be able to use these relations in solving problems, the student must have at his disposal a structured survey of the most important relations. To be more exact: he must select and hold at his disposal the relations that are particularly suitable as starting point in solving problems. These relations are called *Key Relations.* The number of key relations has to be kept as small as possible, because then it is easier to remember both the relations and the conditions for their validity. Key relations must be formulated in such a way as to ensure their usefulness in the transformation of the problem. After a few lectures on a given topic the students are asked to produce a summary of key relations (a KR chart, see Mettes et al. [1980, 1981]) for that topic. Before they start working on problems in class, the teacher discusses these designs. He then hands out his own KR chart and, if necessary, comments on differences between the two. Students use the KR charts continuously during the problem-solving exercise and the teacher refers to these charts regularly when giving feedback. In this way, the students survey the core of the subject matter and use this survey to begin to master it. They also learn to acquire an important study skill: extracting and organizing subject matter.

4.4. THE ORGANIZATION OF THE COURSE

The organization of the course is – as we mentioned in par. 2.3 – based on a group based system of mastery learning. The course which is given in the third term for first-year students consists of 34 hours of lectures and 36 hours of classes, evenly spread over five study units. Each unit is finished by taking

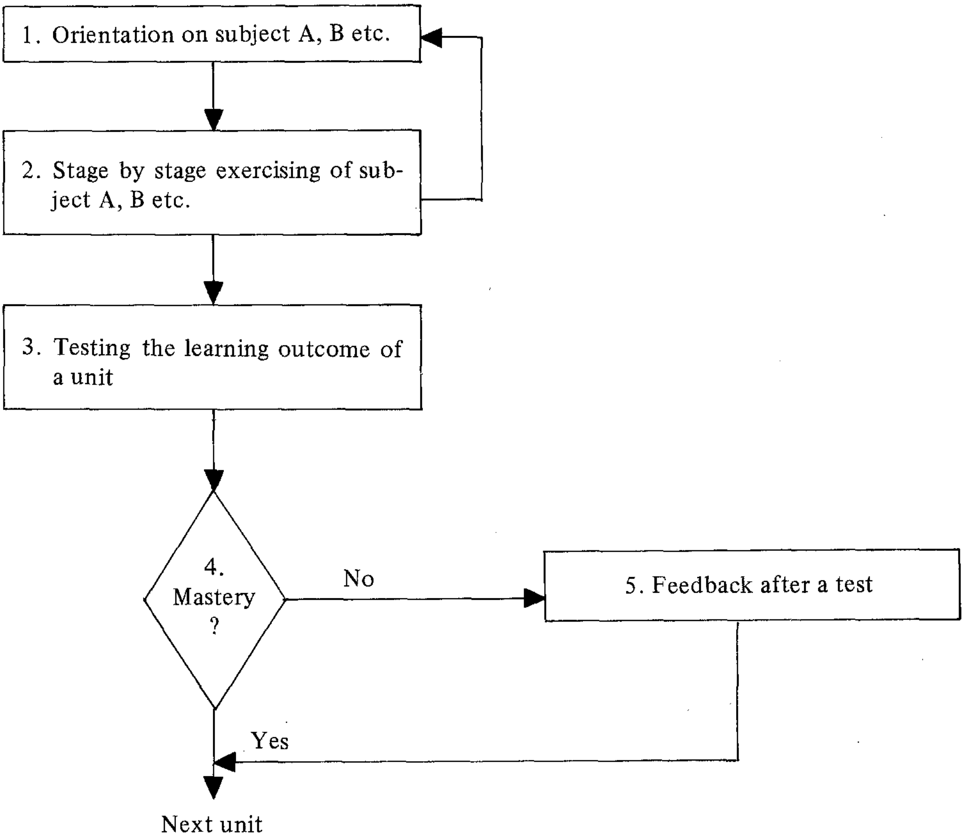


Fig. 6. Model of the instructional process in a unit of the course.

a test. In the class following the test the teacher discusses the main mistakes that have been made. Students who fail this test get extra opportunity to exercise under close supervision of the teacher. Students who do not reach mastery of the unit after this exercising nevertheless should start with the next one. Tests and remedial exercising are set at fixed times. Grading is not based upon the tests but on an examination at the end of the course. Figure 6 shows the model of the instructional process of a unit. This model relates the most important instructional functions (see Fig. 4) in a flow diagram.

5. Formative and Summative Evaluation

This section summarizes the answer to the third set of questions posed in section 1, concerning the results of the experimental course, the kind of criteria and data to be used to judge the worth of the new instructional programme. The evaluation was directed at the two main constituents of the experimental course: the PAM and the instructional plan. The aim of the formative evaluation was to improve both. For this purpose during the first experimental course information was gathered about the way teachers and students executed the planned instructional procedures and handled the instructional materials. This information was used to judge the degree to which the instructional functions had been realized.

After the course the information about the instructional process was related to the results of the course in order to detect the elements of the course that needed improvement. Like the formative evaluation, the summative evaluation contained information about the instructional process and the PAM as well as about the results of the course.

The most important criteria for judging the worth of the instructional process were:

1. The *feasibility* of the instructional plan: was it possible in the experimental course to teach according to the plan we devised? (the feasibility criterion).

2. The *functionality* of the instructional plan: was it possible in the experimental course to fulfil sufficiently the instructional functions? (the functionality criterion).

3. In judging the success of the experimental course we hoped above all that teachers and students would prefer to teach and learn in the way that is recommended in the instructional plan (*satisfaction* criterion).

In assessing the quality of the PAM and the heuristics on the SAP chart the following six criteria are used:

1. the extent to which it contains all the necessary action links and conditions;
2. the appropriateness for all relevant problems of the course;
3. the fitness for promoting the abbreviation and automatization of the performance of the actions;
4. the comprehensibility of the heuristics;
5. the suitability of the design of the charts;
6. the acceptance by teachers and students.

Our criterion variables for judging the results of the course were:

1. the learning outcomes of the students,
2. the time teachers and students spent on the course,
3. the satisfaction of teachers and students.

The original course ran for two years (1975 and 1976) without modification and was replaced by the new course in 1977–1979. We took the first two years as our "control" groups of students. So the summative evaluation involved two control groups (1975–1976) and three experimental groups (1977–1979). The lectures and classes of all courses were observed to gather data for the evaluation of the instructional process, except for the last experimental course in 1979. Because of this the results of this last course are considered to be representative for the results of a course in "normal" circumstances. The control courses were observed intensively to gather data for the construction of the experimental instructional plan and also to minimize differences which would arise from observing just the experimental group.

5.1. THE FORMATIVE EVALUATION

We summarize here the main conclusions of the formative evaluation of the PAM and the instructional process. The central conclusion of the formative evaluation of the Programme of Actions and Methods and the heuristics was that in general the criteria for assessing their quality were met; only minor changes (see Mettes and Pilot, 1980) were necessary.

The conclusions of the formative evaluation of the instructional process are summarized together with the main suggestions for improvement:

1. Most instructional procedures and materials were carried out according to the instructional plan.

2. There were some deviations from the planned process so that both the functionality and the feasibility of the instructional plan needed improvement. The feasibility of the plan had to be improved by training the teachers in supervising the exercising. To maximize adaptation of the exercising procedure by the students, PAM and the instructional plan were implemented in the Introductory Course in Thermodynamics in the first trimester of their first year. We were convinced students who met this at the beginning of their first-year's course would more easily accept and use the PAM and the exercising procedure because in this way the introduction of PAM and exercising procedure was integrated in the introduction of the subject matter. As a consequence the students had little chance of developing a (less suitable) way of problem solving before the Thermodynamics course started.

5.2. SUMMATIVE EVALUATION OF TWO EXPERIMENTAL COURSES

In the summative evaluation the decision had to be taken on whether to continue the experimental course. This decision had to be based on three value judgements:

1. Does the PAM meet its criteria of quality?
2. Does the instructional process meet the criteria of feasibility and functionality?
3. Are the results of the experimental course better than those of the control course?

For the first two judgements data were used from the introductory course and the Thermodynamics course in 1978. As in the formative evaluation we summarize the conclusions below (for more information see Mettes and Pilot, 1980).

PAM

The general conclusion of par. 5.1 was that the PAM and the heuristics derived from it were useful instructional tools. Only minor changes were proposed. The data in the summative evaluation showed that these changes were certainly improvements. The data also indicated that still more explicitness on the SAP chart might be relevant on some minor points:

- the relation between analysis of the problem and the action of replacing general by specific quantities in the key relations;
- "hidden" key relations. Hidden key relations are general relations students know very well but do not think of using in solving a problem, e.g. the relation: the sum of all fractions is one.

We concluded that the PAM and SAP chart meet our original criteria of quality, but like most things could be improved slightly, as indicated.

Instructional process

Again the changes suggested in the formative evaluation appeared to be improvements. Especially successful were the implementation of the experimental instruction into the introductory course in the first trimester: this gave more time to exercise problem solving in the Thermodynamics course in the third trimester because the students were already used to this type of instruction. All instructional procedures and materials were carried out and used sufficiently according to the instructional plan. (As a consequence all functions were sufficiently realized.) It appeared that the extent of the subject matter to be mastered limited the time available for exercising in the materialized form. Also the teachers had barely enough time for diagnosing the mistakes made by the students. From the data of the summative evaluation we concluded that the instructional plan met the criteria of feasibility and functionality.

Results

At the beginning of this section we described three criterion variables for judging the results of the experimental courses: learning outcomes, time spent by teachers and students, and satisfaction with the course. On each of these variables we defined an absolute standard:

- The percentage of 70–75% sufficient marks. We chose this first standard in reference to the mean of 57% sufficient marks in the control courses. A gain of 15–20% seemed the maximum possible gain in view of the high-level objectives of the course.
- The study load or nominal time which indicates the mean time the department expects the students need for a course (110 hours for this course).
- The maximum acceptable percentage of students dissatisfied with the course: this is 20%.

For the learning outcomes a relative standard was used as well: a comparison with the outcomes of the control courses should be in favour of the experimental courses.

Table I shows the mean *exam scores* and the percentage sufficient marks of the experimental and control courses. The scores in the courses 1976, 1977 and 1979 are equated by the equipercentile conversion (Angoff, 1971, p. 564). The examinations in the other two courses are not comparable because they probably vary in range and level of difficulty.

TABLE I

Mean Exam Scores, Standard Deviations, Numbers of Students and Percentages of Sufficient Marks

	Control courses		Experimental courses		
	1975	1976	1977	1978	1979
Mean	5.8	5.7	6.9	6.1	7.3
S.d.	1.6	1.9	1.7	1.9	1.8
n	19	43	32	52	51
% s.m.	54	61	85	69	79
n	22	49	33	52	53

The percentage of students obtaining sufficient marks on the experimental courses in 1977 and 1979 meet the standard of 70–75%, that of the control course in 1976 does not.

Because the entrance qualifications of the students in the courses differed to some extent, we used ANCOVA (analysis of covariance) to assess a treatment or course effect. The covariates in this analysis were the scores for the high school examinations in mathematics, physics and chemistry, which in the Netherlands are controlled by a central examination board. The assumptions involved in analysis of covariance: homogeneity of variance, normality of distributions and homogeneity of regression were met (Mettes and Pilot, 1980). The data of the ANCOVA are shown in Table II. The course effect is significant, but much more variance is explained by the sum of the covariates. The variance explained by course effect and covariates together is less than the error variance.

The students voluntarily noted each day *the time spent* on the course on computer cards that had to be placed weekly in a box in the department building. When a student did not deliver his cards in time he was reminded in person or by phone. This procedure of time measurement functioned quite

TABLE II

Analysis of Covariance of the Exam Scores

Source	Sum of squares	df	Mean square	F	Significance of F
Covariates	177.34	3	59.11	27.37	0.00
Mathematics	32.40	1	32.40	15.00	0.00
Physics	20.70	1	20.77	9.62	0.00
Chemistry	3.60	1	3.60	1.67	0.20
Course effect	26.11	1	26.11	12.09	0.00
Error	261.32	121	2.16		
Total variance	464.77	125	3.72		

well until the 1977 course. In 1977 and 1978 only 56% of the students delivered all their cards in time. This percentage of participants is too small to be representative. For this reason we asked the students of the 1978 course, who had not delivered their cards, to estimate at the end of the course the hours they had spent. By combining these estimates with the data of the students who did hand in their cards, we got data on the time spent by 96% of the students. Table III contains the data of all courses, except the course

TABLE III

Mean Numbers of Hours Spent on the Control and Experimental Courses, Standard Deviations and Percentages of Participants

		Control courses		Experimental courses		Nominal time
		1975 (*n* = 22)	1976 (*n* = 49)	1977 (*n* = 33)	1978 (*n* = 52)	
Hours spent on the course	mean	98	102	115	95	110
	s.d.	26	25	39	26	
Hours spent in the third trimester	mean	368	335	384	315*	370
	s.d.	87	76	91	87	
Percentage of participants		95	73	58	96	

* Percentage of participants : 54.

in 1979, that had to be the first experimental course without any interference by the researchers. Only the mean time in the first experimental course in 1977 exceeds the nominal time. However, this difference is far too small to be of any significance. The mean time in the experimental and control courses does not differ significantly either.

The time spent by the teachers was not measured. We asked every teacher after each experimental course to estimate if the course was more time consuming than the control courses. The general conclusion from these estimations is as follows. The experimental courses do not consume more time or only a little more, and even this is expected to vanish when more experience with this new way of teaching has been gained.

Both students and teachers were satisfied with the lectures, classes and new instructional materials (charts and worksheet). In the questionnaire of the 1978 course – the last course where the students filled out the questionnaire – 85% of them answered that the experimental treatment should be introduced by teachers of similar courses.

5.3. SUMMARY

The examination scores of two experimental courses came up to the desired standard of 70 to 75% sufficient marks; in the other experimental course this criterion was almost met. The means of the exam scores of the experimental courses were significantly higher than those of the control courses. There is no indication that students spent more time in the experimental courses. Although the teachers spent a little more time this time difference is expected to disappear. Both teachers and students prefer the experimental treatment. The results of the experimental course "Introduction in Thermodynamics" given in the first trimester (see 5.1) were the same or even better (Mettes and Pilot, 1980). The quality of the PAM and the feasibility and functionality of the instructional plan were judged favourably by both students and lecturers. Based upon the criteria for the evaluation, our conclusion is that the experimental treatment is superior to the control treatment. This means that the first objective of our project (see section 3) is realized. The next section is devoted to the second objective.

6. Generalizations on Instructional Development and Evaluation

The second objective of this project was the formulation of a set of generalizations on development and evaluation of instruction in problem solving in science. In this section we shall summarize the most important generalizations. These generalizations have to be considered as hypotheses derived from our experiences in this project. As can be expected we describe two sets of generalizations, one concerning the construction of a PAM and one concerning the development of an instructional plan to teach the PAM. Research on these hypotheses has already rendered positive results (Van Weeren et al., 1980; Kramers-Pals et al., 1980).

6.1. THE CONSTRUCTION OF A PAM

This paragraph describes only the principal characteristics of a procedure for the construction of a PAM. The procedure consists of ten steps that are summarized in Fig. 7. At the moment we believe that the validity of this procedure is limited by at least two conditions:

1. The problem solving to be learned must concern specification problems (see par. 3.1.).
2. For solving these problems it is necessary, among other things, to use as transformations a limited set of quantitative relations.

Within these limitations the procedure can be used generally because of the great analogy between specification problems in Thermodynamics and other science subject matter areas. Empirical evidence has been found by Van Weeren et al. (1980) and Kramers-Pals et al. (1980).

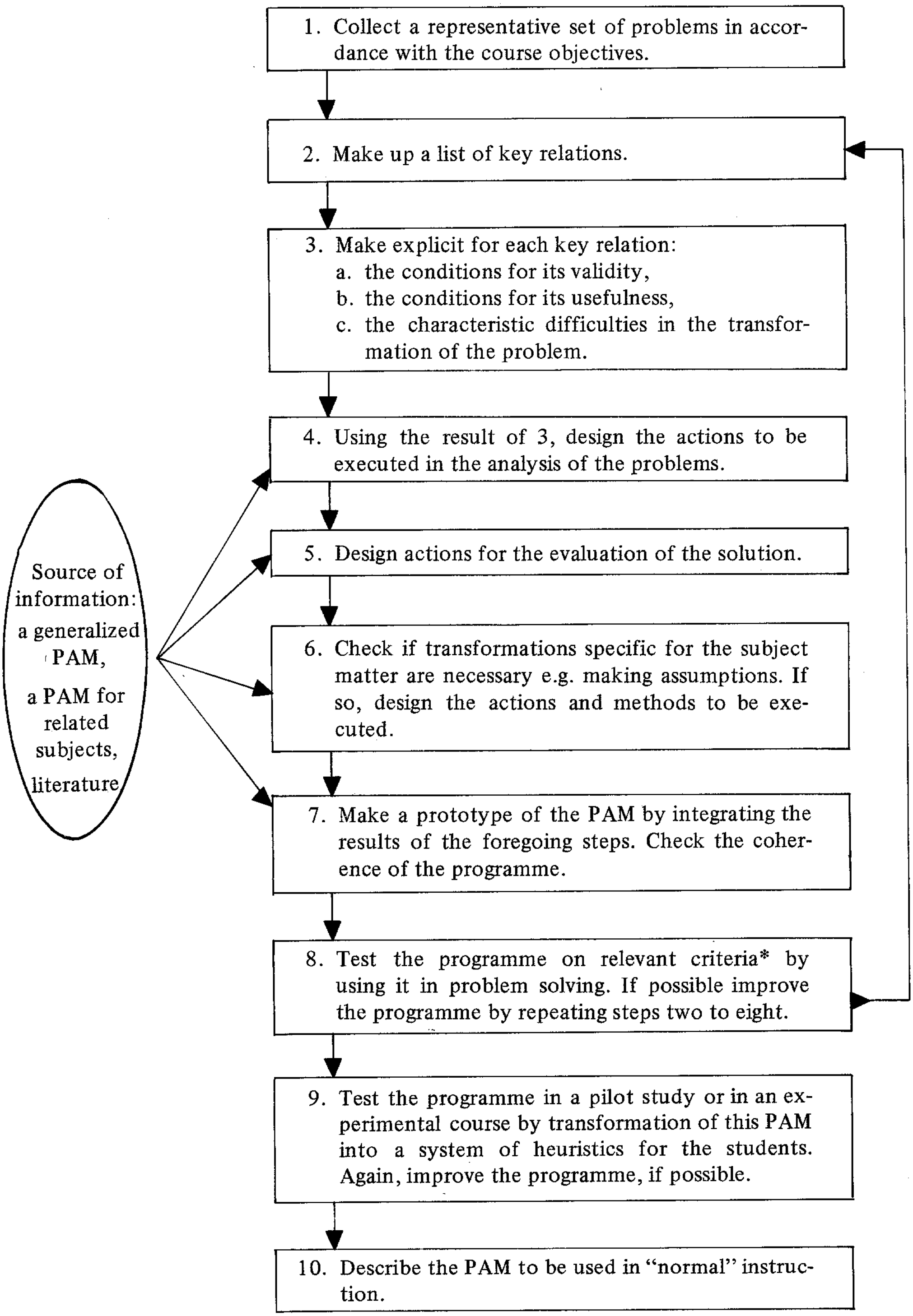
6.2. THE DEVELOPMENT OF AN INSTRUCTIONAL PLAN

The generalizations about the development of the instructional plan are also presented in the form of a procedure. This procedure is only partly new, a number of steps can be found in the literature on development and evaluation of instruction (e.g. Davis et al., 1974; Gagné and Briggs, 1974; Scriven, 1974; Stufflebeam et al., 1971). As far as we can see, there are no limits to the validity of this procedure. Of some relevance might be the condition that the set of transformations to be used in problem solving has to be finite. Because almost all steps have been described in the previous sections, this section ends with Fig. 8 showing the total procedure in one flowchart.

6.3. FINAL REMARKS

Using East European learning theories as our starting point (e.g. De Corte, 1980), we have derived some hypothetical generalizations for the construction of a PAM and the development of an instructional plan for courses in problem solving. Our research and development is now directed to test the effectiveness of these generalizations for quite a different type of course: problem solving in political administration.

If readers are interested in more information about experimental results and generalizations than could be given in this article, they can contact us.



1. Collect a representative set of problems in accordance with the course objectives.
2. Make up a list of key relations.
3. Make explicit for each key relation:
   a. the conditions for its validity,
   b. the conditions for its usefulness,
   c. the characteristic difficulties in the transformation of the problem.
4. Using the result of 3, design the actions to be executed in the analysis of the problems.
5. Design actions for the evaluation of the solution.
6. Check if transformations specific for the subject matter are necessary e.g. making assumptions. If so, design the actions and methods to be executed.
7. Make a prototype of the PAM by integrating the results of the foregoing steps. Check the coherence of the programme.
8. Test the programme on relevant criteria* by using it in problem solving. If possible improve the programme by repeating steps two to eight.
9. Test the programme in a pilot study or in an experimental course by transformation of this PAM into a system of heuristics for the students. Again, improve the programme, if possible.
10. Describe the PAM to be used in "normal" instruction.

* See section 5.

Fig. 7. Summary of the ten steps for the construction of a PAM.

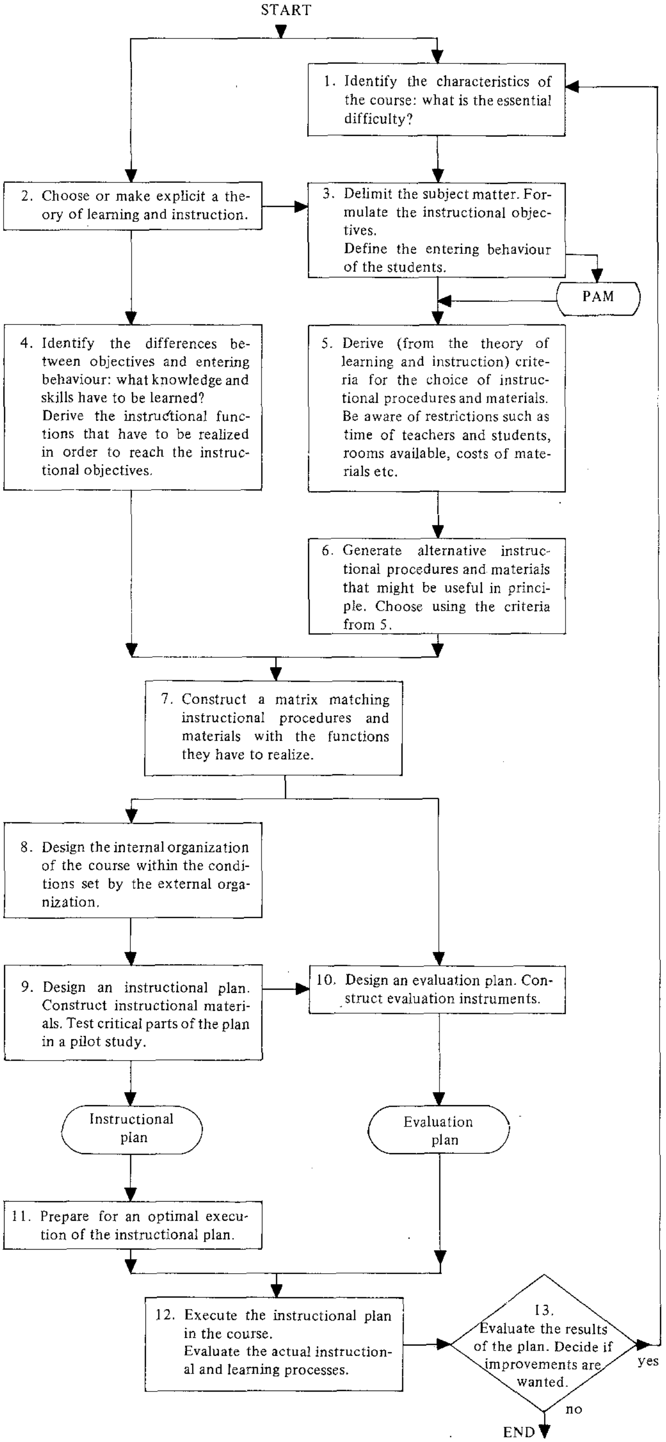


Fig. 8. Summary of the procedure for the development of an instructional plan. The arrows indicate a rational sequence.

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