TOWARDS THE CIRCUIT THEORY OF SOLAR FLARES

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Abstract. It has been shown that the main problems of the circuit theory of solar flares – unlikely huge current growth time and the origin of the current interruption – have been resolved considering the case of magnetic loop emergence and the correct application of Ohm's law. The generalized Ohm's law for solar flares is obtained. The conditions for flare energy release are as follows: large current value, $> 10^{11}$ A, nonsteady-state character of the process, and the existence of a neutral component in a flare plasma. As an example, the coalescence of a flare loop and a filament is considered. It has been shown that the current dissipation has increased drastically as compared with that in a completely ionized plasma. The current dissipation provides effective Joule heating of the plasma and particle acceleration in a solar flare. The ion–atom collisions play the decisive role in the energy release process. As a result the flare loop resistance can grow by 8–10 orders of magnitude. For this we do not need the anomalous resistivity driven by small-scale plasma turbulence. The energy release emerging from the upper part of a flare loop stimulates powerful energy release from the chromospheric level.

1. Introduction

The circuit model for solar flares proposed more than twenty years ago by Alfvén and Carlqvist (1967) is still attractive among the numerous flare models. Following the idea of Alfvén and Carlqvist the problem of flare energy release is equivalent to the problem of electric current interruption in the solar corona–photosphere circuit. The model is based both on the measurements of Severny (1965) indicating vertical currents up to a few times 10^{11} A in the neighborhood of sunspots and on the analogy with a circuit containing a mercury-vapor rectifier. The plasma in the rectifier can, under certain conditions, make a jump from a highly conducting to a highly resistive state. Sen and White (1972) proposed a dynamo mechanism for generating an electromotive force in the corona–photosphere circuit. The energy of the generator is taken from the kinetic energy of neutral atoms in the photosphere. The photospheric dynamo can supply the required power ($\approx 10^{29}$ erg s^{-1}) for a fairly big flare if the plasma velocity in the dynamo region is about 10^5 cm s^{-1} (Kan, Akasofu, and Lee, 1983). Because the current description facilitates the discussion of global constraints of solar flares, and because of the fruitful analogies between a flare loop and equivalent electric circuit as well as between the Earth's magnetosphere–ionosphere circuit and the solar corona–photosphere circuit, the circuit model is still very sound. There were many attempts to develop the circuit theory of solar flares (Sen and White, 1972; Heyvaerts, 1974; Spicer, 1977; Akasofu, 1979; Alfvén, 1981; Kan, Akasofu, and Lee, 1983; Henoux, 1987; Melrose

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and McClymont, 1987). The circuit analog is also a very effective tool to investigate the coronal heating mechanism (see, e.g., Ionson, 1982).

Current interruption in the circuit model was usually attributed to the electric double layer or multiple double layers (Alfvén, 1981; Henoux, 1987). It was proposed that charged particles are accelerated up to high energies in the corona within a very short distance of the order of the Debye length, much less than 1 cm. Meanwhile, from the kinetic theory of the steady-state double layers (Gurevich, Meerson, and Rogachevsky, 1985) it follows that the energy of the charged particles accelerated in a double layer is determined by the layer potential drop and cannot exceed the thermal energy (a few keV) of a flare plasma. Thus, for the flare energy release the multiple double layers must switch on coherently. Moreover, the formation of the double layers produces a plasma density drop. This is in contrast to the observations, since X-ray and microwave data (de Jager, 1979; Zaitsev and Stepanov, 1983) suggest that the plasma density of the energy release volume in the coronal part of a flare loop can be sufficiently high: 10^{11}–10^{12} cm^{-3}.

Hence, the circuit theory of solar flares must be developed a lot because there is no answer today on the key problem of a flare: what is the origin of sudden enhancement of the resistance in the current circuit?

In most solar flare models the current dissipation is used this way or another to explain the energy release (Alfvén and Carlqvist, 1967; Syrovatsky, 1966; Sturrock, 1980; de Jager, 1986). The stationary current density $\mathbf{j}$ is given by the relation

$$\mathbf{j} = \sigma \mathbf{E} \tag{1}$$

or its known modifications. Classical or Coulomb conductivity due to particle collisions σ_{Coul} does not ensure the required energy release. It is therefore common practice to use the anomalous conductivity $\sigma_{\text{an}} \ll \sigma_{\text{Coul}}$, caused by the current instabilities of small-scale waves (Tomozov, 1971; Spicer, 1977; Somov and Titov, 1985). The conditions for current instability onset on the Sun are quite rigid and cause numerous difficulties in the flare theory. Observations show, however, that a solar flare is an ordinary phenomenon and occurs in many various situations.

As for the circuit model it should be noted that the anomalous resistivity due to, for example, electron collisions with the ion-sound turbulence does not supply the required resistance in the coronal part of a flare loop. In order to resolve this problem, Melrose and McClymont (1987) suggested that the current profile in a coronal loop breaks up into many filaments and the filling factor must be about 10^{-5} for the required energy release rate. However, the laboratory experiments do not confirm this idea. According to Fadeev, Kvartshava, and Komarov (1965) the filling factor in the Z-pinch device does not exceed 0.5, as a rule.

Recently, attention was drawn to the fact that Ohm's law, as it stands in Equation (1), is not correct for solar flares (Zaitsev and Stepanov, 1989). A determining role in the current dissipation in flares is played by both the ion–neutral collisions and the non-stationary character of the energy release process. Consequently, the conductivity decreases by 8–10 orders of magnitude as compared with σ_{Coul}. An important factor earlier disregarded is the partial plasma ionization in the primary energy release volume. There are indications based on synchronous X-ray and Hα observations that the cold ($\approx 10^4$ K) partially ionized and hot ($\approx 10^7$) completely ionized plasma components coexist in the coronal part of the flare loop (Zirin *et al.*, 1967; Rust and Webb, 1977; Webb and Jackson, 1981). Essential temperature inhomogeneity in the coronal part of the flare is confirmed also by Švestka (1976).

Another important problem of solar flare physics is the huge inductance of the coronal magnetic arch–photosphere circuit. Because of this inductance the current rise time after the voltage source starts to operate is more than 10^4 years thus being unrealizable for the corona–photosphere circuit.

In Section 2 we will analyze the electric current behavior during the emergence of a current-carrying magnetic loop and will discuss the global constraints of the flare loop–photosphere circuit. Section 3 presents an illustration of the flare energy release using the interaction of a single flare loop with a filament as an example. We have used the generalized Ohm's law for the non-steady-state condition taking into account the neutral plasma component in the energy release volume (Appendix). We will show also that the energy release rate of a solar flare can be explained without anomalous resistivity and that current filamentation is not needed. The main energy release of a flare occurs in the chromosphere rather than in the corona (Section 4). In Section 5 we will briefly describe the flare scenario in the context of the advanced circuit model and summarize the results.

2. The Current-Carrying Flare Loop: Circuit Analog

Consider an equivalent electric circuit composed of a coronal magnetic arch with resistance R_c and inductance L and a photospheric section with resistance R_{ph} and electromotive force (e.m.f.), $\mathscr{E}$, (see, e.g., Alfvén and Carlqvist, 1967; Henoux, 1987). The photospheric e.m.f., $\mathscr{E}$, is caused by the Lorentz force $(e/c)(\mathbf{v} \times \mathbf{H})$, which in turn is created by the photospheric material motion. For this dynamo mechanism to work, the plasma must not be frozen-in. Such conditions do exist in the photosphere where the ion–neutral collision frequency is much more than the ion gyrofrequency (Sen and White, 1972). A quite opposite relation is true for the electrons. Therefore the ions follow the neutral component of the photospheric plasma, a charge imbalance arises, and an e.m.f. sets up:

$$\mathscr{E} = \frac{1}{c} \int \mathbf{v}_a \times \mathbf{H} \, d\mathbf{x} , \tag{2}$$

where $\mathbf{v}_a$ is the velocity of neutral atoms.

An essential feature of the electric circuit under consideration is associated with the high inductance of the flare loop (Alfvén and Carlqvist, 1967):

$$L = 4l \left(\ln \frac{8l}{\pi r} - \frac{7}{4} \right) . \tag{3}$$

Here the magnetic loop is approximated by a slender semi-circular flux tube of radius l/π. At $l = 2 \times 10^9$ cm and $r = 10^8$ cm, Equation (3) yields $L \approx 10^{10}$ cm $\approx 10H$.

An important property of a high-inductance circuit is the large rise time of current when the e.m.f. is switched on. This is apparent from the equation

$$RI + \frac{\mathrm{d}}{\mathrm{d}t}(LI) = \mathscr{E}, \qquad R = R_c + R_{ph}. \tag{4}$$

The resistance of the coronal part of the magnetic loop $R_c = l/\sigma S$ with the Coulomb conductance σ_{Coul} is about $R_c \approx 10^{-11}\ \Omega$ for the loop length $l \approx 2 \times 10^9$ cm, the loop cross-sectional area $S \approx 3 \times 10^{16}$ cm^2, and the plasma temperature $T = 10^6$ K. Kan, Akasofu, and Lee (1983) found that the photospheric resistance is of the same order of magnitude, i.e., $R_{ph} \approx 10^{-11}\ \Omega$. It is seen from Equation (4) that for constant inductance, L, the current rise time is $t = L/R \approx 3 \times 10^4$ years. At the first sight this seems to be an unsurmountable obstacle to the circuit models of flares. However, bearing in mind the magnetic loop emergence increasing the inductance, we have one more time-scale $\tau_L = (\mathrm{d}L/L\,\mathrm{d}t)^{-1}$, which governs the characteristic current rise time. Indeed, if the inductance changes linearly as $L(t) = L_0 + \eta t$ then the solution of Equation (4) will be:

$$I(t) = I_0\left(\frac{L_0}{L}\right)^{R/\eta + 1} + \frac{\mathscr{E}}{R + \eta}\left[1 - \left(\frac{L_0}{L}\right)^{R/\eta + 1}\right]. \tag{5}$$

It is seen from Equation (5) that for $I_0 = 0$, the characteristic rise time of the current is of the same order as the inductance variation time in the circuit, $L\eta^{-1} = (\mathrm{d}L/L\,\mathrm{d}t)^{-1}$. The current formation in flaring loops is attested by the emergence of the magnetic flux, often observed a few days or a few hours before the flare. Assuming, for example, that the emergence time Δt is about seven hours, one has $\eta = L/\Delta t \approx 4 \times 10^{-4}\ \Omega \gg R$. Let us suppose also that $L_0 \ll L(t)$. Then from Equations (2) and (5) it follows that for photospheric material velocity $v_a = 5 \times 10^4$ cm s^{-1}, $H = 10^3$ Oe, and $\Delta x = 10^9$ cm, the maximum value of electric current formed in a flare loop is about $I_{\max} = \mathscr{E}/\eta \approx 10^{12}$ A.

The stored magnetic energy in a single flare loop is then $W = (1/2)LI^2$. For $L = 10H$ and $I = 10^{11}$–10^{12} A, we have $W = 5 \times 10^{29}$–5×10^{31} erg, which is quite enough for a modest flare. However, the power of Joule dissipation in a flare circuit $\dot{W} = (R_c + R_{ph})I^2 = 10^{18}$–$10^{20}$ erg s^{-1} = 10^{11}–10^{13} W, or 8–10 orders less than the required value in the impulsive phase of a solar flare. Hence, the flare can occur in the coronal or chromosphere-photosphere part of the circuit only if, for this reason or another, the resistance grows rapidly up to $R = 10^{-4}$–$10^{-2}\ \Omega$, meaning essentially current circuit interruption. This is one of the key flare problems to elucidate the origin for such an increase in circuit resistance.

3. Energy Release in a Prominence-Loaded Flaring Loop

Observations indicate a close connection between flares and dark filaments located above a magnetic neutral line (Švestka, 1976). About 45% of major flares are preceded by filament activations and flares that occur in old active regions are quite generally connected with filament motions (Smith and Ramsey, 1964; Dodson and Hedeman, 1970). In this section we illustrate the flare energy release driven by the generalized Ohm's law using a current carrying magnetic loop as an example. Particular attention is paid to the possible contribution of the prominence to the energy release process.

3.1. Current Disruption due to Prominence – Flare Loop Interaction

The prominence on the top of a magnetic arch (Figure 1) can be unstable against the flute-type perturbations and can serve as a flare trigger (Pustyl'nik, 1973). The flute

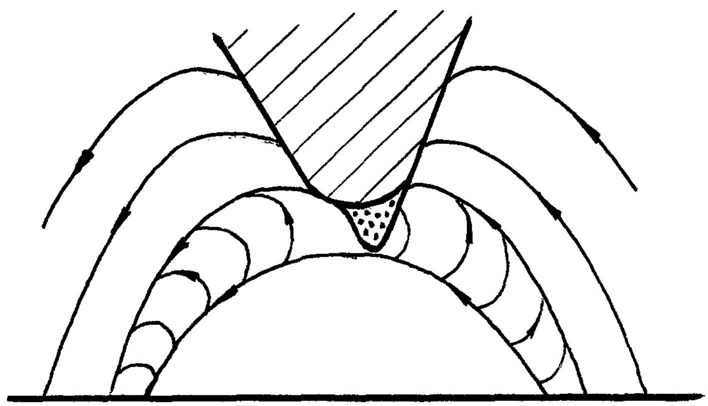

Fig. 1. Interaction of a current-carrying flare loop with a prominence. The prominence tongue (dotted area) penetrates into the flare loop due to the flute instability.

instability of the ballooning mode occurs when the radius of curvature of the trough, having the form of a tongue, is

$$\mathcal{R} < \frac{4\pi n_\Sigma g \mathcal{L}^2}{H^2}\,, \tag{6}$$

where $n_\Sigma = n_a + n$, and $\mathcal{L}$ is the length of the magnetic field line supporting the prominence. Putting $n_\Sigma = 10^{12}\ \text{cm}^{-3}$, $\mathcal{L} = 10^9$ cm, $H = 500$ Oe, from Equation (6) we find $\mathcal{R} < 2 \times 10^6$ cm. If the temperature of the prominence bottom exceeds 3×10^4 K, the gravity force is inferior to the centrifugal force. The gravity acceleration, g, in Equation (6), is substituted by $2\kappa T/m_i\mathcal{R}$ (Zaitsev and Stepanov, 1988), where κ is the Boltzman constant, m_i the ion mass, so that the recessed tongue may have a smaller curvature. For example, at $T = 10^7$ K, we have $\mathcal{R} < 4 \times 10^8$ cm. The active region, heating before the flare up to a temperature $T \geq 10^7$ K, is indicated by precursors in soft X-rays 10–30 min before the flare start (Zhdanov and Charikov, 1985).

The flute instability leads to three important consequences. First, prominence oscillations arise (Pustyl'nik, 1973; Zaitsev and Stepanov, 1988), which produce nonstationary conditions in a flaring loop. Second, the prominence deforms the magnetic flux tube and contracts the current channel, thus increasing eventually the resistance of the coronal part of the circuit and consequently the energy release rate. Third, the flaring loop is filled with a partially ionized plasma and the equilibrium conditions in a flaring loop are violated when the right-hand side of the motion equation (A.8) in the Appendix is not zero.

Note that even if the prominence bottom is hot ($\approx 10^7$ K), the tongue-entrained cold plasma also penetrates into the flare loop due to the flute instability. As shown below, large radiative losses prevent fast heating of prominence tongues in a flare loop by the current. Therefore, the partially ionized cold and completely ionized hot plasma components can coexist in a coronal loop in accord with the observations of Zirin *et al.* (1967), and Webb and Jackson (1981).

We now estimate the value of total energy power which goes into plasma heating:

$$\dot{W} = \int_V q \, \mathrm{d}V \tag{7}$$

in the volume V of that part of the flaring loop, which interacts with the prominence. Here $q = \mathbf{j} \cdot \mathbf{E}^*$ means the work of the electric field $\mathbf{E}^*$ under current in a coordinate system moving with the bulk plasma, which is spent for the plasma Joule heating. Consider for simplicity a horizontal current channel having axial symmetry. With $n_\Sigma = 10^{11}$–10^{12} cm^{-3}, $I = 10^{11}$–10^{12} A, the azimuthal magnetic field component $H_\varphi = 500$ Oe, and the channel cross-sectional area $S = 10^{16}$ cm^2, the Ampère force $\mathbf{j} \times \mathbf{H}$ exceeds the gravity force by about three or four orders of magnitude, so that the latter can be neglected in Equations (A.8) and (A.11). We, therefore, assume that in the equilibrium state the Ampère force in the current channel is balanced by the gas pressure gradient.

The equilibrium state is violated as the flute instability of the prominence develops. The current channel is filled with the dense partially-ionized plasma of the prominence, which smooths out the initial radial pressure distribution. In the case of $c^{-1}\mathbf{j} \times \mathbf{H} \gg \nabla p_\Sigma$ Equation (A.14) can be written as

$$q = \frac{j^2 m_e(\nu_{ei} + \nu_{ea})}{e^2 n} - \frac{F}{c m_i n \nu_{ia}} (\nabla p_a \times \mathbf{H}) - \frac{\nabla p_e \mathbf{j}}{en} + \frac{F^2}{c^2 n m_i \nu_{ia}} (\mathbf{j} \times \mathbf{H})^2 . \tag{8}$$

We then integrate Equation (8) over the cylindrical volume, bearing in mind that in this geometry $\nabla p_e \mathbf{j} = 0$ and disregarding terms of the order of $2\pi p_\Sigma / H_\varphi^2$ as compared to unity (the plasma $\beta \ll 1$) and find the energy release power:

$$\dot{W} = \left[\frac{m_e(\nu_{ei} + \nu_{ea})d}{e^2 n S} + \frac{2\pi F^2 I^2 d}{c^4 n m_i \nu_{ia} S^2} \right] I^2 , \tag{9}$$

where d is the length of the flare loop part interacting with the prominence. It should be readily apparent that the second term in Equation (9), which describes the Joule dissipation of the current due to ion–neutral collisions, exceeds the conventional Joule dissipation described by the first term by many orders of magnitude. A considerable increase of Joule dissipation in partially-ionized gases was noted first by Schlüter and Biermann (1950) (see also Cowling, 1957). This is because the energy of ions moving through the neutral gas due to the Ampere force $\mathbf{j} \times \mathbf{H}$ is much larger than the energy of the relative motion of electrons and ions. The dissipation of the energy of moving ions by ion – neutral collisions implies great additional dissipation of the energy of the electric currents. This effect is absent in a steady-state fully-ionized plasma.

Hence, we avoid using the anomalous resistivity when solving the solar flare problem and consider the energy release in a sufficiently large volume. This fact is usually ignored in the analysis of the flare energy release. Indeed, putting into Equation (9) $n_\Sigma = n_a + n = 10^{12}\ \text{cm}^{-3}$, which corresponds to the dense parts of the prominence material that has penetrated into the flare loop, and assuming that the cold plasma component ensures $F = 0.1$, with $I = 3 \times 10^{11}$ A, $T = 10^7$ K, $d = 5 \times 10^8$ cm, and $S = 10^{16}\ \text{cm}^2$ we find

$$R_{\text{lin}} = \frac{m_e(\nu_{ei} + \nu_{ea})d}{ne^2 S} \approx 4 \times 10^{-13}\ \Omega\,,$$

$$R_{nl} = \frac{2\pi F^2 I^2 d}{c^4 n m_i \nu_{ia} S^2} = 1.6 \times 10^6\ \frac{F}{1 - F}\ \frac{I^2\, dT^{1/2}}{c^4 m_i n_\Sigma^2 S^2}\ \text{CGS} \approx 2 \times 10^{-4}\ \Omega\,. \tag{10}$$

With these parameters, the energy release rate in the circuit $\dot{W} = R_{nl} I^2$ is about $2 \times 10^{26}\ \text{erg s}^{-1}$, thus being sufficient to explain the energetics of an elementary flare burst. One should point out the very important peculiarity of the proposed energy release mechanism: an increase in current I leads to an increase in energy release according to the formula

$$\dot{W} \sim I^4\,. \tag{11}$$

The characteristic time scale of the energy release is determined by the velocity of the prominence tongue penetration into the flaring loop: $\tau_{fl} \approx r/(T/m_i)^{1/2}$ is about 1–10 s, i.e., of the order of an elementary flare burst. Note, that the current value is almost constant during the energy release due to the huge inductance of a flare loop.

For the sake of comparison, we investigate the conditions for anomalous resistivity driven by, for example, Buneman instability in the context of the circuit model. It is known that Buneman instability occurs if the velocity of electrons relative to ions exceeds the plasma electron thermal velocity V_{Te}. For the current channel under consideration it means that the cross-sectional area S of a channel with $T = 10^7$ K, $n = 10^{12}\ \text{cm}^{-3}$, $I = 3 \times 10^{11}$ A must be

$$S_B < \frac{I}{enV_{Te}} = 2 \times 10^9\ \text{cm}^2\,. \tag{12}$$

For anomalous resistivity due to ion-sound turbulence the cross-sectional area of a current channel must be $S_{i-s} < 10^{11}\ \text{cm}^2$. Thus, the current filamentation in a flare loop with a filling factor about 10^{-7}–10^{-5} is a strong requirement for anomalous resistivity. This is in accordance with the result of Melrose and McClymont (1987).

It is important to note that the radiative losses in a cold ($\approx 10^5$ K) and dense ($\approx 10^{12}$ cm^{-3}) plasma $q_r = L_r(T)F(1 - F)n_\Sigma^2 \approx 10^2$ erg cm^{-3} s^{-1} exceeds the Ohmic heating of the prominence material by the electric current: $\dot{W}/Sd \approx 10$ erg cm^{-3} s^{-1}; hence, the cold prominence material in a flare loop is not subjected to fast heating. Here $L_r(T)$ is the radiative cooling coefficient (Cox and Tucker, 1969). This is the reason for the coexistence of cold partially ionized and a hot fully-ionized plasma components in a flare loop required for the flare energy release within the framework of our approach based on the generalized Ohm's law under the non-steady-state condition.

3.2. Particle acceleration

The potential drop U over the length of the current channel due to the nonlinear resistance $R_{nl} = 10^{-4}\ \Omega$ and for the current $I = 3 \times 10^{11}$ A is

$$U = eR_{nl}I = 30\ \text{MeV}\,. \tag{13}$$

With this potential drop, the bulk of the electrons can be accelerated up to an energy $E_e = (l_{fp}^i/d)U \approx 100$ keV, while the bulk of the ions to $E_i = (l_{fp}^i/d)U \approx 5$ MeV. Here l_{fp} is the mean free path of the particles due to classical collisions. The charged accelerated particles can contain a considerable part of the flare energy. Supposing, for example, that the number of accelerated electrons is about $N \approx 10^{33}$. We then find a total energy of fast electrons $W_e = NE_e \approx 10^{26}$ ergs that is quite comparable with the energy released in a single flaring loop.

3.3. Energy release volume: an equivalent circuit

Analyzing the energy release in the coronal part of a flare loop where the current interruption occurs, we have omitted two points which, in principle, can make the energy release less effective. The first one concerns the spurious capacitance driven by the prominence tongue penetrating into a flare loop. To explain this point, let us represent the primary energy release volume R_c by the equivalent LRC circuit indicated in Figure 2 by a dashed box with spurious capacitance $C \approx dc^2/4\pi V_A^2 \approx 4 \times 10^{12}$ cm, where V_A is the Alfvén velocity, the inductance $L_1 \approx d/c^2 \approx 5 \times 10^{-13}$ CGS $\approx 0.5H$, and the resistance $R(t)$ depending on time and growing suddenly from the value of R_{lin} to the value of R_{nl} (see Equation (10)). Because of spurious capacitance the question arises: what part of the total current will flow through nonlinear resistance? To answer this question, let us represent the maximum value of the current through the capacitance C as follows:

$$I_2^{\max} \le CI\,\frac{\mathrm{d}R(t)}{\mathrm{d}t} \approx CI\,\frac{R_{nl}}{\tau_{fl}}\,, \tag{14}$$

where τ_{fl} is the typical time scale of the flute instability. Putting into Equation (14) $C \approx 4 \times 10^{12}$ cm, $R_{nl} \approx (10^{-4}$–$10^{-3})\ \Omega$, we have $I_2^{\max} \approx 4(10^{-5}$–$10^{-3})I$. Thus, the

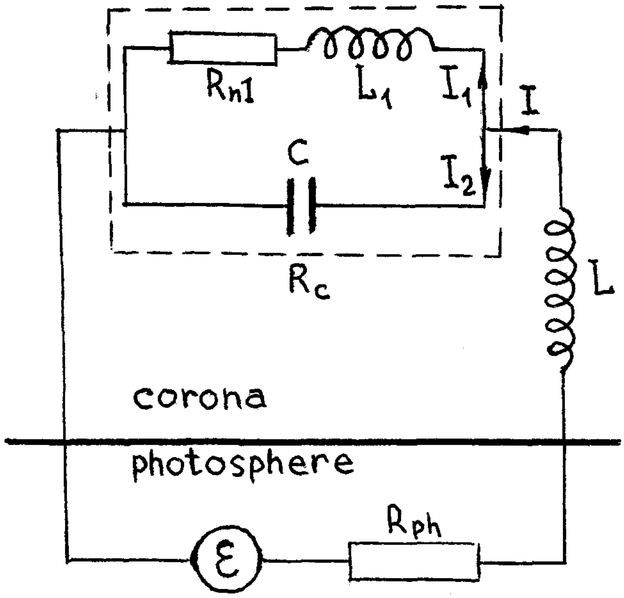

Fig. 2. An equivalent electric circuit for a flare loop interacting with a prominence. The coronal resistance R_c during the energy release process is represented by an LRC circuit with capacitance C, resistance R_{nl}, and inductance L_1. The current in a circuit $I = I_1 + I_2$ with inductance L is driven by e.m.f. $\mathscr{E}$. R_{ph} is the photospheric resistance.

current will flow through nonlinear resistance and effective Joule dissipation takes place.

The second point deals with the excitation of the Alfvén wave propagating along the magnetic flux tube in the current interruption region. Alfvén waves can be generated by the plasma velocity deviation perpendicular to the magnetic field when the prominence tongue penetrates into the flare loop. As a result, a part of the total current belonging to the Alfvén wave can flow around the 'cold' tongue and does not participate in the energy release. It is easy to show that the ratio of Alfvénic current to the total current is

$$I_A/I \approx \frac{1}{2} \frac{H_0 V_\sim^2}{H_\varphi V_A^2} , \tag{15}$$

where $V_\sim$ is the plasma velocity deviation due to the flute instability, H_φ is the nonpotential component of the magnetic field arising due to current I. Taking the relation $V_\sim \leq V_{Ti}$ into account, we find $I_A^{\max}/I \approx \beta H_0/4H_\varphi$, where β is the plasma beta. Hence, for $H_\varphi \approx H_0$ and $\beta \ll 1$ the Alfvénic current is not important. It should be noted also that it is impossible to have any direct current in an LRC circuit with series capacitance as proposed by Ionson (1982).

The shunt capacitance C (Figure 2) introduces a new time scale for the equivalent circuit. Indeed, for an LRC circuit with L_1, C, and R_{nl} we can write the equation

$$\frac{d^2 I_2}{dt^2} + \frac{R_{nl}}{L_1} \frac{dI_2}{dt} + \frac{1}{L_1 C} I_2 = 0 , \tag{16}$$

where $I_2 = I - I_1$. The solution of Equation (16) has the form

$$I_2 = I \frac{CR_{nl}}{\tau_{fl}} \left[\exp\left(-\frac{R_{nl} t}{2L_1} \right) \right] \cos \left[\left(\frac{1}{L_1 C} - \frac{R_{nl}^2}{4L_1^2} \right)^{1/2} t \right]. \tag{17}$$

For $1/L_1 C \gg R_{nl}^2/4L_1^2$ we have dumping oscillation with a period $\tau = 2\pi\sqrt{L_1 C} \approx \approx 2d/V_A \approx 10$ s. The quality of these oscillations under flare loop conditions is sufficiently high: $Q = \sqrt{L_1/C}/R_{nl} \approx (10^2\text{–}10^3)$. The same LRC circuit exists in the chromosphere where energy release is more powerful (see Section 4). Under chromospheric conditions the ratio $I_2^{\max}/I \approx 10^{-1}$. It is not excluded that the manifestation of this 'high Q' resonator can be a weak modulation of the microwave flux with a time scale of roughly several seconds during and after the impulsive phase of the flare observed sometimes for example by the Berne group (Magun *et al.*, 1990).

In the conclusion of this section we summarize the typical time scales for flare energy release in a framework of the circuit approach:

(i) Current rise time before the flare, $\tau_L = (\mathrm{d}L/L\,\mathrm{d}t)^{-1}$, which is equal to the magnetic loop emergence time (a few hours or a few days).

(ii) Time scale of the flute instability, $\tau_{fl} \approx r/V_{Ti} \approx 1\text{–}10$ s.

(iii) Current relaxation time after current disruption, $\tau_r \approx L_1/R_{nl} \approx 500\text{–}5000$ s, which is about the flare duration.

(iv) Period of eigenoscillations of the LRC circuit, $\tau \approx 2\pi\sqrt{L_1 C} \approx 10$ s.

4. Energy Release in the Chromosphere

Heating of the flare loop footpoints by energetic particles produces nonsteady-state conditions in a large current system at the chromospheric level and 'switches' the generalized Ohm's law. It seems reasonable to use the term *chromospheric flare*, recently avoided because of the energy release observations in compact coronal magnetic arches, since at the chromospheric level the energy release is much more powerful than the coronal value. Indeed, for the vertical (with respect to the Sun surface) part of the magnetic flux tube with current $I \approx 3 \times 10^{11}$ A and height $d \approx 2 \times 10^8$ cm, from Equation (10) we find:

$$R_{nl}^{chr} = 10^{12} \frac{F}{1 - F} \frac{I^2 d}{c^4 m_i n_\Sigma^2 S^2 T^{1/2}} \text{ CGS} \approx 3 \times 10^{-2}\,\Omega. \tag{18}$$

In Equation (18) $n_\Sigma \approx 10^{13}$ cm^{-3}, $T \approx 10^4$ K, $F \approx 0.5$ (Švestka, 1976) and we assumed that the tube cross-sectional area is reduced at the footpoints of the flare loop, $S \approx 10^{15}$ cm^2. The energy release rate in the chromosphere $\dot{W}^{chr} = R_{nl}^{chr} I^2$ is about 10^{28} erg s^{-1}, which is quite sufficient to explain a large flare. This fits the conclusion of Stepanov, Urpo, and Zaitsev (1992) based on the analysis of the chromospheric plasma evaporation in the event of June 22, 1989, that the main energy release process occurs in the chromosphere, and not in the coronal part of a flare loop.

5. Conclusions

We have shown that the main problems of the circuit theory of a solar flare which are caused by the non-realizable huge current rise time after the photospheric voltage generator switches on and by the sudden enhancement of resistance have been resolved, bearing in mind the magnetic loop emergence and using the correct form of the generalized Ohm's law.

We propose the following scenario for a solar flare before and during the impulsive phase. The photospheric material motion leads to the generation of e.m.f. producing electric current. The current grows up to 10^{11}–10^{12} A in a flare loop during the period of new magnetic flux emergence (several hours or days). The reason for current disruption in a flare circuit is the flute instability of the prominence located just above the flare loop. The dense, cold matter of the prominence, which fills the flare loop because of the flute instability, ensures non-steady-state conditions and a sufficient number of neutrals for effective Joule current dissipation. It is very important to note that the energy release rate is proportional to the total current to the fourth power. Over a few seconds an energy $> 10^{26}$ ergs is realized in a single flare coronal loop, which is comparable to the energy of an elementary flare burst, and the electrons and ions are accelerated simultaneously. The flare loop footpoint heating by energetic particles and/or by conductive fronts produces non-steady-state conditions at the chromospheric level where the generalized Ohm's law is switched and the major energy release ($> 10^{28}$ erg s^{-1}) occurs. Shock and/or thermal waves, propagating from this region, violate the stationary conditions in neighboring current systems and introduce effective current dissipation over a large area in the chromosphere. Hence, the energy release in the coronal part of a flare loop interacting with a prominence is merely the trigger of a powerful chromospheric flare. Of course, there are other switching mechanisms for the generalized Ohm's law. It is not excluded, for example, that the flare energy release due to the generalized Ohm's law is possible in a current prominence (Carlqvist, 1969; Alfvén, 1981).

In conclusion, it should be noted that the proposed advanced circuit model, based on the generalized Ohm's law of the form of Equation (A.7), can be applied also to the flare energy release in stars. For instance, putting into Equation (14) typical parameters of the chromosphere of red dwarf stars, $n_\Sigma \approx 10^{12}$ cm^{-3}, $T \approx 10^4$ K, $S \approx 10^{18}$ cm^2, $d \approx 10^9$ cm, and supposing $I \approx 10^{13}$ A, one can get the energy release rate $R_{nl} I^2 \approx 10^{31}$ erg s^{-1}, which is quite enough for a modest stellar flare.

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Appendix. Generalized Ohm's Law

We now obtain the generalized Ohm's law taking the partial plasma ionization into account. The equations of motion for electrons (e), ions (i), and atoms (a) can be written as

$$-en\mathbf{E}^* - \frac{en}{c}\,\mathbf{v}_e \times \mathbf{H} - m_e n \nu_{ei}(\mathbf{v}_e - \mathbf{v}_i) - m_e n \nu_{ea}(\mathbf{v}_e - \mathbf{v}_a) - \nabla p_e = 0\,, \quad \text{(A.1)}$$

$$en\mathbf{E}^* + \frac{en}{c}\,\mathbf{v}_i \times \mathbf{H} - m_e n \nu_{ei}(\mathbf{v}_i - \mathbf{v}_e) - m_i n \nu_{ia}(\mathbf{v}_i - \mathbf{v}_a) + m_i n\mathbf{g} - \nabla p_i -$$

$$- m_i n\,\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = 0\,, \quad \text{(A.2)}$$

$$- n m_e \nu_{ea}(\mathbf{v}_a - \mathbf{v}_e) - n m_i \nu_{ia}(\mathbf{v}_a - \mathbf{v}_i) + m_a n_a \mathbf{g} - \nabla p_a - n_a m_a\,\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = 0\,. \quad \text{(A.3)}$$

We have introduced the mean plasma velocity

$$\mathbf{V} = \frac{\sum_k n_k m_k \mathbf{V}_k}{\sum_k n_k m_k} \quad \text{(A.4)}$$

and the diffusion velocities $\mathbf{v}_k$ of electrons, ions, and atoms with respect to the plasma, i.e., $\mathbf{V}_k = \mathbf{V} + \mathbf{v}_k$, and

$$\sum_k n_k m_k \mathbf{v}_k = 0\,. \quad \text{(A.5)}$$

$\mathbf{E}^*$ is the electric field in a coordinate system frozen in a plasma moving with velocity $\mathbf{V}$ in relation to a coordinate system at rest,

$$\mathbf{E}^* = \mathbf{E} + \frac{1}{c}\,\mathbf{V} \times \mathbf{H}\,, \quad \text{(A.6)}$$

and $\mathrm{d}/\mathrm{d}t$ is the mobile operator: $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{V}\,\mathrm{grad}$. In Equations (A.1)–(A.3) the viscosity effects are neglected due to their small magnitude.

Defining the current density as $\mathbf{j} = en(\mathbf{v}_i - \mathbf{v}_e)$, excluding the velocities $\mathbf{v}_k$ from Equations (A.1)–(A.5), and neglecting the terms of the order $(m_e/m_i)^{1/2}$ as compared to unity we obtain the generalized Ohm's law,

$$\mathbf{E}^* = \frac{m_e(\nu_{ei} + \nu_{ea})\mathbf{j}}{e^2 n} + \frac{\mathbf{j} \times \mathbf{H}}{enc} + F\,\frac{\mathbf{f}_a \times \mathbf{H}}{cnm_i\nu_{ia}} + \frac{\mathbf{f}_e}{en} - \frac{F^2}{cnm_i\nu_{ia}}\,\rho\,\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} \times \mathbf{H}\,, \quad \text{(A.7)}$$

and the force-balance equation,

$$\rho \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \frac{1}{c}\,\mathbf{j} \times \mathbf{H} + \sum_k \mathbf{f}_k\,, \quad k = e, i, a\,. \tag{A.8}$$

Here

$$\rho = n_a m_a + n m_i \tag{A.9}$$

is the plasma density,

$$F = \frac{n_a m_a}{n_a m_a + n m_i} \tag{A.10}$$

the relative density of neutrals,

$$\mathbf{f}_e = -\nabla p_e\,, \qquad \mathbf{f}_i = -\nabla p_i + n m_i \mathbf{g}\,, \qquad \mathbf{f}_a = -\nabla p_a + n_a m_a \mathbf{g} \tag{A.11}$$

the forces of nonelectric origin acting on the plasma components, ν_{ek} the collision frequency of electrons with k-species particles, p_k the gas-kinetic pressure, and $\mathbf{g}$ the gravitational acceleration. We also put $n_e = n_i = n$.

In a steady-state homogeneous medium ($\mathrm{d}/\mathrm{d}t = 0$) Equation (A.7) takes on the form of the known expression for Ohm's law in a partially ionized plasma (see, e.g., Gershman, 1974):

$$(\nu_{ei} + \nu_{ea})\mathbf{j} + \frac{e}{m_e c}\,\mathbf{j} \times \mathbf{H} + \frac{Fe^2}{c^2 n m_e m_i \nu_{ia}}\,\mathbf{H} \times (\mathbf{j} \times \mathbf{H}) =$$

$$= \frac{e^2 n}{m_e}\,\mathbf{E}^* + \frac{e}{m_e}\,\nabla p_e + \frac{Fe^2}{c m_e m_i \nu_{ia}}\,(\mathbf{f}_e + \mathbf{f}_i) \times \mathbf{H}\,. \tag{A.12}$$

At equal temperatures of the plasma components, $p_a/p_e = n_a/n = F/(1 - F)$, with the gravity force disregarded, we obtain from Equations (A.7) and (A.8) the well-known expression (Cowling, 1957)

$$\mathbf{j} = \frac{ne^2}{m_e(\nu_{ei} + \nu_{ea})} \left\{ \mathbf{E}^* + \frac{\nabla p_e}{en} - \frac{\mathbf{j} \times \mathbf{H}}{enc} - \frac{F}{cnm_i\nu_{ia}} \times \right.$$

$$\left. \times \left[\nabla p_e \times \mathbf{H} + \frac{1}{c}\,\mathbf{H} \times (\mathbf{j} \times \mathbf{H}) \right] \right\}. \tag{A.13}$$

The Joule dissipation of the electric current, as is well known, is characterized by $q = \mathbf{j} \cdot \mathbf{E}^*$, a quantity which, in view of Equation (A.7), has the form

$$q = \frac{j^2 m_e(\nu_{ei} + \nu_{ea})}{e^2 n} + \frac{F(\mathbf{f}_a \times \mathbf{H})\mathbf{j}}{cm_i n \nu_{ia}} + \frac{\mathbf{f}_e \mathbf{j}}{en} - \frac{F^2}{cm_i n \nu_{ia}}\,\rho \left(\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} \times \mathbf{H} \right) \mathbf{j}\,, \tag{A.14}$$

where $\rho\,\mathrm{d}\mathbf{V}/\mathrm{d}t$ is determined from Equations (A.8) and (A.9).

It should be noted that the Joule current dissipation under non-steady-state condition have been applied in some models of the Earth's magnetospheric substorms for very distant regions of the magnetospheric tail where the ionization degree of a plasma is sufficiently high (Akasofu and Chapman, 1972). The other way the conductivity tensor components in a plasma with neutrals were investigated for the Earth's ionospheric plasma, however, under the steady-state conditions only (Gershman, 1974). The conductivity associated with ion–neutral collisions has been calculated also by Kan and Lyu (1990) in the context of Joule dissipation in a solar flare for the case of a weakly ionized gas when the steady-state approximation is true.

References

Akasofu, S.-I.: 1979, *Solar Phys.* **64**, 333.
Akasofu, S.-I. and Chapman, S.: 1972, *Solar-Terrestrial Physics*, Clarendon Press, Oxford.
Alfvén, H.: 1981, *Cosmic Plasma*, D. Reidel Publ. Co., Dordrecht, Holland.
Alfvén, H. and Carlqvist, P.: 1967, *Solar Phys.* **1**, 220.
Carlqvist, P.: 1969, *Solar Phys.* **7**, 377.
Cowling, T. G.: 1957, *Magnetohydrodynamics*, Interscience Publ. Inc., New York.
Cox, D. P. and Tucker, W. N.: 1969, *Astrophys. J.* **157**, 1157.
de Jager, C.: 1979, *Solar Phys.* **64**, 135.
de Jager, C.: 1986, *Space Sci. Rev.* **44**, 43.
Dodson, H. W. and Hedeman, E. R.: 1970, *Solar Phys.* **13**, 401.
Fadeev, V. M., Kvartzhava, I. F., and Komarov, N. M.: 1965, *Nuclear Fusion* **5**, 202.
Gershman, B. N.: 1974, *Dynamics of Ionosphere Plasma*, Nauka, Moscow.
Gurevich, A. V., Meerson, B. I., and Rogachevsky, I. V.: 1985, *Fizika Plasmy* **11**, 1213.
Henoux, J.-C.: 1987, in V. E. Stepanov and V. N. Obridko (eds.), *Solar Maximum Analysis*, VNU Sci. Press, p. 109.
Heyvaerts, J.: 1974, *Solar Phys.* **38**, 419.
Ionson, J. A.: 1982, *Astrophys. J.* **254**, 318.
Kan, J. R. and Lyu, L. H.: 1990, *J. Geophys. Res.* **95**, 4239.
Kan, J. R., Akasofu, S.-I., and Lee, L. C.: 1983, *Solar Phys.* **84**, 153.
Magun, A., Fuhrer, M., Bruggmann, B., Graeter, M., Eder, S., and Herrmann, R.: 1990, *Berne Solar Observations*, Report No. 57.
Melrose, D. B. and McClymont, A. N.: 1987, *Solar Phys.* **113**, 241.
Pustyl'nik, L. A.: 1973, *Astron. Zh.* **50**, 1211.
Rust, D. M. and Webb, D. F.: 1977, *Solar Phys.* **54**, 403.
Schlüter, A. and Biermann, L.: 1950, *Z. Naturforsch.* **5a**, 237.
Sen, H. K. and White, M. L.: 1972, *Solar Phys.* **23**, 146.
Severny, A. B.: 1965, *Space Sci. Rev.* **3**, 451.
Smith, S. and Ramsey, H. E.: 1964, *Z. Astrophys.* **60**, 1.
Spicer, D. S.: 1977, *Solar Phys.* **53**, 305.
Somov, B. V. and Titov, V. S.: 1985, *Solar Phys.* **102**, 79.
Stepanov, A. V., Urpo, S., and Zaitsev, V. V.: 1992, *Solar Phys.* (in press).
Sturrock, P. A.: 1980, in P. A. Sturrock (ed.), *Solar Flares*, Colorado Associated University Press, Boulder, p. 411.
Švestka, Z.: 1976, *Solar Flares*, D. Reidel Publ. Co., Dordrecht, Holland.
Syrovatsky, S. I.: 1966, *Astron. Zh.* **43**, 430.
Tomozov, V. M.: 1971, *Astron. Zh.* **48**, 556.
Webb, D. F. and Jackson, B. V.: 1981, *Solar Phys.* **73**, 341.
Zaitsev, V. V. and Stepanov, A. V.: 1983, *Solar Phys.* **88**, 297.
Zaitsev, V. V. and Stepanov, A. V.: 1988, *Soviet Astron. Letters* **14**, 193.
Zaitsev, V. V. and Stepanov, A. V.: 1989, 'Generalization of Ohm's Law to Nonstationary Processes in Partially Ionized Plasma: Solar Flare Application', Preprint IPFAN-248, Gorky.
Zhdanov, A. A. and Charikov, Yu. E.: 1985, *Soviet Astron. Letters* **11**, 216.
Zirin, H., Ingham, W., Hudson, H., and McKenzie, D.: 1967, *Solar Phys.* **9**, 269.