ON THE AVERAGE RATE OF GROWTH IN SUNSPOT NUMBER AND THE SIZE OF THE SUNSPOT CYCLE

ROBERT M. WILSON
Space Science Laboratory, NASA Marshall Space Flight Center, Huntsville, AL 35812, U.S.A.

(Received 5 June, 1989; in revised form 31 July, 1989)

Abstract. The average rate of growth during the ascending portion of the sunspot cycle, defined here as the difference in smoothed sunspot number values between elapsed time (in months) t and sunspot minimum divided by t, is shown to correlate ($r \geq 0.78$) with the size of the sunspot cycle, especially for $t \geq 18$ months. Also, the maximum value of the average rate of growth is shown to highly correlate ($r = 0.98$) with the size of the cycle. Based on the first 18 months of the cycle, cycle 22 is projected to have an $R(M) = 186.0 \pm 27.2$ (at the $\pm 1\sigma$ level), and based on the first 24 months of the cycle, it is projected to have an $R(M) = 201.0 \pm 20.1$ (at the $\pm 1\sigma$ level). Presently, the average rate of growth is continuing to rise, having a value of about 4.5 at 24 months into the cycle, a value second only to that of cycle 19 (4.8 at $t = 24$ and a maximum value of 5.26 at $t = 27$). Using 4.5 as the maximum value of the average rate of growth for cycle 22, a lower limit can be estimated for $R(M)$; namely $R(M)$ for cycle 22 is estimated to be $\gtrsim 164.0$ (at the 97.5% level of confidence). Thus, these findings are consistent with the previous single variate predictions that project $R(M)$ for cycle 22 to be one of the greatest on record, probably larger than cycle 21 (164.5) and near that of cycle 19 (201.3).

1. Introduction

Long ago, Waldmeier (1935, 1957) found that the length of rise (the ascent duration) and the respective size (maximum amplitude, expressed in units of smoothed sunspot number; Howard, 1977) of sunspot cycles seem to be inversely correlated: fast risers with large maximum amplitudes and slow risers with small maximum amplitudes (cf. de Jager, 1959). While the association (called the 'Waldmeier effect' by Bracewell, 1988) is statistically important (at >95% level of confidence), one finds that, for predictive purposes, the effect is better described as a 'tendency' to associate (based on a 2 × 2 contingency table analysis) rather than as a rigorous linear correlation (based on linear regression analysis: $r = -0.64$ and standard error of estimate equal to about 5 months, based on the modern era sunspot cycles, cycles 10-present; Wilson, 1988a).

Becasue fast (slow) risers display a tendency to associate with larger (smaller) maximum amplitudes, the rate of rise in sunspot number after sunspot minimum should provide a means whereby the size of the cycle might be reliably predicted. To test this idea, Wilson (1988b) examined the associational aspects of 6-mo sums of monthly mean sunspot number at various intervals during the ascending portion of the sunspot cycle against the later-occurring maximum amplitude. The result of that investigation was that the rate of rise during the earliest portions of the sunspot cycle (prior to about 2 years into the cycle) does not provide a reliable predictor for the size of the cycle.

In this paper, we examine the 'average' rate of growth in sunspot number over selected intervals of time (6, 12, 18, 24, and 30 months from sunspot minimum) and the 'maximum' average value as they both relate to the size of the cycle, in order to predict

Solar Physics **125**: 133–141, 1990.

the size of cycle 22. Also, we compare these predictions with those based on the aforementioned 6-month sum schemes and on the variety of other single variate and bivariate fits (Wilson, 1990), in order to determine if a consensus is apparent.

2. Results

The reliable record of whole sunspot cycles extends from cycle 10 to the present (e.g., McKinnon, 1987). Defining the average rate of growth in sunspot number as the difference in smoothed sunspot number between the value after an elapsed time t from sunspot minimum and the value at sunspot minimum divided by the elapsed time t, one can examine the strength of the relationship between average rate of growth and the size of the cycle. As an example, the following demonstrates the computation of the average rate of growth: For cycle 22, the smoothed sunspot number at sunspot minimum was 12.3 and the value after an elapsed time of 12 months was 39.0; thus, the average rate of growth, as defined above, is computed to be 39.0 minus 12.3 divided by 12 – about 2.2.

TABLE I

Results of linear regression analysis

Parameter	t					
	6	12	18	24	30	Max
r	0.308	0.548	0.777	0.884	0.935	0.983
a	77.378	52.708	35.877	27.828	23.865	12.581
b	68.342	58.776	45.779	38.149	37.373	37.165
s.e.	41.0	36.1	27.2	20.2	15.3	7.9
CL	<80.0%	>90.0%	>99.5%	>99.9%	>99.9%	>99.9%

Table I gives the results of a linear regression analysis between average rates of growth for selected elapsed times t (6, 12, 18, 24, and 30 months) and maximum amplitudes of the sunspot cycle for the modern era sunspot cycles, and between the maximum average rate of growth and the size of the cycle (dubbed MAX; maximum average rate of growth is defined as the maximum value of the average rate of growth). Listed are values of the coefficent of correlation r, the y-axis intercept a, the slope b, the standard error of estimate s.e., and the inferred confidence level for each linear fit CL. Of the linear fits given in Table I, only those for $t \geq 18$ months appear to be statistically important, with the more important relationships being those for $t \geq 24$ months (having $r \geq 0.88$ and s.e. ≤ 20.2), as well as that one based on the maximum average rate of growth (which is the best indicator for the size of the cycle, having $r = 0.98$ and s.e. $= 7.9$).

Table II compares observed and expected values of $R(M)$ for the fits described in Table I, where $R(M)$ is the maximum amplitude of the cycle expressed in terms of smoothed sunspot number. At the bottom of Table II is a listing of the largest observed

TABLE II

Comparison of observed and expected $R(M)$

Cycle	Obs. $R(M)$	Exp. $R(M)$					
		6	12	18	24	30	MAX
10	97.9	89.0	82.7	82.6	80.1	86.7	92.1
11	140.5	145.5	145.6	139.3	127.8	122.5	139.3
12	74.6	122.3	119.1	110.0	93.4	87.8	80.6
13	87.9	123.8	143.2	131.1	118.6	109.1	102.1
14	64.2	110.7	100.3	94.0	80.1	73.9	67.6
15	105.4	117.6	96.2	106.8	102.2	94.1	92.5
16	78.1	131.5	108.0	87.6	93.8	94.9	84.3
17	119.2	118.4	94.4	83.0	96.5	101.6	112.9
18	151.8	123.8	131.5	114.6	122.1	139.0	150.8
19	201.3	135.4	150.9	119.1	210.9	213.7	208.1
20	110.6	129.2	102.1	106.4	120.1	117.3	107.7
21	164.5	110.7	122.1	149.4	150.3	155.4	157.9
LOE		65.9	55.3	43.2	30.7	21.2	14.2
AE		31.4	28.5	19.9	16.7	13.3	6.1

error (LOE) and the average error (AE) for the various predictive schemes. Figure 1 depicts the results tabulated in Table II for elapsed times $t \geq 12$ months and MAX (denoted M in Figure 1), where the prediction intervals are based on the average error. (The fit based on $t = 6$ is not statistically significant.)

From Figure 1, one finds that no one single fit always had the observed $R(M)$ within its respective prediction interval. Likewise, the collective prediction interval was not always large enough to include the observed $R(M)$ values. For example, predicting $R(M)$ from the average rates of growth for elapsed times of 12 or 18 months, one finds that the observed $R(M)$ values for cycles 12, 13, and 14 all lay outside the prediction interval. Likewise, for all the investigated elapsed times and MAX, the observed $R(M)$ for cycle 13 was not well predicted. Thus, for 9 out of 12 sunspot cycles, $R(M)$ was well predicted using average rates of growth for $t \leq 18$ months; for 10 out of 12 sunspot cycles, $R(M)$ was well predicted using average rates of growth for $t \leq 24$ months; and for 11 out of 12 sunspot cycles, $R(M)$ was well predicted using either average rates of growth for $t \leq 30$ months or MAX.

Figure 2 depicts the strong correlation between $R(M)$ and the maximum value of the average rate of growth (MAX). While it is one of the best methods for predicting $R(M)$, having an LOE of only 14.2 units of smoothed sunspot number and an AE of only 6.1, the occurrence of the maximum value has fallen over a wide range of elapsed times – between 25 and 48 months. Thus, it tends to occur during the latter portion of the rise, sometimes within 1–2 months of sunspot maximum (e.g., cycles 11, 15, 17, and 18; for cycle 19, the largest cycle of the modern era, its maximum value of the average rate of growth occurred at $t = 27$ months, or about 20 months prior to sunspot maximum). The

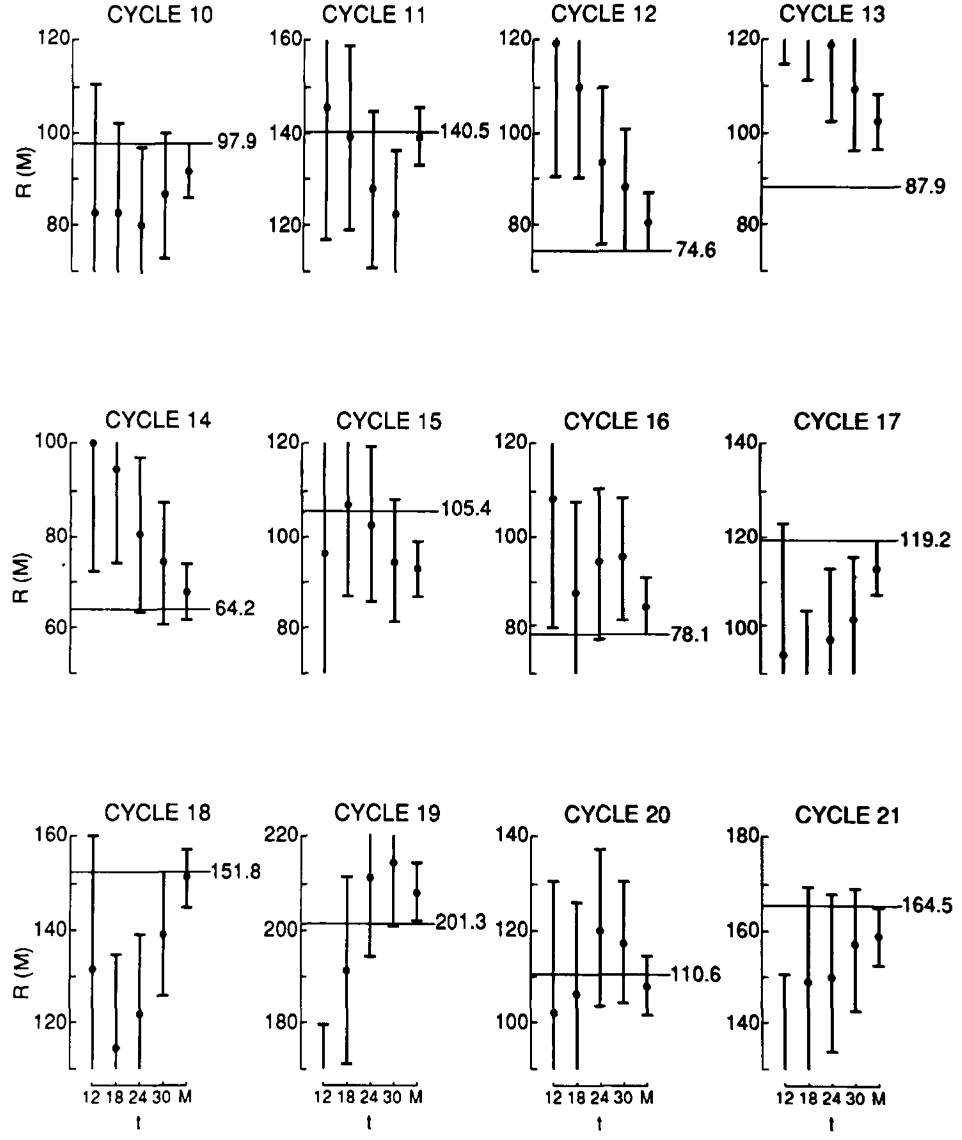

Fig. 1. Comparison of observed $R(M)$ values and the prediction intervals based on selected average rates of growth prediction schemes for cycles 10–21. The observed maximum amplitude for each cycle appears as the horizontal line in each panel. See text for details

importance of Figure 2 becomes apparent if one monitors the average rate of growth for increasing elapsed times, taking the largest observed value to be the potential maximum value of the average rate of growth and calculating the lower limit for the expected $R(M)$. Hence, as an example, if the largest observed average rate of growth for a sunspot cycle is about 4.5 during the rising portion of the cycle, then one expects an $R(M) \gtrsim 171.9$, based on the standard error, or $\gtrsim 173.7$, based on the average error; based on the LOE, one expects $R(M) \gtrsim 165.6$ (For comparison, cycle 19 had a maximum value of the average rate of growth equal to 5.26 and cycle 21 had one equal to 3.91.)

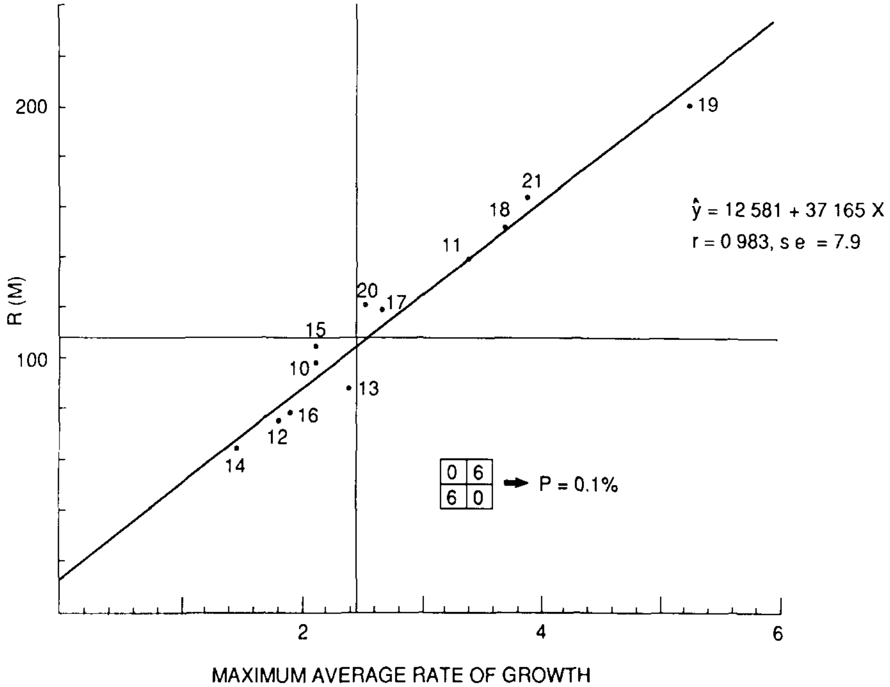

Fig. 2. Linear fit of $R(M)$ versus the maximum value of the average rate of growth. The diagonal line is the regression fit. The vertical and horizontal lines are the median values for maximum average rate of growth and $R(M)$, respectively. The results of a Fisher's exact test for the displayed 2 × 2 contingency table is given Individual cycles are identified by the numbers 10–21.

3. Application to Cycle 22 and Discussion

Figure 3 depicts the history (bottom) of the average rate of growth for cycle 22 from September 1986 (sunspot minimum) through August 1988 ($t = 23$ months) in comparison to the mean average rate of growth based on cycles 10–21, and the deviation (top) in units of the standard deviation. Thus, in August 1988 cycle 22 had an average rate of growth of 4.41 which was about 2.4 standard deviations above the mean. The average rate of growth for cycle 22 has not yet peaked, but is approaching the time when the mean becomes flat and individual cycles have their respective peaks (shown on the bottom-right of Figure 3; the filled triangle denotes the time when the mean is at maximum).

From Figure 3, one finds that the maximum value of the average rate of growth has always occurred between 25–48 months into the cycle, having a median t of 34.5 months. For those having early-occurring maximum average rates of growth (cycles 12, 13, 14, 16, 19, and 20), all have maximum values ≤ 2.56 and $R(M) \leq 110.6$, except one: cycle 19, which had a maximum value of the average rate of growth of 5.26 and an $R(M)$ of 201.3. In contrast, most large amplitude cycles have maximum average rates of growth ≥ 3.41, usually occurring between 36–39 months (cycles 11, 18, and 21). At the present time cycle 22 has a potential maximum value of the average rate of growth

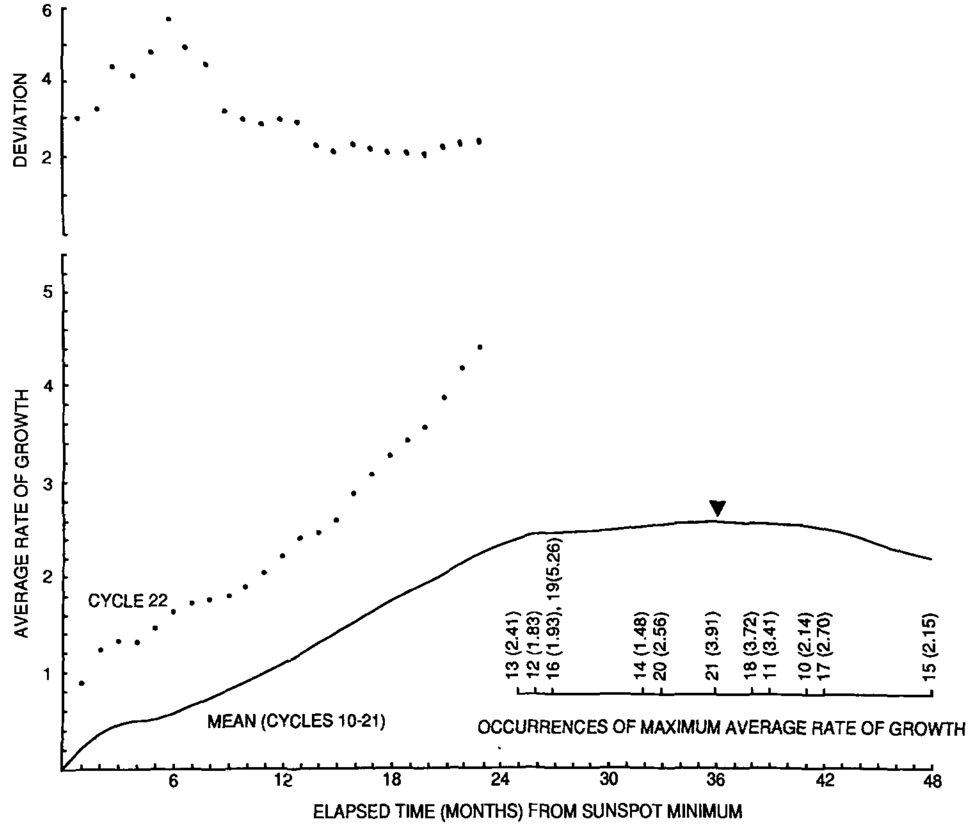

Fig. 3. History of cycle 22. *Bottom*: history of cycle 22 compared to the mean of cycles 10–21, in terms of the average rate of growth. Occurrences of the maximum values of the average rate of growth are identified to the right, as are the actual maximum average values for cycles 10–21. *Top*: history of cycle 22 as compared to the mean of cycles 10–21, in terms of the standard deviation. At present, cycle 22 is a +2.4 standard deviation cycle, in terms of its average of growth; only cycle 19 had a higher average rate of growth for this phase of the sunspot cycle. See text for details.

that is larger than that observed for cycle 21 (the second largest cycle in the modern era), but smaller than that observed for cycle 19 (the largest cycle on record). Thus, if the average rate of growth for cycle 22 is verly close to its maximum value (inferring a behavior more like that of cycle 19), it probably means that cycle 22 will have an $R(M) \gtrsim 164.5$ (and, perhaps, $\lesssim 201.3$ if the maximum average rate of growth does not exceed 5.26). On the other hand, if the average rate of growth for cycle 22 continues to grow, having a peak value larger than 5.26 and occurs later in time like the majority of large amplitude cycles, then cycle 22 clearly will be the record cycle of the modern era.

Table III shows the growth of cycle 22, in terms of monthly mean sunspot number value (denoted r), smoothed sunspot number value (denoted R), selected 6-mo sums of r (denoted sum(r), updated from Wilson, 1988b), and the average rate of growth (denoted ARG, as defined in this study). Also given are predictions of $R(M)$ for cycle 22 based on the linear fits against sum(*r*) and ARG, identified in Wilson (1988b) and in Table I (Section 2), respectively. Associated with each prediction are two prediction intervals, the upper one based on the standard error of estimate for the particular fit and the other based on the average error of the respective fit; the largest observed error

TABLE III

History of cycle 22

Date	*t*	*r*	*R*	Sum (*r*)	*R(M)* [(sum(*r*)]	ARG	*R(M)* [ARG]
Sept. 1986	0	3.8	12.3			0.00	
Oct.	1	35.4	13.2			0.90	
Nov.	2	15.2	14.8			1.25	
Dec.	3	6.8	16.3			1.33	
Jan. 1987	4	10.4	17.6			1.32	
Feb.	5	2.4	19.6		± 42.4	1.46	± 41.0
Mar.	6	14.7	22.1	88.7	137.4 ± 31.9	1.63	188.8 ± 31.4
Apr.	7	39.6	24.4		LOE 89.5	1.73	LOE 65.9
May	8	33.0	26.5		*r* = 0.18	1.78	*r* = 0.31
June	9	17.4	28.4			1.79	
July	10	33.0	31.3			1.90	
Aug.	11	38.7	34.8		± 33.5	2.05	± 36.9
Sept.	12	33.9	39.0	195.6	260.6 ± 24.6	2.22	183.2 ± 28.5
Oct.	13	60.6	43.6		LOE 78.5	2.41	LOE 55.3
Nov.	14	39.9	46.7		*r* = 0.63	2.46	*r* = 0.55
Dec.	15	27.1	51.3			2.60	
Jan. 1988	16	59.0	58.1			2.87	
Feb.	17	40.0	64.5		± 36.6	3.08	± 27.2
Mar.	18	76.2	71.1	302.8	187.7 ± 27.4	3.28	186.0 ± 19.9
Apr.	19	88.0	77.4		LOE 65.5	3.43	LOE 43.2
May	20	59.7	83.7		*r* = 0.53	3.58	*r* = 0.78
June	21	101.8	93.6			3.88	
July	22	112.6	104.2			4.19	
Aug.	23	111.2	113.7		± 24.5	4.41	± 20.2
Sept.	24	120.8	(121.2)	594.1	207.2 ± 17.5	(4.54)	(201.0) ± 16.7
Oct.	25	124.7			LOE 44.2		LOE 30.7
Nov.	26	125.6			*r* = 0.82		*r* = 0.88
Dec.	27	179.4					
Jan. 1989	28	161.6					
Feb.	29	164.5			± 19.9		± 15.3
Mar.	30	(131.0)		(886.8)	(212.2) ± 16.0		? ± 13.3
					LOE 27.9		LOE 21.2
					r = 0.89		*r* = 0.94

(LOE) is also identified, as is the coefficient of correlation. The values shown in parentheses are preliminary values that may slightly change.

From Table III, one finds little consistency between the predictions based on sum(*r*) and ARG prior to 18 months into the cycle. Beginning at 18 months, both techniques are yielding estimates of $R(M)$ for cycle 22 that are close to each other and consistently $> 160–170$. Thus, unless cycle 22 is an anomaly, its $R(M)$ will exceed that observed for cycle 21. Referring back to Figure 2, because at the present time the average rate of growth is about 4.5, one predicts that $R(M)$ for cycle 22 probably will exceed 164.0 (at the 97.5% level of confidence); if the rate eventually exceeds 5, then $R(M)$ for cycle 22 is expected to exceed 182.6 (at the 97.5% level of confidence).

Wilson (1990) has discussed the skill at predicting $R(M)$ from a variety of single variate and bivariate fits (with the dominant determining agent being the geomagnetic minimum value for a cycle). Based on the single variate fits, an $R(M)$ for cycle 22 was estimated to be 170 ± 25, or somewhere between 145–195, with a value >220 being considered unlikely (from the LOE of the single variate fits that are based on geomagnetic data), while based on the bivariate fits, an $R(M)$ for cycle 22 was estimated to be 140 ± 15, or somewhere between 125–155, with a value >160 being considered unlikely. From above, unless cycle 22 proves to be an anomaly, it now seems very likely that $R(M)$ for cycle 22 will lie outside the prediction interval base on the bivariate fits; if so, this will be the first time ever in the modern era of sunspot observations that the bivariate fits have failed to accurately predict the size of a sunspot cycle.

In conclusion, it has been shown that the average rate of growth during the rising portion of the sunspot cycle, indeed, correlates with the size of the sunspot cycle, with the greatest correlation being the one based on the maximum value of the average rate of growth. At present, the maximum value of the average rate of growth for cycle 22 has not yet been observed, although it is very close to the time that it should be seen, if cycle 22 is behaving similarly to that of cycle 19. Thusfar, a potential maximum value of the average rate of growth equal to about 4.5 has been seen for cycle 22, ranking it second only to cycle 19 (5.26). It appears, then, that cycle 22 will have an $R(M)$ that ranks among the greatest on record, either *the* greatest or, more likely, the second greatest, based on the expected value of its maximum average rate of growth.

Acknowledgements

The author thanks John M. Davis (NASA MSFC) for reading the manuscript. This research was supported by NASA's Solar Physics Branch of the Space Physics Division and was performed to support the work of the Prediction of Solar Activity and the Atmospheric Response Working Group, co-chaired by George L. Withbroe (Center for Astrophysics) and Charles R. Chappell (NASA MSFC).

References

Bracewell, R. N.: 1988, *Monthly Notices Roy. Astron. Soc.* **230**, 535.

de Jager, C.· 1959, in S. Flügge (ed.), *Encyclopedia of Physics*, Vol. LII, Astrophysics III: The Solar System, Springer-Verlag, Berlin, p. 51.

Howard, R.: 1977, in A. Bruzek and C. J. Durrant (eds.), *Illustrated Glossary for Solar and Solar-Terrestrial Physics*, Vol. 69, Astrophysics and Space Science Library, D. Reidel Publ. Co., Dordrecht, Holland, p. 7.

McKinnon, J. A.: 1987, *Sunspot Numbers: 1610–1985*, Report UAG-95, World Data Center A for Solar-Terrestrial Physics, Boulder, Colorado.

Waldmeier, M.: 1935, *Astron. Mitt. Zürich* **14**, No. 133.
Waldmeier, M.: 1957, *Z. Astrophys.* **43**, 29.
Wilson, R. M.: 1988a, *J. Geophys. Res.* **93**, 10011.
Wilson, R. M.: 1988b, *Solar Phys.* **117**, 179.
Wilson, R. M.: 1989, *Solar Phys.* **125**, 143.