HELIOSEISMIC IMAGING OF SUNSPOTS AT THEIR ANTIPODES

CHARLES LINDSEY
Institute for Astronomy, University of Hawaii, Honolulu, HI 96822, U.S.A.

and

DOUGLAS C. BRAUN*
National Solar Observatory, Tucson 85726, U.S.A.

(Received 24 July, 1989; in revised form 28 August, 1989)

Abstract. Recent work by Braun, Duvall, and LaBonte has shown that sunspots absorb helioseismic waves. We propose that sunspot absorption causes a seismic deficit that should be imaged at the antipode of the sunspot. If these images are observable, it should be possible to produce seismic maps of magnetic regions on the far side of the Sun. This possibility opens a broad range of synoptic and diagnostic applications. Diagnostic applications would include lifetimes of higher-frequency modes, and possibly rotation of the solar interior and detection of subsurface magnetic structure. We outline elements of the theory of seismic imaging and consider some applications. We propose the extention of acoustic holography to solar interior diagnostics in the context of antipodal imaging.

1. Introduction

Recent work by Braun, Duvall, and LaBonte (1987, 1988) has shown the surprising result that sunspots absorb a substantial fraction of the seismic energy that encounters their cross-section. They find that wave modes, expressed in cylindrical coordinates centered about the sunspot, show considerably more energy flowing into them than out. This phenomenon is so striking that the presence of the sunspot is manifest in wave motion even when the sunspot itself is outside the region observed (Braun, 1988). We propose that the effects of this seismic absorption will manifest themselves at the antipode of the sunspot as a deficit in seismic power. The basic idea is that the spherical symmetry of the Sun as a wave-propagating medium should result in coherent *focusing* of the seismic energy deficit generated by the sunspot at the antipode. Ideally, this would mean that solar p-modes, properly analyzed, could provide us with a sort of seismic absorption map of the far side of the Sun.

It is important to understand that the seismic focusing we are considering is not direct refractive focusing of rays from the source to the antipode by the solar interior. Indeed, the refractive properties of the interior do just the opposite and result in the further divergence of rays emanating from a point, whereby most rays are turned prematurely back to the surface. The attainment of a focus depends on the seismic energy being *trapped* under the surface, repeatedly reflected back inward at each arrival to the surface, for sufficient time for each ray to explore the solar interior extending to the antipode. We will say that the ray trajectory *walks* around the Sun under its surface, where each

* Now at the Institute for Astronomy, University of Hawaii, Honolulu, HI 96822, U.S.A.

encounter with the surface and reflection back into the solar interior is given the analogy of a single step. In a non-dissipative medium this walk carries seismic energy to the other side of the Sun, where spherical symmetry of the Sun results in its eventual coherent convergence at the antipode.

The quality of the resulting image is not as fine as that of a well-designed acoustic lens. Even in the ideal case of a non-dissipative non-rotating medium with perfect spherical symmetry, the point-source response function has broad wings that will degrade contrast. However, the sharpness of the core of the response profile is limited only by the availability of short-wavelength seismic waves to be absorbed by the sunspot and by the spatial resolution of the observations. Given reasonable signal to noise, it should be possible, in principle, to reconstruct the structure of the source to this resolution.

Antipodal imaging has already found terrestrial applications. Rial (1978) and Rial and Cormier (1980) analyzed the seismic image formed in the neighborhood of the antipode, in Europe, of an earthquake with epicenter in New Zealand. Butler, Brocher, and Rial (1986) suggest a variation of those measurements that would take advantage of occasional nuclear tests.

Besides the obvious synoptic utility of detecting sunspots or plage on the far side of the Sun, seismic focusing may offer useful interior diagnostic techniques. We propose the following possibilities:

(1) *P-Mode Lifetimes*

The strength and contrast of seismic images will depend on the lifetimes of the modes that contribute to the image. Thus, image contrast can provide a useful diagnostic of the lifetimes of various p-modes, particularly those that suffer considerable attenuation in only a single trip to the antipode.

(2) *Solar Rotation*

Differential rotation should cause observable aberrations in seismic images at the antipodes of point sources. Observations of these aberrations may, thus, contribute to diagnostics of subsurface rotation.

(3) *Other Sources and Sinks*

Seismic imaging of sunspots onto a region of quiet Sun may show us seismic qualities at a quiet antipode that are difficult to detect in the presence of sunspots and magnetic fields. Magnetic regions, for example, seem to suppress the local surface manifestation of a wave (i.e., the surface velocity- or brightness-amplitude) independent of whether they actually absorb wave energy. This superficial manifestation is eliminated at a quiet antipode, so a seismic deficit there assures us that wave energy has been absorbed. It is possible that antipodal imaging will reveal other localized seismic sinks, even perhaps sources, besides surface magnetic regions themselves.

(4) *Subsurface Structures*

Structures below the visible surface that absorb or emit seismic energy should image seismic disturbances onto the surface at their antipodes. These antipodal images may very well be stronger than the disturbance at the surface directly overlying the sources. In Section 2.2 we will consider holographic reconstruction of the source, including its depth, from observations of the image at its antipode.

A full development of the theory of seismic imaging in the Sun sufficient to address the topics listed above is a formidable task. However, some very useful elementary insight can quickly be gained with relatively simple considerations applied to an idealized non-rotating model of the Sun. The most idealized case assumes that waves, once generated, have an infinite lifetime; this case is considered in Section 2. Practical applications quickly force us to consider waves with limited lifetimes, which are discussed in Section 3. We end with a brief conclusion in Section 4. We now proceed with a discussion of the general considerations that assure focusing at the antipode of a source where perfect spherical symmetry applies in a non-dissipative medium.

2. Wave Optics in an Ideal Spherically-Symmetric Refractor

2.1. Localized Seismic Sources in a Non-Dissipative Medium

We now consider how the disturbance from a localized source forms an image at the antipode in the simple case of a non-dissipative medium in which density and compressibility are functions of radius, r, alone. For such a medium we know that the eigenmodes can be expressed in terms of radial functions, $R_{nl}(r)$, multiplied by spherical harmonics $Y_l^m(\phi, \theta)$. If we denote the eigenfrequencies by $\omega_{nl} > 0$, we can express the general solution of the wave equation in terms of these eigenfunctions:

$$\psi(r, \theta, \phi, t) = \sum_{lmn} a_{lmn} R_{nl}(r)\, Y_l^m(\theta, \phi)\, e^{i\omega_{nl}t} + (c.c.)\,. \tag{1}$$

The term '(c.c.)' denotes the complex conjugate of the foregoing sum. We will let ψ be the pressure perturbation at time t in an element of fluid located at coordinates (r, θ, ϕ) in the unperturbed medium.

We are interested in the response of the medium to a localized excitation, i.e., a point source. For simplicity, we represent this by a single event that perturbs a previously undisturbed medium at a point which we let be the z-axis, i.e., the pole $\theta = 0$, of our spherical-coordinate reference frame. Both the amplitude and its rate must be specified initially. Thus,

$$\psi(r, \theta, \phi, 0) = S(r)\delta_2(\mathbf{p}) \tag{2}$$

and

$$\psi_t(r, \theta, \phi, 0) = \dot{S}(r)\delta_2(\mathbf{p})\,, \tag{3}$$

where $\mathbf{p}$ is the displacement on the unit sphere of the position denoted by the angles θ and ϕ from the initial disturbance. This serves as the argument for a two-dimensional

Dirac delta-function, δ_2, on the unit sphere, whose integral over the entire surface is unity. The radial profile, $S(r)$, of the disturbance and its initial rate, $\partial S/\partial t$ are left arbitrary for now. This disturbance can be expressed in terms of the eigenfunctions as follows:

$$\psi = \sum_l \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta) \sum_n b_{nl} R_{nl}(r) + (c.c.) , \tag{4}$$

by choosing the coefficients b_{nl} so that for each l

$$S(r) = 2 \sqrt{\frac{4\pi}{2l+1}} \operatorname{Re}\{\sum_n b_{nl} R_{nl}(r)\} \tag{5}$$

and

$$\frac{\partial S}{\partial t} = -2 \sqrt{\frac{4\pi}{2l+1}} \operatorname{Im}\{\sum_n \omega_{nl} b_{nl} R_{nl}(r)\} . \tag{6}$$

Following the initial perturbation, the medium is allowed to evolve, which it does by the rule of $\exp(i\omega_{nl}t)$ applied to each mode:

$$\psi = \sum_l \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta) \sum_n b_{nl} R_{nl}(r)\, e^{i\omega_{nl}t} + (c.c.) . \tag{7}$$

We are now interested in how this motion manifests itself in seismic power, ψ^2, at the surface averaged over time. Squaring Equation (7) and averaging over time, we can neglect all cross terms with $n' \neq n$ and $l' \neq l$ whose separation in frequency is large compared to the reciprocal of the time over which we average. For this problem, we will assume no degeneracies, thus, a sufficiently long average will eliminate all cross terms, giving us

$$\langle \psi^2 \rangle = \frac{1}{4\pi} \sum_l (2l+1)\, q_l P_l^2(\cos\theta) , \tag{8}$$

where q_l is the following sum:

$$q_l = \sum_n |b_{nl}|^2 R_{nl}^2(r) . \tag{9}$$

We can only measure seismic power at the surface, whereby q_l must be evaluated at $r = R_0$, the solar radius.

The exact form of the response function expressed by Equations (8) and (9) depends on the depth profile of the initial perturbation, S, and its rate, $\partial S/\partial t$. An example is shown in Figure 1. If large values of l are included in the sum, the response tends toward a narrow peak flanked by relatively broad wings that attenuate in proportion to $1/\sin\theta$.

In practice, we can only average over a finite time. The foregoing results can be accurate only if the average is taken for a period many times the travel time from the disturbance to the antipode. While coherent image will begin to appear at the antipode

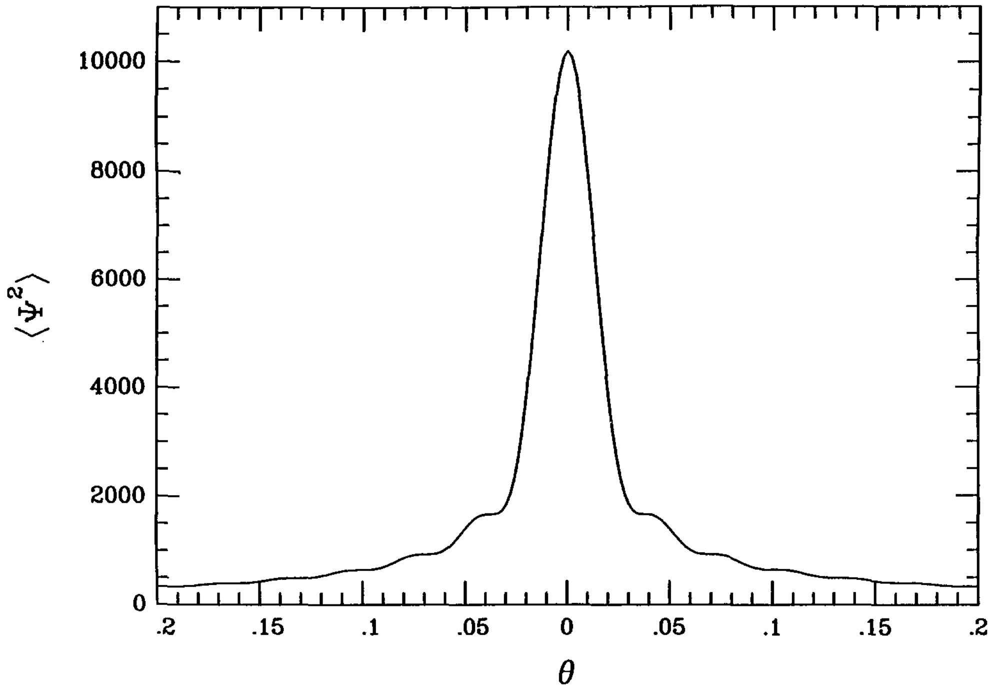

Fig. 1. The point source response function, $\langle \psi^2 \rangle$ (cf. Equation (8)), is plotted for a special case: we assumed q_l to be unity for all $l \leq 100$ and zero otherwise. The ripples seen in the wings will disappear if the transition from unity to zero is made more smoothly; however, the overall amplitude of the wings tends to retain its proportionality to $1/\sin\theta$ for small $\sin\theta$ more or less independently of this.

after the minimal time required for a single trip, approximately one or two days, its full development depends on a time average covering many times this period.

Now, with waves continually being generated in the Sun and absorbed by sunspots, the determinant of image contrast becomes not the time the observation lasts, but rather the *lifetime* of the waves being observed. Longer observations will, as always, provide greater statistics, but the expectation image profile will be stationary. The theory we have developed above is accurate only for waves whose lifetimes are many days.

It is useful to notice that the time-averaged response function for the ideal medium we have been considering is exactly symmetrical about the plane that sits midway between the initial disturbance and its antipode. The *mean seismic power* at any point on the surface (or even in the interior) is equal to that at its antipode. This remains true given any distribution of sources, so long as the waves emitted have a very long lifetime. This result follows quite easily from Equation (1). Any seismic source or sink should manifest itself in seismic power as strongly in the neighborhood of its antipode as in the analogous neighborhood of the source itself.

2.2. Surface sources

Even for a source localized at the surface, the image is not localized at the surface. It will generally be distributed over a broad range of depth below the surface. This is the range of depth explored by the various modes whose participation in the process is set by the initial perturbation. However, the spatial dependence of the seismic power over the surface is the same for all depths. If the initial distribution is localized just at the surface, the redistribution over a range of depth will result in a faded image at the antipode. However, if the source continues to emit further seismic disturbance, this image will accumulate strength continuously with time.

2.3. Absorption

Let us now suppose that the sun contains an initial distribution of seismic power that is statistically independent of ϕ and θ, though it will generally depend on depth. We insert an absorber into some point on the Sun's surface and look for an image at its antipode. The profile of such an image will look exactly like that of a point source, but will appear as a deficit in a constant background of seismic noise.

An absorber placed at $\theta = 0$ should be preferential to spherical harmonics, Y_l^m of relatively low $|m|$. This is expected because the amplitude of the harmonic is proportional to $\theta^{|m|}$ for small θ and, thus, becomes very small for large $|m|$ in the neighborhood of the absorber, resulting in a small interaction. We should expect that absorption of waves will begin to fail when $|m|$ exceeds a certain threshold,

$$|m| > kR_{\text{spot}}, \tag{10}$$

where k is the wave number of the mode and R_{spot} is the radius of the sunspot. This is confirmed in a cylindrical representation of seismic modes with a coordinate system centered on the sunspot and absorption is measured as a function of the analogous index, m, in cylindrical harmonics, namely, the order of the Bessel function representing the radial dependence of the mode (Braun, Duvall, and LaBonte, 1988). It is tempting to simply extrapolate to a point absorber by concluding that only the $m = 0$ mode is absorbed. This is a bit premature, since the absorption process may be complicated. It may depend, for example, on the horizontal velocity gradient or shear rather than simply velocity amplitude, causing it to destroy the modes $m = \pm 1$ and leave the $m = 0$ mode alone. Indeed the resonant absorption theories of Hollweg (1988) and Lou (1989) fail to predict absorption of the $m = 0$ modes. The work of Braun (1988), however, indicates that the $m = 0$ mode is absorbed as strongly as all others. For the present we will consider a *point absorber* to be one that destroys only the $m = 0$ mode, and we will assume that such a thing can somehow exist. Absorbers that destroy modes of higher $|m|$ will be supposed to consist of a distribution of point absorbers on the surface. (Note that this definition of a point absorber localizes the absorber only laterally, not in depth.)

2.4. Surface absorption

An efficient point absorber extending deep under the surface will destroy the entire $m = 0$ mode in the time it takes for the wave to propagate once around the Sun. After only half of that time, i.e., the time it takes for wave energy to propagate just to the antipode, the antipode should show a complete deficit in seismic power, as the $m = 0$ mode is extinguished there.

While we know that sunspots are efficient absorbers at the surface, we do not know that they extend directly downward to great depths nor, if they do, whether they are efficient absorbers at great depths. As Braun, Duvall, and LaBonte (1988) point out, even if we could depend on the former we dare not expect the latter. Until we better understand the mechanism of absorption in sunspots, it may be safer to expect magnetic flux tubes to interact strongly with seismic waves only near the surface. At great depths the gas pressure far exceeds the magnetic pressure, and this would seem to make the magnetic flux tube invisible to seismic waves. If this is actually the case, a point absorber localized at the surface can destroy only a relatively small portion of the wave mode in a single *walk* around the Sun. If the absorber remains in place, however, it should eventually destroy the mode completely, again effecting a complete seismic deficit just at the antipode. Such a destruction must take many days, since in any single walk around the Sun a wave will encounter the surface absorber only by accident.

By averaging seismic power over a long time, we were able to arrive at a relatively simple expression (Equation (8)) for the time-averaged seismic power. We know that the Sun has modes with very long lifetimes. Unfortunately, these modes do not seem to be the ones best absorbed by sunspots. Modes with l up to 100 can be individually resolved in $l - \nu$ power spectra and, therefore, are assured of relatively long lifetimes (Duvall *et al.*, 1988). However, sunspots absorb these inefficiently if at all. They do absorb efficiently waves with $l \sim 200$ or greater. Here absorption is many times more efficient in a sunspot than in the quiet Sun; however, it may not be so strong as to be the dominant mechanism of seismic absorption in the sun. While these waves may well survive a single walk to the antipode, they do not seem asssured of surviving for the many days in the quiet Sun we have assumed up to this point. In the following section we consider how to look for seismic images where wave attenuation is important.

3. Waves with Limited Lifetime

3.1. Source and Image Profiles

We now admit the consideration that in the real Sun waves are continuously being generated in the convection zone and destroyed, not only by magnetic regions but in the quiet Sun as well by gradual loss of coherence due to turbulence and perhaps by other absorption mechanisms. Attenuation of waves by the quiet Sun will further decrease image contrast at the antipode. Attenuation limits spatial discrimination, since the shorter wavelengths needed to resolve smaller features tend to attenuate faster. Thus, it is important that we understand antipodal imaging when wave attenuation is important. In the real Sun, a long lifetime may become a liability, allowing time for differential rotation to destroy the antipodal image. Thus, it may serve to our considerable advantage to purposely select for waves whose lifetimes are relatively short, so that they contribute significantly to the image only on their first walk to the antipode.

For this discussion we will represent the destruction of each mode, (l, m, n), statistically by a simple exponential attenuation of its amplitude at an appropriate rate, γ_{nl}, and its replacement by random noise. We suppose that this is a considerable simplification of what actually happens in the Sun; however, it is useful for the elementary discussion that follows. Equation (1) now takes the form

$$\psi(r, \theta, \phi, t) = \sum_{lmn} a_{lmn} R_{nl}(r)\, Y_l^m(\theta, \phi)\, e^{(i\omega_{nl} - \gamma_{nl})t} + (c.c.)\,. \tag{11}$$

Detection of a seismic image rests on our ability to discriminate this surviving coherent seismic power at the antipode from the noise that surrounds it. If the time required for a wave to reach the antipode is many times the dominant coherence times, γ^{-1}, the antipodal image will be very weak and the statistics required to discriminate it from noise will eventually become prohibitive.

When attenuation is important, a number of important changes occur. A fundamental change is that the time-averaged seismic power at each point is no longer equal to that at its antipode. Not only is the contrast of the image at the antipode reduced, but its profile is changed. Let us again consider a seismic source that takes the form of an initial disturbance localized at the surface. This disturbance propagates outward along the surface and downward into the solar interior. Because the disturbance spreads both outward and downward, its amplitude at the surface decreases quite rapidly with distance from the source point, so that the mean seismic power at the surface will appear strongly localized near the source. When the disturbance arrives at the antipode spherical symmetry assures a coherent image, as before. However, unlike the seismic power in the neighborhood of the source, the wave energy at the antipode makes no particular attempt to convene back to the surface; rather, it remains diluted over the range of depth penetrated by the various modes excited. This range of depth is substantially greater than the smallest features we are interested in resolving, and, therefore, the dilution is considerable.

The overall form of the image profile at the antipode, the width of its core and the relative strength of its far wings, is not particularly different from that for infinite lifetimes, discussed in Section 2. As in the case of infinite lifetimes, the image profile still basically reflects the overall properties of the spherical harmonics (see Figure 1), and is not particularly changed. It is simply weaker, in proportion to its attenuation. What has changed considerably is the quality of the disturbance in the neighborhood of the *source*. The source profile has actually *improved*; it is more localized, sharper. The concept is illustrated in Figure 2. Because the waves being emitted do not live to return to the neighborhood of the source, the seismic disturbance surrounding the source is due entirely to energy radiating outward and downward immediately from the source, energy that was entirely concentrated into the source at the instant it was emitted. When waves have long lifetimes, the disturbance surrounding the source is actually broader, because then it is dominated by energy still remaining from a long time before, which has settled to the same profile with relatively broad wings that characterizes the antipode.

Exactly how the same spherical harmonics that comprise a diluted source profile at the antipode can collaborate to give a sharp one at the source is simply by destructive interference. We actually contrived, by destructive interference, to construct an initial

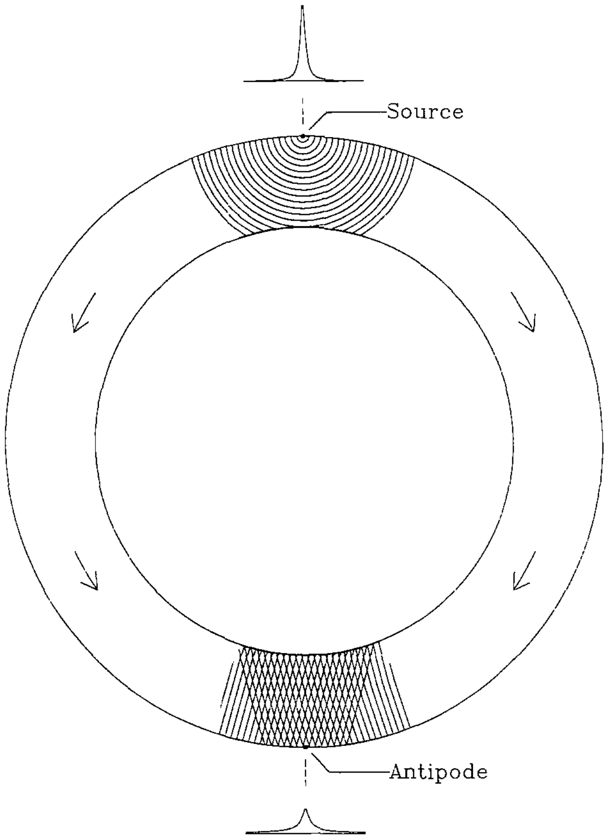

Fig. 2. Conceptual illustration of image dilution at the antipode. Waves emanating from a surface source (*top*) comprise a disturbance highly concentrated about the source itself (*top profile*). At the antipode (*bottom*) waves converge to give rise to a coherent image. However, this image is greatly diluted (*bottom profile*), since the energy is dispersed over a range of depth large compared to the source. The spherical wavefronts emanating from the source in this figure can be correct only for a Sun with a constant refractive index in its interior. This is not nearly the case for the real Sun.

disturbance that was zero everywhere except at the source point (see Dirac delta functions in Equations (2) and (3)). In choosing the mode coefficients, a_{lmn}, of Equation 1 so that the initial coherent sum was totally localized, we greatly improved the power distribution averaged over the early part of the evolution of the disturbance. By the time the disturbance has propagated to the antipode, the coherence that originally kept it localized is lost, resulting in an image that is diluted vertically.

In principle, the sharp profile surrounding the source on the far side of the sun should be restorable to some degree, given sufficient observations of the near surface, preferably including the image at the antipode. There seems to be already an approach to this concept in the science of *acoustic holography*, which makes it possible to reconstruct sub-surface as well as surface sources, from their surface disturbances. Acoustic holography is an active and growing science that has already found powerful terrestrial applications, particularly in medicine, in interior diagnostics of the human body.

Roddier (1975) suggested its use for detecting subsurface acoustic sources in the Sun. The application of acoustic holography to the Sun – we will call it *helioseismic holography* – deserves a detailed development, which we will not attempt here. An outline of the basic concepts is given in the Appendix.

3.2. Filtering

Different global modes contribute differently to the seismic image at the antipode, some contributing strongly to the image and others contributing little or perhaps too much noise. An important part of image enhancement will consist in filtering preferential to modes that contribute in the way we desire. This will be a powerful tool not only for optimizing an image, but for using the image for solar interior diagnostics as well. Filtering is normally accomplished by selecting power from particular desired regions in the $l - \nu$ plane. Selection of these regions will be based on a number of considerations. We enumerate some of the considerations that are likely to be important:

3.2.1. *Mode Lifetime*

Certain modes will probably have too short a lifetime to contribute usefully to an image at the antipode. In place of their contribution will appear seismic noise unrelated to the sources or sinks of interest, so these are best filtered out. Scherrer (1989) notes that waves with a frequency higher than the seismic cutoff cannot be relied on to reflect nicely at the Sun's surface. It seems that these should pass upward through the temperature minimum and into the chromosphere, where they risk destruction. Even if they survive and are reflected, e.g., by the overlying transition layer, the reflection is likely not to be specular; the highly inhomogeneous structure of the chromosphere may destroy its coherence completely in a single encounter. The lifetimes of global modes of lower l begin to decrease suddenly and rapidly for periods of less than 3 minutes (Libbrecht, 1988). Thus, it is clear that proper frequency selection will be of utmost importance in the detection, enhancement and analysis of seismic images.

3.2.2. *Spherical Harmonic Index, l*

Sunspots seem to absorb efficiently modes with l of order 200 or greater. Modes with l much less than this will be of little use in detecting magnetic regions and, again, will offer only noise in place of useful signals. The detailed dependence of sunspot absorption on l and on frequency, ν (or radial index, n) has yet to be thoroughly explored.

3.2.3. *Radial Index, n*

Modes with a higher radial index, n, penetrate deeper into the solar interior and suffer more from dilution of their energy into a greater volume. Contributions from these modes will benefit most from holographic reconstruction. At present the modes that sunspots are known to absorb efficiently and that seem to have sufficient lifetimes to survive a trip to the antipode have radial indices up to about 5 or possibly as high as 10. These modes reach the antipode in a relatively short time, a day or so for l of order 200. Modes with small n and the fundamental mode adhere closely to the surface, thus suffer very little dilution in depth. However, they require several days to reach the antipode. Consequently, the image they show may be considered somewhat out of date. This should not be considered a criticism; indeed, the range of time delays that characterize different modes suggest interesting diagnostic utilities. A disturbance caused by an active region visible to us near the west limb can appear seismically at its antipode when the antipode has become newly visible to us around the east limb.

The filtering appropriate for a given data set will likely depend on many aspects we have not yet considered. In fact, it may vary from one data set to another, depending on the acoustic features in the data to be detected. Finally, it will always be subject to personal preference and depend on particular qualities of the image we wish to isolate. In the end, optimum filtering procedures for the range of diagnostic and synoptic applications that will arise will have to be established by experiment.

4. Conclusion

The purpose of this paper has been to outline some of the considerations most basic to acoustic imaging in the Sun and to open the concept for discussion and new ideas. The theory needs development in many areas. These include all of the areas we have touched on, some we have only mentioned and otherwise left alone (e.g., aberrations due to solar differential rotation) and certainly many that we are totally unaware of.

The possibility of actually observing antipodal images opens the prospect of a very powerful new diagnostic of the solar interior. This prospect is very real with the sensitive high-resolution observations that will be made continuously over long-periods from the Solar Oscillations Imager that will fly aboard SOHO. Applications of antipodal imaging extend far beyond the natural synoptic utility of producing maps of sunspots on the far side of the Sun. Indeed, antipodal imaging would give us the greatest insight if Earth-based observations could be combined with simultaneous observations from the far side of the Sun. Antipodal correlations in the quiet Sun could then be studied with extraordinary precision and detail, and a whole new area of analysis would probably emerge. Prospects for this are, perhaps, rather distant for now.

Acknowledgements

We wish to thank Dr Thomas Duvall for his thoughtful insight and encouragement. We thank Dr Phillip Scherrer for his very useful comments and perspective. We thank Drs Barry LaBonte and Clifford Toner and Mr Matthew Penn for enlivening discussions and useful suggestions. We thank Dr Francois Roddier for his clear perspective. We finally thank the referee for many important corrections and for a considerate, thorough, and knowledgeable review of the manuscript. Part of this work was done while D. C. Braun held a National Research Council – NASA Research Associateship at the Goddard Space Flight Center. This work was also supported by NASA Grant NAGW 723.

Appendix. Helioseismic Holography

(A). Seismic source reconstruction

However valid our interest in attenuated waves, we must admit a 2-fold liability when waves can survive only a single trip to the antipode: while the waves suffer directly due to attenuation, the acoustic power that does survive the trip to the antipode must suffer further by being diluted over a large range in depth under the antipode. Ironically, wave attenuation *improves* the profile of the disturbance in the neighborhood of the *source* while degrading its amplitude at the antipode (see Figure 2).

Now, if it were possible to reconstruct, to some degree, the improved disturbance profile in the neighborhood of the source, working backwards from the antipodal disturbance, this would considerably offset the liabilities of attenuation and image dilution. We now need to make an important point: acoustic power at the antipode is not in any way wasted for being diluted below the Sun's surface. All wave energy manifests itself at the surface *near* the antipode at some time or other. It is simply not as strongly localized at the antipode as it is in the neighborhood of the source. Given an accurate model of the acoustic properties of the solar interior, it should be possible to use observations of surface motion over an appropriate time interval near the antipode to reconstruct the sharper image surrounding the source.

The principle is relatively simple: We consider acoustic properties of the solar interior to be fully characterized by the frequencies, ω_{nl}, and attenuation rates, γ_{nl}, of the various modes, (l, m, n), of Equation (11). For this discussion we will assume that these are known. The mode coefficients, a_{lmn}, are determined by observing one side of the Sun over an appropriate time period. To the extent that the amplitudes a_{lmn} can be accurately and independently determined, we can use them to run Equation (11) in time-reverse on the far side of the Sun to reconstruct the acoustic environment from which the image emanated.

In practice this concept is not entirely complete. For modes of limited lifetime, observations of only one face of the Sun give us only about half of the necessary information to determine all the amplitudes, a_{lmn}. This leaves us with an ambiguity, resulting in a confused picture in which reconstructed sources on the far side of the Sun cannot be separated from acoustic ghosts projected backward from local sources on the near side, a confusion we do not particularly propose to resolve here. Moreover, wave attenuation, when time reversed, results in unstable growth of motion extrapolated back to the source. This by itself does not contribute any further liability that is not inherent in wave attenuation itself. However, the accidental extrapolation of noise that appears late in the time series backwards over an unnecessarily long time will eventually swamp all early contributions to the reconstructed source. So practical application requires some care.

In fact, the concept we are developing here is very much the acoustic analog, in the environment of a spherical refractor, of *holography* in the optical environment of simple diffraction. The basic procedure of acoustic holography is illustrated in Figure (A1): we

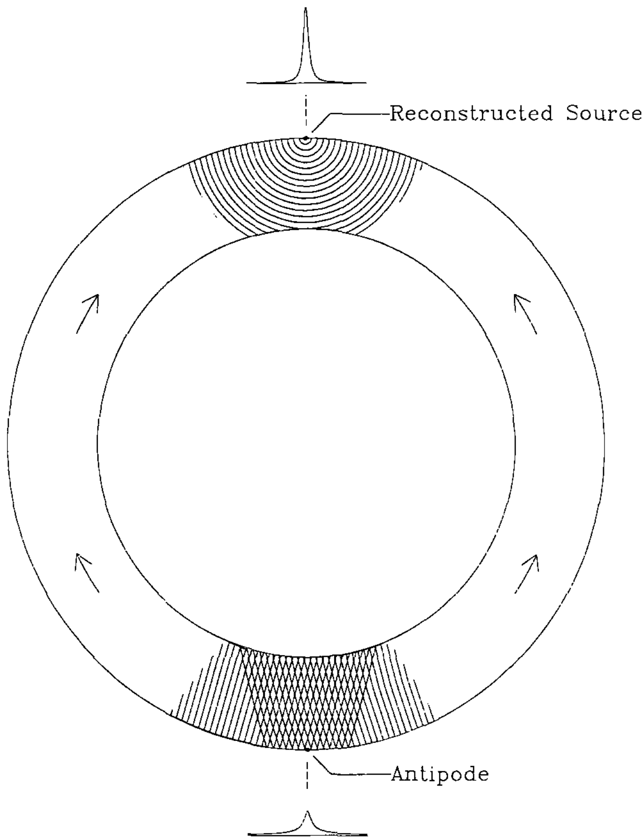

Fig. A1. Conceptual illustration of acoustic holography. The time-reversed surface manifestation of the acoustic disturbance at the antipode is applied to the surface of a solar model (*bottom*), driving it backwards to effect a reconstruction of the surface source (*top*).

simply apply the surface manifestation of the disturbance observed on the near side of the Sun in time-reverse to the surface of an appropriate acoustic model of the Sun, driving it backwards in time to reconstruct the initial source. The analogy to optical holography is best seen by considering that image reconstruction from a hologram can be achieved by passing radiation backwards through it to reconstruct the electromagnetic amplitude at the original source; somewhat the time-reverse of the process that created it. Numerically, image reconstruction from a hologram is accomplished by Fourier analysis, indeed, simply by an inverse Fourier transform when the hologram reduces to a simple Fraunhofer diffraction pattern. It should be possible to formalize acoustic holography in a solar model in a similarly satisfactory way.

In practice it is not usually necessary to obtain the entire hologram, i.e., the amplitude over an entire surface enclosing the source, to reconstruct the basic image. When we view an optical hologram, a piece of the hologram no larger than our pupil is sufficient for the resolution our eye can appreciate. Of course, when only part of the hologram is available, information is incomplete and confusion is introduced, usually in the form of limited spatial discrimination. So, we should appreciate a finer-quality reconstruction if we observe a greater area of the Sun.

It is only fair that we dispel any notion that antipodal images are necessary to acoustic holography in a spherical refractor. It should be straightforward to reconstruct a source by observing any sufficiently large region of the Sun for sufficient time. That region need not include the antipode. However, because an acoustic disturbance emanating from a certain region is accentuated at its antipode, the antipode should constitute a particularly rich part of its acoustic hologram where considerations of signal-to-noise are concerned.

It is also important not to confuse holography with any form of image deconvolution. When a sharp image profile is smeared by a simple convolution, information is irreversibly damaged with respect to signal to noise; the loss cannot be recovered. Waves distorted by a refractor show optical aberrations that may look like smearing. However, when phase coherence is maintained, correcting optics can reconstruct a stigmatic image at no cost in signal to noise. This is the role of holography in acoustic optics as well as in electromagnetic optics.

(B) Focusing and subsurface sources

It is worth noting that standard holographic techniques show not only the lateral location and spatial extent of a source, but its position along the optical path as well. Thus, acoustic holography offers us a powerful tool to detect subsurface sources on the far side of the Sun. Notice that while surface sources are surrounded by a strong localized disturbance at the surface, a submerged source is not. The high concentration of acoustic energy around the source becomes a disadvantage for detecting it on the overlying surface when the source is submerged. However, subsurface sources and sinks will manifest themselves at the antipodal surface approximately as strongly as will surface sources. The position and depth of a source should be evident in its holographic reconstruction. In principle, acoustic holography should reconstruct subsurface sources and sinks approximately as well as those at the surface.

As with normal optical holograms, in acoustic holography the acoustic power can be sampled anywhere in the acoustic medium. We may, for example, choose to compute acoustic power along a surface lying at a certain depth below the Sun's surface. The proper selection of the appropriate surface in which to sample the intensity is the optical equivalent of *focusing* of the image. Focusing techniques analogous to those of optical holography should provide a convenient way to determine the depths of sources. While we have already disclaimed a strong expectation of seeing acoustic sources or sinks far below the surface, experiments with focusing may provide useful insight into the subsurface structure of features apparent on the surface or of structures that are hidden not far beneath the surface.

References

Braun, D. C.: 1988, Ph. D. Thesis, University of Hawaii.
Braun, D. C., Duvall, T., and LaBonte, B. J.: 1987, *Astrophys. J.* **319**, L27.

Braun, D. C., Duvall, T., and LaBonte, B. J.: 1988, *Astrophys. J.* **335**, 1015.
Butler, R., Brocher, T. M., Rial, J. A.: 1986, *EOS* **67**, 89.
Duvall, T. L.: 1989, private communication.
Duvall, T. L., Jr., Harvey, J. W., Libbrecht, K. G., Popp, B. D., and Pomerantz, M. A.: 1988, *Astrophys. J.* **324**, 1158.
Hollweg, J. V.: 1988, *Astrophys. J.* **335**, 1005.
Libbrecht, K. G.: 1988, *Astrophys. J.* **334**, 510.
Lou, Y.-Q.: 1989, preprint.
Rial, J. A.: 1978, *Geophys. J. Roy. Astron. Soc.* **55**, 737.
Rial, J. A. and Cormier, V. F.: 1980, *J. Geophys. Res.* **85**, 2661.
Roddier, F.: 1975, *Comt. Rend. Acad. Sci.* **281**, B993.
Scherrer, P.: 1989, private communication.