HARMONIC RATIOS OF INVERTED-U TYPE III BURSTS*

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Abstract. The harmonic ratios of a large sample of inverted-U bursts are found to be smaller at the turning frequency than at the starting frequency. Ratios <2.0 are explained by postulating that the lowest fundamental frequencies emitted are prevented from escaping from the corona by an evanescent region between the source and the observer. This concept is used to construct a source model for inverted-U bursts where the density is lower inside a magnetic flux tube than it is outside.

1. Introduction

To date only a few measurements of the frequency ratio of harmonic type III burst pairs have been made because of the difficulty of observing clearly defined spectral features within the two components. On the low time-resolution ($1\ \mathrm{cm\ min}^{-1}$) Culgoora and Dapto spectrograms the fundamental and harmonic components tend to merge into a single burst owing to their large instantaneous bandwidths and fast frequency drift-rates. Almost all the measurements made so far refer to variants of the type III burst in which the frequency drift-rate decreases to zero at the lowest observed frequency, the so-called inverted-U or inverted-J burst. The observational results given below confirm earlier measurements (Wild *et al.*, 1964; Stewart, 1963) which showed that the harmonic ratios cover a wide range and are often less than 2.0. A possible explanation for this result is outlined below.

2. Observations

2.1. METHOD OF MEASUREMENT

A careful selection was made from Culgoora spectrograph records of clearly defined harmonic burst pairs. Most of these were inverted-U or inverted J-bursts. Some 28 bursts were selected which, when combined with an earlier sample of 15 bursts from Dapto spectrograph records (Stewart, 1963), gave a total of 43 events for study. All of the bursts occurred within the 25 to 210 MHz frequency range. The harmonic ratio $R=f_H/f_F$ and differential group delay t_F-t_H were measured near two points on the spectrum: At A, the turning frequency (where the drift-rate decreases to zero), and at B, the starting frequency. To do this the following method was used. Outlines of the fundamental and harmonic components were prepared by measuring points along the leading and trailing edges of the burst; the leading edge is usually very well defined and

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has almost peak intensity. The contours were then superimposed so that the leading edges of the fundamental and second harmonics coincided near either A or B. Sometimes (as in the example given in Figure 1) A could not be located precisely. In this case the value of t_F–t_H recorded for A was determined from fitting the contours along the curved portion of the leading edge (where the drift-rate decreases rapidly with decreasing frequency). Usually the harmonic ratio and differential group delay are different at points A and B. In the example of Figure 1, $R = 1.8$ and $t_F - t_H = +2.5$ s at A while $R = 2.0$ and $t_F - t_H \sim 0$ s at B. A positive sign for $t_F - t_H$ means that the fundamental arrives at the Earth after the harmonic.

It should be pointed out that if one simply measures the instantaneous frequency ratio along parts of the leading edge of the burst where the frequency drift-rate is high, then

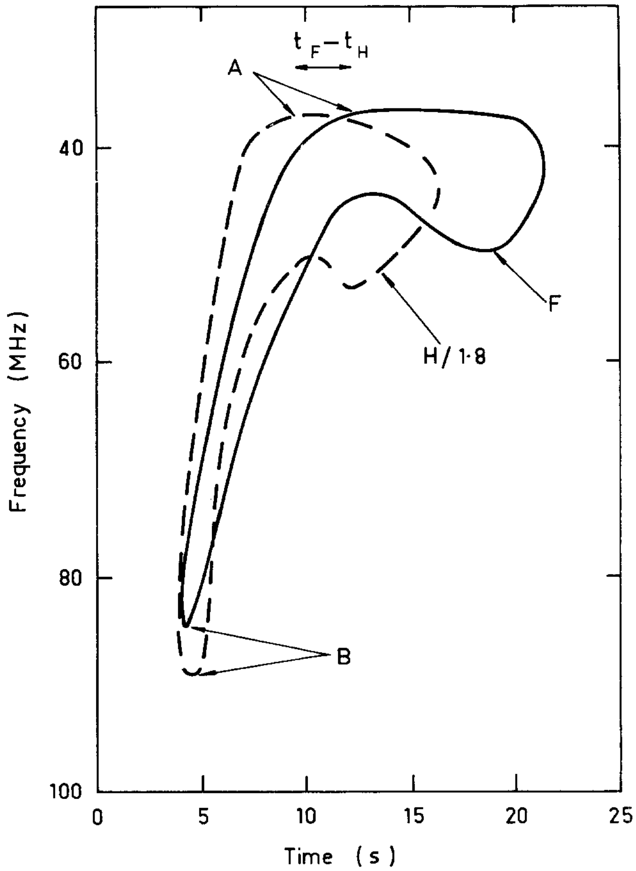

Fig. 1. Example illustrating the method of measuring the harmonic ratio, R, and differential group delay, $t_F - t_H$, between a fundamental and second harmonic inverted-U burst on 1971, February 23 00^h05^m UT. The full line shows an outline of the most intense part of the fundamental burst. The dashed line shows the corresponding outline for the harmonic, redrawn at 1/1.8 of the frequency. This harmonic ratio and a shift in time of the magnitude indicated by ($t_F - t_H$) produces a good fit of the low-frequency edges of the two components near A; to fit the starting points requires division of the harmonic frequencies by 2.0, and a much smaller value of $t_F - t_H$(~ 0 s).

the harmonic ratio is underestimated. Consider, for example, emission from the 80 MHz plasma level in a Newkirk active corona (Newkirk, 1961). Owing to the differential group delay the 80 MHz fundamental emission arrives at the Earth ~0.5 s later than the 2 × 80 MHz second harmonic emission and simultaneously with 2 × × 72 MHz harmonic emission from the 72 MHz plasma level. (In ~0.5 s the electron stream with a velocity $\sim c/3$ travels from the 80 MHz to the 72 MHz plasma level.) Hence the group delay would cause a harmonic ratio measured in this way to be $R = 2 \times 72/80 = 1.8$ instead of $R = 2.0$. This problem was avoided in the present study by measuring harmonic ratios only along the turning arc A where the frequency drift is close to zero and at the starting frequency B. In general, the measurement of R will be more precise at A than at B because the much longer duration of the turning arc allows many more points to be used to ensure a good fit.

2.2. OBSERVATIONAL RESULTS

The harmonic ratio measurements are summarized in Figure 2. At the turning frequency A most values of R lie between 1.6 and 2.0, with an average value 1.80 ± 0.14.

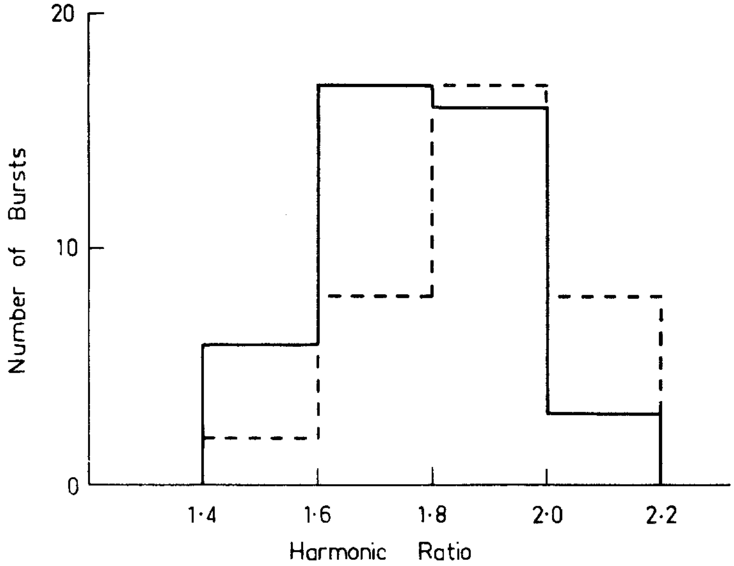

Fig. 2. Histograms of the harmonic ratio of 42 inverted-U bursts. The full line refers to measurements at the turning frequency, the dashed line to measurements at the starting frequency.

At the starting frequency B the average value is somewhat higher, 1.91 ± 0.15. Individual bursts show the same tendency for R to be lower at A than at B (see Figure 1). The differential group delay varies from +1 to 5 s at A with a mean value ~ +2.0 s and from 0 to +2 s at B with a mean value ~ +0.5 s Most bursts have turning frequencies between 30 and 50 MHz and starting frequencies between 60 and 100 MHz.

3. Interpretation

3.1. EARLY INTERPRETATIONS

The original explanation (Wild *et al.*, 1954) for harmonic ratios <2.0 was that the low-frequency edge of the fundamental component cannot escape from the corona. It was assumed that emission occurs over moderately large bandwidths centred on the fundamental plasma frequency and its second harmonic but that refraction effects restrict observation to frequencies $f > f_c$ is the critical escape frequency of the corona. (In a spherically symmetrical corona, such as the Baumbach Allen model, f_c is of the order of $f_p \sec(0.87\,\theta)$, where f_p is the plasma frequency and θ is the angle between the radius vector at the source and the line of sight.)

Roberts (1959) in a study of 19 harmonic type II bursts found that fundamental bursts could be observed almost anywhere on the solar disk and that the harmonic ratio did not decrease with radial distance as Wild *et al.* (1954) predicted. He proposed two modifications to their theory. Firstly, he supposed that small-scale density irregularities in the corona near the plasma frequency level cause radiation in the fundamental band to be scattered through large angles. Detailed calculations of the combined effects of refraction and scattering in the corona by various authors (Steinberg *et al.*, 1971; Riddle, 1972, 1974; Leblanc, 1973) support this supposition. Secondly, Roberts supposed that the observed bandwidth of emission at any one time is determined primarily by the range of densities in the excited region of the corona and not by the natural band width of emission. He assumed, following Wild *et al.* (1954), that the lower half of the fundamental's natural bandwidth of emission will still be removed for each individual source region which is small enough to be considered homogeneous, but that the integrated profile of the many, neighbouring fundamental components will resemble that of the second harmonic except that its peak frequency will exceed half the peak frequency of the second harmonic by a frequency of the order of the natural bandwidth of emission. On this basis he took the observed mean harmonic ratio of type II bursts of ~ 1.95 to imply that the natural bandwidth of plasma emission is only a few percent.

Recent theoretical work (Melrose, 1974) has shown that the above explanation for a low-frequency cut-off of the fundamental band is untenable because the natural bandwidth of plasma ration is extremely small and restricted to frequencies equal to or slightly greater than the fundamental plasma frequency f_p; amplification of plasma emission occurs only for frequencies, f, such that

$$f - f_p \leqslant \tfrac{3}{2}\frac{V_e^2}{V_\phi^2} \approx 10^{-3}$$

(Melrose, 1974). Here V_e is the mean thermal velocity of coronal electrons and V_ϕ, the phase velocity of plasma waves, is approximately equal to the stream velocity of the exciting electrons; the latter has a mean value of $\sim c/3$.

3.2. An alternative interpretation

We accept the theoretical limit that $f - f_p \lesssim 10^{-3}$ and assume that the large instantaneous bandwidth of type III bursts arises because the electron stream at any instant covers a range of plasma frequency levels. It is easily shown that the optical depth for fundamental radiation from the plasma level propagating outwards through the corona decreases with decreasing frequency (see e.g. Figure 9B of Jaeger and Westfold, 1950). Hence attenuation by the medium will not produce a low-frequency cut-off. However, if the radiation encounters a higher density region where f is less than f_p for the lowest frequencies of the fundamental band (so that they become evanescent) then a low-frequency cut-off will occur and the observed harmonic ratio will be <2.0. This concept is used below to postulate a source model for inverted-U bursts.

3.3. A schematic source model for inverted-U bursts

Consider the model of Figure 3. Plasma waves and plasma radiation are excited within a curved magnetic flux tube by a stream of energetic electrons. Because of the finite

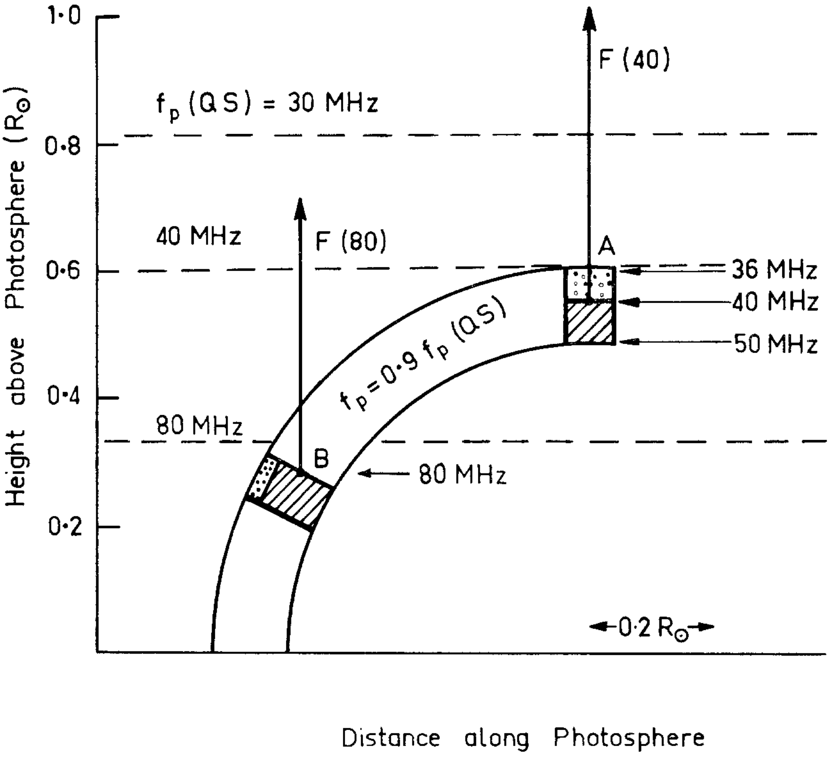

Fig. 3. Schematic drawing of an underdense toroidal flux tube as the source of an inverted-U burst. The dashed lines indicate plasma levels (labelled) in the Newkirk quiet Sun model. The figures and light arrows indicate plasma levels within the tube. The heavy rectangles show the range of plasma frequencies excited at two particular instants of time. Fundamental plasma radiation can escape from the shaded regions but not from the stippled regions (see text).

width and depth of the stream a wide range of plasma frequencies is excited at any instant. We assume for simplicity a toroidal flux tube containing plasma with a density 0.81 of that outside; the density of the corona is assumed to be given by the Newkirk (1961) quiet sun model. The heavy arrows indicate the approximate direction of propagation for radiation (along the radial direction where the density gradient is steepest) from two points, A where the tube is tangential to the Sun, and B where the tube is almost radial; this is an oversimplification, because scattering on density irregularities combined with refraction must bend the rays through large angles before they can reach the Earth.

All emitted harmonic frequencies f_H can escape from A and B because f_H is greater than the plasma frequency f_p along the ray paths. However, the lowest emitted fundamental frequencies f_F from A cannot escape because f_F is less than the plasma frequency just outside the flux tube. For the radial ray from A the predicted harmonic ratio R is equal to $72/40 = 1.8$. Integrating over all possible rays paths from A to the

TABLE I

Model calculations[a]

Calculated values	From the turning point A ($f_p = 40$ MHz)	From the starting point B ($f_p = 80$ MHz)
Differential group delay, $t_F - t_H$ (s)		
(1)	+1.0	+0.6
(2)	+1.1	+0.6
(3)	+1.1	+0.6
Fundamental optical depth, τ_F		
(1)	0.7	1.6
(2)	0.8	1.6
(3)	0.8	1.6
Harmonic optical depth, τ_H		
(1)	0.1	0.2
(2)	0.1	0.2
(3)	0.1	0.2
Predicted harmonic ratio, $R = f_H/f_F$		
(1)	~ 1.8	~ 2.0
(2)	~ 1.6	~ 2.0
(3)	~ 1.4	~ 2.0

[a] Calculated differential group delay and optical depth (accurate to 10 %) of fundamental and second harmonic emission from the turning and starting points (A and B of Figure 3) within a magnetic flux tube of density (1) $N = 0.81\ N_{QS}$, (2) $N = 0.64\ N_{QS}$ and (3) $N = 0.49\ N_{QS}$ is given by the Newkirk quiet Sun density model. An electron temperature of 2×10^6 K was assumed.

observer will not change this value significantly because refraction near the plasma level beams the fundamental radiation outwards along the radial direction. On the other hand, at B the lowest emitted fundamental frequency f_F is equal to the plasma frequency just outside the flux tube. In this case the predicted harmonic ratio will be $R \sim 80/40 \sim 2.0$. Hence the harmonic ratio will tend to be smaller at A than at B, as observed. (Harmonic ratios at the turning frequency as low as 1.5 would occur with this model if the tube density were only 0.55 times the ambient density. This would provide an alternative explanation for those bursts described by Takakura and Yousef (1974) as second and third harmonics. However, the absence of a time delay between the two components supports the latter interpretation.)

To test our model semi-quantitatively calculations for the optical depths and differential group delays of the radial rays from A and B of Figure 3 have been carried out. The Newkirk density model is approximated by an exponential model whose gradient is a slowly varying function of height (see e.g. Appendix, Wild *et al.*, 1959); the results are shown in Table I. The calculated values for optical depth and their increase with frequency support the earlier statement that the low-frequency cut-off cannot be explained by attenuation. The calculated differential group delay $\sim +1$ s at A is smaller than the average observed value $\sim +2$ s. This would be explained if the effects of ray scattering on density irregularities were to increase the ray paths of fundamental radiation relative to those of harmonic radiation. However, current scattering models (Leblanc, 1973; Riddle, 1974) do not predict any appreciable increase.

4. Discussion

There is fairly convincing observational evidence to support the picture of inverted-U bursts being excited by electron streams guided along magnetic loops in the corona (e.g. Labrum and Stewart, 1970; Sheridan *et al.*, 1973). These loops must extend over great distances to explain the low turning frequencies observed (e.g. $f_F = 40$ MHz corresponds to a height $\geqslant 1.6\ R_\odot$). Since loops of this kind have not been observed directly we cannot say whether these flux tubes contain plasma of lower density than the surrounding corona.

The basic argument used above (rather than the specific source model) – i.e. that harmonic ratios < 2.0 can only be explained by the removal of the lowest fundamental frequencies by an evanescent region between source and observer – may also apply to normal type III and type II bursts.

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