BEAM-RETURN CURRENT SYSTEMS IN NONTHERMAL SOLAR FLARE MODELS

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Abstract. Previous investigations of return currents driven by suprathermal electron beams in solar flares have been based both conceptually and mathematically on analyses of electron beams in the laboratory environment. However, the physics of laboratory electron beams is fundamentally different from the physics of solar flare electron beams. Consider first the laboratory beam, which is injected into the plasma from an external source and is, therefore, modeled as a semi-infinite charged rigid rod. The longitudinal electrostatic field of such a charged rod has no preferred direction and therefore cannot drive a return current. Consequently, in the laboratory the return current is established inductively through the appearance of the changing magnetic field associated with the rising beam current, there being no offsetting displacement current term in such a geometry. It subsequently decays on the resistive time-scale; because of this decay, the net current of the system increases, and the lifetime of the electron beam becomes limited by self-pinching effects. Therefore, in the laboratory, the beam/return current system cannot reach a steady state.

By contrast, the electron beam in the solar flare forms *in situ* and the longitudinal electrostatic field is produced by charge separation. Such an electrostatic field *does* have a preferred direction and so *can* drive a cospatial return current. Further, the magnetic field generated by the beam current is always close to being offset by either the magnetic field associated with the displacement current ($\partial E/\partial t$) or the electrostatically-driven return current; hence, inductive fields are never important. Thus, in the solar flare the return current is principally established by *electrostatic* fields; the return current is continuously driven and does *not* decay resistively. Thus, if the acceleration mechanism drives a steady beam current, then the beam/return current system rapidly achieves a steady state. We present in this paper analytic expressions for the approach to this state.

1. Introduction

At present, analyses of beam-driven return currents in astrophysical plasmas such as solar flares (Brown and Bingham, 1984; Spicer and Sudan, 1984), are based on the plasma physics developed to describe electron beams in the laboratory environment (for comprehensive reviews see, e.g., Miller, 1982 and Sudan, 1983). However, the acceleration and propagation of electron beams in the astrophysical environment occurs under conditions that are very different from the laboratory, and this leads to qualitative differences in the physics involved, as we show in this paper.

In the laboratory, electrons are accelerated externally and injected into the plasma. Therefore the acceleration and propagation regions are independent and uncoupled. The beam is modeled as a charged rigid rod initially extending spatially from the origin to negative infinity (see Hammer and Rostoker, 1971; Miller, 1982). In this model the beam electrostatic self-field (created by the excess beam charge introduced into the plasma) has no preferred longitudinal direction and, therefore, cannot drive a return current. In fact, unless this electrostatic field is neutralized it will *prevent* the formation of a return current (Lee and Sudan, 1971). In the laboratory, the beam radius (typically

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1–10 cm) is usually orders of magnitude smaller than the surrounding plasma. Hence, the excess beam charge can be neutralized by transverse, radial motion of the background plasma electrons or ions (Lee and Sudan, 1971; Chu and Rostoker, 1973; Spicer and Sudan, 1984). Once the electrostatic field is neutralized the return current can be driven by the inductive electric field associated with the rising beam current. Given that the return current is established inductively, it must subsequently decay on the resistive time-scale $\tau_d = 4\pi R^2 \sigma / c^2$, where R is the beam radius, σ the electrical conductivity, and c the speed of light. For solar flare electron beams (which have radii of order 10^6–10^9 cm) this time-scale would be, for classical resistivity, of order 10^7–10^{13} s, orders of magnitude longer than any observed timescale associated with energetic beams. However, an essential point made by Brown and Bingham (1984) is that even though the decay of the return current is extremely slow the *absolute* value of the beam current in a nonthermal thick-target interpretation of solar flare hard X-ray bursts is so large (e.g., Hoyng, Brown, and van Beek, 1976) that even a small deviation from complete neutralization would result in a significant net current on observable time-scales. Once this net current exceeds the Alfvén–Lawson limit, or if the self-magnetic field of this current exceeds the background guide magnetic field, the beam self-pinches and stops propagating (see Sudan, 1983). The lifetime of the electron beam, therefore, depends on the rate of decay of the return current.

A detailed application of these ideas to solar flares was carried out by Spicer and Sudan (1984). They argued that since the only acceleration mechanisms known operate on a spatial scale of 10^6 cm or less, beam radii are also of order 10^6 cm. For such small beams the current density must be large and consequently unstable to ion-acoustic wave generation. In such a situation, the resistivity becomes anomalously high and the time-scale τ_d can be reduced by perhaps four orders of magnitude from the classical value. On this basis Spicer and Sudan (1984) calculated that the electron beam self-pinches on flare time-scales. Thus they suggested that the steady-state hypothesis assumed by previous authors (e.g., Knight and Sturrock, 1977) is not valid. However, their analysis, which may have validity in the small-scale laboratory environment, does not apply to solar flares, as we show here.

The first notable difference between the laboratory and flare environments is that in the solar flare electrons are accelerated *in situ*, not injected into the plasma by an external source; this produces a coupling of the acceleration and propagation regions which cannot be overlooked. As electrons are accelerated out of a region they leave behind unbalanced positive charge that must be neutralized to avoid the build up of large electrostatic potentials. Therefore, the acceleration and propagation regions are coupled by the return current *only along the electron path* (no 'ground wires' are present)*. Hence,

* Those acceleration mechanisms that operate within a closed circuit do not increase the net current of the preflare plasma-magnetic field configuration and so do not require a return current. Note, however, that any such mechanism would be incompatible with a nonthermal thick-target interpretation of solar hard X-ray bursts in that the number of electrons that can be accelerated is limited by inductive considerations to the preflare current I_0, i.e., to $I_0/e \simeq 10^{31}$ electrons s^{-1} (unless the acceleration region is highly filamented; Holman, 1985). This is at least five orders of magnitude smaller than the number of electrons required by a nonthermal thick-target interpretation of solar hard X-ray bursts (e.g., Spicer, 1982).

the beam cannot be modeled as a semi-infinite rigid rod. The second qualitative difference between the laboratory and astrophysical environments is that radical motion is not important in the large-scale astrophysical environment.

We assume, as in all other investigations of return current formation (Knight and Sturrock, 1977; Brown and Bingham, 1984; Spicer and Sudan, 1984), the accelerated electrons form one (uni-directional) beam (i.e., the acceleration mechanism does not produce many fine-scale oppositely directed beams). Given the number of electrons required by the nonthermal thick target model of hard X-ray bursts (e.g., Hoyng, Brown, and van Beek, 1976), the beam radius must be a large fraction of the radius of the flaring loop to avoid unacceptably large electron fluxes and current densities (Emslie, 1981). If the current density exceeds the threshold for various plasma instabilities, the solar plasma will be rapidly heated, producing a thermal hard X-ray emitting plasma, whose existence is at variance with both the hypothesis of the nonthermal model and observations that point to a nonthermal mechanism for the hard X-ray emission (see, e.g., Canfield *et al.*, 1986; Emslie, 1988; LaRosa and Emslie, 1988). A 'pepper-pot' picture of the beams may be valid, but the total filling factor of such beams must be of order unity to account for the total electron injection rates implied by the hard X-ray observations. Thus considering the flaring arch as a sum of discrete acceleration and propagation regions does not alter the fact that since the beam fills the entire plasma, transverse, radial motion of the background charges (as discussed, in reference to the laboratory problem, by Chu and Rostoker, 1973), is an insignificant edge effect occurring only within a Debye length (≈ 1 cm) of the beam edge. Therefore, in the solar flare electrostatic fields cannot be shorted out by radial motion of charge. Note that radial expansion of the beam due to the radial electrostatic self-field must be balanced by a longitudinal guide magnetic field. The condition for radial equilibrium is $\Omega_c^2 > 2\omega_{pe}^2$, where the cyclotron frequency $\Omega_c = eB/m_e c$ and the plasma frequency $\omega_p^2 = 4\pi N e^2/m_e$. The nonthermal model therefore implicitely requires that this inequality is satisfied in a solar flare loop.

We thus see that results valid for the laboratory may have at best limited, and possibly no, applicability to the solar flare plasma. It is purpose of this study to re-examine the generation of return currents in large-scale astrophysical plasmas, with particular emphasis on the solar flare environment.

2. Return Current Generation

The essential differences between the laboratory and solar flare environments are (i) that the electrostatic field in the solar flare is produced by *charge separation* (not accumulation) and (ii) that radial motion of background charge, important in the laboratory, is rendered insignificant in the solar flare due to the large scale. Thus, in the solar flare the electrostatic field *does* have a preferred direction, is *not* shorted out, and *can* drive a cospatial return current. The electrostatic field generated by a solar flare electron beam is analogous to that generated by a parallel plate capacitor. As electrons are accelerated out of a region (by, e.g., stochastic electron acceleration by lower hybrid waves (Benz and Smith, 1987)) they leave behind unbalanced positive charge. Hence, the acceleration site is analogous to the positive plate and the beam the negative plate. The electrostatic field generated by the charge separation is simply $E = 4\pi\Sigma$, where Σ is the surface charge density on the equivalent plates. Integrating the charge continuity equation over space gives Σ in terms of the current density j:

$$\frac{\partial \Sigma}{\partial t} + j = 0\,. \tag{1}$$

The electrostatic field is, therefore, given by

$$\mathbf{E}_{es} = -4\pi \int_0^t \mathbf{j}\,\mathrm{d}t\,. \tag{2}$$

Note that $\mathbf{j}$ is the *net* current of the system, the beam current minus the return current. If we now consider Ampère's law, substituting from Equation (2) for the (time-varying) electrostatic field we obtain

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \mathbf{0}\,, \tag{3}$$

showing that $\nabla \times \mathbf{B} = \mathbf{0}$ for all times. Initially the magnetic field generated by the beam current is exactly balanced by that associated with the displacement current. At later times, the now significant $\mathbf{E}$ drives ohmically a return current that balances the beam current, causing both the net $\mathbf{j}$ and the time variation of $\mathbf{E}$ to vanish. This result is fundamentally different from the semi-infinite beam model: consider Ampère's law in its integral form

$$\oint \mathbf{B}\cdot \mathrm{d}\mathbf{l} = \frac{4\pi}{c}\int\!\!\int \mathbf{j}\cdot \mathrm{d}\mathbf{S} + \frac{1}{c}\frac{\partial \Phi_e}{\partial t}\,, \tag{4}$$

applied at a point within the beam. In the case of a semi-infinite beam, any surface that intersects the beam has the same net current passing through it, while surfaces that do not intersect the beam see no current but a growing electrostatic flux. Hence, $\nabla \times \mathbf{B} \neq \mathbf{0}$ for a semi-infinite beam and inductive effects will be important. However, in a solar flare, the acceleration site defines the point of origin of the beam. An Ampèrian surface that encompasses the acceleration site does *not* enclose a net current. Moreover, neither does it enclose a changing electrostatic flux: the electric field of an infinite parallel plate capacitor is zero outside the plates, thus to the extent that the beam radius is large the electric field outside the beam is effectively zero insuring that not only is $\nabla \times \mathbf{B} = \mathbf{0}$ but also $\partial \mathbf{B}/\partial t = \mathbf{0}$ (cf. Equation (3)). To put it another way, in the case of infinite beam radius $\nabla \times \mathbf{j} = \mathbf{0}$ so that by taking the curl of Equation (2) and comparing with Faraday's law, we find that $\partial \mathbf{B}/\partial t = \mathbf{0}$*. In the solar flare, therefore, inductive fields do not arise and so the return current is established purely electrostatically.

* We thank S.K. Antiochos for this succinct derivation.

Brown and Bingham (1984) have also argued that the return current in a solar flare is established electrostatically. Although conceptually their thinking was correct, they employed the mathematical model of Miller (1982), which assumes a semi-infinite beam. Given their mathematical model did not correspond to their physical interpretation, they were led to several inconsistent and misleading results: for example, they found that the electrostatic field decayed on a resistive time-scale and that the electrostatic field possessed a finite curl – an impossible result. We present in the next section a much simpler time-dependent mathematical model that describes self-consistently an electrostatically driven return current.

3. The Time Evolution of the Return Current

The time evolution of the return current can be studied via a generalized Ohm's law, neglecting the cross-field Hall and Pedersen currents, viz.,

$$\frac{\partial j_{ret}}{\partial t} + \nu_{ei} j_{ret} = \frac{\omega_{pe}^2}{4\pi} E \,, \tag{5}$$

where ν_{ei} is the electron-ion collision frequency and ω_{pe} is the electron plasma frequency. Substituting from Equation (2) for the electrostatic field, we obtain (noting that $\mathbf{E}$ is antiparallel to $\mathbf{j}$)

$$\frac{\partial j_{ret}}{\partial t} + \nu_{ei} j_{ret} = \omega_{pe}^2 \int_0^t (j_b - j_{ret}) \, \mathrm{d}t \,. \tag{6}$$

Equation (6) is a mathematical statement of the fact any imbalance between j_b and j_{ret} results in the growth of residual positive charge in the acceleration region producing an electrostatic field that accelerates the return current. This electrostatic field also decelerates the beam current and leads to a complicated feedback between the beam acceleration and propagation regions. In essence a second equation coupled to Equation (6) describing the time evolution of j_b is required to completely specify the problem. However, in order not to obscure the essential physical point made in this paper (i.e., that the electric field responsible for driving the return current is produced by charge separation and not inductive effects) we do not consider the feedback of the electrostatic field on the acceleration mechanism. In any case, to properly characterize this feedback, knowledge of the (presently unknown) micro-physics and structure of the acceleration mechanism would be required. Given the uncertainties in this area we shall assume for purposes of illustration that the acceleration mechanism simply drives a steady current j_b. For a constant j_b, and noting the initial conditions

$$j_{ret}\big|_{t=0} = \frac{\partial j_{ret}}{\partial t}\Big|_{t=0} = 0 \,,$$

the solution to Equation (6) is

$$j_{ret} = j_b \left\{ 1 - e^{-(\nu_{ei}/2)t} \left[\cos(\omega_{pe}\alpha t) - \frac{(\nu_{ei}/2)}{\omega_p \alpha} \sin(\omega_{pe}\alpha t) \right] \right\}, \tag{7}$$

where $\alpha = (1 - \nu_{ei}^2/4\omega_{pe}^2)^{1/2}$.

We see from the result (7) that for times $t \gg 2\nu_{ei}^{-1}$ the beam-return current system reaches the steady-state solution $j_{ret} = j_b$. For the short time that $j_{ret} \neq j_b$ the resulting current imbalance leads to a growing charge separation that establishes the electrostatic field responsible for driving the return current (cf. Equation (2)). The asymptotic electric field reduces to $E(t \to \infty) = j_b/\sigma$ where σ is the conductivity. Given the assumption of a steady acceleration mechanism, this solution justifies the original Knight and Sturrock (1977) steady-state analysis. We note that Knight and Sturrock determined the electric field, beam current and return current as functions of position; therefore, we do not consider further the spatial evolution of the return current. We emphasize that this steady-state solution is due to the fact that in the solar flare the beam acceleration and propogation regions are coupled by the return current. Laboratory analyses based on the semi-infinite beam model of essence ignore this coupling that is characteristic of the solar flare problem.

4. Summary and Conclusions

We have shown that the generation of return current in large scale astrophysical plasmas such as solar flares is quite different from the laboratory environment. Whereas in the laboratory the return current is established inductively and subsequently decays on a resistive timescale, the return current in a solar flare is established electrostatically and, therefore, does not decay.

This difference between the laboratory and astrophysical environments is due to the nature of the laboratory circuit. In the laboratory, an apparatus external to the plasma generates the electron beam, hence, charging of the acceleration region is not an issue. In the solar flare, the flow of the return current back into the acceleration region is the only mechanism by which the acceleration region can remain charge neutral. The electric field generated by charge separation as electrons are propelled out of the acceleration region continuously drives the return current. This results in a steady-state beam-return current system if the acceleration mechanism is steady. This result has the important implication that the observed temporal variations in the hard X-ray intensity profiles do *not* reflect the dynamics of the beam-return current system, as has been argued by Spicer and Sudan (1984). These variations must instead be intrinsic to the acceleration mechanism(s).

This study may be regarded as a first step in incorporating the coupling between the acceleration region and the rest of the flaring loop; further analysis of the problem must address the issue of the feedback on the return current and its associated fields on the acceleration mechanism(s).

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