ASYMMETRIC FLUX LOOPS IN ACTIVE REGIONS, II

K. PETROVAY[1], J. C. BROWN[2], L. VAN DRIEL-GESZTELYI[3,4], L. FLETCHER[2], M. MARIK[1], and G. STEWART[2]

(Received 2 June, 1989; in revised form 23 January, 1990)

Abstract. We propose that magnetic flux loops in the subphotospheric layers of the Sun are seriously asymmetrical as a consequence of the drag force exerted on them because of the different rotational rate of the surrounding plasma. In numerical models of stationary slender flux loops in the plane parallel approximation we show that a serious tilt is both possible and probable. Observational facts (see van Driel-Gesztelyi and Petrovay, 1989; Paper I) strongly support the case for high asymmetry. The different stability of p and f spots may also be related to such an asymmetry.

The tilts are very sensitive to the rotational profile and to the magnetic field structure. Nevertheless the characteristic maximal tilts can be tentatively estimated to be 20° for thin flux tubes and 5° for thick tubes.

For two of the five observational consequences of such a tilt (described in detail in Paper I) order-of-magnitude estimates of the effects are given. The estimates are in reasonable accord with observations.

We also explore the possibilities of an inverse treatment of the problem whereby subphotospheric rotation and/or flux tube shapes can be inferred from observations of velocities of photospheric spot motions. In particular we demonstrate how analytic inverse solutions can be obtained in special cases.

1. Introduction

Available observational data seem to show that the average rotational rate of young sunspot groups is about 5% (100 m s^{-1}) faster than the rotational rate of nonmagnetic plasma from spectral Doppler measurements (see the detailed account and references of Paper I). This means that the $v(z)$ average relative rotational velocity of the surrounding plasma and the flux loops forming the spots does not identically vanish (z is the depth measured from photosphere; so $v(0) \neq 0$). Any flux loop in the solar convective zone must be distorted to some degree by the drag force arising because of this horizontal flow. This effect was already mentioned by Foukal (1977) who also estimated its order of magnitude in the special case of a stationary tube artificially anchored at a great depth, and found the distortion to be negligible (as in this case the F_d drag force must counteract both the F_m magnetic curvature force and the component F_b of the buoyancy perpendicular to the tube).

In this paper we estimate the significance of such a distortion for the quite different case of loop geometry. In line with the currently most widely held 'buoyant loop picture' of active region formation, we are going to neglect convective motions. In this picture, which is at present the only more or less coherent and quantitatively elaborated explanation of the basic mechanism of solar activity, the driving force of kink instability

[1] Department of Astronomy, Eötvös University, Kun B. tér 2, H-1083 Budapest, Hungary.
[2] Department of Physics and Astronomy, University of Glasgow, Glasgow G 12 8 QQ, Scotland.
[3] Heliophysical Observatory of the Hungarian Academy of Sciences, P.O. Box 30, H-4010 Debrecen, Hungary.
[4] Sterrenkundig Instituut, Postbus 80.000, NL-3508 TA Utrecht, The Netherlands.

is buoyancy: convective motions have only a secondary role. Supposing that the relative position of the loops and the convective cells is random, the convective drag will cause only a scatter in active region properties, not influencing the averages. (For the general features of, and the considerations underlying this 'buoyant loop picture' we refer to the fundamental paper by Moreno-Insertis, 1986.)

Unfortunately, the estimate in the case of loop geometry is not so simple as in the case examined by Foukal (1977), as in a loop F_b alaways acts *against* F_m (Figure 1), so

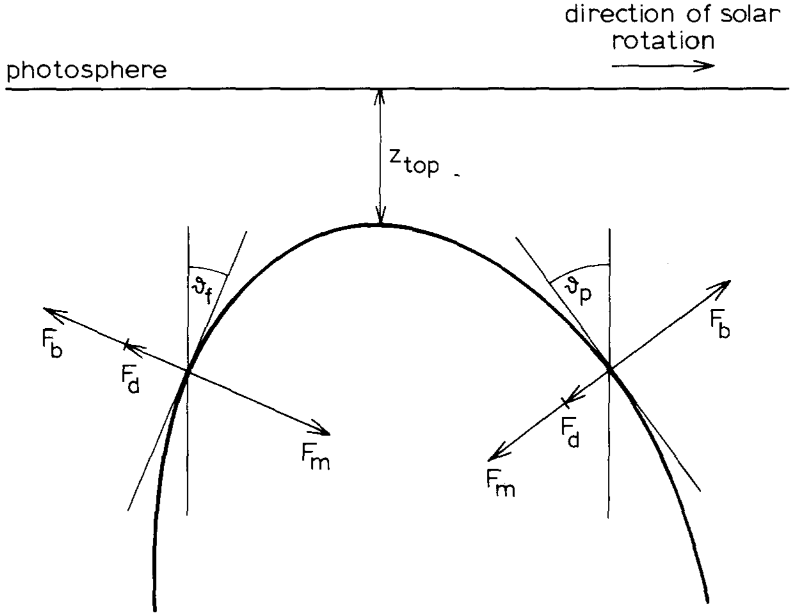


Fig. 1. Forces acting on a stationary flux loop in the convective zone. (Plane parallel geometry; in a reference frame where the horizontal velocity of the loop is zero.)

in a stationary state the absolute value of the F_d drag force should be equal to the *difference* and not the sum of $|F_b|$ and $|F_m|$.

This means that the real significance of the distorting effect of different surrounding rotation rate on flux loops can only be appreciated by building detailed models of buoyant flux tubes in a horizontally shearing environment. On the other hand, this effect means that emergent flux tubes carry important information on subphotospheric rotation and flux tube dynamics.

The simplest possible models describing flux loops are those with plane parallel geometry and in a stationary state, i.e., $F_b + F_m + F_d = 0$. (Such a state, although not realistic, is in principle always possible for a flux tube rooted in the top of the radiative zone, see, e.g., van Ballegooijen, 1982.) In Section 2 of this paper we present such static models for several different types of vertical differential rotation profiles and of magnetic field structure. The results are highly sensitive to the chosen velocity profiles and field structures: in some cases the tilt is small, in other, perhaps more realistic, cases the tilt is several degrees or for thin tubes even more; in extreme cases the asymmetry of thin tubes can be as high as 50°. So from the theoretical point of view the significant asymmetry of a loop is quite possible.

The order of magnitude of the observable consequences of the asymmetry, described in Paper I, is estimated in Section 3. As discussed in Paper I, five major effects are expected, all of them being confirmed observationally. In cases where an order of magnitude estimate of the expected effect is possible, this estimate is consistent with the observed values, thus lending support to the high tilts obtained in our theoretical models.

In Section 4, we examine the problem from the quite different viewpoint by exploring what could be learned inferentially about subphotospheric radial differential rotation and loop dynamics from observations of photospheric motions of loop emergence points. In general this inverse problem is complicated, with loop formation and dynamics and solar radial differential rotation all entering as unknows. We restrict ourselves here to showing how the differential rotation function could be inferred for specific assumptions about the other unknowns. However, this suffices to illustrate how a future combined study of inferential loop kinematics and deductive loop dynamics could provide a valuable new tool for subphotospheric studies.

2. The Models

Let us take a plane parallel stratification where every physical quantity depends on the z depth only; $z = 0$ in the photosphere. In our reference frame the horizontal velocity of the loop is zero. We are going to use the slender flux tube (SFT) approximation (see, e.g., Parker, 1979). For a stationary tube model the relevant forces per unit volume are:

$$F_b = -\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{B^2(z)}{8\pi}\right)\sin\theta\,, \tag{1}$$

$$F_m = -\frac{B^2(z)}{4\pi}\,\frac{\mathrm{d}\sin\theta}{\mathrm{d}z}\,, \tag{2}$$

$$F_d = \pm\frac{C_d}{2a}\rho(z)v^2(z)\cos^2\theta = \pm\frac{C_d}{2}\rho(z)v^2(z)\cos^2\theta\,\frac{B^{1/2}(z)\,\pi^{1/2}}{\Phi^{1/2}}\,, \tag{3}$$

with $\rho(z)$ the density, $B(z)$ the field strength, $v(z)$ the horizontal velocity (relative to the tube!), a the tube radius, $\Phi = B\pi a^2$ the magnetic flux, C_d the drag coefficient, and θ the angle of the tube with the vertical (see van Ballegooijen, 1982, for details). The equation to be solved is

$$F_b + F_m + F_d = 0\,, \tag{4}$$

or with $q = \sin\theta$:

$$\frac{\mathrm{d}q}{\mathrm{d}z} = -\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}z}q \pm 2\pi^{3/2}C_d\rho(z)v^2(z)\Phi^{-1/2}B(z)^{-3/2}(1-q^2)\,, \tag{5}$$

the + sign belonging to the preceding, the − to the following half of the loop (as we know from observations that in the photosphere, and consequently also in a certain layer under it, the spots rotate about 100 m s^{-1} faster than their surroundings, i.e., the direction of F_d is that seen in Figure 1).

$\rho(z)$ can be taken from mixing-length theories; a good approximation is the one used by Meyer *et al.* (1979):

$$\rho[\mathrm{g\,cm^{-3}}] = (0.00129 + 0.0022z/10^3\,\mathrm{km})^{2.25}\,. \tag{6}$$

According to Parker (1979) the C_d drag coefficient is of order unity. Inasmuch as the thicker tubes consist of a loose bundle of thin ones, the effective cross-section of the tube will be higher and C_d is also higher: this possibility is taken into account in some of the models.

The Φ flux is about 10^{18} Mx for thin tubes and of order 10^{22} Mx for all of the active region; we will use 10^{21} Mx for a moderate spot.

According to nonlinear MHD simulations of loop formation (Moreno-Insertis, 1986; Fisher, Chou, and McClymont, 1988), in the bulk of the convective zone $B(z)$ is determined by the adiabatic change of state of the plasma contained in the loop. In the upper 10% or so of the convective zone the adiabatic field strength would decrease under the equipartition value defined by

$$\frac{B_{eq}^2}{8\pi} = \frac{1}{2}\rho v_c^2 \tag{7}$$

with v_c the convective velocity. In these heights the work done by turbulent motions on the tube cannot be neglected any more (it is in this layer that the loop fragments into thinner ones), and this 'turbulent pumping' will keep $B(z)$ near the equipartition value (except at the uppermost 1–2 Mm, where convective collapse sets in, strengthening the field). Such a $B(z)$ can be approximated crudely by a simple quadratic model used by Meyer *et al.* (1979):

$$B(z) = B_0[1 + (z/10\,\mathrm{Mm})^2] \tag{8}$$

with $B_0 = 1500$ G. Here the parameters are chosen so that the equipartition field strength is approximately reproduced at depths of 10–20 Mm.

As a comparison we also investigated the case examined by van Ballegooijen (1982) where $B(z)$ is adiabatic throughout (therefore it must be much stronger than the quadratic approximation). This case is highly unrealistic, but it may give us an impression of the sensitivity of the results to $B(z)$.

Recent oscillation measurements show that at low heliographic latitudes the plasma rotational velocity is almost constant with depth, so the surface value of $v(z)$ of 100 m s^{-1} will be kept high also in deeper, denser layers. Nevertheless here we are going to examine a more moderate case where $v(z)$ linearly decreases to zero at a depth of 25 Mm:

$$v(z) = 4 \times 10^2\,\mathrm{cm\,s^{-1}}(z - 25) \tag{9}$$

with z in megameters (1 Mm = 1000 km). In this way we get a lower estimate for the asymmetry, but for the sake of interest we also investigate the case $v(z) \equiv 10^4$ cm s^{-1}.

So our 'representative' (but lower estimate) model will be the quadratic tube model with a linear $v(z)$. The depth of maximal rotational velocity in Equation (9) was taken to be 25 Mm, and we supposed that the tubes preserve their individuality down to this layer (i.e., the fragmentation penetrates as deep as this layer).

Equation (5) was integrated with the Euler–Cauchy method; the step in z was chosen to be $H(z)/20$ with $H(z)$ the local pressure scale height. Some of the results are presented on Figures 2(a)–(c).

In the model with quadratic $B(z)$ and linear $v(z)$ the 10^{18} Mx thin flux tubes show a tilt of 20°. 10^{21} Mx spots with a conventional treatment of the drag (i.e., $C_d = 1$) show only very small asymmetry: however, taking into account the enhanced drag arising from the 'flower bundle' character of the tube with $C_d = 10$ the tilt is crudely 5° (Figures 2(a)–(c)).

How does the alteration of $B(z)$ and $v(z)$ affect these results? In the case of the same thin and thick tube models, but with adiabatic $B(z)$ the strong downward growth of the field causes a more than ten times decrease of the tilt. For again the same tubes but now in a $v(z)$ = const. velocity field the tilt is several times higher than in Figures (2(a) and

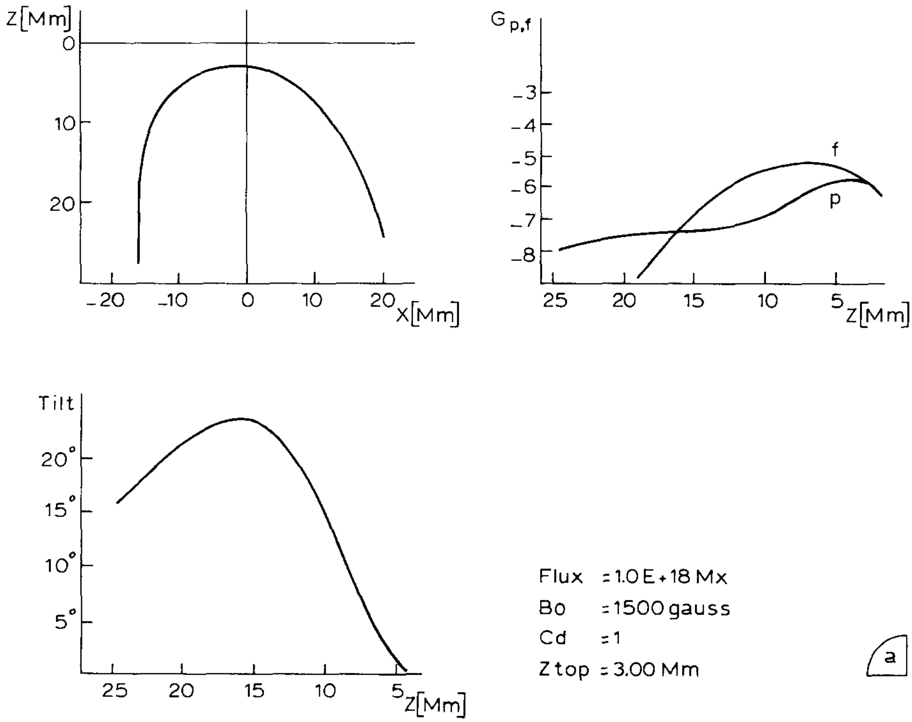
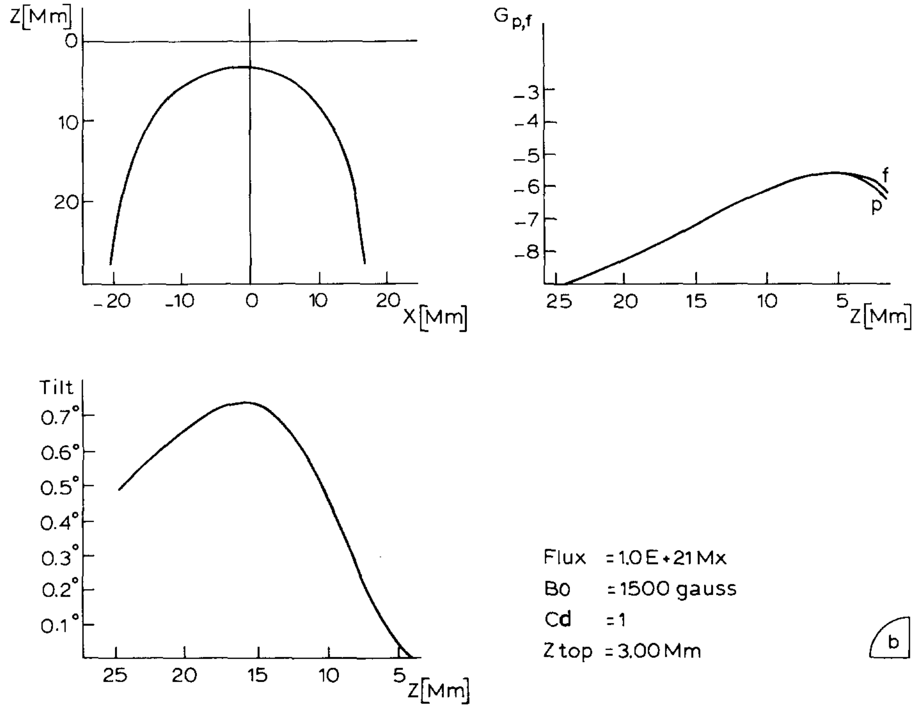
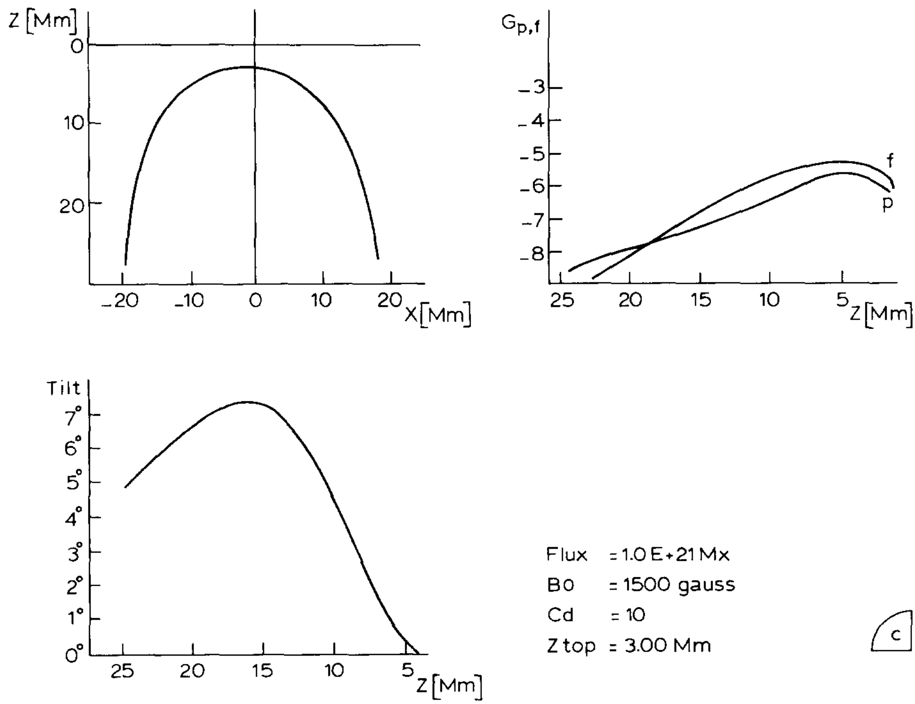


Fig. 2a–c. Models of stationary slender flux loops distorted by a horizontal flow, for different values of the magnetic flux and C_d. *Top left*: the tube shape; *bottom left*: the $\theta_p - \theta_f$ tilt as a function of depth; *top right*: the $G_{p,f}$ instability parameter as a function of depth.


Z[Mm]
X[Mm]
$G_{p,f}$
f
p
Z[Mm]
Tilt
Z[Mm]
Flux = 1.0 E+21 Mx
B0 = 1500 gauss
Cd = 1
Z top = 3.00 Mm
b


Z [Mm]
X[Mm]
$G_{p,f}$
f
p
Z[Mm]
Tilt
Z[Mm]
Flux = 1.0 E+21 Mx
B0 = 1500 gauss
Cd = 10
Z top = 3.00 Mm
c

2(c) because the $v(z)$ velocity difference between the tube and its surroundings is kept high, even in deeper, denser layers. This shows that dropping the assumption $\partial\Omega/\partial r < 0$ in the upper convective zone will only increase the asymmetry.

We can summarize the results as follows. The thin tubes always show a much higher asymmetry than thick ones. The quantity

$$G_{p,f} = \ln\left(\frac{\mathrm{d}\sin\theta_{p,f}}{\mathrm{d}z}\right)^2 \tag{10}$$

characterizing the curvature is different for the two halves of the loop (in general $G_f > G_p$). The maximal tilt for 10^{18} Mx tubes is always higher than 1°, in extreme cases reaching 60°; the maximal tilt of 10^{21} Mx tubes (with $C_d = 10$) is between 0.2° and 10°. Although we think that the most realistic values are 20° and 5° for thin and thick tubes, respectively, practically any value in the above regimes is possible. So while a serious asymmetry of the loops turns out to be both possible and probable, this large asymmetry cannot be firmly established on purely theoretical grounds, because of the uncertainty of the parameters involved.

3. Estimates for the Observational Consequences

The question naturally arises whether we can decide between symmetry and asymmetry and perhaps quantify the asymmetry with the help of observations (and hence constrain the parameters arising in our theory). For a loop to be observed, it must first rise to the photosphere, so for quantitative predictions dynamical models would be needed instead of the stationary ones of Section 2. As, however, the value of the maximal tilt can hardly be seriously affected by the emergence, qualitative arguments can also be of much help. As described in detail in Paper I, at the emergence of an asymmetrical loop five major observational effects are expected:

(a) Although in the layers at and just above the photosphere the tubes are practically vertical because of the high buoyancy, some slight tilt can be preserved even in these layers, especially in the plage field between the spots. It is unfortunately not possible to give a quantitative estimate for this photospheric tilt; in any case the 0.8° tilt observed by Howard (1974) seems reasonable. The low observed tilts confirm that the tubes are strongly buoyant (i.e., evacuated) at the photosphere (which was already suspected from semi-empirical flux tube models).

(b) Thin flux tubes are much more tilted than thick ones, so the plage field consisting of thin tubes must be asymmetrical compared to the main loop (which causes the spots). As a result of this the magnetic inversion line will be, on average, nearer to the f-spot than the p (Figure 3). The expected value of the asymmetry parameter

$$AP = \frac{X_p}{X_p + X_f} \tag{11}$$

can be calculated by integrating $\tan(\theta_p - \theta_f)$ from $z = z_0$ (say, 20 Mm) to $z = 0$ for both thick and thin tubes, and then taking the difference, dividing it with $(X_p + X_f)$ and adding

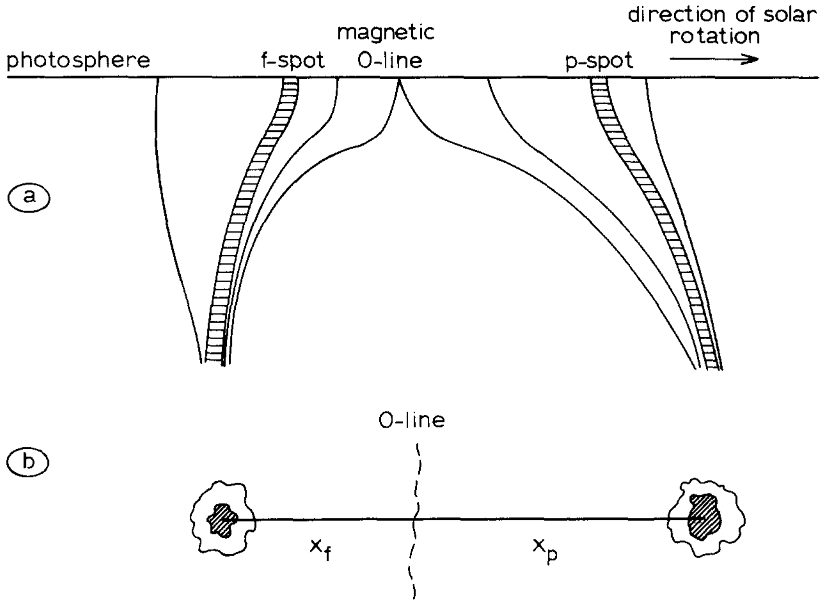


Fig. 3a–b. Geometry of the spot and plage fields in a bipolar sunspot group with a high subsurface asymmetry (a, side-view; b, top-view).

0.5. A simple estimate yields

$$\frac{20\ \mathrm{Mm}\tan 20^\circ - 20\ \mathrm{Mm}\tan 3^\circ}{100\ \mathrm{Mm}} + 0.5 = 0.56 \tag{12}$$

(10^2 Mm is the typical size of a bipolar spot group.) This is in agreement with the observed value of 0.57 ± 0.01 (Paper I). Of course a different combination of the parameters might also give the right result.

(c) The asymmetry will also influence the proper motions of the spots. This influence is schematically shown in Figure 4; if for the sake of simplicity we regard the tubes to be always straight and account for the high buoyancy in the upper layer by 'breaking' them to be vertical at $z \approx 1$–2 Mm depth, then in a time Δt, while both halves rise by $v_e \Delta t \equiv \Delta z$ (with v_e the emergence velocity), the different tilts will cause a higher horizontal displacement of the p-tube than of the f-tube. The apparent expansion velocity of the group is

$$v_p + v_f = v_e(\tan\theta_p + \tan\theta_f)\,. \tag{13}$$

The apparent rotational velocity (relative to the real one) is

$$v_p - v_f = v_e(\tan\theta_p - \tan\theta_f)\,. \tag{14}$$

Dividing the two:

$$\frac{v_p - v_f}{v_p + v_f} = \frac{\tan\theta_p - \tan\theta_f}{\tan\theta_p + \tan\theta_f}\,. \tag{15}$$

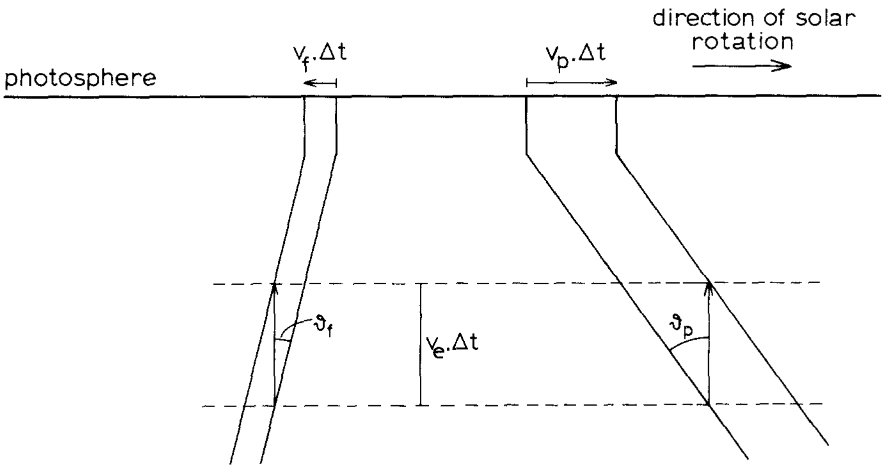


Fig. 4. The influence of asymmetry of the flux tube on sunspot motions: the p-spot seems to move faster westward than the f-spot eastward, so the group seems to rotate faster than its real rotation rate.

In the layer where the difference $\theta_p - \theta_f = 5°$ for a thick tube, $\theta_f \cong 25°$ (Figure 2(c)), so $(v_p - v_f)/(v_p + v_f) \approx 0.12$. The observed value is typically 0.5 ($v_p \approx 0.3$ km s^{-1}, $v_f \approx -0.1$ km s^{-1} at the beginning of the sunspot group development, see references in Paper I); the agreement is reasonably good for such a crude estimate.

(d) The dependence of the tilt on the Φ magnetic flux should lead to a dependence of the apparent rotational rate of sunspots on their sizes. This effect is also known from observations (cf. Paper I).

(e) The higher curvature of the f-branches of the loops, illustrated on the top right diagrams of Figures 2(a)–(c), will cause a higher vulnerability of the f-tubes against fluting instability (magnetic exchange instability). This instability is caused by the inhomogenity of the F_m curvature force. From Equations (2) and (10)

$$\frac{\mathrm{d}F_m}{\mathrm{d}R_{p,f}} = \frac{B^2}{4\pi R_{p,f}^2} = \frac{B^2}{4\pi}\exp[G_{p,f}] \tag{16}$$

and this is generally larger for the f-tubes (Figure 2, top right). So we can expect the following half of a loop to be more liable to fragment into thin tubes; thick tubes should be less frequent and shorter-lived in the following polarities of active regions. A similar, but not identical explanation was suggested earlier by Meyer, Schmidt, and Weiss (1977). The order of magnitude of this effect is, however, very difficult to estimate.

4. The Inverse Problem

Here we consider the loop shape/emergence problem from a different (kinematic) viewpoint, namely through the inverse problem approach of asking what we can learn about subphotospheric conditions from observations of the motions of emergent loop footpoints. We have already demonstrated that the tilting of subphotospheric loops by radial differential rotation can explain various asymmetric features of emerging spot pair motions and that the degree of loop asymmetry depends on the magnetic structure of the loop and on the law of radial differential rotation assumed. Though we have thus far considered only stationary loop dynamics it is also clear that the time evolution of the position of the loop emergence points will depend on the vertical velocity of the emergent loop. It is at once evident therefore that, in the inverse problem, it is not in general possible to determine three unknown functions of subphotospheric depth (characterizing respectively loop structure, loop rise speed, and solar radial differential rotation) from observations of two emergent spot positions as functions of time, without input of assumptions, or theory. However, it may in principle be possible to utilize the spot motions to infer constraints on combinations of these unknown subphotspheric functions which, when combined with the constraints of dynamics, allow estimates of all of them. Exploration of this possibility is one of our long-term goals. (If achieved, the inference of the subphotospheric rotation law near the top of the convection zone would be of particular value in complementing inferences from helioseismology data.) For the moment we content ourselves with illustrating what would be achieved in an idealized situation, recognizing that there remain many uncertainties in the problem to be solved before realistic inversion could be practicable.

Suppose we consider an element of a thin flux tube located at time t at depth z below the photosphere and at horizontal position y relative to a system of axes following the Carrington rotation. Next suppose that the horizontal drift speed imparted to the element by the radial differential rotation is V, that the buoyant vertical rise speed of the element is U and that both V and U are functions only of depth z. (In reality these will depend also on the magnetic structure of the tube and its evolution. Further, we emphasize that V is not equal to $v(r)$ (equivalent to the local solar rotation speed $\Omega(r)r$) but is related to it by the physics of the tube dynamics in the flow, as discussed further in a later paper.) Then the trajectory of the tube element will be described by

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -U(z), \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = V(z), \tag{17}$$

from which it follows that

$$\frac{\mathrm{d}y}{\mathrm{d}z} = -\frac{V(z)}{U(z)}, \tag{18}$$

where $y = y(y_0, z_0, t)$ and $z = z(y_0, z_0, t)$, suffix 0 denoting positions at $t = 0$. The location of the element at time t is then defined by

$$y(z(t), y_0, z_0) = y_0 - \int_{z_0}^{z(t)} \frac{V(z')}{U(z')}\,\mathrm{d}z' \tag{19}$$

and $z(t)$ is the solution of

$$t(z, z_0) = \int_{z}^{z_0} \frac{dz'}{U(z')} \, . \tag{20}$$

We now restrict ourselves further, for simplicity, to the simplest special case of constant vertical rise speed $U = U_0$ for which (20) gives

$$z = z_0 - U_0 t \tag{21}$$

and becomes

$$y = y_0 - \frac{1}{U_0} \int_{z_0}^{z_0 - U_0 t} V(z') \, dz' \, . \tag{22}$$

(See Appendix for generalization of the treatment of this section to linear and quadratic $v(z)$.) If the shape of the tube (locus of all tube elements) at time $t = 0$ is described by

$$y_0 = f_0(z_0) \, , \tag{23}$$

then it follows that the tube shape at time t is described by

$$y + \frac{1}{U_0} \int_{z + U_0 t}^{z} V(z') \, dz' = f_0(z + U_0 t) \, , \tag{24}$$

which can be used to compute the tube shape $y = y(z, t)$ at time t given any $U_0, f_0(z_0)$, and $V(z)$. We have computed results from this equation for a variety of values of U_0 and forms of $f_0(z_0)$ and $V(z)$ and find results showing tube tilting qualitatively similar to those found in the previously described dynamical models, as would be expected.

For the inverse problem approach, we want to consider the evolution of the tube emergent points at $z = 0$. These points $y_*(t)$ are the solutions of (from (24))

$$y_*(t) - \frac{1}{U_0} \int_{0}^{U_0 t} V(z') \, dz' = f_0(U_0 t) \, , \tag{25}$$

from which, with $\cdot = d/dt$, we obtain the solution

$$\dot{y}_*(t) = V(U_0 t) + U_0 f_0'(U_0 t) \, . \tag{26}$$

This may be interpreted as allowing inference of the tube rotation law $V(z)$ (related physically to $\Omega(r)r$) from

$$V(z) = \dot{y}_* \left(\frac{z}{U_0} \right) - U_0 f_0'(z) \tag{27}$$

for the tube of prescribed initial shape f_0, or as yielding the tube shape as a function of depth,

$$y(z) = f_0(z) = y_*\left(\frac{z}{U_0}\right) - \frac{1}{U_0}\int_0^z V(z')\,\mathrm{d}z' , \tag{28}$$

where the rotation law $V(z)$ is prescribed. In both cases of course, $V(z)$ is not the actual solar rotation speed but the tube drift speed imparted by the action of the solar rotation, as governed by the (non-steady) dynamical equations for the tube motion. More generally, Equation (26) can be regarded as providing a joint constraint on the unknown $V(z)$ and $f_0(z)$ functions based on the assumption of constant vertical rise speed U_0. Though this assumption is restrictive, it allows exploration of many of the properties of the inverse problem.

In particular we have performed model calculations, details of which will be presented elsewhere (Fletcher, Brown, and van Driel-Gesztelyi, 1990, Paper III), which reveal that for any given $y_*(t)$, bounds exist on the permissible values of V_0 for solutions to be physically acceptable (e.g., $V < 0$). Likewise (27) has been used to investigate the accuracy and frequency needed in spot location measurements in order for numerical noise in the computation of $y_*(t)$ not to swamp the real values of V being sought. To generalize the treatment to non-constant $U(z)$ requires either that $U(z)$ be such as to allow analytic integration of (20) to obtain $Z(z_0, t)$ explicitly (cf. Appendix) or an implicit treatment for more general trajectory equations.

5. Conclusions

In numerical models of stationary flux loops with the SFT approximation and in plane parallel geometry we have shown that a serious tilt of the magnetic flux tubes caused by the different rotation rate of nonmagnetic surrounding plasma is both possible and probable. As described in Paper I in detail, the observational facts strongly support the proposed high asymmetry. Although the maximal tilts are very sensitive to rotational profiles and field structures, the characteristic values can be estimated to be 20° for thin tubes and 5° for thick ones.

For two of the five important observational consequences of such a tilt (described in Paper I) order of magnitude estimates were given on the basis of stationary models. The estimates are in reasonable accordance with observations.

An analytic, kinematical description of the emergence of an asymmetrical flux loop (with some simplifying assumptions) was also presented. In this formulation, the analytic inversion of the problem becomes possible, offering the possibility of inferring the involved unknown parameters from observations.

The most severe deficiency of these calculations is obviously their stationary nature on the one hand, and their kinematical character on the other. Thus the construction of nonstationary dynamical models of emerging flux tubes in a differentially rotating environment can be the next logical step to the quantitative investigation of flux loop asymmetry. Combining such models with the inversion method may prove to be a new, useful tool for the investigation of solar radial differential rotation.

Acknowledgements

L. van Driel-Gesztelyi gratefully acknowledges the support of the Royal Society of Edinburgh and of the U.K. Science and Engineering Research Council during part of this work. We thank E. Landré for the final drawings.

Appendix. Generalization of the Inverse Problem

In Section 4 we showed how the inverse problem of determining $V(z)$ from spot motions (for a given initial tube shape) could be formulated for a constant $U(z)$. While not attempting a complete generalization for arbitrary $U(z)$ we here show that the analysis is not restricted to constant U. In particular for a linear form

$$U(z) = U_0\left(1 + \frac{z}{H}\right). \tag{A1}$$

Equation (21) becomes

$$t(z, z_0) = \frac{1}{U_0}\int_z^{z_0} \frac{\mathrm{d}z'}{1 + z'/H} = \frac{H}{U_0}\ln\left(\frac{1 + z_0/H}{1 + z/H}\right) \tag{A2}$$

so that (22) is replaced by

$$z(t) = H[(1 + z_0/H)\,e^{-U_0t/H} - 1]$$

or

$$z_0(t) = H[(1 + z/H)\,e^{U_0t/H} - 1]\,. \tag{A3}$$

Then (25) becomes

$$y(z, t) + \frac{1}{U_0}\int_{H[(1+z/H)\,e^{U_0t/H} - 1]}^{z} \frac{V(z')\,\mathrm{d}z'}{1 + z'/H} = f_0[H\{(1 + z/H)\,e^{U_0t/H} - 1\}] \tag{A4}$$

and the emergent points $y_*(t) = y(0, t)$ are located at

$$y_*(t) - \frac{1}{\mathrm{U}_0}\int_0^{H[e^{U_0t/H} - 1]} \frac{V(z')\,\mathrm{d}z'}{1 + z'/H} = f_0[H(e^{U_0t/H} - 1)]\,. \tag{A5}$$

Differentiating with respect to time then gives

$$\dot{y}_*(t) - U_0 f_0'[H(e^{U_0 t/H} - 1)]\, e^{U_0 t/H} = e^{U_0 t/H} \left\{ \frac{V[H(e^{U_0 t/H} - 1)]}{e^{U_0 t/H}} \right\}, \tag{A6}$$

the interpretation of which is that (analogously to (28)), $V(z)$ may be inferred from $\dot{y}_*(t)$ according to

$$V(z) = \dot{y}_* \left[\frac{H}{U_0} \ln\left(\frac{z}{H} + 1 \right) \right] - U_0 f_0'(z)\,(1 + z/H), \tag{A7}$$

which reduces to (26) as $H \to \infty$.

Analytic generalization is likewise possible for a quadratic approximation instead of (A1), for which integral (A2) is also analytic, and for higher orders numerically.

References

Fisher, G. H., Chou, D.-Y., and McClymont, A.: 1988, 'Emergence of Anchored Flux Tubes through the Convection Zone', Univ. Hawaii preprint, to be published in an *American Geophysical Union Monograph.*
Fletcher, L., Brown, J. C., and van Driel-Gesztelyi, L.: 1990, in preparation (Paper III).
Foukal, P.: 1977, *Astrophys. J.* **218**, 539.
Howard, R.: 1974, *Solar Phys.* **39**, 275.
Meyer, F., Schmidt, H. U., Weiss, N. O.: 1977, *Monthly Notices Roy. Astron. Soc.* **179**, 741.
Meyer, F., Schmidt, H. U., Simon, G. V., and Weiss, N. O.: 1979, *Astron. Astrophys.* **76**, 35.
Moreno-Insertis, F.: 1986, *Astron. Astrophys.* **166**, 291.
Parker, E. N.: 1979, *Cosmical Magnetic Fields*, Clarendon Press, Oxford.
van Ballegooijen, A. A.: 1982, *Astron. Astrophys.* **106**, 43.
van Driel-Gesztelyi, L. and Petrovay, K.: 1990, *Solar Phys.* **126**, 285 (Paper I).