HORIZONTAL COHERENCE OF WIND FLUCTUATIONS

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Abstract. The coherence for streamwise and cross-stream wind components is studied at four meteorological sites and compared with a representative wind-tunnel experiment. The coherence is approximated by a negative exponential in terms of a non-dimensional frequency, *Af,* and a decay parameter, *a.* Theoretical guidelines are developing to aid in identifying the pertinent variables affecting the decay parameters. These theoretical discussions indicate that for longitudinal separations, both the streamwise and cross-stream decay parameters are functions of roughness; the cross-stream decay parameter is a strong function of stability while the streamwise component is not. For lateral separations, it is found that both the streamwise and cross-stream decay parameters are functions of stability.

Isopleths of the decay parameter are drawn on graphs with coordinates of angle and Richardson number for both the streamwise and cross-stream decay parameters of coherence. These empirical curves give an indication of the behavior of the decay parameters of coherence for a range of stabilities given by $-0.9 < Ri < 0.08$, and a range of angles between zero and ninety degrees.

1. Introduction

In recent years, meteorologists have become increasingly interested in the cross correlations of the turbulent wind components between points separated in space, and their Fourier transforms, the cross-spectra. Such interest is motivated largely by the desire to get a clearer picture of the three-dimensional statistical structure of turbulence. A useful statistic related to the spectra and cross-spectra is the coherence, which may be thought of as a correlation in frequency space. More precisely, the coherence is, according to Lumley (1970), a measure of the square of the correlation between the Fourier components of two records. It is given by the expression (Panofsky and Brier, 1965)

$$
\cosh(n) = \frac{\cosh^{2}(n) + Q^{2}(n)}{\phi_{1}(n)\phi_{2}(n)}
$$
(1)

where *n* is the frequency, $\phi_1(n)$ and $\phi_2(n)$ are the spectral estimates of the two time series, $\cos p(n)$ is the cospectrum and $Q(n)$ is the quadrature spectrum. The cospectrum is the in-phase, or real, part of the cross-spectrum and the quadrature spectrum is the out-of-phase, or imaginary, part of the cross-spectrum. Therefore if the coherence is computed between wind components at two places separated by a distance Δx in the direction of the mean wind, the coherence removes the phase change due to the translation of the patterns by the wind and just measures the correlation that would be experienced by air moving with the mean wind. The coherence has a maximum value

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of one and a minimum of zero. A coherence of one for all *n* means that all frequencies are perfectly correlated across some separation, Δx_i . If the coherence is small, near zero, it means that the frequencies of the turbulent fluctuations across a separation are poorly correlated.

Davenport (1961) has shown that the coherence of wind records from *vertically* separated instruments can be approximated by an exponential in the form

$$
\coh\left(n\right) = e^{-a\Delta f} \tag{2}
$$

where '*a*' is a non-dimensional parameter which will be called the decay parameter. *Jf* is defined by $n\Delta z/U$ where Δz is the vertical separation of the measurements and U the mean wind speed in the layer Δz . If Taylor's hypothesis is accepted, Δf also measures the ratio of vertical separation to horizontal wavelength.

This concept was generalized by Pielke and Panofsky (1970) to include horizontal separations so that

$$
\cosh_j^i(n) = \exp(-a_j^i \Delta f_j) \tag{3}
$$

where a_j^i is a matrix of decay parameters, Δf_j is a non-dimensional frequency defined by $\Delta f_i = n \Delta x_i/U$, $i = 1, 2, 3$ is an index that refers to the streamwise, cross-stream, and vertical wind components, respectively, and, $j = 1, 2, 3$ is an index that refers to longitudinal, lateral and vertical instrument separations with respect to the mean wind. We will limit the discussion in this paper to the cases of horizontal instrument separations and the streamwise and cross-stream components of the wind. The geometry used in defining the indices is illustrated in Figure 1.

Fig. 1. Illustration of the geometry used to define the fluctuating wind component and separation with respect to the mean wind (mean wind from left to right).

It can be shown that the coherence and the correlation function are related by the expression

$$
r^{i}(\Delta x_{j}, t) = \int_{-\infty}^{\infty} \left[\phi_{1}^{i}(\Delta f_{j}) \phi_{2}^{i}(\Delta f_{j}) \cosh^{i}(n) \right]^{1/2} \cos 2\pi n \left(t - P h_{j}^{i} \frac{\Delta x_{j}}{U} \right) \mathrm{d} n \tag{4}
$$

where $r^i(\Delta x_i, t)$ is the cross correlation between the *i*th components of the wind across a separation Δx_i , and $\phi^i(\Delta f_i)$ and Ph_i^i are the appropriate spectral estimates and phases, respectively. Thus if the spectral estimates and phases are known and if the behavior of the decay parameter matrix can be ascertained, (3) can be substituted into the above expression and values of the correlation functions calculated. The purpose of this note is to identify the variables affecting the decay parameter matrix. Further it will be shown that empirical curves can be drawn for various components of the decay parameter matrix based on existing data and simple dimensional analysis arguments.

2. Data

Existing data from several meteorological sources were examined for suitability for this study. Of these, data obtained at four locations were selected. Only those experiments in which the coherence, stability and mean wind directions were known, or could be calculated, were chosen. The four locations and their sources are: (1) O'Neill, Nebraska, *Geophys. Res. Paper #* 59, Vol. III (1959), Haugen (ed); (2) White Sands, New Mexico, Armendariz and Rider (1971), and Armendariz, unpublished; (3) Hanford, Washington, Elderkin *et al.* (1971); (4) Shikoku Island, Japan, Shiotani (1968). In addition, wind-tunnel data from a paper by Champagne *et al.* (1970) are included in the study as an example of laboratory work.

Details as to the site characteristics and original data processing may be found in the cited references. General site and data characteristics are listed in Tables I and II. For the O'Neill data, coherence was calculated from spectral and cross-spectral estimates according to (1).

Summary of site characteristics				
Location	Number of towers	Tower array configuration	Instrument height (m)	Separations (m)
O'Neill	5	In line	2	6, 12, 18, 24, 36, 42
White Sands	9	T array	1.5	25, 50, 100, 150
White Sands	9	T array	4	25, 50, 75, 100, 125, 150, 175, 225, 275, 300
White Sands	9	T array	16	75, 225, 300
Shikoku Island	5	In line	40	12, 23, 35, 45,
Hanford	2	In line	58	223
Wind Tunnel	2	In line	0.15	0.051, 0.127, 0.203

TABLE I

a Assumed to be neutral.

b Near zero Ri.

For the remaining meteorological sites, the coherence had already been calculated. The wind-tunnel data consisted of correlation curves which were Fourier-transformed to get spectra and cross-spectra and eventually coherence through (1). In all cases, the natural logarithm of coherence was plotted versus non-dimensional frequency; straight lines were then fitted by eye to obtain values of the decay parameters.

The gradient Richardson number is used to characterize the stability for all atmospheric wind data except the Shikoku Island runs. Data at this location were obtained when the wind speeds were very high, greater than 20 m s^{-1} , with cloudy skies. These conditions correspond to neutral stability, Richardson number near zero, as suggested for example by a scheme given by Pasquill (1962).

For White Sands, stability was given in terms of the ratio of height to Monin-Obukhov length, *z*/*L*. Both Pandolfo (1966) and Businger (1966) have found, independently, that the Richardson number and z/L are approximately equal for unstable stratification, at least near the ground. For stable air, an empirical expression similar to one given by McVehil (1964) was used to calculate the gradient Richardson number.

The wind tunnel was assumed to be neutrally stratified since no significant temperature gradient was induced during the experiment.

3. Theoretical Considerations

The behavior of a statistical quantity, such as the coherence, is, in general, very complicated for atmospheric data. Dimensional analysis arguments, coupled with physical reasoning, are employed below to give results that are useful as a guide to interpreting the experimental values of the streamwise and cross-stream decay parameters.

A. LONGITUDINAL SEPARATIONS

For simplicity, the discussion is first limited to longitudinal separations and the decay parameter of coherence for the streamwise wind component. The separation between instrument towers, Δx , is in the direction of the mean wind. The mean wind speed is *U.* In this configuration, an eddy that first appears at the upstream tower appears at the downstream tower at a time $\Delta x/U$ later. Thus the coherence corresponds to a lagged correlation following an 'eddy' with velocity U written as

$$
\coh\left(n\right) = \exp\left(-\frac{t}{T}\right) \tag{5}
$$

where $t = \frac{\Delta x}{U}$ is the time it takes the eddy to be advected downstream and T is a time scale associated with the decay of the eddy. To be correct, (5) and all of the expressions below should contain arbitrary constant multipliers. These constants will not be written explicitly until the final expression for the decay parameter is obtained.

The time, T, may be thought of as the 'eddy turnover time' or Lagrangian time scale. T may be estimated by the ratio of the characteristic eddy size A_1 in the streamwise direction to the characteristic velocity of the turbulence $(T = A_1/U')$. The characteristic velocity associated with the turbulence, *U',* is approximated here by the root-mean-square velocity in the direction of the mean wind, σ_{u} .

The frequency, n , associated with an eddy of size A_1 , being advected past a stationary observer at a mean velocity U, can be written as $n = U/A_1$ according to Taylor's hypothesis.

With these estimates and the definition of the non-dimensional frequency in *(5),* the expression for the coherence becomes:

$$
\cosh\left(n\right) = \exp\left(-\frac{n\Delta x}{U}\frac{\sigma_u}{U}\right) = \exp\left(-\Delta f \frac{\sigma_u}{U}\right). \tag{6}
$$

Thus the decay parameter of coherence for the streamwise wind component, longitudinal separations, may be written as

$$
a_1^1 = C \frac{\sigma_u}{U} \tag{7}
$$

where *C* is an unknown non-dimensional constant.

Baldwin and Johnson (1973) have suggested that a_1^1 is proportional to T_E/T_S , where T_E is the ordinary Eulerian integral time scale of turbulence, and T_S is the time scale of turbulence for a sensor moving with the mean flow. It is easy to see from qualitative considerations that this ratio is also proportional to the relative intensity of turbulence, σ_u/V , so that Baldwin and Johnson's conclusion agrees with the present argument.

As a consequence of (7), the decay parameter, a_1^1 , is a function of the intensity of the turbulence, σ_u/U . According to Monin-Obukhov similarity theory, the relative intensity of turbulence, σ_u/U , can be written in the form

$$
\frac{\sigma_u}{U} = \frac{\phi_1 \text{(Ri)}}{\ln \left(\frac{z}{z_0}\right) - \psi \text{(Ri)}}
$$

(see, e.g., Lumley and Panofsky, 1964). Here ϕ_1 and ψ are universal functions of the Richardson number Ri, z is the height and z_0 the roughness length. Thus, the longitudinal decay parameter is a strong function of roughness and would be expected to be much larger in the atmosphere over land than over water, or than in the wind tunnel. σ_u/U also depends on the Richardson number.

The range of stabilities in the present investigation is however relatively small; Richardson numbers range between -0.4 and $+0.08$. According to experimental work by Monin as cited by Lumley and Panofsky (1964), the variation of intensity with stability in this limited range of Richardson numbers is observed to be small. Thus the streamwise decay parameter of coherence for longitudinal separations is not expected to be a strong function of stability in this range.

The development of an expression for a_1^2 , the cross-stream decay parameter, with longitudinal separations, follows in a manner analogous to that given above. The main difference is that the appropriate turbulent velocity becomes σ_p and

$$
a_1^2 = D \frac{\sigma_v}{U} \tag{8}
$$

where *D* is an arbitrary constant.

In contrast to the intensity σ_u/U , there is a large variation of σ_u/U with stability even over the narrow range of Richardson numbers being considered here (Lumley and Panofsky, 1964). Thus the cross-stream decay parameter of coherence is expected to be a relatively strong function of stability for longitudinal separations. In addition, of course, a_1^2 depends strongly on terrain roughness.

B. LATERAL SEPARATIONS

For lateral separations, the mean wind makes a ninety-degree angle with the anemometer line. Since there are no lag times involved for this configuration, the coherence may be written, in complete analogy with Davenport's (1961) original expression, as

$$
\coh\left(n\right) = \exp\left(-\frac{\Delta y}{\Lambda_2}\right) = \exp\left(-\frac{\Delta y}{\Lambda_1}\frac{\Lambda_1}{\Lambda_2}\right) \tag{9}
$$

where Δy is the separation between the measurement points, A_2 is the appropriate transverse length scale, and A_1 is the longitudinal length scale. The arbitrary constants are again suppressed. Noting that A_i is equal to U/n and using the definition of nondimensional frequency, the expression for the coherence becomes

$$
\cosh\left(n\right) = \exp\left(-\frac{n\Delta y}{U}\frac{A_1}{A_2}\right) = \exp\left(-\Delta f \frac{A_1}{A_2}\right). \tag{10}
$$

Thus, for lateral separations, the decay parameter of coherence is given by

$$
a_2^1 = C' \frac{A_1}{A_2}, \qquad a_2^2 = C'' \frac{A_1}{A_2}
$$
 (11)

where *C'* and *C*["] are unknown non-dimensional constants. It should be noted that the discussion given above is equally valid for the streamwise and cross-stream wind fluctuations.

It has been shown from observations (Panofsky, 1962) that the ratio of the length scales is a function of stability. It follows that the decay parameters (a_2^1, a_2^2) of coherence for lateral separations are also functions of stability. Physically the fact that the ratio A_1/A_2 is a function of stability can be interpreted to mean that the shapes of the eddies are functions of stability. For relatively stable stratifications the eddies will tend to be long and narrow $(A_1/A_2 \ge 1)$ so that according to (1) the decay parameters for lateral separations will tend to be large. Conversely, for unstable stratifications, the eddies tend to be more nearly circular $(A_1/A_2 \approx 1)$ so that the decay parameters for lateral separations will be relatively smaller.

C. SUMMARY OF THE THEORETICAL DISCUSSION

The results of this section are summarized in Table III below. This table shows the meteorological variables that are considered to be relevant in explaining the variations of the decay parameter. The corresponding decay parameters are indicated in parentheses above the relevant parameters.

To summarize in words, for longitudinal separations, the cross-stream decay parameter is a strong function of stability, and the streamwise decay parameter is not, over the range of stabilities analyzed. Both depend strongly on roughness. For lateral separations, decay parameters for both wind components should be functions of the stability but not roughness. Similar relationships have been found in a recent study by Berman (1972). Finally, since the decay parameters of coherence for both wind components have a different functional dependence for longitudinal and lateral separations, we expect the decay parameters to be a function of the angle between the mean wind and the anemometer line.

4. Complete Analysis of the Decay Parameter for Coherence

Since the quantity In (z/z_0) does not change significantly from site to site in the data analysed here, the decay parameters for coherence can be thought of as being functions of at least two variables, the stability and angle between the mean wind and the anemometer line. Ideally, then, it would be desirable to study the variation of the decay parameters at fixed angles for varying stabilities and, conversely, the variation of the decay parameters at fixed stabilities for varying angles. It is, unfortunately, all but impossible to obtain a systematic series of experimental data in the atmosphere. Both the stability and the angle of the mean wind relative to the anemometer line tend to change from one experimental run to another. Thus, in order to maximize the usefulness of the available data, graphs were constructed with angle and Richardson number as coordinates, containing isopleths of the decay parameter. Two graphs are presented, one for the decay parameters of coherence for the streamwise wind component and the other for decay parameters of coherence for the cross-stream wind component. The angle between the mean wind and the anemometer line is plotted as the ordinate on a linear scale. The Richardson number, Ri, is plotted as the abscissa, x, on a modified logarithmic scale given by $x = -\ln(1-10 \text{ Ri})$.

Before discussing the isopleths, however, it is interesting to examine the simplest case, the streamwise decay parameter of coherence with a zero angle between the mean wind and the anemometer line. In this case, the decay parameter, a_1^1 , should depend on σ_u/U only, according to (7). The validity of this relationship can be checked by comparing values of the decay parameter, a_1^1 , obtained from two independent experiments in which the intensities differ significantly. If the Richardson numbers are approximately the same for the two experiments, the slight dependence of σ_u/U on stability can be ignored completely. The ratio of decay parameters should then be equal to the ratio of the intensities. For the wind-tunnel experiments of Champagne *et al.* (1970), which are representative of laboratory data, the intensity is approximately 0.02 and the calculated average value of the decay parameter is of order one. Baldwin and Johnson's paper, mentioned above, contains measurements from many additional laboratory experiments which show that the decay parameter obtained by Champagne *et al.* are typical for flow with small turbulence intensities. From Pielke's (1969) analysis of the O'Neill data, an average value for the decay parameter is eight. A representative value of the intensity for these runs is 0.2. Using these figures, the ratio of intensities and the ratio of decay parameters are both of order 10. Thus, the dependence of the decay parameter, a_1^1 , on intensity is a possible explanation of the slower decay of coherence in the wind tunnel than in the atmosphere.

The decay parameters for arbitrary angles and stabilities are discussed next.

A. THE ISOPLETHS OF THE DECAY PARAMETER OF COHERENCE, STREAMWISE WIND COMPONENT

The isopleths of the decay parameters of coherence for the streamwise wind component are shown in Figure 2. The solid portions of the isopleths are drawn to provide optimum agreement with the data. The dashed portions of the isopleths are inferred from the theoretical discussion of the previous section. According to this discussion, the decay parameter of coherence for the streamwise wind component is only a very weak function of stability for zero angle between the mean wind and the anemometer

Fig. 2. Isopleths of the decay parameter of coherence of streamwise component, as function of Richardson number and angle between wind and line separating anemometer (Circled values are data points).

line. Consequently, the isopleths, in Figure 2, are drawn so as to become nearly parallel to the x axis as zero is approached. Conversely, for lateral separations in which the mean wind makes a 90-degree angle with the anemometer line, the streamwise decay parameter is a strong function of stability. As instability increases, however, the angle becomes less important and the isopleths are drawn as shown in Figure 2.

Qualitatively, Figure 2 indicates that for Richardson numbers less than some value, $Ri < -0.3$ say, the decay parameters of the coherence for the streamwise wind component will be less than 10 regardless of the angle between the mean wind and the anemometer line. For stable stratifications, however, the angle of the mean wind to the anemometer line becomes critical in determining the value of the decay parameter for the streamwise wind component.

B. THE ISOPLETHS OF THE DECAY PARAMETER OF COHERENCE, CROSS-STREAM WIND COMPONENT

The isopleths of the decay parameter of coherence for the cross-stream wind component are shown in Figure 3. As before, the solid portions of the isopleths are drawn to fit the data. The dashed portions of the isopleths are implied from the discussions of the previous section. According to (11), the decay parameters of coherence for the cross-stream wind component are functions of the ratio A_1/A_2 , and hence of the stability, for all values of the angle of the mean wind to the anemometer line. Thus the

Fig. 3. Isopleths of the decay parameter of coherence of cross-stream component, as function of Richardson number and angle between wind and line separating anemometers (circled values are data points).

isopleths of the cross-stream decay parameter of coherence do not become parallel to the stability axis. This is the main difference between the behaviour of the crossstream and streamwise isopleths of the decay parameters. For very unstable conditions, however, the ratio A_1/A_2 is expected to approach unity and thus the angle of the mean wind to the anemometer line is expected to become less important. This behavior is illustrated in Figure 3.

On the stable side, $R_i > 0$, it is difficult to obtain values for the decay parameter. This difficulty is a reflection of the fact that the coherence of the cross-stream wind component falls off extremely rapidly for positive Richardson numbers for the data analyzed in this study. This extremely rapid decay of the coherence of the crossstream wind component is consistent with (11) since A_1/A_2 is thought to be large in stable conditions. The isopleth values, $a=30$ and 40, are drawn to indicate that the decay of coherence for the cross-stream wind component is large for positive Richardson numbers regardless of the angle between the mean wind and the anemometer line.

5. Final Remarks

This preliminary study indicates the behavior of the decay parameters of coherence as functions of stability and angle of the mean wind to the anemometer line. The theoretical discussions indicate that the decay parameters are also functions of the roughness, z_0 , for longitudinal separations but not for lateral separations. The dependence of the decay parameters on roughness will be investigated by members of the Meteorology Department at The Pennsylvania State University by comparing the present results with data taken over Lake Ontario in connection with the International Field Year program.

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