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Applying Efficiency Measurement Techniques to the Measurement of Productivity Change

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Abstract

Deterministic frontier analysis (DFA), stochastic frontier analysis (SFA), and data envelopment analysis (DEA) are alternative analytical techniques designed to measure the efficiency of producers. All three techniques were originally developed within a cross-sectional context, in which the objective is to compare the efficiencies of producers. More recently all three techniques have been extended for use in a panel data context. In the latter context it is possible to measure productivity change, and to decompose measured productivity change into its sources, one of which is efficiency change. However when efficiency measurement techniques, particularly SFA, have been applied to panel data, it has infrequently been made clear what the objective of the analysis is: the measurement of efficiency, which may vary through time as well as across producers, or the measurement and decomposition of productivity change. In this paper I explore the use of each technique in a panel data context. I find DFA and DEA to have achieved a more satisfactory reorientation toward productivity measurement than SFA has.

Keywords. Efficiency measurement, productivity change, deterministic frontier analysis, stochastic frontier analysis, and data envelopment analysis

1. Introduction

Nearly 40 years ago Farrell (1957) showed how to measure the relative technical and economic efficiency of a sample of producers. More recently, Farrell's innovative work has been extended and refined, and we now have three empirical methodologies for the measurement of relative efficiency. Deterministic frontier analysis (DFA) (Aigner and Chu 1968) measures efficiency relative to a deterministic parametric frontier. Stochastic frontier analysis (SFA) (Aigner, Lovell and Schmidt 1977, Meeusen and van den Broeck 1977) measures efficiency relative to a stochastic parametric frontier. Data envelopment analysis (DEA) (Charnes, Cooper, and Rhodes 1978) measures efficiency relative to a deterministic nonparametric frontier. Each methodology has its strengths and weaknesses, but it is probably fair to say that SFA and DEA are generally preferred to DFA, which is neither stochastic nor nonparametric.[1]

Farrell illustrated his ideas with an application to U.S. agriculture, using cross-sectional data to compare the efficiency of producing units. Since then many hundreds of studies have appeared, applying DFA, SFA, or DEA to cross-sectional data to measure the relative efficiency of producing units. In a cross-sectional context the objective of the exercise is clear, and the interpretation of the results is relatively straightforward, regardless of the methodology used. However, the drawback of a cross-sectional analysis is that it provides only a snapshot of a process which evolves through time. Consequently a cross-sectional analysis provides only a partial, and possibly a misleading, evaluation of the relative performance of the producers under investigation.

For this reason all three methodologies have recently been applied to panel data. DFA was perhaps first applied to panel data by Førsund and Hjalmarsson (1979a, 1979b), SFA by Pitt and Lee (1981) and Schmidt and Sickles (1984), and DEA by Charnes et al. (1985) and, in a much more informative way, by Färe et al. (1994). The great advantage of having access to panel data is that they offer the opportunity of providing a much more detailed evaluation of the relative performance of the producers under investigation. In a panel data context it is possible to measure the productivity change of each producer, and to decompose measured productivity change into its sources, one of which is efficiency change. Efficiency measurement techniques would therefore appear to be ideally suited to this task. Unfortunately, while they have been used to measure efficiency, and occasionally efficiency change, they have rarely been used to measure and decompose productivity change. Not only has an opportunity been lost, it is difficult to interpret the results of such an exercise, since the measured effects of efficiency change are apt to be confounded with the unmeasured effects of other sources of productivity change.

In this paper I review the extension of each methodology from an application to cross section data for the purpose of measuring relative efficiency to an application to panel data for the purpose of measuring and decomposing productivity change. I find surprisingly few instances in which panel data have been fully exploited to measure and decompose productivity change. More typically, panel data have been used simply to produce better measures of efficiency or of efficiency change. The problem with this approach is that these measures may not be better after all, since they are likely to incorporate the unmodeled effects of other sources of productivity change.

The paper is organized as follows. In Section 2 I briefly present an analysis of productivity change and its decomposition. In Section 3 I discuss the application of DFA to panel data, in Section 4 I discuss the application of SFA to panel data, and in Section 5 I discuss the application of DEA to panel data. In Section 6 I summarize and conclude.

2. A Model of Productivity Change and its Decomposition

I assume for simplicity that producers use J inputs $x = (x_1, \ldots, x_J)$ to produce a single output y. The extension to multiple outputs is straightforward. Producers are not required to be technically efficient, and so

$$y \leq f(x, t), \tag{1}$$

where $f(x, t) = \max\{y\colon (x, y) \in S^t\}$ is the production frontier; S^t is the set of all technologically feasible input-output combinations at time t; $t = 1, \ldots, T$; and t is a time index which serves as a proxy for technical change. An output-oriented measure of the technical efficiency of a producer is given by

$$TE_o(x, y, t) = y/f(x, t) \leq 1. \tag{2}$$

If only (x, y, t) are observed, then a measure of productivity change is provided by

$$\dot{TFP} = \dot{f}(x, t) + \dot{TE}_o(x, y, t) + [\Sigma_j e_j(x, t) - 1]\Sigma_j \hat{e}_j(x, t)\dot{x}_j, \tag{3}$$

where a dot over a function or a variable indicates a time rate of change, $e_j(x, t) = \partial \ln f(x, t)/\partial \ln x_j$ is the output elasticity of input x_j, $\Sigma_j e_j(x, t)$ is the scale elasticity, and $\hat{e}_j(x, t) = e_j(x, t)/\Sigma_j e_j(x, t)$. Thus productivity change decomposes into three sources: technical change, output-oriented technical efficiency change, and scale economies.

The expression above exploits only (x, y, t) information. If information on input prices is available, and if producers seek, not necessarily successfully, to utilize their inputs in an allocatively efficient manner, then Bauer (1990) has shown that an alternative measure of productivity change is given by

$$\dot{TFP} = \dot{f}(x, t) + \dot{TE}_o(x, y, t) + \Sigma_j [e_j(x, t) - s_j]\dot{x}_j, \tag{4}$$

where $s_j = w_j x_j/\Sigma_j w_j x_j$ is the actual cost share of input x_j, w_j being the exogenously determined price of input x_j. Productivity change again decomposes into three sources: technical change, output-oriented technical efficiency change, and a term that combines the effects of scale economies and input allocative inefficiency. If inputs are utilized in an allocatively efficient manner, the third team becomes a pure scale effect equal to $(e_{Cy}^{-1} - 1)\Sigma_j s_j(y, w, t)\dot{x}_j$, where the cost elasticity $e_{Cy} = \partial \ln C(y, w, t)/\partial \ln y$ provides a dual measure of scale economies, $s_j(y, w, t) = w_j x_j(y, w, t)/C(y, w, t)$ is the allocatively efficient cost share of input x_j, and $C(y, w, t)$ is the cost frontier dual to the production frontier $f(x, t)$. Thus under input allocative efficiency expressions (3) and (4) coincide, and productivity change is attributed to technical change, output oriented technical efficiency change, and scale economies. Under input allocative efficiency and constant returns to scale, the third terms in (3) and (4) vanish, and productivity change consists solely of technical change and output-oriented technical efficiency change.

When input price information is available, and a behavioral objective of cost minimization is imposed on producers, a more detailed decomposition of productivity change can be obtained. The cost efficiency of a producer can be expressed as

$$\begin{aligned} CE(y, x, w, t) &= C(y, w, t)/\Sigma_j w_j x_j \\ &= TE_i(y, x, t) \cdot AE_i(y, x, w, t), \end{aligned} \tag{5}$$

where $TE_i(y, x, t) \leq 1$ is the input-oriented technical efficiency of a producer and $AE_i(y, x, w, t) \leq 1$ is the input allocative efficiency of a producer. Again following Bauer, productivity change can be expressed in terms of the cost frontier as

$$\begin{aligned} \dot{TFP} = {} & [1 - e_{Cy}(y, w, t)]\dot{y} + \dot{TE}_i(y, x, t) + \dot{AE}_i(y, x, w, t) \\ & - \dot{C}(y, w, t) + \Sigma_j [s_j - s_j(y, w, t)]\dot{w}_j. \end{aligned} \tag{6}$$

Productivity change now decomposes into five sources: a term incorporating the effect of scale economies, a pair of terms reflecting changes in input-oriented technical efficiency and input allocative efficiency, a term reflecting the effect of technical change, and an input price effect term which is zero if the producer is allocatively efficient, or if all input prices change at the same rate. Thus, under input allocative efficiency the third and fifth terms vanish, and productivity change is once again attributable to scale economies, technical efficiency change, and technical change.

If panel data are available, it is possible to evaluate the performance of each producer in the sample in terms of productivity change. It is also possible, at least in principle, to decompose each producer's measured productivity change into three general sources: technical change, efficiency change (technical, or technical and allocative), and scale economies. If only input and output quantity data are available, the evaluation would be based on the production frontier, using equation (3), while if input price data were also available, the evaluation could be based either on the production frontier, using equation (4), or on the cost frontier, using equation (6). In any case, efficiency change would be a potential source of productivity change.

3. The Application of DFA to Panel Data

Let the scalar output of producer i in period t be y_{it}, and let the jth input of producer i in period t be x_{jit}, where $i = 1, \ldots, I$; $t = 1, \ldots, T$; and $j = 1, \ldots, J$. Then, following Førsund and Hjalmarsson (1979a, 1979b, 1987), a deterministic homothetic Cobb-Douglas production frontier may be written as

$$(a_1 - b_1 t)\ln y_{it} + (a_2 - b_2 t)y_{it} \leq a_o + b_3 t + \Sigma_j(c_j - d_j t)\ln x_{jit}, \tag{7}$$

where $\Sigma_j(c_j - d_j t) = 1, t = 1, \ldots, T$. Despite the Cobb-Douglas kernel common to all producers, this formulation is remarkably flexible. The parameters a_1 and a_2 transform the linearly homogeneous kernel frontier into a homothetic frontier, and allow for output-dependent returns to scale. The parameters b_1 and b_2 allow the scale elasticity, and hence the magnitude of technically optimal scale, to be time-dependent. The parameter b_3 allows for neutral technical change, and the parameters d_j ($j = 1, \ldots, J$) allow for input-biased technical change.

The parameters may be calculated by solving the linear programming problem

$$\min \Sigma_i \Sigma_t [a_o + b_3 t + \Sigma_j(c_j - d_j t)\ln x_{jit} - (a_1 - b_1 t)\ln y_{it} - (a_2 - b_2 t)y_{it}], \tag{8}$$

or the analogous quadratic programming problem $\min \Sigma_i \Sigma_t [\cdot]^2$. Either program is solved subject to the $N \cdot T$ nonnegativity constraints $[a_o + b_3 t + \Sigma_j(c_j - d_j t)\ln x_{jit} - (a_1 - b_1 t)\ln y_{it} - (a_2 - b_2 t)y_{it}] \geq 0$, which guarantee that all producers operate on or beneath the production frontier; the T linear homogeneity constraints $\Sigma_j(c_j - d_j t) = 1$, $t = 1, \ldots, T$; and whatever other parameter restrictions are deemed appropriate. The relative technical efficiency of each producer in each period is calculated from the optimal slacks in the $N \cdot T$ nonnegativity constraints. The magnitude of scale economies and the magnitude of technical change, as well as its scale bias and the nature of its input bias, are all determined by the calculated parameter values. This provides all the information required to measure and decompose producitivity change along lines suggested in equation (3) above.

If input price data are available, and if cost minimization is an appropriate behavioral objective, it is possible to use DFA techniques to calculate the parameters of a cost frontier, and to calculate the contribution of the change in cost efficiency to productivity change, as in equation (6) above. I am not aware that this has been accomplished. Alternatively, if producers use a single input to produce multiple outputs, it is possible to use DFA techniques to calculate the parameters of an input requirements frontier, and to calculate and decompose productivity change. Bjurek (1994; Ch. 3) has done so for a panel of Swedish social insurance offices. Finally, if producers use multiple inputs to produce multiple outputs, it is possible to use DFA techniques to calculate the parameters of a distance function, and to calculate and decompose productivity change. Althin, Färe and Grosskopf (1994) have done so for a panel of Swedish pharmacies.

The great strength of the DFA approach is its ability to calculate efficiency change and technical change, to distinguish one from the other, and to assess their separate contributions to productivity change. This is, after all, a fundamental objective of the analysis of panel data. However the DFA approach does have a drawback: it is deterministic, and so is capable of confusing the unfortunate but likely effects of omitted variables and measurement error with the desired effects of efficiency change and technical change. This is an old complaint made of both parametric and nonparametric deterministic models, and I shall not dwell on it except to say that confidence in the findings would be enhanced by resort to some type of sensitivity analysis. Førsund and Hjalmarsson (1987) recommend a sensitivity analysis consisting of the removal from each cross section of the producer having the largest dual variable corresponding to the nonnegativity constraint in that time period, and then resolving the programming problem with $(N - 1) \cdot T$ observations. Bjurek recommends bootstrapping as a means of conducting a sensitivity analysis. Bjurek's calculations are fairly robust, with 50 resamples generating approximately the same calculated rates of productivity change.

4. The Application of SFA to Panel Data

For many years econometricians have been using panel data to estimate flexible production or cost functions, and to measure and decompose productivity change. A good illustration of the methodology is Gollop and Roberts (1981). However since it is *functions* rather than *frontiers* which have been estimated, producers are implicitly assumed to be efficient, and so efficiency change makes no contribution to productivity change, which is attributed entirely to technical change and scale economies. For almost as many years, other econometricians have been using panel data to estimate production or cost frontiers, for the purpose of estimating efficiency (and occasionally efficiency change), but rarely for the purpose of measuring and decomposing productivity change.

There are many different SFA panel data models. All are based on a relationship of the general form

$$y_{it} = f(x_{it}; \beta) \exp\{v_{it} - u_i\}, \tag{9}$$

where $x_{it} = (x_{1it}, \ldots, x_{Jit})$. the function $f(x_{it}; \beta)$ is the deterministic kernel of the stochastic production frontier $[f(x_{it}; \beta)\exp\{v_{it}\}]$, where β is a vector of technology parameters to be estimated and v_{it} is a normally distributed error term intended to capture the effects of statistical noise. The term $\exp\{-u_i\} = y_{it}/[f(x_{it}; \beta)\exp\{v_{it}\}] \leq 1$, and corresponds to $TE_o(y, x, t)$ in equation (2) above. In this formulation technical inefficiency is producer-specific but time-invariant, and no provision is made for technical change. This model is suitable only for very short panels.

Perhaps the simplest way to generalize (9) is to allow the error component representing technical inefficiency to be time-varying, and to make some assumptions concerning its structure. Following Kumbhakar (1990), Battese and Coelli (1992) proposed replacing u_i with

$$u_{it} = \exp\{-a(t - T)\}u_i. \tag{10}$$

The model given by (9) and (10) can be estimated using maximum likelihood techniques, once a functional form for $f(x_{it}; \beta)$ is specified and distributional assumptions on v_{it} (normal) and u_i (typically half-normal or truncated normal) are made. In this formulation u_{it} increases (decreases) through time toward its terminal value of u_i according as $a < 0$ (>0). The simplicity of the parameterization in (10) implies that, although each producer has its own terminal level of technical efficiency, $\exp\{-u_i\}$, every producer has the same time pattern of technical efficiency change, $\exp\{-a(t - T)\}$. Moreover, no behavioral or institutional motivation has been offered for the time variance of technical efficiency.

The model given by (9) and (10) is restrictive in that productivity change is attributed exclusively to scale economies and efficiency change, the time path of which is monotonic and common to all producers. Monotononicity can be relaxed by generalizing the function $\exp\{-a(t - T)\}$, although at a cost of increased complexity of estimation. It is also possible to add a time index to (9), in an effort to incorporate the effects of neutral technical change. However in this case identification of the separate effects of neutral technical change, which is common to all producers, and neutral technical efficiency change, which is also common to all producers, would be problematic. Interacting the time index with other regressors would help, by deneutralizing technical change, but also at a cost of additional parameters to be estimated.

Cornwell et al. (1990) adopt a different approach. Rather than model technical inefficiency through an error component, they model it through the intercept of the production frontier. If the intercept in (9) is β_o, then it is possible to specify

$$\beta_{it} = \beta_o - u_{it} = a_{i1} + a_{i2}t + a_{i3}t^2. \tag{11}$$

In this formulation each producer has its own intercept, which is allowed to vary quadratically through time at producer-specific rates. The technical inefficiency of a producer in a time period is obtained from the estimated intercepts by means of the normalization $u_{it} = [\max_i\{\beta_{it}\} - \beta_{it}]$, $i = 1, \ldots, I$, $t = 1, \ldots, T$. Cornwell et al. discuss various estimation strategies, but the problem is not so much one of estimation as it is one of interpretation. One interpretation is that the parameters in (11) represent producer-specific levels of, and trends in, technical efficiency; in this case productivity change is producer-specific, but is attributed entirely to scale economies and efficiency change. An alternative interpretation is that the $u_{i1} = [\max_i\{\beta_{i1}\} - \beta_{i1}]$ represent producer-specific initial (or persistent) levels of inefficiency, and that the u_{it} ($t = 2, \ldots T$) represent producer-specific technical change;

in this case productivity change is also producer-specific, but it is attributed entirely to scale economies and technical change. Writers have offered contradictory interpretations; see, for example, Cornwell et al., and Good, Röller and Sickles (1995). There is no easy way to empirically distinguish between the two interpretations.

Kumbhakar, Heshmati and Hjalmarsson (1995) provide an empirical comparison of the performance of the Battese and Coelli and the Cornwell et al. models using panel data on Colombian cement plants. They find substantial variation in estimated patterns of technical change and technical efficiency change across the two models, but they offer no insight into the possible causes of this variation that would help in slecting between the two models.

Heshmati and Kumbhakar (1995) and Heshmati, Kumbhakar and Hjalmarsson (1995) specify a stochastic production frontier model like that in equation (9), with a trend variable included among the regressors. They also add a producer-specific error component to capture time-invariant heterogeneity among producers. It is not possible to determine if this effect measures the contribution of inputs which vary across producers but not through time, or if it measures producer-specific persistent technical inefficiency. Fortunately, the interpretation of this effect is irrelevant for the measurement and decomposition of productivity change. Finally, they assume that u_{it} has a truncated normal distribution. By including a trend variable among the regressors, they resolve the interpretation problem in the Cornwell et al. specification, since the trend variable is associated with technical change and the error component u_{it} is associated with technical efficiency, which is allowed to vary across producers and through time. This specification thus enables estimation of the contributions of technical change, technical efficiency change and scale economies to productivity change.

Each of the above approaches can be applied to a cost frontier context, by replacing technical efficiency with cost efficiency, and my remarks on each approach would stand. However cost functions are typically estimated as part of a system consisting of the cost function and all but one of its associated input share equations, and it would be desirable to estimate a cost frontier as a part of the same system. Bauer attempted to estimate a full-blown translog cost system incorporating the effects of scale economies, technical change (by including a trend term among the regressors) and cost efficiency change (modeled with a nonnegative truncated normal error component on the cost frontier), and then to decompose measured productivity change as in equation (6) above. He was able to distinguish the separate effects of scale economies, technical change, and cost efficiency change, among others, but he was unable to distinguish the effects of technical efficiency change from those of input allocative efficiency change. Moreover, he was forced to assume that the cost inefficiency error component was independent of the share equation errors, which is inconsistent with the notion that a part of cost inefficiency is allocative in nature. However Kumbhakar (1995) has recently developed an analytically consistent formulation of the model, in which an exact relationship is derived between input allocative inefficiencies and their cost. Färe and Primont (1996) have demonstrated that this exact relationship must hold for any cost function. This development breathes new life into what was once considered a promising approach to the use of SFA to measure technical and input allocative efficiency, and to measure and provide a more complete decompositoon of productivity change.

The great virtue of SFA, and its main advantage over the other two approaches, is that it is stochastic. Only this approach is capable of distinguishing the effects of statistical noise from those of inefficiency. The primary drawback of SFA is that it is parametric; not only is the structure of technology parameterized, frequently so too is the structure of inefficiency. When faced with the inevitable trade-off between parsimonious but inflexible parameterizations, and flexible parameterizations which consume degrees of freedom and create collinearity problems, scholars have tended to opt for the former alternative. But this leads to a confounding of inefficiency with specification error caused by the use of overly restrictive functional forms. The singular advantage of being able to distinguish noise from inefficiency is offset by the disadvantage of being unable to distinguish inefficiency from inappropriate functional forms.

5. The Application of DEA to Panel Data

Let $Y_{it} = [y_{1it}, \ldots, y_{Kit}]'$ be a $K \cdot 1$ vector of outputs of producer $i = 1, \ldots, I$ in period $t = 1, \ldots, T$, and let $X_{it} = [x_{1it}, \ldots, x_{Jit}]'$ be a $J \cdot 1$ vector of inputs of producer $i = 1, \ldots, I$ in period $t = 1, \ldots, T$. Then, in keeping with the output orientation in Sections 3 and 4, an output-oriented DEA envelopment problem for producer o in period t can be expressed as

$$\begin{aligned} \max \emptyset \quad \text{subject to} \quad \emptyset Y_{ot} &\leq \Sigma_i \lambda_{it} Y_{it} \\ \Sigma_i \lambda_{it} X_{it} &\leq X_{ot} \\ \Sigma_i \lambda_{it} &= 1 \\ \lambda_{it} &\geq 0. \end{aligned} \tag{12}$$

Solving this linear programming problem I times in period t generates technical efficiency scores for each producer in period t. A producer is judged to be technically inefficient unless $\emptyset = 1$ and no slacks are present in the functional constraints to problem (12). Solving this problem I times separately for each time period does not generate information on efficiency change for each producer, since comparison sets change from time period to time period. Pooling the data and solving this problem $I \cdot T$ times does generate information on efficiency change for each producer, because there is only one comparison set, the pooled data set. However in this case technology is assumed to be unchanging, and so productivity change is attributed entirely to technical efficiency change.

A method for detecting trends in efficiency scores is provided by the window analysis methodology of Charnes et al. (1985). In this approach the entire set of T time periods is divided into a sequence of overlapping subperiods of equal length. The first subperiod might consist of periods $\{1, \ldots, S\}$, the second subperiod would then consist of periods $\{2, \ldots, S + 1\}$, and so on through periods $\{T - S + 1, \ldots, T\}$. The DEA problem is solved $I \cdot S$ times in each subperiod, after which the average efficiency score of each producer can be tracked through the sequence of overlapping subperiods. This provides evidence on the trend in efficiency for each producer relative to a technology that is changing through the sequence of overlapping subperiods. However it provides no evidence on the nature of the technical change, and little information on productivity change.

The nonparametric strength of DEA was exploited to a much greater degree by Färe et al. (1994), who used linear programming techniques to calculate and decompose a Malmquist (1953) productivity index. There are several approaches to constructing such an index; here I follow Färe, Grosskopf and Lovell (1994). One variant of an output-oriented Malmquist productivity index can be written as

$$M_o^G(X_{it}, Y_{it}, X_{it+1}, Y_{it+1} C) = \left[\frac{TE_o^t(X_{it}, Y_{it} \mid C)}{TE_o^t(X_{it+1}, Y_{it+1} \mid C)} \cdot \frac{TE_o^{t+1}(X_{it}, Y_{it} \mid C)}{TE_o^{t+1}(X_{it+1}, Y_{it+1} \mid C)} \right]^{1/2} \tag{13}$$

where $TE_o^t(X_{it}, Y_{it} \mid C)$ is the output-oriented technical efficiency of producer i in period t, defined relative to a constant returns to scale technology prevailing in period t, $TE_o^{t+1}(X_{it}, Y_{it} \mid C)$ is the output-oriented technical efficiency of producer i in period t, defined relative to a constant returns to scale technology prevailing in period $t + 1$, and so on. $M_o^G(\cdot)$ is the geometric mean of a pair of adjacent-period Malmquist productivity indexes; the first measures productivity change between periods t and $t + 1$ using period t technology as a reference, and the second measures productivity change between periods t and $t + 1$ using period $t + 1$ technology as a reference. $M_o^G(\cdot)$ is greater than, equal to, or less than unity according as productivity growth, stagnation, or decline has occurred for producer i between periods t and $t + 1$.

One virture of $M_o^G(\cdot)$ is that it decomposes into the product of three components of productivity change, as follows

$$\begin{aligned} M_o^G(X_{it}, Y_{it}, X_{it+1}, Y_{it+1} \mid C) = {} & \frac{TE_o^t(X_{it}, Y_{it} \mid V)}{TE_o^{t+1}(X_{it+1}, Y_{it+1} \mid V)} \\ & \cdot \frac{S_o^t(X_{it}, Y_{it})}{S_o^{t+1}(X_{it+1}, Y_{it+1})} \\ & \cdot \left[\frac{TE_o^{t+1}(X_{it+1}, Y_{it+1} \mid C)}{TE_o^t(X_{it+1}, Y_{it+1} \mid C)} \cdot \frac{TE_o^{t+1}(X_{it}, Y_{it} \mid C)}{TE_o^t(X_{it}, Y_{it} \mid C)} \right]^{1/2} \end{aligned} \tag{14}$$

The first component of $M_o^G(\cdot)$ provides a measure of the change in the technical efficiency of producer i between periods t and $t + 1$, measured relative to variable returns to scale technologies prevailing in each period. The second component provides a measure of the change in the scale efficiency of producer i between periods t and $t + 1$. It shows whether producer i is moving closer to or farther away from technically optimal scale, and provides a measure of the contribution of scale economies to productivity change. The third component provides a measure of technical change between periods t and $t + 1$ in a region of the data occupied by producer i. Färe et al. (1995) showed that the technical change component can be decomposed into a magnitude component, an output bias component and an input bias component. Thus the Malmquist index provides a measure of productivity change, and provides an attribution of productivity change to components associated with technical efficiency change, scale economies and the magnitude and biases of technical change. Each component is greater than, equal to, or less than unity according as it contributes positively, not at all, or negatively to productivity growth.

The Malmquist productivity index $M_o^G(\cdot)$ and its components are based on output-oriented measures of technical efficiency. DEA provides a nonparametric method of calculating these measures. Implementation requires that a total of six efficiency scores be calculated for each producer; this number rises to eight if the technical change component is decomposed. $TE_o^t(X_{it}, Y_{it} | V)$ is the objective in equation (12). The remaining efficiency scores are obtained either by deleting the convexity constraint $\Sigma_i \lambda_{it} = 1$ or by modifying the comparison set.

The DEA-based Malmquist productivity index has a number of virtues. First and foremost, it is nonparametric, unburdened by the functional forms required in DFA and SFA. Consequently it avoids the risk of confounding the effects of each component of productivity change with those of an inappropriate functional form. Second, being based on DEA, it naturally accommodates multiple outputs as well as multiple inputs without having to resort to price data and a value dual representation of technology. Although the Malmquist framework is entirely general, and can be implemented using DFA, SFA, or DEA, the fact of the matter is that DFA and SFA rarely have been used within a Malmquist framework. Third, the DEA-based Malmquist productivity index is a local index, and so productivity change and each of its sources are allowed to be producer-specific as well as time-varying, and the temporal patterns are totally unrestricted. Fourth, it decomposes into three (or five) measurable sources of productivity change. The only real drawback of the index is that it is deterministic; see my comments above concerning DFA, and see Atkinson and Wilson (1995) and Burgess and Wilson (1995) for recent applications of bootstrapping to DEA-based Malmquist productivity indexes.

6. Summary and Conclusions

DFA, SFA, and DEA each gained popularity as alternative methodologies for obtaining evidence concerning the relative efficiency of a sample of producers, based on a single cross section of data. Given the obvious limitations of cross-sectional data, and in light of the growing availability of panel data, applications of each technique based on panel data have recently appeared. Unfortunately (from my perspective), the large majority of SFA panel data applications have been oriented toward providing better estimates of efficiency and its change, rather than toward providing new estimates of the components of productivity change. This is less true of the relatively few DFA panel data applications, and not at all true of the rapidly growing number of DEA panel data applications.

In this paper I have attempted to evaluate the three approaches, in terms of their respective abilities to exploit the panel nature of the data to provide evidence concerning the magnitude and sources of the productivity change of producers. Despite its evident flexibility, and its appealing and longstanding orientation toward productivity measurement, I find the DFA approach to be heavily burdened by being both deterministic and parametric. This has no doubt contributed to its relative lack of popularity. The SFA approach has the great advantage of being the only stochastic approach among the three, but parameterization has been a problem, and it has not yet oriented itself toward productivity measurement. When the inevitable reorientation arrives, I believe that SFA will achieve equal status with DEA, which has exactly the opposite advantages and disadvantages, but for which the reorientation arrived in 1989.

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Note

1. A fourth approach to the measurement of relative efficiency, stochastic DEA, is in its infancy, and was widely discussed at the workshop. Since it is the subject of other contributions to this volume, I do not cover it in this paper.

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