PERIODICITIES IN SOLAR ACTIVITY*

T. W. COLE
Division of Radiophysics, CSIRO, Sydney, Australia

(Received 5 February, 1973)

Abstract. The techniques of power spectral analysis are used to determine significant periodicities in the annual mean relative sunspot numbers. The main conclusion is that a period of 10.45 yr is very basic and can be associated with an excitation of new solar cycles. When combined with a period of 11.8 yr, associated here with the free-running length of a solar cycle, the mean cycle length of 11.06 yr and a phase variation of 190 yr are explained. Similarly the amplitude variations with periods 88 and 59 yr (previously described as the 80-yr cycle) are due to an amplitude modulation of the solar cycle by a period of 11.9 ± 0.3 yr. The results dispute several associations of planetary position and solar activity.

1. Introduction

The activity of the Sun is well known to follow a cycle with an 11-yr period. Quantitative indices of activity introduced by Wolf in 1847 form the basis of most discussions about solar variability, and reviews of these variations are given by Waldmeier (1957, 1966). For times earlier than 1600, Schove (1955) has collected data on aurorae and large sunspots and used them to list the dates of solar cycle maxima back to 649 BC. The estimates of cycle intensities given by Schove were used to discover a 200-yr variation in solar activity (Zhukov and Muzalevskii, 1969). Gleissberg (1952) has suggested a 1000-yr variation and the literature very often refers to an 80-yr cycle.

Despite widespread reference to periodicities, it appears that current techniques of power spectral analysis (Blackman and Tukey, 1959) have not so far been widely used in a detailed investigation of the solar variations. The aim of this paper is to use these methods to search for significant periodic behaviour. With knowledge of the periodicities in the sunspot activity it is possible to derive a simple model of them and to critically test some previously suggested associations of planetary positions and solar activity.

2. The Power Spectrum of the Sunspot Numbers

The annual mean values of the relative sunspot number between the years 1700 and 1969 are plotted in Figure 1(a). The 11-yr cycle is clearly apparent while the variation in peak values is the basis for the suggestion of an 80-yr cycle.

The power spectrum is obtained as the Fourier transform of the autocorrelation of the relative sunspot numbers (Blackman and Tukey, 1959). The autocorrelation is approximated by the mean, lagged product of the finite number of samples in Figure 1(a). In this function, shown in Figure 1(c), the values at larger lags are increasingly unreliable because of the small number of products defining them and the diagram

---

* Radiophysics Publication RPP 1647, January, 1973.

Solar Physics **30** (1973) 103–110. *All Rights Reserved*

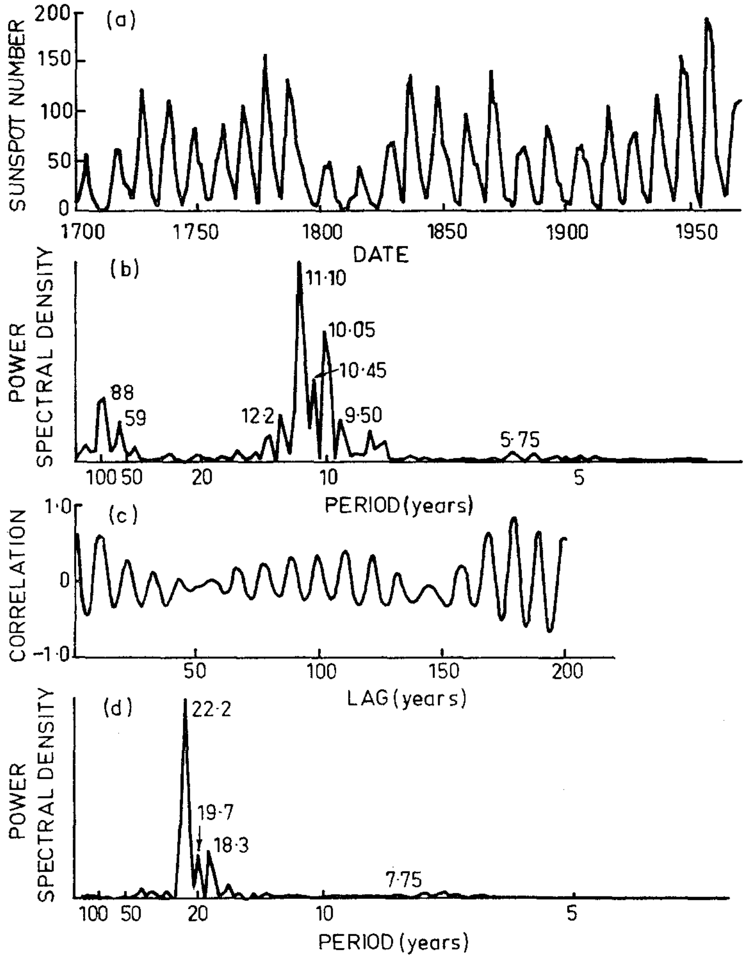

Fig. 1. (a) The mean annual Wolf relative sunspot numbers plotted from 1700 until 1969. (b) The power spectral density of the annual relative sunspot numbers from 1700 until 1969 plotted against the periodicity in years. (c) The autocorrelation of the series in (a) plotted against lag in years up to a maximum lag of 200 yr. (d) The power spectrum of the 22-yr solar magnetic cycle obtained by inverting the sign of alternate peaks in (a).

is therefore of only the first 200 points. The power spectral estimates are made reliable by applying a tapering function to the autocorrelation before Fourier transformation. A triangular weighting was used, tapering to zero at the maximum lag of the series (270 yr). In this way, any periodicity in the data would appear as a peak in the power spectrum with a $(\sin f / f)^2$ shape as a function of the frequency f. The width of the peak between half-intensity points is, in this case, (1/270) yr^{-1} and the height of the first confusing sidelobe or unwanted response is 4.5% of the main peak. The power spectrum of the series of Figure 1(a) is plotted as Figure 1(b). The significant peaks are marked on the figure; they consist of three groups, centred on 10.45, 88 and 5.75 year periods.

By considering the polarity of the solar magnetic field, the sunspot cycle can also be represented as a 22-yr cycle in which every second peak in Figure 1(a) is reversed in sign (Bracewell, 1953). The power spectrum of this 22-yr cycle is shown in Figure 1(d), and in this case the strongest and significant periodicities are the group near the 22-yr period and another near the 7.75-yr period.

3. The Phase of the Sunspot Cycle

The phase of the solar cycle can be studied by comparing the dates of the solar cycle maxima or minima with those expected for a constant period of 11.06 yr, the mean spacing of the cycles. The residual phase so obtained is plotted in Figure 2(a) for solar cycle maxima between 300 AD and 1968 (Waldmeier, 1961; *Astronomische Mitteilungen*; Schove, 1955).

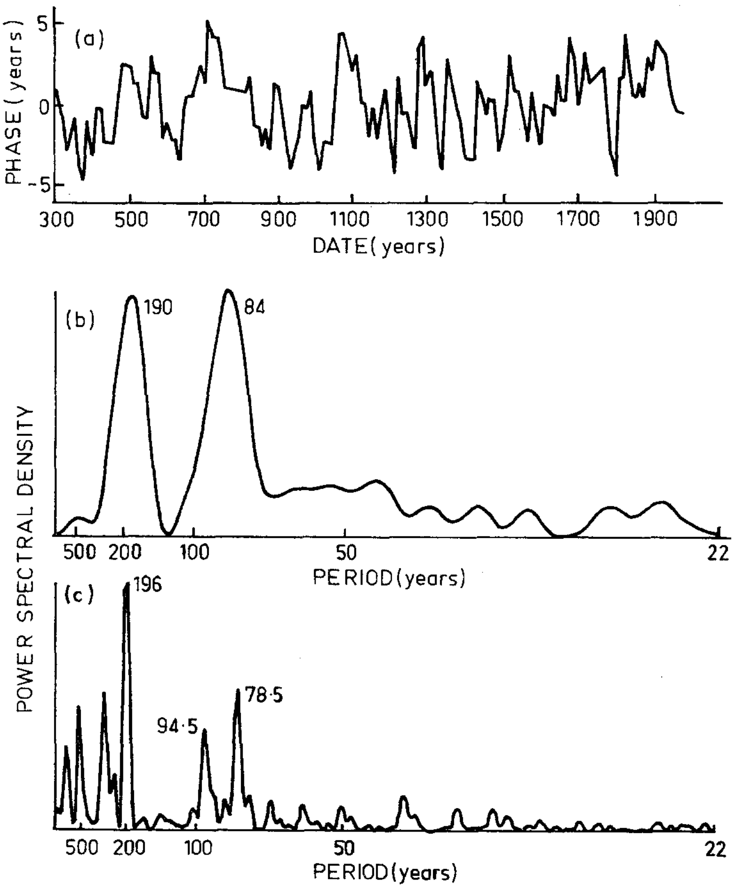

Fig. 2. (a) The relative phase of the dates of solar cycle maxima relative to a constant 11.06-yr period, plotted from 300 AD until 1968 using the data of Schove (1955) for maxima before the year 1600. (b) The power spectrum of the section of the curve in (a) between 1626 and 1968. (c) The power spectrum of the residual phase for the whole 152 cycles of Figure 2(a).

The power spectrum of this residual phase over the period between 1626 and 1968 (where the data are more accurate) is shown in Figure 2(b); it reveals two significant peaks at 190- and 84-yr periods. Both peaks are broadened, indicating that they consist of components which cannot be resolved by the 340-yr length of data. Some extra information can be obtained from the less accurate data of Schove (1955), and Figure 2(c) is a plot of the power spectrum of the residual phase for the complete 1668-yr interval of Figure 2(a). It indicates three main periodicities in the phase of periods 196, 94.5 and 78.5 yr. The phase also contains periodicities of 280, 560 and 1050 yr but it is difficult to place accuracy limits on these longer-term variations derived from the data of Schove (1955).

The periodicities can now be interpreted, but first they will be used to predict the dates of the next few solar cycle maxima and minima.

4. The Predicted Solar Cycle Activity

The simplest description of the residual phase, based upon the power spectra in Figure 2, is that it consists of the summation of three periodicities of periods 190, 94.5 and 78.5 yr. The amplitudes and phases of components of these periods (as contrasted with the *power* spectrum) can then be found by direct Fourier transformation of the residual phase of Figure 2(a).

If one assumes that the past behaviour of the residual phase is also representative of the future, then extrapolation using just these three frequency components gives an estimate of the future sunspot cycle behaviour. The projected dates of sunspot cycle maxima and minima for the cycles up to the year 2000 are listed in Table I. The intensity of the cycles was also predicted by using the relationship between solar cycle length and intensity found for previous cycles (Waldmeier, 1966).

TABLE I

Predicted solar cycles

Cycle number	20	21	22	23
Minimum date	(1964.8)	1975.3	1988	1999
Maximum date	(1968.9)	1981.2	1993	2004
Peak Wolf number	(110)	60	70	90

5. The 190-yr Periodicity

The strongest feature in the phase of the 11-yr cycle over the last 270 yr is a 190-yr periodicity. The component of period 94.5 yr is most likely a second harmonic of the 190-yr component. The form of the 190-yr variation is therefore not sinusoidal. Further understanding of this periodicity can be gained through the spectra of Figure 1.

Given a 190-yr phase modulation, the power spectrum in Figure 1(b) should show this as a series of peaks about the basic frequency of the cycle with a separation corresponding to a 190-yr period. Indeed, the peaks about the 10.45-yr peak are separated, on average, by this amount. Further, modulation theory (see, for example, Everitt and Anner (1956)) indicates that the ratio of the various peaks in Figure 1(a) is due to a phase modulation of ±90° and the non-symmetry about the centre of the group of peaks at the 10.45-yr period indicates a non-sinusoidal form to the phase modulation.

The autocorrelation function and the spectrum of Figure 1(d) add to the argument in support of the real existence of a 190-yr phase modulation of the solar cycle.

Figure 3(a) illustrates the mean behaviour of the 190-yr phase variation. It is obtained by superimposing 190-yr segments of the residual phase of Figure 2(a). Also drawn on the diagram are solid lines which are those which would be obtained from a periodicity of 10.45 yr in the data. A discussion of the 10.45-yr period is left to a later section, but it can be seen that the phase is strongly modulated and that for almost half of the time the sunspot cycle is, on average, closer to 10.45 yr than the mean, 11.06-yr period.

The form of the 78.5-yr phase modulation is apparently sinusoidal, as is shown in the superimposed residual phase plotted in Figure 3(b).

6. An Amplitude Modulation

Figures 1(b) and 1(d) show significant modulation of the amplitude of the solar cycle, including the peak at 88 yr, close to that expected from the well-known 80-yr cycle. On closer observation, however, it is possible to interpret these extra peaks in terms of a single amplitude modulation.

It is significant that the groups of peaks near the 88-yr and 5.75-yr periods in Figure 1(b) and near the 7.75-yr period in Figure 1(d) are similar in structure to the group of peaks associated with the 11- or 22-yr cycles respectively. Further, the frequency separations of the 88- and 5.75-yr groups from the 11-yr group are equal, both to each other, and to the group separation in Figure 1(d).

A simple and adequate explanation of these groups of peaks is that the 11-yr cycle is being amplitude-modulated (Everitt and Anner, 1956) by a periodicity represented by the separation between the 'carrier' at the 11-yr period and the 'sidebands' at the 88- and 5.75-yr periods. The spectra indicate modulation with a period of 11.9 ± 0.3 yr, and this is then the only significant amplitude modulation found in the data.

7. The Significance of a 10.45-yr Period

Mention has been made several times of a period of 10.45 yr in the data and, on the evidence available from the sunspot numbers, this period seems to be significant in any description of the solar cycle.

The period first appeared as the apparent centre of the main group of peaks in the spectrum of Figure 1(b). That it is, indeed, the basic and central period for this group is made clear in the autocorrelation function plotted in Figure 1(c). This function is composed of a 10.45-yr sinewave whose envelope follows the 190-yr period and which reverses phase at the point at which its amplitude is zero. If one is to consider then that 10.45 yr is the basic period of the series plotted in Figure 1(a), it is necessary to study the way in which the mean cycle length becomes 11.06 yr.

The 10.45-yr period relates closely to the mean behaviour of the phase over the 190-yr cycle. This is illustrated in Figure 3(a), where both the mean phase behaviour and a 10.45-yr periodicity are compared with the mean cycle length of 11.06 yr. For almost half of the 190-yr period the cycle is 10.45 yr; for the other half the cycle lengthens, such that at the end of the 190-yr period the cycle is in phase again with the 10.45-yr periodicity. However, it is now in phase exactly one period of 10.45 yr later than before. That is, the 190-yr period contains exactly one period more of 10.45 yr than it does of the mean cycle length of 11.06 yr. The exact relationship is

$$18 \times 10.45 \simeq 17 \times 11.06 \simeq 190\,\mathrm{yr}.$$

This is illustrated more clearly in the actual data between 1626 and 1968. Using the dates of the well-defined minima (rather than maxima) for this period, Figure 3(c) shows the residual phase of the minima relative to a 10.45-yr periodic waveform. In this figure the section after the year 1790 has been displaced exactly one cycle to show more clearly that the sections marked from 1710 to 1790 and from 1890 to the present are at the same phase and are approximately straight, horizontal lines. This implies that for these times, the period between sunspot minima was 10.45 yr and that between these times the dates of the minima have been retarded in phase exactly one cycle of 10.45 yr.

The length of the last seven cycles has been very close to 10.45 yr and, on the basis of the mean behaviour of the 190-yr phase variation, it could be expected that this length will abruptly increase within the next cycle. This is a similar conclusion to that already discussed in Section 4 and shown in the table.

There are therefore several different ways to describe the phase behaviour, dependent upon which of the 10.45-, 11.06- or 190-yr periods is considered the most fundamental. It is suggested here that the 10.45-yr period is more exactly defined, and

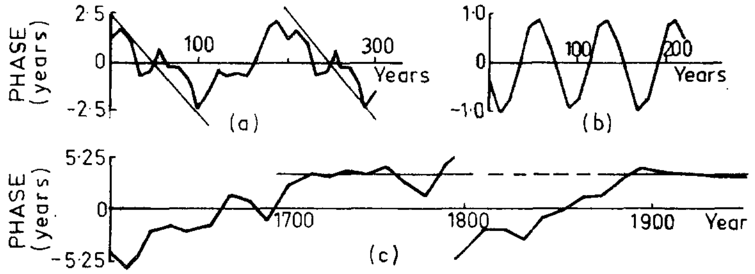

Fig. 3. (a) The mean shape of the 190-yr variation in phase derived by superimposing 190-yr segments of Figure 2(a) for the interval 300 AD until 1968 AD. The sloping lines represent the phase of a 10.45-yr periodicity. Note that the section between 190 and 300 yr is merely a repeat of the section from 0 to 110 yr. (b) The mean 78.5-yr variation in phase derived by superimposing 78.5-yr segments of Figure 2(a). (c) The relative phase between dates of solar cycle minima and a 10.45-yr period for the interval 1626 to 1968. The section from 1790 onwards has been displaced by one cycle. The two sections shown by a solid, thin line are those where the period is close to 10.45 yr.

therefore more basic, than the 190-yr period. The support for this suggestion is Figure 2(c), where there is an apparent variation in the 190-yr period throughout the data of Schove (1955), as indicated by the broadening of the peak centred now on 196 yr. In comparison, the plot in Figure 3(c) defines the 10.45-yr period to within one part in 200.

The assumption that 10.45 yr is the basic periodicity in the cycle suggests an interpretation based upon a triggering mechanism of period 10.45 yr and a free-running solar cycle period corresponding to the other half of the 190-yr cycle in Figure 3(a). A new solar cycle is triggered every 10.45 yr until the phase difference between this period and the free-running solar period has increased to such an extent that a new cycle is unable to be triggered at the next 10.45-yr interval. As a result, the cycle 'free runs' at its natural, longer rate until the phase once again matches that of the 10.45 yr cycle. In this way it is the 10.45-yr and the free-running periods which define both the mean cycle length and the exact length of the 190-yr phase variation. This free-running period of the solar cycle is therefore somewhere near 11.8 yr.

A further discussion of this interpretation and its relationship to other phenomena on the Sun is left to the conclusion.

8. The Association between Solar Activity and the Planets

The only suggestion in the data of a possible association between solar activity and a planet is the coincidence of the orbital period of Jupiter and the 11.9-yr period of the amplitude modulation of the solar cycle. Since only annual mean sunspot numbers were used, the analysis above is not able to repeat the detection of a periodicity associated with the planet Mercury (Bigg, 1967). However, the analysis does not support the association found between a 178.8-yr periodicity in the orbital positions of the planets and solar cycle phase (Jose, 1965). The simple periodogram analysis used to suggest this association failed to recognize 178.8 yr as the first confusing sidelobe of the strong 190 ($\simeq 178.8 + 11.1$)-yr periodicity in the data.

A recent letter (Wood, 1972) has revealed a general agreement between sunspot numbers and the fluctuation in planetary tidal effects at the Sun. The tidal fluctuation is shown to have a periodicity which matches the 11-yr solar cycle. But the deviations of the sunspot data from this tidal effect, as represented by the errors of prediction of peak and valley dates (Wood, 1972), are almost identical with the phase deviations shown in Figure 2(a). Tidal fluctuation is unable to explain the 10.45- and 190-yr phase variations in the solar data. An influence of tidal fluctuation on the mean solar cycle activity is not, however, precluded by the analysis of this paper.

9. Conclusions

Power spectral analysis is a means of detecting the significant periodicities in the relative sunspot numbers. The resultant spectra reveal a relatively simple system of modulation which includes the previously discussed 80- and 190-yr variations. Basically, it is suggested that the mean cycle length of 11.06 yr and the 190-yr phase modulation are a result of two periods of 11.8 and 10.45 yr representing a characteristic, free-running time for the solar cycle, and a well-defined periodicity which excites the new solar cycle.

In addition, the cycle is subject to an amplitude modulation of period 11.9 ± 0.3 yr and it is the sidebands of this which appear at periods of 88 and 59 yr. This modulation, if not associated with the 11.8-yr period discussed above, is the only period found which appears possibly related to the orbital period of a planet (Jupiter).

The only other significant periodicity in the data is a 78.5-yr variation in phase, and although there is a peak in Figure 2(c) corresponding to Gleissberg's suggestion of a 1000-yr variation it is difficult to place a limit to its statistical reliability.

But the most radical conclusion of the analysis is the suggestion of the 10.45-yr exciting mechanism. The last seven solar cycles have been of this length and the nature of any triggering mechanism would be more apparent if detailed observations were available over the period when the cycle length abruptly increases. It would not be expected that the general magnetic polarity of the 22-yr cycle would be disrupted but it could be expected that the detailed differences in behaviour between the two hemispheres of the Sun (Waldmeier, 1967, 1966) would be affected. Detailed observations of such behaviour exists only for 10 or 11 solar cycles. They show that for the last seven cycles, when the mean period has been locked to 10.45 yr, solar activity has become progressively stronger in the northern hemisphere as compared with the southern, and the peak activity of a new cycle is triggered progressively earlier in the southern hemisphere as compared with the northern. It is a change in this behaviour in relationship to the 10.45-yr period which might be used to formulate a model of any internal effects of the Sun upon solar activity.

References

Astronomische Mitteilungen der Eidgenössischen Sternwarte, Zürich, No. 273, 279, 283, 284.
Bigg, E. K.: 1967, *Astron. J.* **73**, 463.
Blackman, R. B. and Tukey, J. W.: 1959, *The Measurement of Power Spectra*, Dover, N.Y.
Bracewell, R. N.: 1953, *Nature* **171**, 649.
Everitt, W. L. and Anner, G. E.: 1956, *Communication Engineering*, McGraw-Hill, N.Y.
Gleissberg, W.: 1952, *Die Häufigkeit der Sonnenflecken*, Akademie-Verlag, Berlin.
Jose, P. D.: 1965, *Astron. J.* **70**, 193.
Schove, D. J.: 1955, *J. Geophys. Res.* **60**, 127.
Waldmeier, M.: 1957, *Z. Astrophys.* **43**, 149.
Waldmeier, M.: 1961, *The Sunspot-Activity in the Years 1610–1960*, Schulthess & Co. AG, Zürich,
Waldmeier, M.: 1966, *Astron. Mitteilungen der Eidgenössischen Sternwarte*, Zürich, No. 274.
Wood, K. D.: 1972, *Nature* **240**, 91.
Zhukov, L. V. and Muzalevskii, Yu. S.: 1969, *Astron. Zh.* **46**, 600. (English translation in *Soviet Astron AJ* **13**, 473.)