THE FIVE-MINUTE PERIOD OSCILLATION IN MAGNETICALLY ACTIVE REGIONS

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Abstract. The magnetohydrodynamic frequency-wavelength relation, derived by McLellan and Winterberg (1968), has been evaluated for an isothermal atmosphere. In particular, the effect which an inclined magnetic field and a finite horizontal wavelength have on the critical sonic and internal-gravity cut-off frequencies has been examined, in which it has been assumed that the magnetic field vector, wave vector, and gravity vector are coplanar. It is shown that the frequency band in which vertical wave propagation is impossible in the non-magnetic photosphere, becomes smaller when an inclined uniform magnetic field is introduced, and that low frequency magnetically coupled internal-gravity waves do not propagate vertically if the horizontal wavelengths associated with this mode are greater than a critical wavelength which decreases with field strength.

It is also demonstrated that an inclined magnetic field will inhibit the resonance that occurs at the critical frequency ω_g in the non-magnetic atmosphere which is a result consistent with recent observations of the 'wiggly line structure' in active regions.

At the boundary of supergranular cells where the magnetic field is approximately vertical and concentrated into knots of flux at photospheric and low chromospheric levels, wave motion across the magnetic lines of force will become complicated by the extra restoring forces which the field imposes on the wave at depths where the magnetic energy density is comparable to the gas kinetic energy density. We shall use the term β, defined as

$$\beta = NkT \Big/ \frac{B^2}{8\pi} \tag{1}$$

throughout this paper to indicate the relative importance of the forces involved in the motion of a wave which travels vertically in the photosphere. At depths where $\beta \gg 1$, the magnetic forces are small compared to the kinetic forces, and wave motion is not affected by the presence of the field at these levels.

1. Frequency and Wavelength Dependence on the Strength and Orientation of the Magnetic Field

We can now examine the frequency response of the photosphere in the presence of a magnetic field and examine the frequency and wavelength dependence on β. McLellan and Winterberg (1968) have derived the hydromagnetic dispersion equations from a linearized system of equations. These equations relate the frequency and wavelength of a small amplitude wave perturbation to magnetic field strength, and to the density

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and sound speed in an exponential, isothermal atmosphere. The relations for Alfvén mode waves, and for slow and fast-mode waves are respectively,

$$\omega^2 - \frac{(\tilde{B}_0 \cdot \tilde{K})}{4\pi\varrho_0} = 0\,, \tag{2}$$

$$\omega^4 + \omega^2 \left[- ig\gamma k_z - k^2\left(V_s^2 + \frac{B_0^2}{4\pi\varrho_0}\right)\right] + g^2 (\gamma - 1)(k^2 - k_z^2) + + \frac{V_s^2}{4\pi\varrho_0} (\tilde{B}_0 \cdot \tilde{K})^2 k^2 + ig\gamma B_z \frac{(\tilde{B}_0 \cdot \tilde{K})}{4\pi\varrho_0} k^2 = 0\,, \tag{3}$$

where V_s is the constant sonic velocity, $\tilde{B}_0$ is the magnetic field strength which is assumed uniform, $\tilde{K}$ is the total wave vector so that $\tilde{K} = |k|$, ϱ_0 is the equilibrium density. B_z and k_z are the components of the magnetic field and wave vector respectively in the direction of the gravitational acceleration, which is taken as $(0, 0, -g_z)$.

Equations (2) and (3) represent a local relationship between frequency and wavelength, because the variation of density over the vertical wavelength of the perturbation is assumed negligible. Therefore, the validity of these equations is restricted to perturbations whose wavelengths are small compared to the scale height of the atmosphere, i.e. $\lambda_{\text{wave}} \ll V_s^2/g\gamma$. Furthermore, Equations (2) and (3) are derived from the fundamental equation of motion and the equations of continuity, entropy, the equation of state, and the equation of magnetic induction. Additionally, the gas has been assumed to be compressible, and the dissipative effects arising from viscosity, heat conductivity, and electrical resistivity are assumed negligible. The linearization of the equation of motion further requires that the perturbed density, pressure, and magnetic terms are small compared to their equilibrium values.

In active regions of the solar photosphere, the precise value of the magnetic field strength at the line forming levels is still uncertain for two reasons. Firstly, if the magnetic field is concentrated into knots which occupy a small fraction of the area of the magnetometer slit, then the observed field will represent an averaging of the magnetic field over the individual elements from the entire slit area. Recent observational work by Howard and Stenflo (1972) indicates that the boundary of supergranular cells is composed of filamentary structures, in which the magnetic field is highly concentrated deep in the photosphere, and spreads rapidly with height.

Secondly, Harvey and Livingston (1969) have pointed out that many of the photospheric lines which are used for magnetograph measurements are temperature sensitive and weaken in active regions. The corresponding change in profile of these lines introduces errors in calibration which could be as large as a factor two or three. Therefore, estimates of the magnetic field strength at the boundary of supergranular cells range from 60 G to 250 G. In the present computations, the dispersion relation (3) has been evaluated for magnetic field strengths in the range 1 G to 300 G.

Bel and Mein (1971) have used McLellan and Winterberg's relations (2) and (3), and evaluated these equations for the active photosphere in the case of purely vertical propagation, so that the horizontal wave number $k_x = 0$. However, as we are examining the modification of the frequency response of the photosphere in the resonant range of 300 s in the presence of a magnetic field, it is essential that we allow $\tilde{K}$ to have a horizontal component. From observations, the five–minute period oscillation is typically associated with long but finite horizontal wavelengths of the order of several thousand kilometers (Ulrich, 1970). We would like to consider the general case and examine the effects which a finite horizontal wavelength and an inclined magnetic field have on the frequency response of the photosphere.

If we assume $\tilde{B}_0$, $\tilde{g}$, and $\tilde{K}$ have components $(B_x, 0, B_z)$, $(0, 0, -g_z)$ and $(k_x, 0, k_z)$ respectively, for which we assume the form $k_z = \pm a \pm ib - ig\gamma/2V_s^2$, we may expand the terms of Equation (3) and obtain a complex quadratic polynomial in ω. We have restricted this analysis to coplanar solutions by assuming B_y and k_y are zero in order to simplify the rather cumbersome algebraic expressions. However, computations which have been performed for the more general case of (B_x, B_y, B_z), (k_x, k_y, k_z) and $(0, 0, g_z)$ modify the important results of this and the following section only slightly.

If we set $a = 0$ in these equations and solve for ω, we will obtain the frequencies for which vertical wave propagation does not occur. For the non-magnetic case ($\tilde{B} = 0$) vertical wave propagation does not occur in the frequency band $\omega_s > \omega > \omega_g$.

The terms of Equation (3) have been evaluated using the values of density and temperature for which $\varrho_0 = 3.54 \times 10^{-9}$ gm/cm^{-3} and $T = 4170$ (K) at temperature minimum in the Harvard-Smithsonian Reference Atmosphere (Gingerich *et al.*, 1971).

The value $\gamma = 1.3$ has been selected as the mean ratio of specific heats for the photosphere because, in the first approximation, lowering γ from its adiabatic value of $\frac{5}{3}$, reflects the energy losses due to radiation. The solutions of the dispersion of Equation (3) have been plotted as frequency ω against horizontal wave number k_x in Figure 1 for selected magnetic field strengths and field inclinations.

In Figure 1a and 1c the curves drawn correspond to the solution of Equation (3) for a one gauss magnetic field, for which $\beta = 2 \times 10^4$ at $\tau_{5000} = 10^{-4}$. A one gauss field at this depth has a negligible effect on the frequency response of the photosphere, and therefore, at levels where $\beta \gg 1$, the solution of Equation (3) corresponds closely to the solution of the dispersion relation when $\tilde{B}_0 = 0$. The solution for the one gauss case when $b = -g\gamma/2V_s^2$ (thick curves) divides the diagnostic diagram into three distinct regions. Region I corresponds to vertically propagating sonic-gravity waves which travel upwards, provided the frequency ω is greater than the critical sonic frequency ω_s. If $\omega < \omega_g$, vertical wave propagation is restricted to the internal-gravity mode (Region III). Waves with frequencies in Region II are essentially horizontal waves which are non-propagating in the vertical direction, and are so termed evanescent.

However, a relatively weak magnetic field modifies somewhat the response of the photosphere in this frequency band. Since the magnetic terms in the dispersion equation provides for solutions for which $b = \pm k_x + g\gamma/2V_s^2$ (thin curves in Figure 1), we find that the magnetic field imposes a horizontal spatial dependence on the condition for energy conservation in vertically travelling waves. Specifically, if k_x is small compared to $g\gamma/2V_s^2$, i.e. if λ_x is large compared to the scale height of the atmosphere,

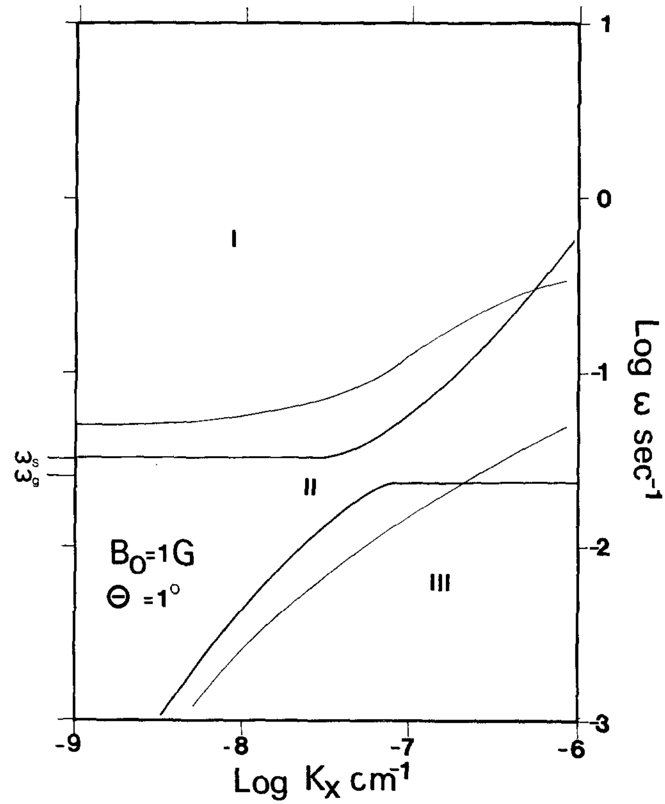

Fig. 1. Region I of the diagnostic diagram consists of vertically propagating magnetosonic-gravity waves, and Region III is comprised of magnetically coupled internal-gravity waves. Both Regions I and III are bounded by the solution of the dispersion equation in which the real component of $k_z(a)$ is set equal to zero, and the imaginary component (b) is $b = -g\gamma/2_s{}^2$ (thick curves). Waves in these regions are both vertically propagating and energy conserving. The thin curves correspond to waves in which $b = \pm k_x + g\gamma/2V_s{}^2$. This solution corresponds to waves which are energy conserving when propagating vertically if $k_x \ll g\gamma/2V_s{}^2$ ($\lambda_x \gg 2V_s{}^2/g\gamma$). If $k_x \gg g\gamma/2V_s{}^2$ ($\lambda_x \ll 2V_s{}^2/g$) the waves are attenuated rapidly and are non-energy conserving.

Fig. 1a. $B_0 = 1$ G and the field is inclined 1 deg with respect to $\bar{g}$.

then magnetically coupled waves can propagate vertically, and expand in amplitude with increasing height, thereby conserving energy. In Figure 1 these wave solutions are represented by the thin curves, and occur in Regions I and III. This wave solution is of particular physical importance since waves with growing amplitude will eventually steepen sufficiently to form shock waves, and the rapid dissipation of mechanical energy once a finite amplitude discontinuity has developed can heat upper levels of the chromosphere and corona. From this analysis we find that if an inclined magnetic field is present, vertically travelling magnetically-coupled waves will be energy conserving modes if the horizontal scale associated with these waves is large compared to the scale height of the atmosphere, i.e. if $\lambda_x > V_s^2/g\gamma$.

If λ_x is small compared to the scale height of the atmosphere, we find that magneti-

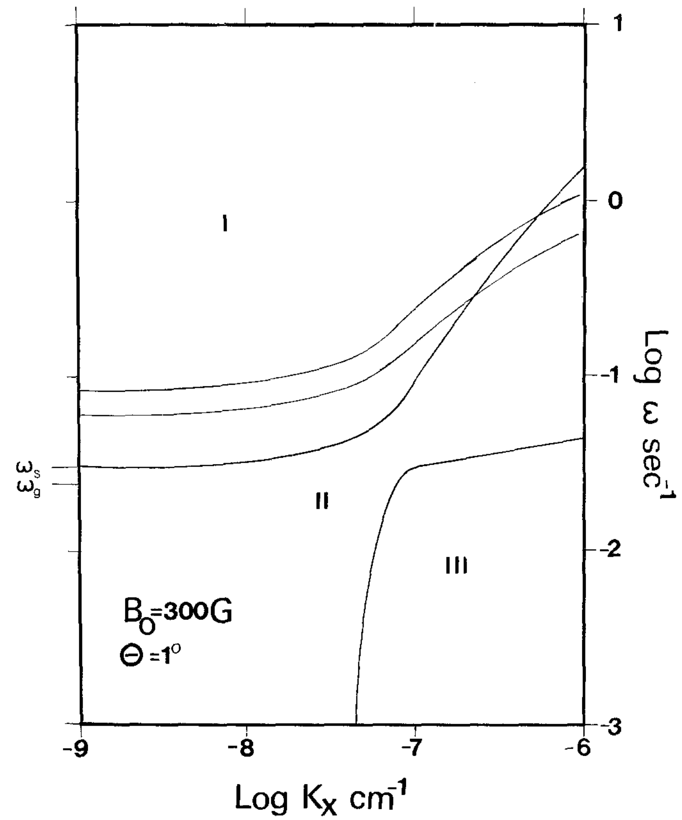

Fig. 1b. $B_0 = 300$ G and the field is inclined 1 deg with respect to $\vec{g}$.

cally coupled waves are no longer energy conserving. This case is not of physical importance considering the transmission of mechanical energy to upper levels of the solar atmosphere, since waves with horizontal scale sizes $\lambda_x < V_s^2/g\gamma$ will be attenuated quite severely. In this respect, this case is not of physical importance when formulating a model of chromospheric heating in magnetic regions.

If we now increase the magnetic field strength to 300 G, so that $\beta = 0.261$ at $\tau_{5000} = 10^{-4}$, we note that the solution of Equation (3) (Figures 1b and 1c) is substantially altered. The effect of an approximately vertical magnetic field (Figure 1b) is to alter the $\omega(k_x)$ dependence in such a way that perturbations with horizontal wavelengths which are large compared to the scale size $2V_s^2/g\gamma$, i.e. for horizontal wavelengths $\lambda_x \gg 2V_s^2/g\gamma$, vertical wave propagation at high frequencies takes place in the sonic-gravity mode. However, as the beta of the gas is less than unity in this case, a sonic-gravity wave is coupled to the magnetic field, because the magnetic restoring forces in Equation (3) are large compared to the gas kinetic forces. In this case, vertical wave propagation occurs in the magnetosonic-gravity mode, and as the curve bounding Region I in Figure 1b is asymptotic to ω_s as k_x tends to zero, fast and slowmode waves propagate vertically in Region I provided $\omega > \omega_s$.

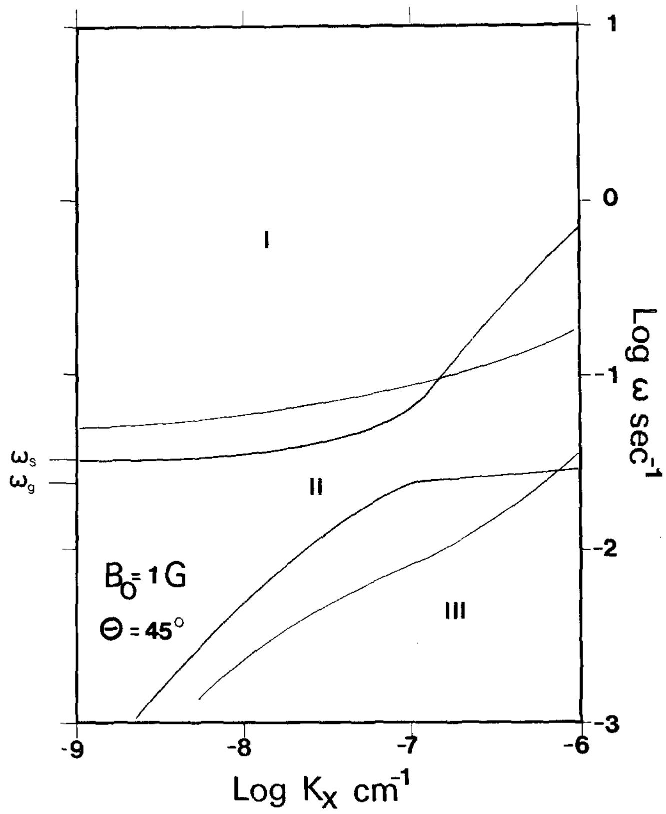

Fig. 1c. $B_0 = 1$ G and the field is inclined 45 deg with respect to $\bar{g}$. A magnetic field strength of 1 G has a negligible effect on the frequency response of the atmosphere because $\beta \gg 1$.

If we incline the magnetic field (with respect to g) through 45 degrees in Figure 1d, we note that in Region I, $\omega(k_x)$ is no longer asymptotic to ω_s as k_x tends to zero. Therefore, for an inclined magnetic field, magnetosonic waves may propagate vertically at frequencies $\omega < \omega_s$. If in Equation (3) we set $a = 0$ and $k_x = 0$, and let $b = -g\gamma/2V_s^2$ we will obtain the critical magnetosonic-gravity frequency ω_c, where

$$\omega_c^2 = \omega_s^2 \left(\frac{1}{2} - \frac{1}{\gamma\beta}\right) + \omega_s^2 \left[\left(\frac{1}{\gamma\beta} - \frac{1}{2}\right)^2 + \frac{2\cos^2\theta}{\gamma\beta}\right]^{1/2}, \tag{4}$$

and $\theta = \arccos(B_z/B_0)$. Therefore, at levels where $\beta < 1$, the critical magnetosonic-gravity frequency is less than the critical sonic-gravity frequency ω_s when the field is inclined from the vertical.

If the horizontal wavelength is small compared to the scale height of the atmosphere, so that $\lambda_x \ll V_s^2/\gamma g$, a vertically travelling low frequency wave will propagate as a magnetically coupled internal-gravity wave when $\beta < 1$. Magnetically coupled internal-gravity waves propagate vertically at frequencies greater than the critical frequency ω_g, because the limiting curve $\omega(k_x)$ in region III is not asymptotic to ω_g as k_x tends

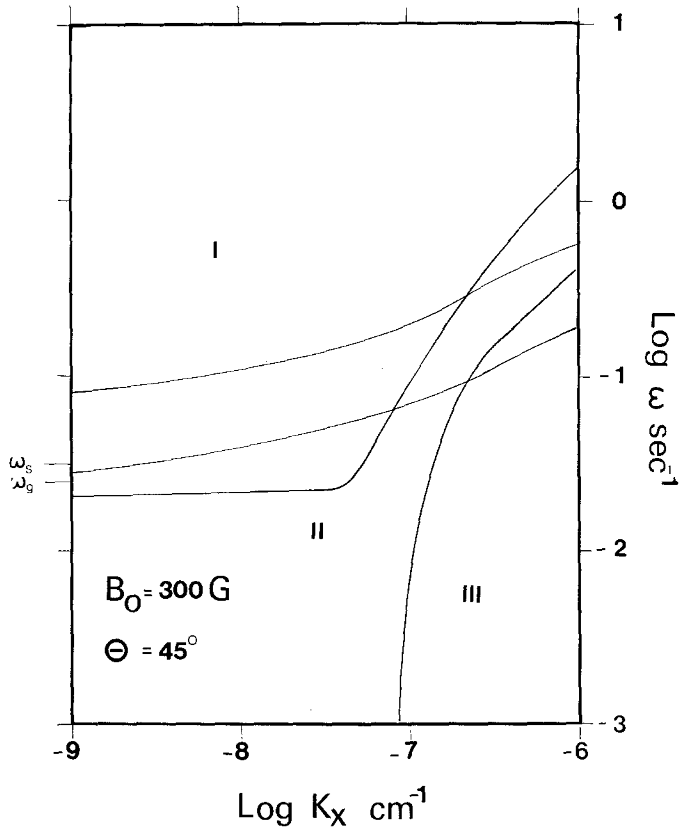

Fig. 1d. $B_0 = 300$ G and the field is inclined 45 deg with respect to $\vec{g}$. Both curves bounding Regions I and III are no longer asymptotic to ω_s or ω_g.

to infinity when $\beta < 1$. Setting $a = 0$ in Equation (3) and solving for the frequency ω in the limit as $k_x \to \infty$, we find that ω is unbounded by k_x, and the frequency tends to infinity in the limit as the horizontal wavelength approaches zero.

The effect which an inclined magnetic field has throughout all solutions of Equation (3) is to decrease the frequency band in which vertical wave propagation does not occur, and introduce vertically propagating fast and slow modes in place of the evanescent waves in the frequency band $\omega_s > \omega > \omega_g$. This indicates that the levels where $\omega < \omega_s$ or $\omega > \omega_g$, a wave need not be reflected when an inclined magnetic field of sufficient strength permeates the photosphere. At levels where $\beta \sim 1$, magnetically coupled waves may appear which can propagate energy vertically for a larger range of frequencies than can the magnetically uncoupled modes.

2. The Resonant Frequency Response of the Active Photosphere

Howard *et al.* (1968), and more recently Blondel (1971) have observed the 'wiggly line structure' in active and non-active regions in photospherically formed spectral lines. Blondel has found that in active regions, in the resonant range of 300 s, the lobe of the power spectrum is reduced by approximately 20% to 25%. He also finds that the velocity amplitudes for waves of periods less than 240 s is strengthened in magnetically active regions.

In order to examine the effect which an inclined magnetic field has on the resonant frequency response of the active photosphere, we will derive an expression for the vertical velocity amplitude in terms of the magnetic field strength and the wave vector $\tilde{K}$.

The maximum frequency response of an isothermal atmosphere, which is permeated by an inclined magnetic field can be obtained by first deriving a relation for the rate of variation of pressure with time (Equation (8)) from the equations of continuity (5), entropy (Equation (6)), and the equation of state (Equation (7)), which are

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \cdot \tilde{V}) = 0, \tag{5}$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\partial s}{\partial t} + \tilde{V} \cdot \nabla s = 0, \tag{6}$$

$$\mathrm{d}\varrho = \left.\frac{\partial \varrho}{\partial s}\right|_P \mathrm{d}s + \left.\frac{\partial \varrho}{\partial P}\right|_s \mathrm{d}P, \tag{7}$$

and

$$\frac{\partial P'}{\partial t} = V_z \varrho_0 g - \varrho_0 V_s^2 \nabla \cdot \tilde{V}, \tag{8}$$

where P' is the time-dependent pressure perturbation at the height $z=0$, given as

$$P' = P(\omega, K)\, e^{i(k_x x - \omega t)}. \tag{9}$$

Likewise, we can represent the vertical and horizontal velocity fields as

$$V_x = u_x e^{i(\tilde{K}\cdot\tilde{r} - \omega t)}, \tag{10}$$

and

$$V_z = u_z e^{i(\tilde{K}\cdot\tilde{r} - \omega t)}, \tag{11}$$

where u_x and u_z are the horizontal and vertical velocity amplitudes respectively.

In order to simplify the algebraic expressions in this paper, we have again restricted ourselves to solutions for which the magnetic field vector, velocity vector, and gravity vector are coplanar. In fact, it has been found that an extension of these equations to the more general case where $\tilde{V}$, $\tilde{B}$ and $\tilde{g}$ have components (V_x, V_y, V_z), (B_x, B_y, B_z) and $(0, 0, -g_z)$ respectively, does not significantly affect the conclusions obtained for the coplanar solutions.

It is now possible to solve for the vertical velocity amplitude u_z by substituting relations (9) through (11) into Equation (8), for which we obtain

$$\frac{u_z}{P(\omega, K)} = \frac{\omega}{\varrho_0 \left\{ i V_s^2 \left(\frac{k_x}{v_z/v_x} + k_z \right) - g \right\}}. \tag{12}$$

An algebraic equation which relates the ratio v_z/v_x in Equation (12) to the frequency, wave number, and magnetic field strength can be obtained by expanding the terms of Equation (13)* which gives the velocity amplitudes $\tilde{v} = (v_x, 0, v_z)$.

We take the magnetic field $\tilde{B}_0$ and wave vector $\tilde{K}$ to have components $(B_x, 0, B_z)$ and $(k_x, 0, k_z)$ respectively.

$$\omega^2 \tilde{v} - V_s^2 (\tilde{K} \cdot \tilde{v}) \tilde{K} + i (\tilde{v} \cdot \tilde{g}) \tilde{K} + i (\gamma - 1) (\tilde{K} \cdot \tilde{v}) \tilde{g} + \\ + \frac{1}{4\pi\varrho_0} \tilde{B}_0 \times \{ \tilde{K} \times [\tilde{K} \times (\tilde{v} \times \tilde{B})] \} = 0. \tag{13}$$

If we expand the terms of Equation (13) and resolve the equation in the vertical direction (unit vector $\tilde{n}$) and horizontal direction (unit vector $\tilde{l}$) we will obtain

$$\tilde{l} \{ \omega^2 v_x - i v_z g k_x - V_s^2 (k_x^2 v_x + k_z k_x v_z) + \\ - \frac{1}{4\pi\varrho_0} [B_z (k_x^2 + k_z^2)(B_z v_x - B_x v_z)] \} + \tilde{n} \{ \omega^2 v_z - i v_z g k_z + \\ - i (\gamma - 1) k_x v_x g - i (\gamma - 1) k_z v_z g - V_s^2 (k_z k_x v_x + k_z^2 v_z) + \\ + \frac{1}{4\pi\varrho_0} [B_x (k_x^2 + k_z^2)(B_z v_x - B_x v_z)] \} = 0, \tag{14}$$

where $\tilde{v}$ is the velocity vector defined by

$$\tilde{v}(\tilde{r}, t) = \tilde{v} = u e^{i \, (\tilde{K} \cdot \tilde{r} - \omega t)}. \tag{15}$$

Setting both bracketed expressions equal to zero, we obtain two simultaneous equations, each of which is in terms of v_x and v_z. Upon solving for the ratio of v_z/v_x in each of the simultaneous equations, we obtain the relations

$$f(\omega, K, B) = \frac{v_z}{v_x} = - \left\{ \frac{\omega^2 - V_s^2 k_x^2 - \frac{B_z^2}{4\pi\varrho_0} (k_x^2 + k_z^2)}{\frac{B_x B_z}{4\pi\varrho_0} (k_x^2 + k_z^2) - V_s^2 k_x k_z - i g k_x} \right\}, \tag{16}$$

and

$$f(\omega, K, B) = \frac{v_z}{v_x} = \left\{ \frac{V_s^2 k_x k_z + i (\gamma - 1) k_x g - \frac{B_x B_z}{4\pi\varrho_0} (k_x^2 + k_z^2)}{\omega^2 - V_s^2 k_z^2 - \frac{B_x^2}{4\pi\varrho_0} (k_x^2 + k_z^2) - i g k_z \gamma} \right\}. \tag{17}$$

* For derivation of Equation (13) see Equation (12) of McLellan and Winterberg (1968).

Worrall (1972) has shown for the non-magnetic case ($\tilde{B}=0$) that an isothermal atmosphere which is subjected to pressure fluctuations at its base will respond singularly at the internal-gravity frequency ω_g. Worrall finds that in the absence of dissipative viscosity and heat conductivity, the vertical velocity amplitude for oscillations at ω_g varies exponentially rather than sinusoidally with height, i.e. the vertical wave number associated with oscillations at ω_g is *purely* imaginary ($k_z = \mathrm{i}g/V_s^2$). Furthermore, the singular response of the atmosphere at ω_g is independent of the horizontal scale of the exciting fluctuations, i.e. this particular solution is independent of λ_x. In a manner of speaking, the atmosphere heaves in phase at the internal frequency ω_g when no magnetic field is present.

Therefore, as a check for consistency of Equations (16) and (17) in the non-magnetic case, we set $\tilde{B}_0=0$ in both equations, and examine the value of ω and k_z for which the ratio v_z/v_x diverges. From Equations (16) and (17) we cannot examine the divergence of v_x or v_z separately, since by solving one of these equations in terms of the other, both v_z and v_x cancel. This leaves us with only the dispersion relationship for a non-magnetic isothermal atmosphere. However, if we substitute the quantities $\omega=\omega_g=g/V_s(\gamma-1)^{1/2}$ and $k_z=ig/V_s^2$ into Equations (16) and (17) we find that the ratio v_z/v_x diverges. This truly indicates that the vertical velocity field v_z becomes infinitely large for this particular case, since from Worrall (1972) it is known that the response of an isothermal atmosphere at the critical frequency ω_g when $k_z=ig/V_s^2$ provides for waves which are propagating *only* horizontally. Therefore, v_x will be finite and v_z will be infinite for this particular solution.

This is supported by observations of Tanenbaum *et al.* (1969) of the five minute period oscillation in 'quiet' regions. Their work suggests that the photosphere responds in phase at a single frequency which is independent of the horizontal wavelength. If this model of the frequency response of the photosphere is correct for 'quiet' regions, the addition of an inclined magnetic field will be such as to prevent resonance from occurring at the critical frequency ω_g.

It would be of interest now to examine the maximum frequency response of the atmosphere in the presence of an inclined magnetic field. We can use either Equations (16) or (17) as the relation $f(\omega, K, B)$ in Equation (12). It is possible to obtain the total differential of Equation (12) with respect to ω, and set the resulting expression equal to zero, in order to find the maximum frequency response of the atmosphere.

$$\frac{\mathrm{d}}{\mathrm{d}\omega}\left\{\frac{u_z}{P(\omega, K)}\right\} = \frac{\partial}{\partial\omega}\left\{\frac{u_z}{P(\omega, K)}\right\} + \frac{\partial}{\partial k_z}\left\{\frac{u_z}{P(\omega, K)}\right\}\frac{\partial k_z}{\partial\omega}. \tag{18}$$

However, obtaining the total differential product $(\partial\omega/\partial k_z)\,(\partial k_z/\partial\omega)$ from the dispersion relation is algebraically tedious. Furthermore, this equation cannot be solved in closed form, so that we shall examine the divergence of the vertical velocity amplitude directly from Equation (12).

Therefore, as an approximate treatment, if the vertical velocity amplitude $[u_z/P(\omega, K)]$ is a maximum, then $[u_z/P(\omega, K)]^{-1}$ approaches zero. Putting $f(\omega, K, B)$ equal to Equation (16), we can solve for the frequency $\omega=\omega_{\max}$ in Equation (12) at which the atmosphere responds most efficiently. Doing this we obtain

$$\omega_{\max}^2 = \frac{g^2}{V_s^2}(\gamma - 1) + (k_x^2 + k_z^2)\left[\frac{B_x^2}{4\pi\varrho_0} + \frac{B_x B_z}{4\pi\varrho_0}\left(k_z - \frac{g}{iV_s^2}\right)\right]. \tag{19}$$

In order to make Equation (19) exact, we would in principle have to replace the product term involving the magnetic field by the expression

$$\frac{\partial}{\partial k_z}\left\{\frac{u_z}{P(\omega, K)}\right\}\frac{\partial k_z}{\partial \omega}.$$

From this approximate treatment, it is clearly shown in Equation (19), by setting the magnetic terms equal to zero, that the singular response of an isothermal atmo-

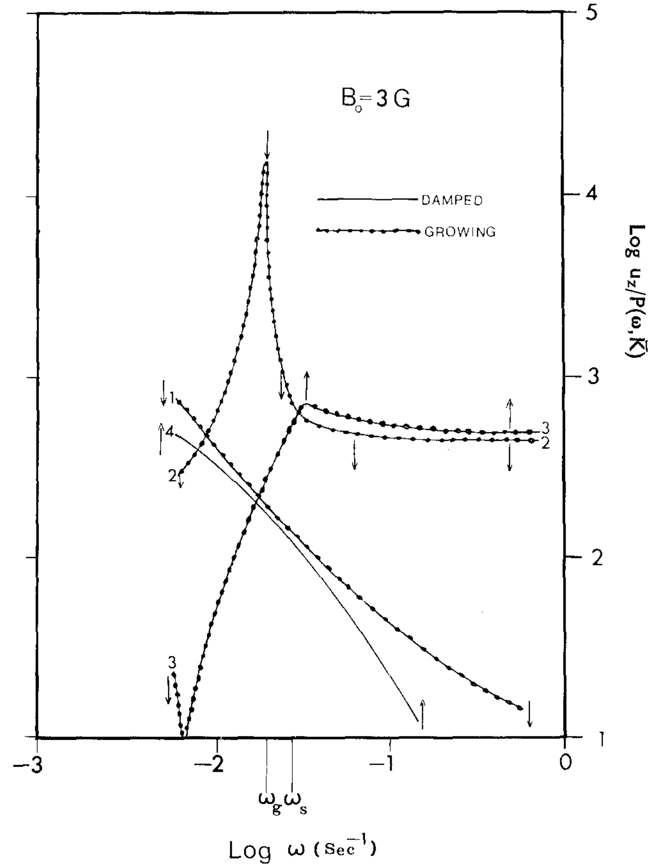

Fig. 2. The vertical velocity amplitude $[u_z/P(\omega, K)]$ plotted against the frequency ω for selected magnetic field strengths. The horizontal wavelength $\lambda_x = 5000$ km, and the magnetic field is inclined 45 deg with respect to $\bar{g}$. The solutions corresponding to expanding wave amplitudes are shown by the dotted curves, and the damped waves are indicated by the solid curve.

Fig. 2a. Magnetic field intensity is 3 G, and a maximum in the frequency response of the atmosphere occurs at the internal frequency ω_g (solution 2).

sphere occurs at the critical internal-gravity frequency, as has been shown previously by Worrall (1972). In a steady-state, the non-magnetic photosphere will amplify fluctuations at the resonance frequency ω_g, and the energy imparted by fluctuations at the base of the atmosphere will be propagated essentially as a horizontal wave.

It is now possible to solve Equation (12) numerically, using various values of magnetic field strength, and determine at which frequencies the atmosphere will amplify pressure fluctuations which are imparted at its base. This has been accomplished using the IBM 360/44 computer at the University of Cambridge. The results of these computations for a 3 G and 300 G field are shown in Figures 2a and 2b.

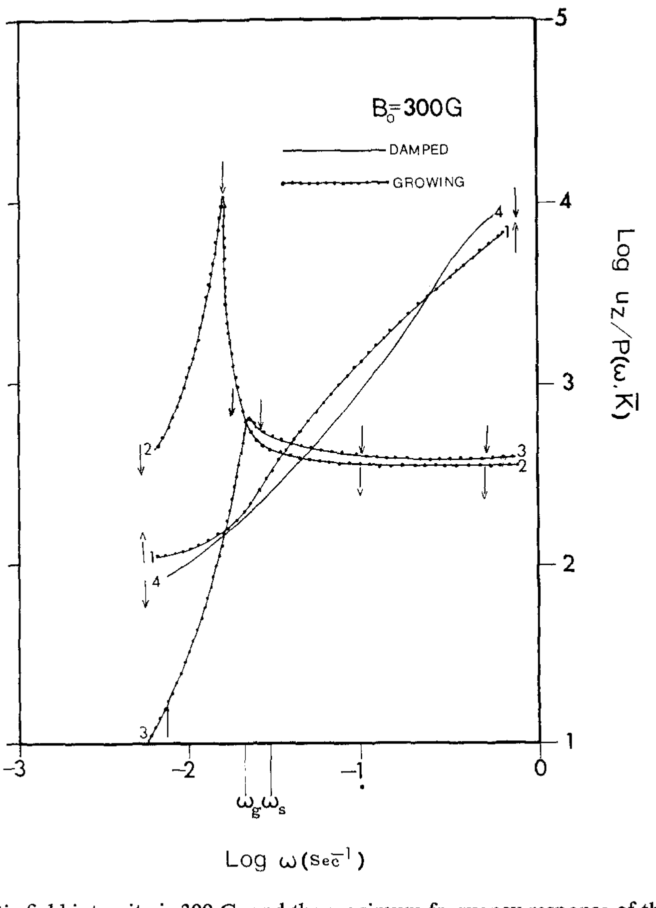

Fig. 2b. Magnetic field intensity is 300 G, and the maximum frequency response of the atmosphere (solution 2) is shifted to lower frequencies. Additionally, high frequency waves ($\omega > \omega_s$) carry a higher energy flux in magnetic regions (solutions 1 and 3).

From Equation (12) we obtain four solutions for $[u_z/P(\omega, K)]$ which have been plotted against frequency ω. In these computations we have assumed that the horizontal wavelength associated with the five minute period oscillation in the photosphere is of the order of 5000 km. For the 3 G solution, a maximum in the vertical velocity amplitude $[u_z/P(\omega, K)]$ occurs at approximately the critical internal frequency ω_g (solution (2)). In fact, if the magnetic field was identically equal to zero $[u_z/P(\omega, K)]$ would be infinite at this frequency. However, an inclined magnetic field makes the vertical velocity amplitude finite at this frequency. Furthermore, solution (2) in Figure 2 corresponds to downward propagating waves, which expand in amplitude (expanding wave amplitude indicated by dotted curve; diminishing amplitude by dashed curve). Solution 1, 3 and 4 provide for both upward and downward propagating waves, which expand and diminish in amplitude. However, in relation to the relative magnitude of $[u_z/P(\omega, K)]$ of these solutions, and of solution 2, it is evident that the preferred response of the atmosphere occurs at frequencies near ω_g.

If we increase the magnetic field intensity to 300 G (see Figure 2b), it is clear from solution 2 that the maximum frequency response of the atmosphere is shifted to lower frequencies, and that high frequency waves (see solution 1 and 3) in the range $\omega > \omega_s$ carry a higher energy flux. This qualitatively substantiates the observations of Blondel (1971), who has observed an enhancement in the velocity amplitude of long period waves in the range of 700 s, and short period oscillations with periods of 240 s and smaller, in active regions of the photosphere.

It is of further interest to note that the long period oscillations correspond to downward propagating waves. From Figure 2b it is evident that only high frequency waves, i.e. at frequencies $\omega > \omega_s$, are upwardly propagating. This makes magneto-sonic-gravity waves an important mechanism for transmitting mechanical energy vertically from the active photosphere to the active chromosphere.

3. Discussion and Summary of Results

From the results of Section 1, it has been shown that the frequency response of an isothermal atmosphere is substantially altered when permeated by an inclined magnetic field. In applying these results to active regions of the solar photosphere, it should be emphasized that several mathematical simplifications were necessary in order to obtain solutions. In particular, the assumption that the density is constant over the vertical wavelength will lead only to qualitative results when the vertical wavelength becomes large as compared to the local scale height of the atmosphere.

Additionally, the degree to which the magnetic field interferes with wave motion is dependent on height, so a complete treatment of this problem would have to include the variation of β with optical depth, as well as the effects of a non-homogeneous magnetic field.

As applied to the active photosphere, the results of these computations indicate that when $\beta < 1$, vertically travelling low frequency waves, with horizontal wavelengths which are large compared to the mean scale height of the photosphere, propagate in the magneto-acoustic gravity mode. Magnetically coupled internal-gravity waves do not propagate if the horizontal wavelength is greater than $2V_s^2/g\gamma$.

Furthermore, the frequency band from which vertically propagated waves in non-magnetic regions are excluded, i.e. frequencies ω for which $\omega_s > \omega > \omega_g$, is made smaller in active regions having a magnetic field such that $\beta < 1$.

This result indicates that at levels where $\beta \sim 1$, vertically propagating internal-gravity waves need not be reflected upon reaching a level where $\omega_s > \omega > \omega_g$. For sufficiently large magnetic field strengths, say 300 G, for which $\beta = 0.261$ at temperature minimum, the mechanical energy associated with wave perturbations will be channelled into a transverse motion of the magnetic lines of force. At levels where $\beta \sim 1$, some of the energy contained in the internal modes and sonic modes are propagated in magnetic modes, and the characteristics of wave motion above active regions are determined essentially by the configuration and strength of the magnetic field at heights where $\beta \ll 1$.

One of the most important features of diagnostic diagram 1d is that the cut-off frequency for magnetically coupled internal-gravity waves is increased substantially when an inclined magnetic field of moderate strength permeates the atmosphere. This result is of particular importance since it indicates that waves which are either trapped standing internal waves below the temperature minimum, or purely vertical evanescent modes in the photosphere, become vertically travelling waves in the active photosphere. As a consequence, the mechanical energy associated with long period oscillations in the 'quiet' photosphere can penetrate to the chromosphere, where the dissipation of this energy by shock formation can provide an enhancement of heating at upper levels of the solar atmosphere.

Thomas *et al.* (1971) suggest that the five-minute oscillation in 'quiet' regions is the result of standing internal-gravity waves which they assume are driven from below by the 'piston' action of rising granules. They find that at low chromospheric levels where hydrogen becomes predominantly ionized, the critical frequency ω_g decreases, trapping the internal resonant modes inside the photosphere and low chromosphere. From the results of these computations, if a magnetic field of sufficient strength permeates the photosphere, then internal-gravity waves may continue to propagate vertically. The magnetic field provides a mechanism by which energy, which is originally contained in the trapped resonant modes of the photosphere and low chromosphere, may 'leak' into the upper chromosphere and corona, by channelling along the magnetic lines of force. The energy which is removed from the five-minute period oscillation in the internal-gravity mode of the photosphere and placed in the magnetic modes is propagated to the upper chromosphere and corona, where the dissipation of this energy produces localized heating around the magnetic tubes of flux.

Additionally, it has been shown that the resonant response of the photosphere at the critical internal frequency ω_g is not affected when the magnetic field is strictly vertical. Accordingly, this model indicates that the field should have an appreciable horizontal component at photospheric depths. This result is in fact consistent with recent observations by Howard and Stenflo (1972).

In Section 2 it has been shown that pure resonance at the critical frequency ω_g

does not occur when a moderately strong inclined magnetic field is present. The solution of Equation (4) in the magnetic case provides for magnetically coupled waves which travel either upwards or downwards, with growing or decreasing amplitudes. If the 'quiet' photosphere responds in phase at the critical frequency ω_g (Worrall, 1972; Deubner, 1972), then in magnetically active regions one should observe a reduction in power contained in the resonance lobe of the power spectrum at the critical frequency ω_g, and a phase change in the 'wiggly line structure' in the wings and cores of photospherically formed spectral lines, if the wavelength of magnetically coupled oscillations are greater than the depths of the line forming levels.

We may conclude that for moderately strong magnetic fields in the photosphere, where β can be of the order of unity, the modification of the oscillations which cause the 'wiggly line structure' becomes important at high levels in the solar atmosphere. Furthermore, magnetic modes which grow in amplitude in the photosphere could form shocks, for which the dissipation of mechanical energy from the waves could explain the temperature enhancement of several hundred degrees which has been observed by Chapman and Sheeley, Jr. (1968). Further investigations are required which incorporate a non-linear theory in order to obtain a model of wave mode coupling, for which an accurate estimate of the amount of acoustic energy which is channelled into magnetic shocks and dissipated in the active solar atmosphere can be obtained.

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References

Bel, N. and Mein, P.: 1971, *Astron. Astrophys.* **11**, 234.
Blondel, M.: 1971, *Astron. Astrophys.* **10**, 342.
Chapman, G. A. and Sheeley, Jr., N. R.: 1968, *Solar Phys.* **5**, 442.
Deubner, F.-L.: 1972, *Solar Phys.* **22**, 263.
Gingerich, O., Noyes, R. W., Kalkofen, W., and Cuny, Y.: 1971, *Solar Phys.* **18**, 347.
Harvey, J. and Livingston, W.: 1969, *Solar Phys.* **10**, 283.
Howard, R., Tanenbaum, A., and Wilcox, J.: 1968, *Solar Phys.* **4**, 268.
Howard, R. and Stenflo, J. O.: 1972, *Solar Phys.* **22**, 402.
McLellan IV, A. and Winterberg, P.: 1968, *Solar Phys.* **4**, 401.
Tanenbaum, A., Wilcox, J. M., Frazier, E. N., and Howard, R.: 1969, *Solar Phys.* **9**, 328.
Tanenbaum, A., Wilcox, J. M., and Howard, R.: 1971, in R. Howard (ed.), 'Solar Magnetic Fields', *IAU Symp.* **43**, 348.
Thomas, J. H., Clark, P. A., and Clark, Jr., A.: 1971, *Solar Phys.* **16**, 51.
Ulrich, R. K.: 1970, *Astrophys. J.* **162**, 993.
Worrall, G.: 1972, *Astrophys. J.* **172**, 749.