SOLAR DIFFERENTIAL ROTATION DERIVED FROM SUNSPOT OBSERVATIONS

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Abstract. Sunspot drawings obtained at National Astronomical Observatory of Japan during the years 1954–1986 were used to determine the differential rotation of the Sun. From the limited data set of three solar cycles it was found that three factors (the level of cycle activity, the cycle phase, and sunspot type) affect the solar rotation rate. The differential rotation varies from cycle to cycle in such a way that the rotation velocity in the low activity cycle (cycle 20) is higher than in the high-activity cycle (cycle 19). The equatorial rotation rate shows a systematic variation within each cycle. The rate is higher at the beginning of the cycle and decreases subsequently. Although quite small, the variation of solar differential rotation with respect to Zürich sunspot type was found. The *H* and *J* types show the slowest rotation among all the sunspot types.

1. Introduction

The solar rotation velocity depends on solar latitude. The velocity is highest at the equator and decreases with increasing latitude. This phenomenon is, therefore, called the solar differential rotation. The differential rotation velocity in the photospheric layer can be determined by two different ways: Doppler shift method (plasma rotation measurement) and from the tracing of solar features (mostly sunspots). The tracer method has the advantage over the Doppler shift method, in that one can in principle measure two components of the velocity field. However, the tracing of sunspots can only determine the solar velocity within the sunspot belt (latitude up to 45 deg).

The modern era of plasma rotation measurement has started in the sixties. Livingston (1969) published comprehensive measurements of solar differential rotation taken at the Kitt Peak Observatory, covering the period 1966–1968. Soon afterwards Howard and Harvey (1970) presented Mt. Wilson data for the years 1966 through the end of 1968. They found that the solar rotation as measured by Doppler shift method is 7% slower than that measured by sunspot proper motion technique (Newton and Nunn, 1951). This difference may be partly caused by the difference in the measuring techniques, but in general it is believed that this reflects the difference in the physical conditions between photospheric (non-magnetic) plasma and sunspots.

Although Doppler shift measurements are equally important, the solar differential rotation can be derived most directly from the measurement of daily sunspot positions. There are two methods in finding the sunspot positions. One is based on photographs of the solar disk, and the other utilizes the projection of the solar image on the screen

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paper (i.e., sketches). Newton and Nunn (1951), Ward (1966), Balthasar and Wöhl (1980), Arevalo *et al.* (1982) and Balthasar, Vazquez, and Wöhl (1986) used Greenwich Photoheliographic Results (GPR) for different kinds of Zürich sunspot types and for different time intervals. Newton and Nunn (1951) used only recurrent single spots. Arevalo *et al.* (1982), Ward (1966), Balthasar and Wöhl (1980) used all types of sunspots but only for restricted time intervals. Balthasar, Vazquez, and Wöhl (1986) used the complete sample of GPR in the full interval 1874–1976. Howard, Gilman, and Gilman (1984) analyzed the Mt. Wilson white-light plate collection from 1921 through 1982.

Lustig (1982) studied the sunspot drawings obtained at Kanzelhöhe solar observatory in the interval 1970–1979. Later, Lustig (1983) analyzed carefully all the data from 1947 to 1981. In this paper we take the same approach as Lustig (1982, 1983) and study the sunspot drawing data obtained at National Astronomical Observatory of Japan during the period 1954–1986. Special attention is paid to:

(1) whether the differential rotation changes from cycle to cycle,

(2) whether the differential rotation changes systematically within a cycle as a function of cycle phase, and

(3) whether the magnitude of the differential rotation is different for different sunspot types.

2. Observation and Data Reduction

The sunspot telescope of National Astronomical Observatory at Mitaka which had been used to obtain the present data has a focal length of 3600 mm and an aperture of 200 mm. The observation consisted of the drawings of the Sun with a diameter of 240 mm. The daily sunspot position can be determined by overlaying the Stonyhurst disk on the drawings, with the expected accuracy of 0.5 deg in heliographic coordinates. In the case of large groups or bipolar groups the definition of the spot position has some ambiguity. From 1954 to 1980 the position of a large sunspot group was defined by following the same particular spot in the group. From 1981 to 1986, the center of gravity was used for defining the position of a sunspot group. The data cover three activity cycles, namely cycle 19 (1954–1964), cycle 20 (1965–1975), and cycle 21 (1976–1986).

Rotation velocities (deg day^{-1}) as a function of latitude were calculated by the difference of daily positions and the time interval between the two observations. For this purpose only sunspots with longitudes smaller than 70 deg and latitudes less than 40 deg were selected. We eliminated the single *A* type spots from the data that were not observed in two consecutive days. For other types of sunspots, two observations separated by more than a day were included in the data if the shape of the sunspots on the two days is similar. All of these data were divided into 6 classes: *A*, *AA* (non-single *A* type), *B*, *C*, *DEFG*, and *HJ* types of Zürich classification of sunspots. Generally sunspot groups change their type as they evolve. When a sunspot group on the two observing days belongs to different classes, such a group was excluded from the data. There are 8531 spots remained in total: 482, 710, 854, 1979, 2068, and 2438 of *A*, *AA*,

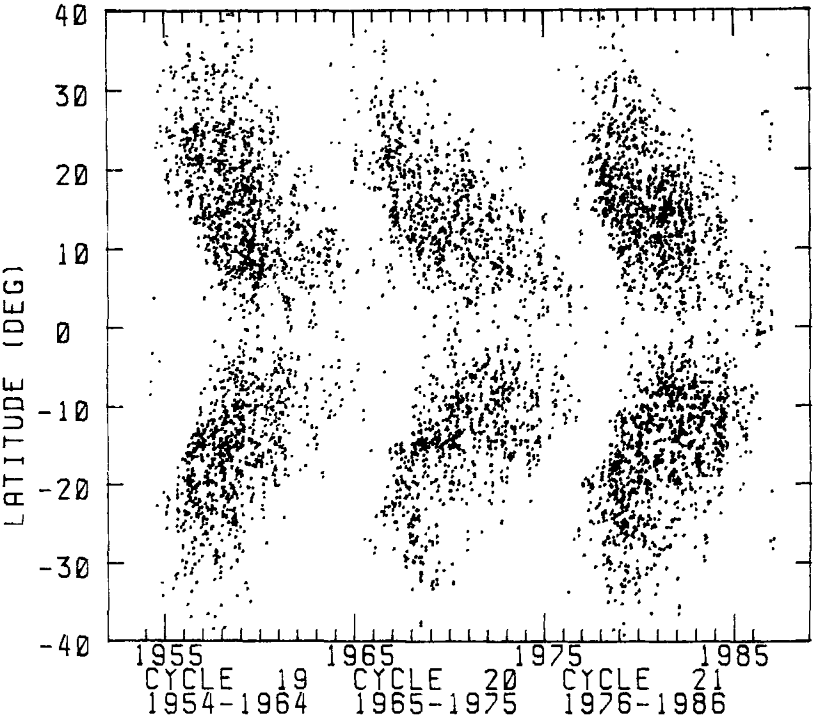

Fig. 1. The latitude distribution of the sunspots in 1954–1986 which were used for determining the solar differential rotation.

B, *C*, *DEFG*, and *HJ* types, respectively. These 8531 spots are plotted in Figure 1 in the form of the butterfly diagram. Figure 1 shows that more than 99% of sunspots in our data belong to cycles 19, 20, and 21, with only a few sunspots in cycles 18 and 22. The distribution of sunspots in northern and southern hemispheres are shown separately in Figure 2. There are no significant differences between the two.

The latitude dependence of solar rotation is usually represented by the formula

$$\omega(\phi) = A + B \sin^2 \phi \,, \tag{1}$$

where ϕ stands for the latitude, ω, A, and B are given in unit of deg day^{-1}, and ω is considered as the sidereal angular velocity. The coefficients A and B are determined from measurements by the least square method as

$$A = \frac{[\Sigma X_j^2 \omega_j - \Sigma X_j \Sigma (X_j \omega_j)]}{[N \Sigma X_j^2 - (\Sigma X_j)^2]} \,, \tag{2}$$

$$B = \frac{[N \Sigma (X_j \omega_j) - \Sigma X_j \Sigma \omega_j]}{[N \Sigma X_j^2 - (\Sigma X_j)^2]} \,, \tag{3}$$

where $X = \sin^2 \phi$, j ($= 1, 2, 3 \ldots N$) distinguishes each measurement, and N stands for the total number of measurements. ΔA and ΔB are the expected errors in A and B defined

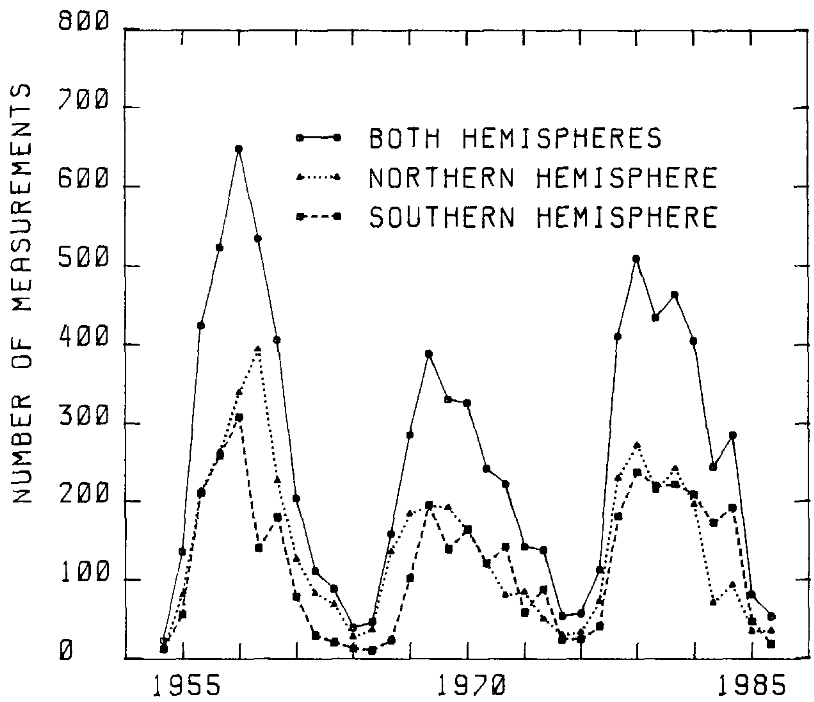

Fig. 2. Distribution of the sunspots in each hemisphere of the Sun which were used in the present analysis.

by the formulas

$$\Delta A = \frac{[\Delta^2(\Sigma X_j^2)]^{1/2}}{[N\Sigma X_j^2 - (\Sigma X_j)^2]}, \tag{4}$$

$$\Delta B = \frac{[\Delta^2(\Sigma N)]^{1/2}}{[N\Sigma X_j^2 - (\Sigma X_j)^2]}, \tag{5}$$

with

$$\Delta^2 = \frac{\Sigma[\omega_j - A - B\sin^2\phi]^2}{N-2}. \tag{6}$$

3. Results and Discussion

3.1. DIFFERENTIAL ROTATION PROFILES

By applying Equations (2) and (3) to all the data, we found that the equatorial rotation rate $A = 14.44$ and gradient $B = -2.56$. The fitted curve is plotted in Figure 3 together

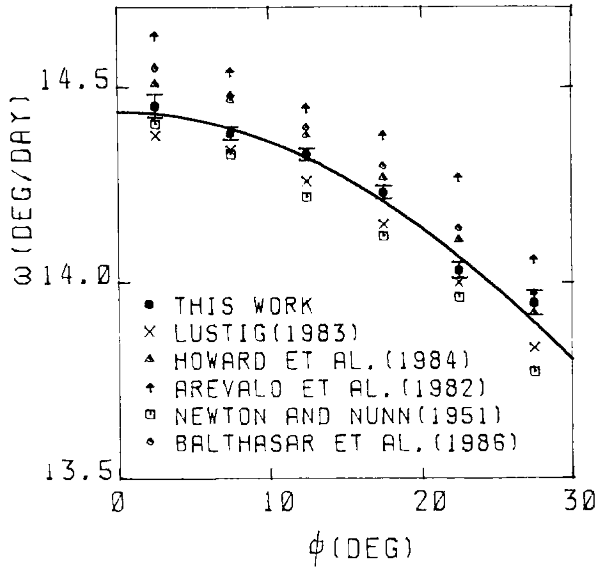

Fig. 3. Differential rotation profiles obtained by various authors together with the present work.

with the mean values and their error bars of the rotation rate in every 5 deg in latitude. The results of Newton and Nunn (1951), Arevalo *et al.* (1982), Lustig (1983), Howard, Gilman, and Gilman (1984), and Balthasar, Vazquez, and Wöhl (1986) are also shown in Figure 3. When we use all the data over three solar cycles to calculate the solar equatorial rotation velocity, the accuracy is found to be around 0.01 deg day^{-1} in this work. Our curve lies between the results of Balthasar, Vazquez, and Wöhl (1986), Howard, Gilman, and Gilman (1984) and Lustig (1983). The results of Arevalo *et al.* (1982) and Newton and Nunn (1951) are, respectively, higher and lower than the other four results. A possible source of such discrepancy is that those works were based on the data in different time intervals. Arevalo *et al.* (1982) used sunspot groups from GPR data in the period from 1874 through 1902 (cycles 12 and 13). Solar rotation rate derived from sunspot groups in that time interval is 1% higher than in other cycles, as was found by Balthasar, Vazquez, and Wöhl (1986) and Balthasar and Fangmeier (1988) (see Figure 5). The slowest rotation rate was produced by Newton and Nunn (1951). They used recurrent single sunspots (*H* and *J* types) only from 1934 through 1944. It is well known that *H* and *J* types show the slowest rotation among all the sunspot types (Balthasar and Wöhl, 1980; Balthasar, Vazquez, and Wöhl, 1986).

Balthasar and Fangmeier (1988) applied both photographic and sunspot drawing methods to obtain the rotation velocity in the same time interval (1883–1893 and 1948–1976). They found that the rotation velocity derived from sunspot drawing data is 1% smaller than that derived from photographic data. This tendency is in agreement with the fact that our curve as well as Lustig's (1983) data in Figure 3 lies below the results of Balthasar, Vazquez, and Wöhl (1986) and Howard, Gilman, and Gilman (1984). The reason for the difference between Lustig (1983) and the present study, both of which used sunspot sketches, could be that our definition of sunspot positions may be different from Lustig's (1983).

3.2. DIFFERENTIAL ROTATION IN THREE SOLAR CYCLES

By applying the least-square formulae (2)–(3) separately to the three cycles, we obtained the fitted curve for each cycle (Figure 4). The mean values of the rotation rate in every 5° in latitude are also shown in Figure 4 with the error bar for cycle 21. The values of rotation velocity at the equator (A) and the gradient of solar differential rotation (B) are tabulated in Table I. We can see that the differential rotation varies from cycle to cycle in such a way that the rotation velocity in the low activity cycle (cycle 20) is higher than in the high activity cycle (cycle 19). Figure 4 shows that this systematic variation of solar rotation velocity is seen up to solar latitudes of 30°. This tendency is not found in higher

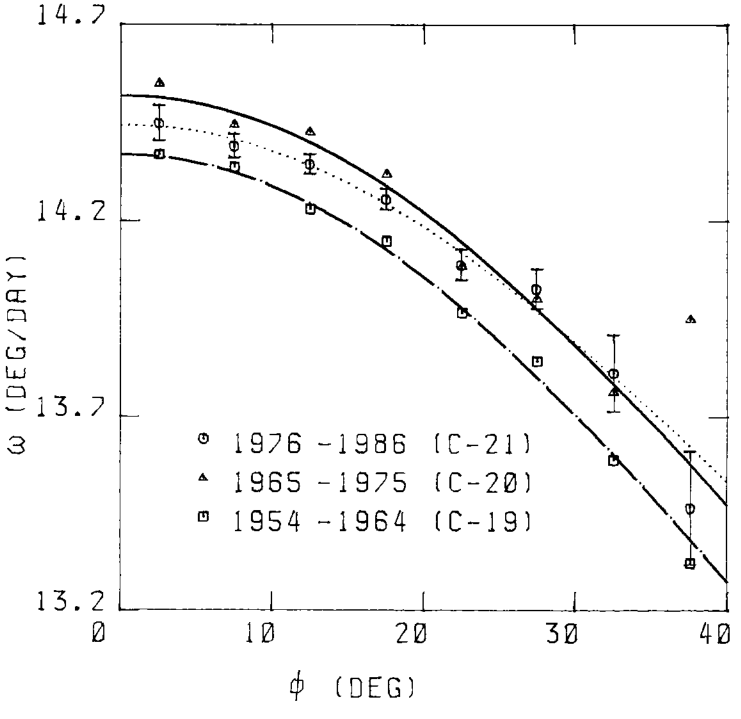

Fig. 4. Differential rotation in cycles 19, 20, and 21.

TABLE I

The values of A and B and their error ΔA and ΔB for different cycles

Cycle	A	ΔA	$-B$	ΔB
19	14.37	0.02	2.65	0.16
20	14.52	0.02	2.53	0.22
21	14.44	0.02	2.22	0.18

latitudes (up to 40°). For cycles 20 and 21, the anti-correlated variation of B with respect to A makes the rotation rate at higher latitudes almost identical. This is not the case for cycle 19, and therefore, we conclude that the variation of B is not correlated with the level of activity like the variation of A.

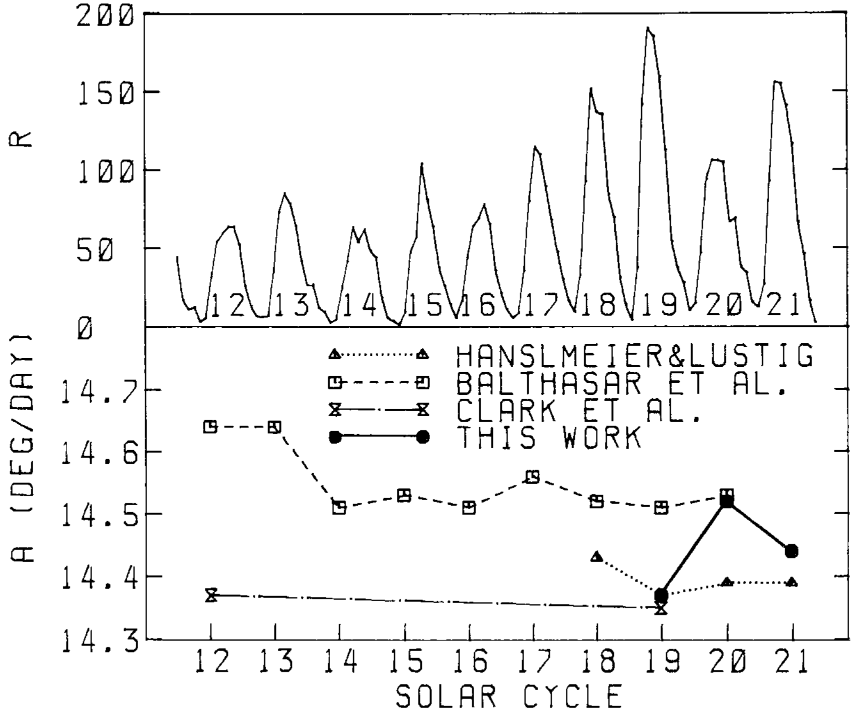

Fig. 5. Variation of A from cycle to cycle, taken from Balthasar *et al.* (1986), Clark *et al.* (1979), Hanslmeier and Lustig (1986) and from our results.

The variation of A we obtained is plotted in Figure 5 together with the results of Balthasar, Vazquez, and Wöhl (1986), Clark *et al.* (1979), and Hanslmeier and Lustig (1986). Our result is different from Balthasar, Vazquez, and Wöhl (1986), Clark *et al.* (1979) and Hanslmeier and Lustig (1986). Balthasar, Vazquez, and Wöhl (1986) claimed that the only significant variation was the decrease in A from cycle 13 to 14. They found no variations in A for other cycles between 12 and 20. Balthasar and Fangmeier (1988) suggested that no definite answer can be given whether the decrease in the rotation velocity from the GPR during the early years of our century is of solar origin or is produced by systematic effects in the data. Clark *et al.* (1979) also found that A was independent of sunspot activities by comparing cycles 12 and 19. They used long-lived unipolar spots whose area is 30 to 300 millionths of the visible solar disk and whose latitude is less than 30°. It is clear that the result of Clark *et al.* (1979) as shown in Figure 5 lies below the other results. Similar values of A were also found by Hanslmeier and Lustig (1986) for cycles 18, 19, 20, and 21. They used all sunspot types and derived

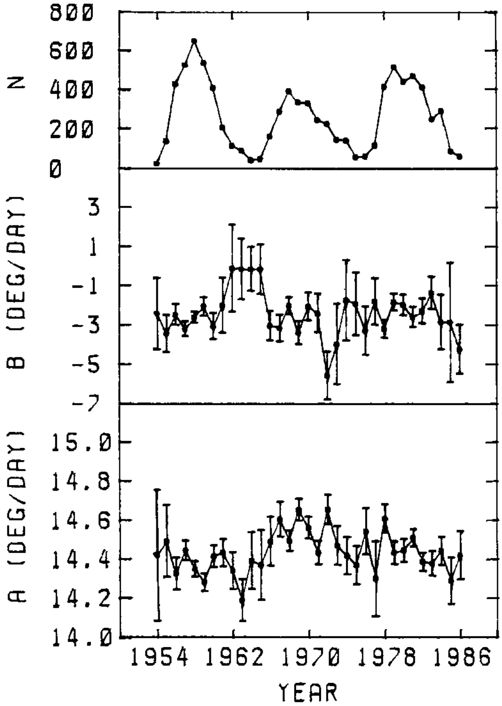

Fig. 6. Rotation velocity at equator (*A*) and gradient of solar differential rotation (*B*) in every year from 1954–1986.

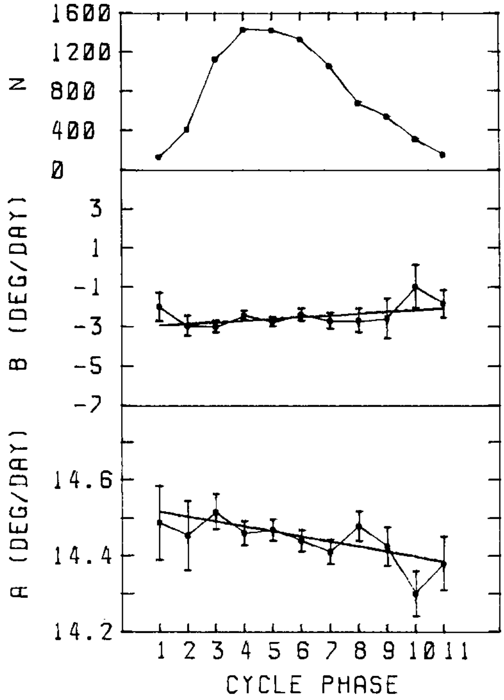

Fig. 7. The variation of *A* and *B* with the phase of solar cycle by superposing three cycles with the period of eleven years.

the value of A which is systematically smaller than the result of Balthasar, Vazquez, and Wöhl (1986) by approximately 1%. This is in agreement with the result of Balthasar and Fangmeier (1988), who found that the rotation velocity derived from sunspot drawing data is 1% smaller than that from sunspot photographic data. In contrast to these, for cycle 19 our result agrees with Hanslmeier and Lustig (1986) and Clark *et al.* (1979), whereas for cycle 20 we recover the result of Balthasar, Vazquez, and Wöhl (1986), and the rate of cycle 21 lies between the two cycles.

TABLE II

Rotation velocity at equator (A) and gradient of differential rotation (B) in every year from 1954–1986

Year	A	ΔA	$-B$	ΔB	Total number of measurements
1954	14.42	0.34	2.42	1.81	23
1955	14.49	0.19	3.45	0.97	136
1956	14.32	0.08	2.48	0.54	425
1957	14.44	0.05	3.20	0.38	523
1958	14.35	0.04	2.60	0.30	649
1959	14.28	0.05	2.06	0.47	534
1960	14.41	0.06	3.06	0.67	406
1961	14.43	0.07	2.01	1.42	204
1962	14.34	0.10	0.09	2.21	111
1963	14.19	0.11	0.13	1.55	88
1964	14.39	0.15	0.13	1.14	39
1965	14.37	0.18	0.16	1.28	46
1966	14.49	0.13	3.04	0.76	159
1967	14.60	0.09	3.15	0.70	286
1968	14.50	0.05	2.03	0.45	389
1969	14.65	0.06	3.40	0.60	331
1970	14.56	0.06	2.05	0.72	327
1971	14.43	0.06	2.40	0.99	242
1972	14.65	0.08	5.58	1.24	222
1973	14.47	0.10	3.98	2.05	142
1974	14.42	0.10	1.75	2.07	138
1975	14.37	0.10	1.92	1.62	54
1976	14.54	0.12	3.30	1.23	57
1977	14.30	0.19	1.81	1.20	113
1978	14.61	0.07	3.20	0.46	411
1979	14.43	0.06	1.84	0.44	510
1980	14.44	0.06	1.99	0.53	434
1981	14.51	0.05	2.60	0.52	464
1982	14.38	0.05	2.29	0.64	405
1983	14.38	0.07	1.36	0.84	244
1984	14.44	0.07	2.85	1.39	285
1985	14.29	0.12	2.87	3.06	81
1986	14.42	0.13	4.24	1.27	53

3.3. DIFFERENTIAL ROTATION AS A FUNCTION OF CYCLE PHASE

We also applied the formulae (2) and (3) to the data year by year to find a yearly variation of the solar equatorial rotation rate, A and the gradient of solar rotation, B. These values are plotted in Figure 6 and tabulated in Table II. The variation of A shows a systematic variation within each cycle. The value of A is higher at the beginning of the cycle and decreases subsequently. Figure 7 is the plot of A versus cycle phase (t = 1–11) and it shows an approximately linear decrease in A. The least-square fit gives $A = \dot{A}t + A_0$ with $\dot{A} = -0.013 \pm 0.005$. This phenomenon is not found for the value of B. Balthasar and Wöhl (1980) analyzed the Greenwich sunspot data from 1940 through 1968 and found faster rotation around sunspot minimum. Arevalo *et al.* (1982), using the same data for different cycles from 1874 through 1902, found a similar variation. Lustig (1983) concluded from an analysis of sunspot drawings of Kanzelhöhe Observatory covering 4 solar cycles (1947–1981), that the equatorial rotation rate is significantly higher and the rotation is slightly more rigid during sunspot minima. Gilman and Howard (1984) analyzed Mt. Wilson white-light plate collection obtained in the period 1921 through 1981 (6 solar cycles). They used individual spots that appeared only within the equatorial belt (within ±30°) and found strong peaks of rotation rate near sunspot minima and somewhat weaker ones near sunspot maxima. Balthasar, Vazquez, and Wöhl (1986) found that (1) the highest velocity occurs around the activity minima, (2) at the beginning of the activity maxima there is a secondary maximum of velocity, and (3) the lowest velocity is found after the activity maxima.

The results of Balthasar, Vazquez, and Wöhl (1986), Lustig (1983), and ours are compared in Figure 8. In the present work we found a tendency different from the previous results, in that at the beginning of the cycle the equatorial velocity is higher than

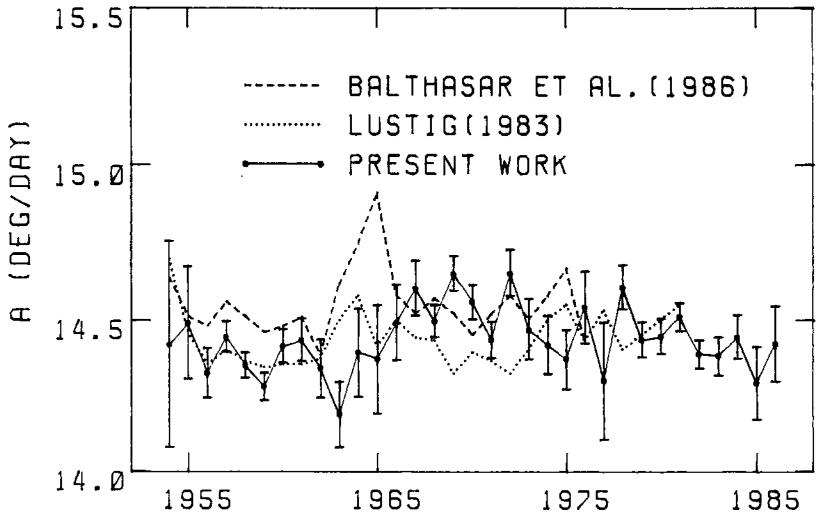

Fig. 8. The variation of A from year to year, taken from Balthasar *et al.* (1986), Lustig (1983), and from our results.

at the end of the cycle, and the equatorial rotation velocity decreases with time in-between.

3.4. DEPENDENCE OF SOLAR DIFFERENTIAL ROTATION ON SUNSPOT TYPES

The values of A calculated for various sunspot types are plotted in Figure 9. For comparison we also plotted the result of Balthasar, Vazquez, and Wöhl (1986) in the same figure. The variation of the solar differential rotation with respect to Zürich sunspot type is similar in tendency to Balthasar, Vazquez, and Wöhl (1986), but the dependence of A on the Zürich type found in our analysis is smaller than that of Balthasar, Vazquez, and Wöhl (1986).

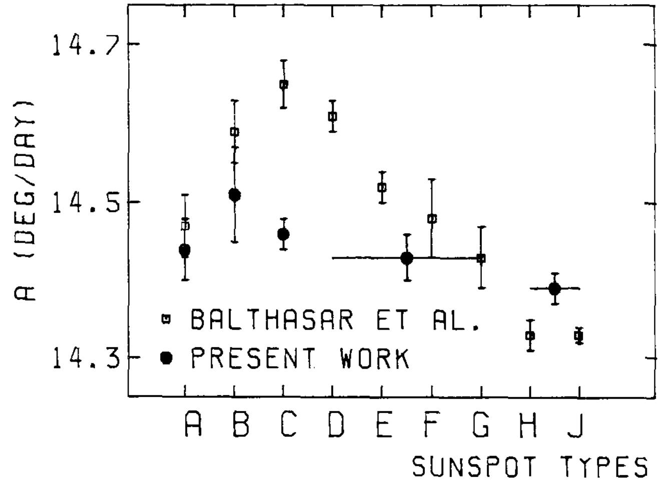

Fig. 9. The variation of A with respect to sunspot types.

4. Conclusions

We have found that three factors (the level of cycle activity, the cycle phase and sunspot type) affect the solar rotation rate. The differential rotation varies from cycle to cycle in such a way that the rotation velocity in the low activity cycle (cycle 20) is higher than in the high activity cycle (cycle 19). This result is different from Balthasar, Vazquez, and Wöhl (1986), Clark *et al.* (1979), and Hanslmeier and Lustig (1986). Although it may be caused by our data selection and the definition of sunspot position, such systematic variation of A can also be understood as the reaction of magnetic fields against the differential rotation.

In the present work we found that the rotation velocity at the equator (A) shows a systematic variation within each cycle. The value of A is higher at the beginning of cycle and decreases subsequently. On the contrary Balthasar and Wöhl (1980), Arevalo *et al.* (1982), Lustig (1983), Gilman and Howard (1984), and Balthasar, Vazquez, and Wöhl (1986) found faster rotation around sunspot minima. It should be cautioned however that a large discrepancy among these results appears only in the beginning and the end of cycle where fewer sunspots are included in the data.

Although quite small, the variation of solar differential rotation with respect to Zürich sunspot types was found. *HJ* type sunspots, which are presumably old, decaying spots, show the slowest rotation among all the sunspot types. In other words old spots rotate at the rate closer to the speed of non-magnetic gas. Our result is similar in tendency, but is reduced in magnitude, as compared to the result of Balthasar, Vazquez, and Wöhl (1986).

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