A THEORY FOR THE SOLAR TYPE-I RADIO CONTINUUM

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Abstract. The type-I radio continuum may arise from the combination of two electrostatic waves, both directed nearly normal to the magnetic field. One wave, near the upper-hybrid frequency, is generated by gyroresonance with superthermal electrons and comes into equilibrium with these electrons. The other wave, at the lower-hybrid frequency, is generated by the loss-cone instability of trapped superthermal protons in those wave directions for which the lower-hybrid frequency is an exact multiple of the proton gyrofrequency. The brightness temperature of the continuum indicates both the energy of the superthermal electrons and the existance of at least a small number of superthermal protons.

1. Introduction

The solar type-I radio continuum occurs roughly in the range 100 to 300 MHz. Its high polarization in the *o*-mode suggests emission at the plasma frequency. The brightness temperature, of order 10^8 ranging up to 10^{10} K (Kerdraon and Mercier, 1983) suggests a relation to superthermal electrons trapped in magnetic fields of the upper corona. The observed location somewhat away from the associated optically active region implies that these fields are of very large scale (Elgarøy, 1977; Mercier *et al.*, 1984).

Radio emission at the plasma frequency is usually expected to arise from the combination of Langmuir waves and some low-frequency waves. No obvious low-frequency wave has been identified. This led Melrose (1983) to consider the merging of Langmuir waves with two low-frequency waves.

The Langmuir waves have usually been assumed roughly parallel to the ambient magnetic field. When Langmuir waves travel nearly normal to the magnetic field, they are essentially upper-hybrid waves, or 'generalized Langmuir waves' in the usage of Melrose (1980). Such waves can combine with lower-hybrid waves to yield the observed radio emission. I discuss the appropriate wave parameters in Section 2. I show in Section 3 that the appropriate upper-hybrid waves can be emitted spontaneously by trapped superthermal electrons and come into equilibrium with them. Thus the brightness-temperature of the waves, T_u, will equal the temperature, T_h, of the hot trapped electrons, which are taken to be Maxwellian for computational purposes. In Section 4 I evaluate the minimum brightness temperature of the lower-hybrid waves, T_l, needed for the radio emission to reach optical depth at least unity. The result is $T_l \gtrsim 10^{10}$ K. If this very modest condition is satisfied, then the brightness-temperature of the radiation, T_r, is simply $T_r = T_u = T_h$. The observed range in T_r (Kerdraon and Mercier, 1983) corresponds to electrons of 10 keV to a rare maximum of about 1 MeV.

The uniformity of the continuum requires $T_l > 10^{10}$ K over a large volume in the upper corona. This implies generation of the lower-hybrid waves by an instability that occurs

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in the same volume where the fast electrons are trapped. I propose in Section 5 that the instability is due to trapped superthermal protons with a loss-cone. The instability occurs when the lower-hybrid frequency is an exact multiple of the proton gyrofrequency. Mathematically, this is closely analogous to the generation of upper-hybrid waves by trapped electrons with a loss-cone when the plasma frequency is a multiple of the electron gyrofrequency (Berney and Benz, 1978). Physically, it is quite different because the lower-hybrid resonance is possible everywhere, while that of the upper-hybrid waves can occur only on thin sheets. In Section 6 I discuss the relation of this theory to the polarization of the continuum, the scattering of type-I and type-III bursts, and the generation of type-I bursts. A summary appears in Section 7.

There are two qualitatively important features of this theory: (i) the highest observed continuum brightness temperatures $T_r \simeq 10^{10}$ K require trapped electrons that are mildly relativistic; and (ii) the radio continuum is to be considered a signature of trapped superthermal protons as well as electrons.

2. Wave Parameters

An essential parameter is the ratic of electron gyrofrequency to plasma frequency, Ω/ω_e. This ratio is proportional to the Alfvén speed, which has been estimated in the upper corona in terms of observed velocities of type-II radio bursts and coronal transients (Dulk and McLean, 1978). I adopt in the following $\Omega/\omega_e = 0.1$, corresponding to an Alfvén speed of 800 km s^{-1}. At the 100 MHz level ($n_e = 10^8$ cm^{-3}), the magnetic field is 3 G.

The dispersion relation for the lower-hybrid waves is

$$\omega_l^2 = \Omega^2(m/M + \cos^2\theta)\,, \tag{1}$$

where m and M are the electron and proton masses, respectively, and $\theta = 0$ in the magnetic direction. We are interested in $\cos^2\theta < m/M$. The generation of these waves is possible because even this very small range in $\cos\theta$ includes over a dozen values of $\cos\theta$ at which ω_l is an exact multiple of the proton gyrofrequency, namely $\omega_l/\Omega_i = 43$ to about 60.

A convenient measure for the several wavenumbers that we shall encounter is the electron gyroradius $\rho_e = v_e/\Omega$. With a temperature $T_e = 2 \times 10^6$ K and $B = 3$ G at the 100 MHz level, $\rho_e = 10$ cm.

Landau damping by thermal electrons is strong unless the phase velocity, $\omega_l/k_l \cos\theta_l$, of the lower-hybrid waves along the magnetic field exceeds $4v_e$, or

$$k_l\rho_e \cos\theta_l < \tfrac{1}{4}(m/M)^{1/2}\,. \tag{2}$$

We shall be interested in a range of approximately $0.2 < k_l\rho_e < 0.5$.

The upper hybrid or generalized Langmuir waves in a cold plasma have the dispersion relation (Melrose, 1980)

$$\frac{k^2c^2}{\omega_u^2} = \frac{\Omega^2\sin^2\theta}{\omega_e^2 + \Omega^2\sin^2\theta - \omega_u^2}\,. \tag{3}$$

A thermal correction is important for our purposes. Then the dispersion relation becomes

$$\omega_u^2 = \omega_e^2 + \Omega^2 \sin^2\theta + 3k^2 v_e^2 - \Omega^2 \sin^2\theta\, \omega_e^2/(k^2c^2)\,, \tag{4}$$

which merges smoothly into ordinary Langmuir waves when the last term is negligible (Melrose, 1980). Since Equation (7) will yield a small value for $\cos\theta$, I set $\sin\theta = 1$ in Equation (4). The ratio of the last two terms in Equation (4) is about $10^2(k\rho_e)^4$. Thus the thermal term dominates for $k\rho_e > 0.3$, a limit that is within the range of interest. (When $k\rho_e < 0.3$, the wave is no longer really electrostatic. The changes due to electromagnetic corrections will be ignored since they would alter only amply satisfied requirements.)

The electromagnetic waves created by the merging of the upper- and lower-hybrid waves must satisfy the conditions on conservation of energy and momentum,

$$\omega_r = \omega_u + \omega_l\,, \qquad \mathbf{k}_r = \mathbf{k}_u + \mathbf{k}_l\,. \tag{5}$$

Close to the plasma frequency, where only o-mode is possible, this radiation satisfies

$$k_r = (\omega_e\Omega)^{1/2}/c\,, \qquad k_r\rho_e \simeq 0.06\,. \tag{6}$$

I assume at least roughly isotropic emission, since the continuum is observed with rather little diminution well past the solar limb (Elgarøy, 1977). With $\cos\theta_r$ of order unity, Equation (5) for the wave vectors is easily satisfied in the direction normal to the magnetic field, $k_u \simeq k_l$ with roughly oppositely directed $\mathbf{k}_u$ and $\mathbf{k}_l$. The direction along the field demands a finite value for $\cos\theta_u$,

$$\cos\theta_u = k_r \cos\theta_r/k_u = 0.06 \cos\theta_r/(k_u\rho_e)\,. \tag{7}$$

We are interested in the range $0.2 < k_u\rho_e < 0.5$, $\cos\theta_u \simeq 0.2$. The finite value of $\cos\theta_u$ will be important in Section 3 for the generation of the upper-hybrid waves.

3. Generation of Upper-Hybrid Waves

We are interested in upper-hybrid waves generated by gyroresonance with superthermal electrons, satisfying

$$\omega = s\Omega + k_\parallel v_\parallel\,. \tag{8}$$

Let these electrons have a density n, $n \ll n_e$, and a normalized Maxwellian distribution function $f(\mathbf{v})$ with rms velocity v_h, $T_h = mv_h^2$. These electrons emit upper-hybrid waves spontaneously at a rate $P(\mathbf{k})$ (energy per unit volume per unit wavenumber per s) and re-absorb them at the rate $\gamma(\mathbf{k})$, where $T_h = P(\mathbf{k})/\gamma(\mathbf{k})$. The upper-hybrid waves come into equilibrium with the electrons, $T_u = T_h$, unless they interact more rapidly with the thermal electrons. Therefore, one must evaluate the absorption coefficient $\gamma(\mathbf{k})$ and compare it with the collisional dissipation rate. The absorption coefficient for electrostatic waves is

$$\gamma(k) = -16\pi^3 e^2 R\, \frac{\omega^2}{k_\parallel k^2}\, n \sum_s m^2 \int \mathrm{d}v_\perp\, \frac{\partial f}{\partial v_\perp}\, J_s^2\left(\frac{k_\perp v_\perp}{\Omega}\right), \tag{9}$$

where $v_\parallel$ in the integral is selected by the resonance condition (8) and R is the ratio of electric to total energy of the wave. For the emission $P(\mathbf{k})$, $\partial f/\partial v_\perp$ in Equation (9) is replaced by $mv_\perp f$. A small loss-cone in the trapped electrons makes no difference because the resonance condition selects values of $v_\parallel$ away from the loss-cone (Wentzel, 1985). Equation (9) can be integrated in terms of I_s, the modified Bessel function,

$$\gamma = (2\pi)^{1/2} \frac{n}{n_e} R \frac{\omega^2 \omega_e^2}{k_\parallel k^2 v_h^3} \times$$
$$\times \sum_s I_s(k_\perp^2 \rho_h^2) \exp[-(k_\perp^2 \rho_h^2 + v_\parallel^2/2v_h^2)]\,. \tag{10}$$

We shall be interested in rather large s, namely $s \simeq \omega_e/\Omega \simeq 10$. The quantity I_s rises steeply until $k_\perp \rho_h \simeq s$. When only one value of s contributes and $k_\perp \rho_h > s$,

$$\gamma = \frac{n}{n_e} R \frac{\omega^2 \omega_e^2}{k_\parallel k^2 v_h^3} \frac{1}{k_\perp \rho_h} \exp[-v_\parallel^2/2v_h^2]\,. \tag{11}$$

The resonance condition (8) together with $k_\parallel = k_r \cos\theta_r$ and Equation (6) for k_r yields

$$\frac{v_\parallel}{v_h} = \frac{\omega_e - s\Omega}{\Omega \cos\theta_r} \frac{c}{v_h} \left(\frac{\Omega}{\omega_e}\right)^{1/2}. \tag{12}$$

For 20 keV electrons, $c/v_h = 3$. It is clear from Equations (11) and (12) that the favoured s is the integer closest to ω_e/Ω. No special values are required for ω_e/Ω. Some suitable s always exists such that Equation (12) yields $v_\parallel < v_h$.

Equation (11) can be reduced to a more convenient form upon taking $R = \frac{1}{2}$, setting $k_\parallel = k_r$, $k_\perp = k$, using Equation (6), and setting $v_\parallel < 0.5 v_h$ in Equation (12),

$$\gamma = \frac{1}{2} \frac{n}{n_e} \omega_e \left(\frac{\Omega}{\omega_e}\right)^{1/2} \frac{c}{v_h} \left(\frac{\omega_e}{k v_h}\right)^3. \tag{13}$$

The last factor must be less than unity because $k\rho_h \gtrsim \omega_e/\Omega$. But it cannot be very small because this same condition demands

$$v_h/v_e \gtrsim (\omega_e/\Omega)/(k\rho_e)\,. \tag{14}$$

For example, $v_h/v_e = 20$ corresponds to 60 keV electrons.

I adopt $n/n_e = 10^{-5}$. Such a value was also adopted by Benz (1980). Then Equation (13) yields $\gamma \sim 10^4\ \mathrm{s}^{-1}$. Collisional damping rates are at least two orders of magnitude less rapid (see Equation (21)). The waves have slow group velocities. It follows that they saturate and come into equilibrium with the hot electrons. The brightness temperatures equalize, $T_u = T_h$. Since γ is far above the rate of collisional dissipation, the condition $k\rho_h \geq s$ can be relaxed. A useful condition for the onset of the continuum is that the hot electrons satisfy

$$v_h/v_e \gtrsim 3/(k\rho_e)\,. \tag{15}$$

With $k\rho_e \simeq 0.3$ as adopted below, the continuum requires electrons of at least 15 keV.

4. Brightness Temperatures

I wish to estimate the minimum brightness-temperature in the lower-hybrid waves that is needed to convert upper-hybrid waves into radiation with $T_r = T_u$.

The radiation is due to the nonlinear current created by the two interacting waves. The lower-hybrid wave provides the density fluctuations, the upper-hybrid wave the electron velocity. Following standard procedures (Melrose, 1980), one finds that the power radiated per s per cm^3 and per unit radiation wavenumber space is

$$P = 4\pi^2 e^2 \int |\delta n_l|^2 \, |v_u|^2 \, |\hat{\mathbf{e}} \cdot \hat{\mathbf{v}}_u|^2 \times$$
$$\times \, \delta(\omega_r - \omega_u - \omega_l) \, \delta(\mathbf{k}_r - \mathbf{k}_u - \mathbf{k}_l) \, \mathrm{d}\mathbf{k}_u \, \mathrm{d}\mathbf{k}_l/(2\pi)^3 \,. \tag{16}$$

The wave amplitudes are expressed most conveniently in terms of brightness temperatures

$$T_u = n_e m \, |v_u|^2, \qquad T_l = \frac{m\Omega^2}{k^2 n_e} \, |\delta n_l|^2 \left(1 + \frac{M}{m} \cos^2\theta_l\right)^2 , \tag{17}$$

where the Boltzmann constant is suppressed here and through Equation (19). I shall take these two temperatures as uniform within the angular ranges specified above and ignore the term involving $\cos\theta_l$. Integration of the directional factor in Equation (16) over the azimuth angles yields $\pi \cos^2\theta_r$. The delta function of the wave vectors is conveniently used to eliminate $k_{u_\parallel}$ and $k_{l_\perp}$. The power P is proportional to the temporal change of the radiation brightness temperature, T_r, along a ray:

$$(2\pi)^3 P = \frac{\mathrm{d}T_r}{\mathrm{d}t} = \pi^2 \cos^2\theta_r \, \frac{\omega_e^2}{\Omega^2} \, T_u \, \frac{T_l}{mc^2} \int \frac{k_u^3}{n_e} \, \frac{c^2 \, \mathrm{d}k_{\parallel l}}{\partial\omega_u/\partial k_u} \,. \tag{18}$$

The growth of T_r ceases when T_r approaches T_u. Thus the physically important quantity is $\gamma_r = T_u^{-1} \, \mathrm{d}T_r/\mathrm{d}t$. With the thermal term in the dispersion relation (4) determining the group velocity in Equation (18), we have

$$\gamma_r = \frac{\pi^2}{3} \cos^2\theta_r \, \omega_e \, \frac{\omega_e^2}{\Omega^2} \, \frac{c^2}{v_e^2} \, \frac{T_l}{mc^2} \int \frac{k_u^2}{n_e} \, \frac{\mathrm{d}k_{\parallel l}}{k_u} \,. \tag{19}$$

Here, k_u is a function of $k_{\parallel l}$ through Equation (5): the frequencies of the two electrostatic waves, which are functions of k_u and $k_{\parallel l}$, must add to the fixed radio frequency implicit on the left side of Equations (18) and (19). Since we wish to establish mainly that γ_r is adequate, I replace $\mathrm{d}k_{\parallel l}/k_u$ by $\cos\theta_l$, use its limit given by relation (2), and introduce the Debye length $\lambda_\mathrm{D} = v_e/\omega_e$, with the final result

$$\gamma_r = \frac{\pi^2}{12} \left(\frac{m}{M}\right)^{1/2} \cos^2\theta_r \, \Omega \, \frac{T_l}{T_e} \, \frac{(k_u \rho_e)^2}{n_e \lambda_\mathrm{D}^3} \simeq 10^{-3} \, \frac{T_l}{T_e} \, \mathrm{s}^{-1} \,, \tag{20}$$

where the numerical estimate is appropriate to the 100 MHz level, with $n_e = 10^8\ \mathrm{cm}^{-3}$, $T_e = 2 \times 10^6$ K, $k_u \rho_e = 0.3$, and $\cos\theta_r = 1$.

There are two conditions to be met. First, γ_r should exceed the rate of collisional damping of the radiation, which is half the electron-ion collision rate,

$$\gamma_d = 30\, T_e^{-3/2}\, n_e \simeq 1\ \mathrm{s}^{-1}\,. \tag{21}$$

It follows that $\gamma_r > \gamma_d$ requires

$$T_l/T_e \gtrsim 10^3\,. \tag{22}$$

A more substantial requirement is that the radiation grows before the group velocity $c(\Omega/\omega_e)^{1/2}$ carries the radiation to a height where it can no longer interact with the upper-hybrid waves. If the thermal term in the dispersion relation (4) determines the bandwidth, the radiation must grow significantly over a fraction $k_u^2 v_e^2/\omega_e^2 = 10^{-2}(k_u\rho_e)^2$ of the density scale height measured along the ray. If α is the angle between the ray and the local density gradient, of scale height L, then saturation of the radiation requires

$$\frac{\gamma_r}{c}\left(\frac{\Omega}{\omega_e}\right)^{3/2} (k_u\rho_e)^2\, \frac{L}{\cos\alpha} > 1\,. \tag{23}$$

I adopt $L = 10^5$ km for the extended coronal regions where type-I continuum is observed. Then with $k_u\rho_e = 0.3$ and Equation (20), we require

$$\gamma_r > 10^3 \cos\alpha\ \mathrm{s}^{-1}, \qquad T_l/T_e > 10^6 \cos\alpha\,. \tag{24}$$

At first sight, the requirement $T_l > 10^{12}$ K seems quite large. But the energy density involved,

$$W = \int KT_l\, \mathrm{d}^3\mathbf{k}/8\pi^3\,, \tag{25}$$

is really quite small. Let the solid angle in $\mathrm{d}^3\mathbf{k}$ be $2\pi(m/M)^{1/2}$ and assume T_l is uniform in $\mathbf{k}$ up to a value k_l. Then adequate opacity requires

$$W = 2 \times 10^{-4}\, k_l^3\, KT_l > 10^{-2} \cos\alpha\, KT_e\ \mathrm{erg\ cm}^{-3}\,, \tag{26}$$

with $k_l = 0.3/\rho_e = 0.03$ and T_l given by condition (24). In comparison, the thermal energy density $n_e KT_e$ is some ten orders of magnitude larger. In the following, I shall argue that the lower-hybrid waves are generated by a loss-cone instability among superthermal protons, in analogy to the generation of upper-hybrid waves treated by Berney and Benz (1978). Benz (1980) estimated the saturation of the latter at an energy density $10^{-2} n/n_e \simeq 10^{-7}$ of the thermal energy density, and commented that even that value was an unusually low value for saturation. Spicer *et al.* (1981) estimate a ratio of 10^{-6} for the rather weak shocks they propose as the source for type-I bursts. Thus it appears that an instability which generates lower-hybrid waves is likely to grow to at least the minimum (24).

The condition (24) implies that $T_r = T_u$, no matter how much energy resides in the lower-hybrid waves. Therefore, the uniform nature of the observed radio continuum implies uniformly distributed superthermal electrons, but the lower-hybrid waves might be quite inhomogeneous.

When condition (24) is satisfied, it is also possible to obtain stimulated decay of the upper-hybrid wave, satisfying $\omega_r = \omega_u - \omega_l$. This is an instability that can drive $T_r \gg T_u$. However, the radiation must later pass through a region where $\omega_r = \omega_u + \omega_l$. There it is probably reabsorbed until $T_r = T_u$. Conceivably, conditions leading to type-I bursts may allow escape of this radiation.

5. Generation of Lower-Hybrid Waves

Superthermal electrons with a loss-cone become unstable to upper-hybrid waves if the plasma frequency is nearly an integer multiple of the gyrofrequency (Berney and Benz, 1978; and references therein). The behaviour of the observed continuum suggests that it is caused by trapped electrons. I postulate that superthermal protons exist together with superthermal electrons and that their loss-cone yields the lower hybrid waves.

The electrons acquire a loss-cone when their mean free path becomes comparable to the length of the magnetic trap. The mean-free path depends on T_e and v/v_e. Protons have a mean free path comparable to that of the electrons when $T_i = T_e$ and when v/v_i for the protons is the same as v/v_e for the electrons. Therefore, one expects the protons to acquire a loss-cone similar to that of the electrons if the distributions in v/v_i and v/v_e are similar.

The dispersion relation appropriate to the proton loss-cone instability is a sum of the dispersion relation for the lower-hybrid waves, derived in the limit of cold electrons and protons and assuming $k_\parallel = 0$ (hydrodynamic approximation), plus one term for the protons involving both a loss-cone and a harmonic of the proton gyrofrequency which makes $\omega - s\Omega_i$ small or zero,

$$0 = 1 + \frac{\omega_e^2}{\Omega_e^2 - \omega^2} + \frac{\omega_i^2}{\Omega_i^2 - \omega^2} - \cos^2\theta\, \frac{\omega_e^2}{\omega^2} + \\ + \frac{\omega_i^2}{k^2} \int \mathrm{d}^3\mathbf{v}\, J_s^2\left(\frac{kv_\perp}{\Omega_i}\right) \frac{1}{v_\perp} \frac{\partial f}{\partial v_\perp} \frac{\omega}{\omega - s\Omega_i}. \tag{27}$$

The closely equivalent problem for upper-hybrid waves and a hot super-thermal Maxwellian electron distribution has been worked out in much detail (Berney and Benz, 1978). For the moment, let us isolate the poorly known parameters for the protons in the integral

$$I = \int \mathrm{d}^3\mathbf{v}\, J_s^2\left(\frac{kv_\perp}{\Omega_i}\right) \frac{\omega^2}{k^2} \frac{1}{v_\perp} \frac{\partial f}{\partial v_\perp}. \tag{28}$$

Since the Bessel function contributes mainly when $kv_\perp = s\Omega_i \simeq \omega$, the magnitude of I is roughly $J_s^2(s) \simeq 0.02$ times n/n_i, the ratio of superthermal to thermal protons. Of main interest will be the sign of I, which is positive if the loss-cone dominates, negative if

$\partial f/\partial v^2$ is sufficiently negative. In particular, in the case of a hot Maxwellian, the loss-cone dominates if J_s emphasizes $v < v_h$; thus instability is restricted to $kv_h/\Omega_i > s$. This is evident in the equivalent results of Berney and Benz (1978; especially their Figure 7).

To determine conditions for instability, define

$$\omega_l = (s + \varepsilon_l)\Omega_i, \qquad \omega = \omega_l + \varepsilon\Omega_i \tag{29}$$

and assume ε_l and ε are small. Then the dispersion relation (27) reduces to Equation (1) in lowest order and also to

$$0 = 2\varepsilon(1 + \cos^2\theta M/m) + Is^2/(\varepsilon + \varepsilon_l)\,. \tag{30}$$

The term involving $\cos^2\theta$ is very important in Equation (1) in allowing ω_l to be an exact integer multiple of Ω_i, but it is rather unimportant in Equation (30) and will be omitted there. Equation (30) is a quadratic in ε yielding instability if

$$I > \varepsilon_l^2/2s^2\,, \qquad s^2 \simeq M/m\,, \tag{31}$$

with a growth rate of the order $\omega_l I^{1/2}$. The growth rate exceeds collisional damping if $I^{1/2} > 10^{-6}$.

Equation (31) guarantees instability if I is positive, since $\varepsilon_l = 0$ at several very small values of $\cos\theta$. If $I > 6 \times 10^{-5}$, then instability occurs even for $\varepsilon_l = 0.5$, that is, for all values of $\cos\theta$, but there is no expectation that I is this large.

This discussion must now be related to that of the previous sections by asking: for what k_l is $I > 0$, and do these values of k_l match those of k_u derived earlier, in particular condition (15)? Let us start with a hot Maxwellian distribution of protons. Define κ through $kv_h/\Omega_i = s\kappa$. In terms of the gyroradius of thermal electrons, used earlier,

$$k\rho_e = \kappa v_i/v_h\,. \tag{32}$$

We have already seen that $\kappa > 1$ for instability. For the parameter $s = 30$ used by Berney and Benz (1978), instability occurs for $2 < \kappa < 4.5$ (their Figure 7). A similar range is expected for $s \simeq (M/m)^{1/2}$. A secondary condition on instability is that the thermal protons provide negligible damping. Table IV in Berney and Benz (1978), extrapolated to $s = 43$, suggests $v_h/v_i \geq 12$ guarantees the dominance of the superthermal protons, almost whatever their density. It appears, therefore, that the combination $\kappa = 4$, $v_h/v_i = 12$, $k\rho_e = 0.3$ is indeed unstable. Conditions (15) and (32) look remarkably similar, but they are physically unrelated.

The superthermal electrons and protons need not be Maxwellians. If the protons constitute a power-law tail, instability extends to higher velocities. In general, instability occurs for smaller k than for the Maxwellian protons. If the protons have a distribution with $\partial f/\partial v^2 > 0$ in some range of v, that is, a 'ring' distribution, then the loss-cone instability is certain to occur for those velocities. In the extreme of a delta-function ring-distribution at $v = v_h$, $k\rho_e = v_i/v_h$. Although the instability is then not restricted significantly by the thermal distribution, one would expect $v_h \geq 4v_i$, thus $k\rho_e \leq 0.25$. Therefore, Maxwellian hot protons are preferable.

6. Relation to Other Observations

Benz and Zolliker (1985) analyzed observations of a highly polarized continuum with $T_r \simeq 10^8$ K. They determined that the ratio of harmonic (unpolarized) to fundamental (fully polarized) emission is 0.019 ± 0.012, consistent with zero. What harmonic emission does one expect if $T_r = T_u$ for upper-hybrid waves with $\cos\theta_u \simeq 0.2$? If the upper-hybrid waves are just a part of an essentially isotropic distribution of waves, all satisfying $T_u = T_h = 10^8$ K, then the predicted harmonic emission is far less than the observed upper limit (Wentzel, 1985; Equation (13)). Possibly, the hot electrons have a gap-distribution. In that case, the waves with $\cos\theta_u < 0.7$ still have $T_r \simeq 10^8$ K, but waves with $\cos\theta_u > 0.7$ may achieve $T_u \simeq 6 \times 10^9$ K and yield a harmonic radiation with $T_b \simeq 10^6$ K (Wentzel, 1985; Equation (14)). This is comparable to the observational upper limit. (Benz and Zolliker, 1985, also derive a harmonic flux comparable to the upper limit, using somewhat different assumptions about the plasma waves.)

One attractive feature of lower-hybrid waves is their ability to scatter radio waves by large angles when the wavelengths of the lower-hybrid and radio waves are comparable. Momentum conservation for scattering of the radio wave by an angle α requires

$$k_l = 2k_r \sin\alpha/2\,, \tag{33}$$

where $k_r = \omega_r/c$ since the scattering occurs far above the plasma level. If k_l is known, the scattering height of a given radio wave is a function of the scattering angle. The height is most usefully expressed in terms of ω_e. In terms of the known parameters v_e/c and Ω/ω_e,

$$\frac{\omega_r}{\omega_e} = \frac{\Omega}{\omega_e}\,\frac{c}{v_e}\,k_l\rho_e\,\frac{2}{\sin\alpha/2} \simeq 7k_l\rho_e\,, \tag{34}$$

where $\alpha = 90°$ is used for the numerical estimate. The estimate $k_l\rho_e = 0.3$ of Section 5 places the scattering somewhat outside the level of the plasma harmonic. This result is attractive for explaining the apparent heights of fundamental type-III bursts (Wentzel, 1982) and for the decrease in the polarization of type-I bursts toward the solar limb (Wentzel *et al.*, 1986). Observations indicate scattering at least out to the 50 MHz level. However, closed magnetic fields, trapped protons and their associated lower-hybrid waves extend only to about the 100 MHz level, where the type I continuum tends to be replaced by storm-type III bursts. Other sources of lower-hybrid waves may exist at greater heights.

The upper-hybrid waves invoked in Section 2.5 are in equilibrium with superthermal electrons. An additional source of upper-hybrid waves occurs where ω_e is close to an integer multiple of Ω. There, the loss-cone of the superthermal electrons yields an instability of upper-hybrid waves directed normal to the magnetic field with a bandwidth of the order of $\omega_e(n/n_e)^{1/2}$ (Berney and Benz, 1978). If the Alfvén speed and ω_e/Ω are strictly uniform in the upper corona, then the instability occurs either everywhere or, most probably, nowhere. More realistically, if the Alfvén speed varies by 10 to 50% across the region, then the instability occurs in 1 to 5 thin sheets, whose shape may be quite unrelated to the shapes of the magnetic fields or equal-density contours. Why do we not see radiation, generated by these upper-hybrid and the proton-generated lower hybrid waves, emitted steadily from these sheets? If the waves saturate at an energy $10^{-7} n_e K T_e$ (Benz, 1980), an argument equivalent to Equations (25) and (26) yields $T_u \sim 10^{15}$ K. Even a very small opacity would yield observable radio emission. Possibly, the wavevectors of the upper- and lower-hybrid waves do not match. More probably, the radiation actually exists, but only within the sheets. Its bandwidth is so narrow that is is reabsorbed once it leaves these sheets, just like the stimulated emission mentioned at the end of Section 4. Conceivably, type-I bursts occur when this radiation is permitted to escape temporarily, for instance when shocks alter the local density distribution.

7. Summary

I showed in Section 2 that upper- and lower-hybrid waves satisfying $k\rho_e \simeq 0.3$ can combine to yield electromagnetic radiation if the upper-hybrid waves are nearly but not quite normal to the magnetic field, $\cos\theta_u \simeq 0.2$. In Section 3 I demonstrated that such upper-hybrid waves are easily generated by gyroresonance, $s \simeq \omega_e/\Omega$, with superthermal electrons, and that the waves comes into equilibrium with the electrons so that $T_u = T_h$. The minimum energy of electrons needed to generate $k_u\rho_e \simeq 0.3$ is about 15 keV ($T_h \simeq 2 \times 10^8$ K). Radio emission from the two electrostatic waves requires a sufficient intensity of lower-hybrid waves. I showed in Section 4 that $T_l > 10^{12}$ K is adequate for the growth of the radio waves to saturate, i.e., $T_r = T_u = T_h$. This minimal value of T_l is quite small compared to estimates of energies at which instabilities generating lower-hybrid waves would saturate. A specific instability is discussed in Section 5, namely the loss-cone instability due to superthermal protons. The instability is efficient because the lower-hybrid frequency can be an exact multiple of the proton gyrofrequency. Only very few superthermal protons are needed. The minimum energy of protons needed to generate $k_l\rho_e \simeq 0.3$ is about 20 keV ($T_h \simeq 3 \times 10^8$ K).

A possible new interpretation of type-I bursts emerges: stimulated emission can occur very near the plasma level, causing locally very high T_r. Normally the radiation is reabsorbed on traveling outwards. A type-I burst may represent a condition when this radiation can escape. A similar situation may occur within the sheet-like regions of the corona where $\omega_e/\Omega \simeq$ integer. There the electron loss-cone instability causes a very high T_u and thus probably high T_r. Again the radiation is local and normally reabsorbed, but might sometimes be allowed to escape.

The type-I radio continuum may be taken as a signature of super-thermal electrons and protons. Normally $T_h \gtrsim 2 \times 10^8$ K, but the most intense observed continuum requires mildly relativistic electrons.

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