ON THE EXISTENCE OF THE CONTINUOUS SPECTRUM OF IDEAL MHD IN A 2D MAGNETOSTATIC EQUILIBRIUM

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Abstract. The continuous spectrum of a 2D magnetostatic equilibrium with y-invariance is derived. It is shown that the continuous spectrum is given by an eigenvalue problem on each magnetic surface and is related to the different behaviour of the equilibrium quantities in different magnetic surfaces. The special case of a uniform poloidal magnetic field in a 1D equilibrium that is stratified with height, has been considered in detail and it is found that there is no continuous spectrum. It is shown that this result relies completely on the artificial property that the behaviour of the equilibrium quantities along a magnetic field line is independent of the field line considered. As a consequence the non-existence of a continuous spectrum in a 1D equilibrium with a uniform magnetic field cannot be used to argue that the continuous spectrum has no physical relevance.

1. Introduction

The linear and adiabatic oscillations of a vertically stratified plasma with a horizontal magnetic field are governed by an ordinary second order differential equation for the vertical component of the displacement vector. It is well-known that this differential equation possesses a singularity at the level where the local Alfvén frequency or the local cusp frequency equals the frequency of the oscillation (see e.g., Appert *et al.*, 1975; Chen and Hasegawa, 1974; Adam, 1977; El Mekki *et al.*, 1978; Rae and Roberts, 1982). The level where the singularity occurs is called the critical level and the range of frequencies for which there is a singularity is the continuous spectrum. For a compressible plasma the continuous spectrum consists of an Alfvén continuum and a cusp continuum. In the incompressible limit the Alfvén and cusp continua coincide. The solutions that correspond to the continuum frequencies are non-square integrable and have been discussed for instance by Goedbloed (1983).

The continuous spectrum has received ample attention in plasma physics because it is realized that the continuous spectrum is an integral part of the spectrum of ideal MHD and also because resonant absorption of Alfvén waves is a possible means of heating plasmas (see e.g., Chen and Hasegawa, 1974; Tataronis, 1975). Recently, Schwartz and Bell (1984) have argued that the existence of critical levels, and as a consequence the continuous spectrum (although Schwartz and Bell do not mention the continuous spectrum) is an artifact of the assumption that the magnetic field is purely horizontal. Schwartz and Bell use the result that there are no critical levels in a uniform magnetic field of an arbitrary direction even if the magnetic field is almost horizontal, to suggest that the singularities and as a consequence the continuous spectrum of the exactly

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horizontal case are of no physical relevance for example in the solar atmosphere where the planar horizontal field approximation is seldom adequate.

A magnetic field of an arbitrary direction is likely to be non-uniform and to have components that depend on two spatial coordinates rather than to be constant. We therefore conclude that the suggestion by Schwartz and Bell (1984) is premature. The continuous spectrum of the exactly horizontal case is of no physical relevance if it can be shown that there is no continuous spectrum for 2D equilibria.

The present paper is concerned with the continuous spectrum of a static two dimensional equilibrium in which all physical variables depend on two cartesian coordinates x and z but are invariant with respect to y. Gravity is included. The continuous spectrum of a static 2D axisymmetric and toroidal plasma has been derived independently by Goedbloed (1975) and Pao (1975). Ample attention has since then been given to the continuous spectrum and effects like plasma flow (Hameiri and Hammer, 1979; Hameiri, 1983; Hellsten and Spies, 1979) and plasma anisotropy (Hellsten, 1979; Hellsten and Scheffel, 1984) have been taken into account. A rigorous determination of the continuous spectrum has been given by Hameiri (1985). All the papers referred to so far in this paragraph concern laboratory plasmas and neglect gravity. The continuous spectrum of a static 2D axisymmetric equilibrium with gravity included has been derived for a purely poloidal magnetic field by Goossens *et al.* (1984) and Hermans *et al.* (1984) and for a mixed poloidal and toroidal magnetic field by Poedts *et al.* (1985).

The present paper is arranged as follows. The equations that describe the equilibrium state and the linear motions about this equilibrium state are recalled in Section 2. The continuous spectrum of a 2D magnetostatic equilibrium with y-invariance is derived in Section 3. The special case of a 1D magnetostatic equilibrium with a uniform magnetic field of arbitrary direction is considered in Section 4.

2. Basic Equations

A self-gravitating magnetostatic plasma is governed by the magnetostatic equations

$$\nabla p + \rho \nabla \Phi - \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla^2 \Phi = 4\pi G \rho, \tag{3}$$

where p, ρ, and Φ are the pressure, density and gravitational potential, and $\mathbf{B}$ the magnetic field. The equilibrium state which is of interest to us, is a 2D equilibrium with all physical variables functions of the cartesian coordinates x and z but not of y. The magnetic field is split up in a poloidal and a toroidal magnetic field, $\mathbf{B}_p$ and $\mathbf{B}_t$, and the poloidal magnetic field is written in terms of a magnetic flux function $\psi(x, z)$ as

$$\mathbf{B}_p = -\nabla \psi(x, z) \times \mathbf{1}_y = \frac{\partial \psi}{\partial z} \mathbf{1}_x - \frac{\partial \psi}{\partial x} \mathbf{1}_z, \tag{4}$$

where $\mathbf{l}_x$, $\mathbf{l}_y$, and $\mathbf{l}_z$ are the unit vectors in the x-, y-, and z-direction. The toroidal magnetic field cannot be chosen at will. The assumption of y-invariance implies that

$$\mathbf{B}_t = B_y \mathbf{l}_y = f(\psi)\mathbf{l}_y \tag{5}$$

with f an arbitrary function of its argument.

It is known in the literature that there exist 2D-equilibria with nested magnetic surfaces even when gravity is included (see e.g. Low, 1980). Consider such an equilibrium with nested magnetic surfaces.

We then define a local orthogonal system of flux coordinates $\psi(x, z)$, $\chi(x, z)$, y with χ the poloidal variable. All equilibrium quantities are now functions of χ and ψ but not of y, and the equilibrium magnetic field has components $(0, B_\chi, B_y)$ in the (ψ, χ, y) system of coordinates. We use the abbreviation

$$B^2 = B_\chi^2 + B_y^2. \tag{6}$$

We need to have expressions for the operators ∇, ∇^2, div and rot in terms of χ, ψ, and y for the study of the equilibrium equations and the linearized equations of MHD. We have

$$\nabla f = \frac{1}{JB_\chi}\frac{\partial f}{\partial \chi}\mathbf{l}_\chi + B_\chi \frac{\partial f}{\partial \psi}\mathbf{l}_\psi + \frac{\partial f}{\partial y}\mathbf{l}_y, \tag{7}$$

$$\nabla^2 f = \frac{1}{J}\left[\frac{\partial}{\partial \chi}\left(\frac{1}{JB_\chi^2}\frac{\partial f}{\partial \chi}\right) + \frac{\partial}{\partial \psi}\left(JB_\chi^2 \frac{\partial f}{\partial \psi}\right) + \frac{\partial}{\partial y}\left(J\frac{\partial f}{\partial y}\right)\right], \tag{8}$$

$$\nabla \cdot \mathbf{u} = \frac{1}{J}\left[\frac{\partial}{\partial \chi}\left(\frac{u_\chi}{B_\chi}\right) + \frac{\partial}{\partial \psi}(JB_\chi u_\psi) + \frac{\partial}{\partial y}(Ju_y)\right], \tag{9}$$

$$\nabla \times \mathbf{u} = B_\chi\left[\frac{\partial u_y}{\partial \psi} - \frac{\partial}{\partial y}\left(\frac{u_\psi}{B_\chi}\right)\right]\mathbf{l}_\chi + \frac{1}{JB_\chi}\left[\frac{\partial}{\partial y}(JB_\chi u_\chi) - \frac{\partial u_y}{\partial \chi}\right]\mathbf{l}_\psi + \\ + \frac{1}{J}\left[\frac{\partial}{\partial \psi}\left(\frac{u_\psi}{B_\chi}\right) - \frac{\partial}{\partial \psi}(JB_\chi u_\chi)\right]\mathbf{l}_y. \tag{10}$$

J is the Jacobian of the (ψ, χ, y) coordinate system and $\mathbf{l}_\psi$ is the unit vector perpendicular to the flux surfaces and $\mathbf{l}_\chi$ is the unit vector along the magnetic field lines.

The ψ- and χ-component of the force balance equation and Poisson's equation take the form

$$\frac{\partial p}{\partial \psi} + \rho\frac{\partial \Phi}{\partial \psi} + \frac{1}{J}\frac{\partial}{\partial \psi}\left(\frac{JB_\chi^2}{4\pi}\right) = 0, \tag{11}$$

$$\frac{\partial p}{\partial \chi} + \rho\frac{\partial \Phi}{\partial \chi} = 0, \tag{12}$$

$$\frac{1}{J}\frac{\partial}{\partial \chi}\left[\frac{1}{JB_\chi^2}\frac{\partial \Phi}{\partial \chi}\right] + \frac{1}{J}\frac{\partial}{\partial \psi}\left[JB_\chi^2\frac{\partial \Phi}{\partial \psi}\right] = 4\pi G\rho. \tag{13}$$

The equations that govern the linear adiabatic oscillations about a static equilibrium are obtained by linearization of the equations of ideal MHD. We denote the Lagrangian displacement by ξ and the Eulerian perturbation of a physical quantity f by f'. Since the equilibrium state is independent of y and of course of t, we can Fourier decompose the perturbed quantities with respect to time t and y and make them proportional to $\exp(i(\sigma t + k_y y))$. The equations for the linear oscillations about a self-gravitating static equilibrium are

$$-\rho\sigma^2\xi = -\nabla p' - \rho'\nabla\Phi - \rho\nabla\Phi' + \frac{1}{4\pi}[(\nabla\times\mathbf{B})\times\mathbf{B}' + (\nabla\times\mathbf{B}')\times\mathbf{B}], \quad (14)$$

$$\rho' = -\nabla\cdot(\rho\xi), \quad (15)$$

$$p' = -\xi\cdot\nabla p - \Gamma_1 p \operatorname{div}\xi, \quad (16)$$

$$\mathbf{B}' = \nabla\times(\xi\times\mathbf{B}), \quad (17)$$

$$\nabla^2\Phi' = 4\pi G\rho. \quad (18)$$

We now rewrite Equations (14)–(18) so that we can identify the magnetic surfaces as characteristic surfaces without the use of any particular system of coordinates (see also Hameiri and Hammer, 1979; Hameiri, 1985). Equations (14)–(18) can be written in the following form:

$$-\rho\sigma^2\xi + \nabla P' + \rho'\nabla\Phi + \rho\nabla\Phi' - \frac{1}{4\pi}(\mathbf{B}'\cdot\nabla)\mathbf{B} - \frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}' = 0, \quad (19)$$

$$\rho' + \rho(\nabla\cdot\xi) + (\xi\cdot\nabla)\rho = 0, \quad (20)$$

$$p' + (\xi\cdot\nabla)p + \Gamma_1 p(\nabla\cdot\xi) = 0, \quad (21)$$

$$\mathbf{B}' - (\mathbf{B}\cdot\nabla)\xi + \mathbf{B}(\nabla\cdot\xi) + (\xi\cdot\nabla)\mathbf{B} = 0, \quad (22)$$

$$\nabla^2\Phi' - 4\pi G\rho' = 0. \quad (23)$$

It is important to note here the part-hyperbolicity of the system. The existence of a hyperbolic part can be seen from the property of the system that all spatial derivatives are within ψ-surfaces, except in the terms with $\nabla\cdot\xi$, $\nabla\Phi'$, and $\nabla P'$, where $P' = p' + (\mathbf{B}\cdot\mathbf{B}'/4\pi)$ is the disturbed total pressure. Thus, every ψ-surface is a characteristic surface of multiplicity six. If Cauchy data are prescribed on such a surface, only the normal derivatives of P', ξ_ψ, and Φ' are known while all other quantities cannot be continued off the surface. The discussion in the following sections is indeed based on singling out these quantities.

3. The Continuous Spectrum

Modes belonging to the continuous spectrum are recognized by their singular behaviour at a magnetic surface. In the case of a one-dimensional magnetostatic equilibrium (e.g. the plane slab, the diffuse linear pinch) two spatial coordinates are ignorable, the magnetic surfaces coincide with the coordinate surface of the remaining non-ignorable coordinate and the Equations (14)–(18) can be reduced to one ordinary second order differential equation for the component of the displacement that corresponds to the non-ignorable coordinate. The values of σ^2 that correspond to the mobile regular singular points of this differential equation are associated with non-square integrable solutions and define two continuous parts of the spectrum, namely the Alfvén continuum and the cusp continuum. The Alfvén and cusp continua consist of the frequencies $\sigma^2 = \sigma_A^2$, and $\sigma^2 = \sigma_c^2$ where $\sigma_A^2 = (\mathbf{k} \cdot \mathbf{B})^2/4\pi\rho$, and $\sigma_c^2 = \sigma_A^2 c^2/(c^2 + V_A^2)$, with $\mathbf{k}$ the horizontal wave vector, and V_A^2 the square of the Alfvén velocity, $V_A^2 = B^2/4\pi\rho$.

The continuous spectrum of a static one-dimensional magnetic equilibrium is obtained by simply putting the coefficient in front of the highest order derivatives equal to zero. This procedure cannot be followed to obtain the continuous spectrum of a two-dimensional equilibrium. The equations that describe the linear motions are now partial differential equations and we have to redefine the continuous spectrum as those frequencies for which the corresponding solutions show non-square integrable singularities at a flux surface $\psi = \psi_0$ (Pao, 1975). Therefore, we have to rewrite Equations (14)–(18) giving special attention to derivatives across the flux surface, i.e. the ψ-derivatives. Further, the solutions that correspond to the Alfvén continuum and the cusp continuum in the linear diffuse pinch are characterized by motions in the flux surface that are polarized either perpendicular or parallel to the magnetic field lines. This polarization property also holds for a static axisymmetric equilibrium with a purely poloidal magnetic field (Goossens *et al.*, 1984; Hermans *et al.*, 1984), but not for a static axisymmetric 2D equilibrium with a mixed poloidal and toroidal magnetic field (Poedts *et al.*, 1985). Although this polarization property cannot be expected to hold in a mixed poloidal and toroidal magnetic field, it is sometimes convenient to have vectors in the flux surfaces decomposed in components parallel and perpendicular to the magnetic field lines.

We here closely follow Pao (1975) to derive the continuous spectrum. We use Equation (15) to eliminate ρ' from Equation (14) and we rewrite Equations (14), (16)–(18) in the form that we can abbreviate as

$$A_1(\psi, \chi)\frac{\partial R}{\partial \psi} + A_2(\psi, \chi)S = A_3(\psi, \chi)R. \tag{24}$$

A_1 is an algebraic matrix operator, and A_2 and A_3 are differential matrix operators in χ, but algebraic in ψ. R is the column vector that contains the perturbed quantities that are differentiated with respect to ψ and S is the column vector that contains the remaining perturbed quantities.

The vectors R and S are in the present case

$$R = [X, P', \Phi']^t, \qquad S = [\xi_\chi, \xi_y, B'_\psi, B'_\chi, B'_y]^t, \tag{25}$$

where P' is the perturbed total pressure

$$P' = p' + \frac{\mathbf{B} \cdot \mathbf{B}'}{4\pi} \tag{26}$$

and

$$X = B_\chi \xi_\psi. \tag{27}$$

Equation (24) shows again that the magnetic surfaces are characteristic surfaces since only three of the eight unknown functions, namely X, P', and Φ', are differentiated across the magnetic surfaces. It is convenient to have the equations of the spectrum of ideal MHD written in the form (24) since we obtain the continuous spectrum by taking the limit $\partial/\partial\psi \to \infty$. This implies that Q and $\partial Q/\partial\chi$ are neglected compared to $\partial Q/\partial\psi$ for any perturbed quantity Q that is differentiated with respect to ψ.

The explicit form of the set of Equations (24) is the following. The components of the equation of motion:

$$\frac{\partial P'}{\partial\psi} + \rho\frac{\partial\Phi'}{\partial\psi} - \rho\frac{\partial\Phi}{\partial\psi}F^*\left(\frac{\xi_\chi}{B_\chi}\right) - \frac{1}{JB_\chi}\frac{\partial\Phi}{\partial\psi}\frac{\partial\rho}{\partial\chi}\xi_\chi + \frac{\rho}{B_\chi}\frac{\partial\Phi}{\partial\psi}{}'_\chi + $$

$$+ \frac{1}{2\pi J}\frac{\partial}{\partial\psi}(JB_\chi)B'_\chi = g_1, \tag{28}$$

$$\rho\sigma^2\xi_\chi + \frac{\rho}{JB_\chi}\frac{\partial\Phi}{\partial\chi}F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{J^2B_\chi^2}\frac{\partial\Phi}{\partial\chi}\frac{\partial\rho}{\partial\chi}\xi_\chi + \frac{1}{4\pi B_\chi}F^*(\mathbf{B}\cdot\mathbf{B}') - $$

$$- \frac{\rho}{JB_\chi^2}\frac{\partial\Phi}{\partial\chi}B'_\chi - \frac{B_y}{4\pi B_\chi}F^*B'_y = g_2, \tag{29}$$

$$\rho\sigma^2\xi_y + \frac{1}{4\pi}F^*B'_y = g_3; \tag{30}$$

the energy equation:

$$F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{\Gamma_1 p}\frac{1}{J}\frac{\partial p}{\partial\chi}\frac{\xi_\chi}{B_\chi} - \frac{B'_\chi}{B_\chi} - \frac{\mathbf{B}\cdot\mathbf{B}'}{4\pi\Gamma_1 p} = g_4; \tag{31}$$

the components of the induction equation:

$$B_\chi B'_\psi = g_5, \tag{32}$$

$$\frac{\partial}{\partial\psi} + \frac{B'_\chi}{B_\chi} + ik_yQ = 0, \tag{33}$$

$$F^*Q - \left(B'_y - \frac{B_y}{B_\chi}B'_\chi\right) = g_6; \tag{34}$$

and the Poisson equation:

$$\frac{1}{J}\frac{\partial}{\partial\psi}\left(JB_\chi^2\frac{\partial\Phi'}{\partial\psi}\right) + 4\pi G\left[-\rho F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{J}\frac{\partial\rho}{\partial\chi}\frac{\xi_\chi}{B_\chi} - \rho\frac{B'_\chi}{B_\chi}\right] = g_7, \tag{35}$$

where

$$g_1 = \frac{\rho\sigma^2}{B_\chi^2} X + F^*\left(\frac{F^*X}{4\pi B_\chi^2}\right) + \frac{\partial\Phi}{\partial\psi}\frac{1}{J}\frac{\partial(\rho J)}{\partial\psi}X,$$

$$g_2 = \frac{1}{B_\chi}F^*P' - ik_y\frac{B_y}{B_\chi}P' - \frac{\partial\Phi}{\partial\chi}\frac{\partial(\rho J)}{\partial\psi}\frac{X}{JB_\chi} -$$

$$- \frac{1}{4\pi\rho}\frac{\partial}{\partial\psi}(JB_\chi^2)\frac{1}{B_\chi}F^*(X) + \rho\frac{\partial\Phi'}{\partial\chi},$$

$$g_3 = ik_yP' - \frac{1}{4\pi}\frac{dB_y}{d\psi}F^*X + ik_y\rho\Phi',$$

$$g_4 = -\frac{1}{\Gamma_1 p}\frac{\partial p}{\partial\psi}X - \frac{1}{J}\frac{\partial J}{\partial\psi}X - \frac{P'}{\Gamma_1 p},$$

$$g_5 = F^*X,$$

$$g_6 = \frac{1}{J}\frac{\partial(JB_y)}{\partial\psi}X,$$

$$g_7 = F^*\left(\frac{1}{B_\chi^2}F^*\Phi'\right) - \frac{2ik_yB_y}{B_\chi^2}F^*\Phi' - \frac{k^2B^2}{B_\chi^2}\Phi' -$$

$$- i\frac{k_yB_y}{JB_\chi^4}\frac{\partial}{\partial\chi}(B_\chi^2) - 4\pi G\frac{1}{J}\frac{\partial}{\partial\psi}(\rho J)\frac{X}{J}. \tag{36}$$

Now

$$Q = \xi_y - \frac{B_y}{B_\chi}\xi_\chi \tag{37}$$

is a component in the flux surfaces perpendicular to the magnetic field lines and

$$F^* = \mathbf{B}\cdot\nabla = \frac{1}{J}\frac{\partial}{\partial\chi} + ik_yB_y \tag{38}$$

is a differential operator along the magnetic field lines.

The equations that govern the continuous spectrum are obtained by taking the limit $\partial/\partial\psi \to \infty$. This leads to putting the right-hand members of Equations (28)–(35) equal to zero. The continuous spectrum is governed by the Equations (28)–(35) with g_1 up to g_7 put equal to zero. From Poisson's equation it follows that $\partial^2\Phi'/\partial\psi^2$ is of the same order as ξ_χ and/or B'_χ so that Φ' and $\partial\Phi'/\partial\psi$ can be neglected compared with ξ_χ and/or B'_χ. This means that in fact the perturbation of the gravitational potential has no effect on the continuous spectrum.

The set of equations that determines the continuous spectrum is

$$\frac{\partial P'}{\partial\psi} - \rho\frac{\partial\Phi}{\partial\psi}F^*\left(\frac{\xi_\chi}{B_\chi}\right) - \frac{1}{JB_\chi}\frac{\partial\Phi}{\partial\psi}\frac{\partial\rho}{\partial\chi}\xi_\chi + \frac{\rho}{B_\chi}\frac{\partial\Phi}{\partial\psi}B'_\chi + $$
$$+ \frac{1}{2\pi J}\frac{\partial}{\partial\psi}(JB_\chi)B'_\chi = 0, \tag{39}$$

$$\rho\sigma^2\xi_\chi + \frac{\rho}{JB_\chi}\frac{\partial\Phi}{\partial\chi}F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{J^2B_\chi^2}\frac{\partial\Phi}{\partial\chi}\frac{\partial\rho}{\partial\chi}\xi_\chi + \frac{1}{4\pi B_\chi}F^*(\mathbf{B}\cdot\mathbf{B}') -$$
$$- \frac{\rho}{JB_\chi^2}\frac{\partial\Phi}{\partial\chi}B'_\chi - \frac{B_y}{4\pi B_\chi}F^*B'_y = 0, \tag{40}$$

$$\rho\sigma^2\xi_y + \frac{1}{4\pi}F^*B'_y = 0, \tag{41}$$

$$F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{\Gamma_1 p}\frac{1}{J}\frac{\partial p}{\partial\chi}\frac{\xi_\chi}{B_\chi} - \frac{B'_\chi}{B_\chi} - \frac{\mathbf{B}\cdot\mathbf{B}'}{4\pi\Gamma_1 p} = 0, \tag{42}$$

$$B'_\psi = 0, \tag{43}$$

$$\frac{\partial X}{\partial\psi} + \frac{B'_\chi}{B_\chi} + ik_yQ = 0, \tag{44}$$

$$F^*Q - \left(B'_y - \frac{B_y}{B_\chi}B'_\chi\right) = 0, \tag{45}$$

$$\frac{1}{J}\frac{\partial}{\partial\psi}\left(JB_\chi^2\frac{\partial\Phi'}{\partial\psi}\right) + 4\pi G\left[-\rho F^*\left(\frac{\xi_\chi}{B_\chi}\right) + \frac{1}{J}\frac{\partial\rho}{\partial\chi}\frac{\xi_\chi}{B_\chi} - \rho\frac{B'_\chi}{B_\chi}\right] = 0. \tag{46}$$

Equations (39)–(46) are eight equations for eight unknowns; namely $\partial X/\partial\psi$, ξ_χ, ξ_y, B'_ψ, B'_χ, B'_y, $\partial P'/\partial\psi$, and $\partial^2\Phi'/\partial\psi^2$. Equation (43) implies that $B'_\psi = 0$. Since $\xi_\psi \approx 0$, the solutions corresponding to the continuum frequencies have displacements and perturbed magnetic fields in the magnetic surfaces. Equation (39) determines $\partial P'/\partial\psi$ in terms of ξ_χ and B'_χ and Equation (44) determines $\partial X/\partial\psi$ in terms of B'_χ, ξ_χ, and ξ_y. Equation (46) determines $\partial^2\Phi'/\partial\psi^2$ in terms of ξ_χ and B'_χ. Note also that in addition to $\xi_\psi \approx 0$ and $B'_\psi \approx 0$ we have also $P' \approx 0$, $\Phi' \approx 0$, and $\partial\Phi'/\partial\psi \approx 0$. The continuous spectrum is thus governed by the Equations (40), (41), (42), and (45), which are four first order differential equations for the unknowns ξ_χ, ξ_y, B'_χ, and B'_y. We can solve Equation (41) for ξ_y and Equation (40) and (42) for ξ_χ in terms of B'_χ and B'_y as

$$\xi_y = \frac{-1}{4\pi\rho\sigma^2}F^*B'_y, \tag{47}$$

$$\xi_\chi = \frac{-1}{4\pi\rho(\sigma^2 - N_\chi^2)}\left[\frac{1}{B_\chi}F^*(\mathbf{B}\cdot\mathbf{B}') + \frac{\rho}{\Gamma_1 p}\,\frac{1}{JB_\chi}\,\frac{\partial\Phi}{\partial\chi}\mathbf{B}\cdot\mathbf{B}' - \frac{B_y}{B_\chi}F^*B_y'\right], \tag{48}$$

where

$$N_\chi^2 = -\frac{1}{JB_\chi}\,\frac{\partial\Phi}{\partial\chi}\left(\frac{1}{JB_\chi}\,\frac{1}{\rho}\,\frac{\partial\rho}{\partial\chi} - \frac{1}{\Gamma_1 pJB_\chi}\,\frac{\partial p}{\partial\chi}\right) \tag{49}$$

can be considered as the square of the Brunt–Väisälä frequency along the magnetic field lines. N_χ^2 can be negative or positive depending on the variation of density, pressure and gravity in the magnetic surfaces.

Elimination of ξ_y and ξ_χ with the aid of (47) and (48) from the four first order differential equations that govern the continuous spectrum leads to two coupled second order differential equations for $B'_\perp$ and $B'_\parallel$:

$$\begin{aligned}
&\frac{B'_\perp}{B_\chi} + F^*\left[\frac{\sigma^2B^2 - N_\chi^2B_\chi^2}{4\pi\rho\sigma^2(\sigma^2 - N_\chi^2)}F^*\left(\frac{B_\chi}{B^2}B'_\perp\right)\right] + \\
&\quad + B_yF^*\left\{\frac{N_\chi^2}{4\pi\rho\sigma^2(\sigma^2 - N_\chi^2)B^2}F^*B'_\parallel + \frac{\sigma^2B^2 - N_\chi^2B_\chi^2}{4\pi\rho\sigma^2(\sigma^2 - N_\chi^2)}\,\frac{F^*(B^2)}{B_\chi^2B^4}B'_\parallel + \right. \\
&\quad \left. + \frac{1}{\sigma^2 - N_\chi^2}\,\frac{1}{4\pi\Gamma_1 pB_\chi}B'_\parallel\,\frac{1}{JB_\chi}\,\frac{\partial\Phi}{\partial\chi}\right\} = 0,
\end{aligned} \tag{50}$$

$$\begin{aligned}
&B_y\left\{\frac{B'_\perp}{B^2B_\chi} + \left(F^* - \frac{\rho}{\Gamma_1 p}\,\frac{1}{J}\,\frac{\partial\Phi}{\partial\chi}\right)\frac{1}{4\pi\rho(\sigma^2 - N_\chi^2)}F^*\left(\frac{B_\chi}{B^2}B'_\perp\right)\right\} + \\
&\quad + \frac{1}{4\pi}\left\{\frac{c^2 + V_A^2}{\rho c^2V_A^2}B'_\parallel + \left(F^* - \frac{\rho}{\Gamma_1 p}\,\frac{1}{J}\,\frac{\partial\Phi}{\partial_\chi}\right)\frac{1}{\rho(\sigma^2 - N_\chi^2)}\right. \\
&\quad \left.\times\left[\frac{1}{B^2}F^*B'_\parallel + \frac{B_y^2F^*(B^2)}{B_\chi^2B^4}B'_\parallel + \frac{\rho}{\Gamma_1 p}\,\frac{1}{JB_\chi^2}\,\frac{\partial\Phi}{\partial\chi}B'_\parallel\right]\right\} = 0.
\end{aligned} \tag{51}$$

Equations (50) and (51) are the governing equations for the continuous spectrum in terms of the variables $B'_\perp$ and $B'_\parallel$ that are defined as

$$B'_\parallel = \mathbf{B}\cdot\mathbf{B}', \qquad B'_\perp = B_yB'_\chi - B_\chi B'_y, \tag{52}$$

and are the components parallel and perpendicular to the magnetic field lines. Equations (50) and (51) are two ordinary differential equations for the variables $B'_\perp$ and $B'_\parallel$ not involving derivatives across the magnetic surfaces. They can be written in compact form

$$L(\psi, \chi, \sigma^2)\mathbf{V} = 0, \tag{53}$$

where

$$\mathbf{V} = [B'_\parallel, B'_\perp]^t \tag{54}$$

and the operator $L(\psi, \chi, \sigma^2)$ has the important property of being a differential operator in χ but algebraic in ψ. This allows us to separate in the solutions the improper normal dependence from the proper tangential behaviour.

We now restrict the analysis to the neighbourhood of a particular magnetic surface $\psi = \psi_0$. On that magnetic surface

$$L(\psi_0, \chi, \sigma^2)\tilde{\mathbf{V}}_0(\chi) = 0 \tag{55}$$

is a nonsingular eigenvalue problem when supplemented with proper boundary conditions. It can be shown that a solution $\tilde{\mathbf{V}}_0(\chi)$ to the nonsingular eigenvalue problem (55) corresponds to a solution

$$\mathbf{V}(\psi, \chi) = \delta(\psi - \psi_0)\tilde{\mathbf{V}}_0(\chi) \tag{56}$$

of the singular eigenvalue problem. The continuous spectrum can then be found as follows. The nonsingular eigenvalue problem is solved on each magnetic surface. For the fixed value of the wave number k_y, a discrete set of eigenvalues $\{\sigma^2(\psi_0)\}$ is found. Now any equilibrium quantity $f(\psi, \chi)$ depends on the two coordinates ψ and χ and the variation of f in a magnetic surface (i.e. the variation of f with respect to χ for fixed $\psi = \psi_0$) depends on the magnetic surface considered. This implies that the differential matrix operator $L(\psi_0, \chi, \sigma^2)$ depends of the magnetic surface considered and as a consequence that each eigenvalue of the discrete set $\{\sigma^2(\psi_0)\}$ depends on the magnetic surface considered. When the magnetic surface $\psi = \psi_0$ is varied each eigenvalue of the discrete set spreads out a continuous spectrum. The continuous spectrum arises because of the different behaviour of the equilibrium quantities in different magnetic surfaces.

Before we proceed to the discussion of the continuous spectrum defined by Equations (40), (41), (42), and (45) or Equations (50)–(51) we first consider the case of a purely poloidal field ($B_y \equiv 0$). The Equations (40), (41), (42), and (45) can be simplified by dropping the terms that contain B_y. One can now eliminate B'_χ and B'_y or ξ_χ and ξ_y. Elimination of ξ_χ and ξ_y leads to

$$\sigma^2 B'_y + F^*\left(\frac{1}{4\pi\rho} F^* B'_y\right) = 0, \tag{57}$$

$$\left\{\frac{c^2 + V_A^2}{c^2 V_A^2} + \frac{N_\chi^2}{c^2(\sigma^2 - N_\chi^2)} + \frac{1}{J}\frac{\partial}{\partial\chi}\left[\frac{1}{c^2(\sigma^2 - N_\chi^2)}\frac{1}{JB_\chi^2}\frac{\partial\Phi}{\partial\chi}\right]\right\}\frac{B_\chi B'_\chi}{\rho} + \\ + F^*\left[\frac{1}{\rho(\sigma^2 - N_\chi^2)B_\chi^2} F^*(B_\chi B'_\chi)\right] = 0, \tag{58}$$

where now

$$F^* = \frac{1}{J}\frac{\partial}{\partial\chi}.$$

Equations (57) and (58) correspond to (50) and (51) and can actually be derived from

(50) and (51) directly. Elimination of B'_χ and B'_y leads to

$$\sigma^2 \xi_y = -\frac{1}{4\pi\rho}(F^*)^2 \xi_y, \tag{59}$$

$$\sigma^2 \xi_\chi = \left[\frac{c^2}{c^2 + V_A^2} N_\chi^2 + \frac{1}{JB_\chi^2}\frac{\partial}{\partial\chi}\left(\frac{V_A^2}{c^2 + V_A^2}\frac{1}{J}\frac{\partial\Phi}{\partial\chi}\right)\right]\xi_\chi - $$

$$- \frac{1}{\rho B_\chi} F^* \left[\frac{\rho c^2 V_A^2}{c^2 + V_A^2} F^* \left(\frac{\xi_\chi}{B_\chi}\right)\right]. \tag{60}$$

Equations (57)–(58) and Equations (59)–(60) are two uncoupled second-order differential equations for respectively B'_y and B'_χ and ξ_y and ξ_χ. They allow solutions $\xi_y \neq 0$, $B'_y \neq 0$, $\xi_\chi = 0$, $B'_\chi = 0$ and $\xi_y = 0$, $B'_y = 0$, $\xi_\chi \neq 0$, $B'_\chi \neq 0$, respectively. The solutions correspond to motions in the magnetic surfaces that are polarized either perpendicular or parallel to the magnetic field lines. The solutions have the classical polarization property of the solutions associated with the Alfvén and the cusp continua of the linear diffuse pinch. The continuous spectrum of a 2D y-invariant equilibrium with a purely poloidal magnetic field consists of two uncoupled parts which can be called an Alfvén continuum and a cusp continuum. Gravity and compressibility only affect the cusp continuum.

The location of the continuum in the spectrum can be derived with the aid of a variational principle. The variational principle can be derived for the mixed poloidal and toroidal magnetic field case but the expressions involved are rather lengthy. We prefer to derive the variational principle for the case of a purely poloidal magnetic field where the cusp and Alfvén continua are uncoupled so that there is a relatively simple variational principle for each continuum. The operator $F = -iF^*$ satisfies the similar property

$$\int \mathbf{v}^* \cdot F(\mathbf{w}) J \, \mathrm{d}\chi = \int [F(\mathbf{v})]^* \cdot \mathbf{w} J \, \mathrm{d}\chi - i[\mathbf{v}^* \cdot \mathbf{w}], \tag{61}$$

where * denotes the complex conjugate. For functions $\mathbf{v}$ and $\mathbf{w}$ that are periodic in χ, the second term in the right-hand side of Equation (1) vanishes. We then define an inner product in the Hilbert space of one-dimensional functions $\tilde{V}(\chi)$ as

$$(\tilde{\mathbf{V}}_1, \tilde{\mathbf{V}}_2) = \pi \int \tilde{\mathbf{V}}_1 \cdot \tilde{\mathbf{V}}_2 J \, \mathrm{d}\chi, \tag{62}$$

and write Equations (59)–60) in the following form:

$$A(\psi) \cdot \tilde{\mathbf{V}}(\chi) = \sigma^2 \tilde{\mathbf{V}}(\chi). \tag{63}$$

We use (63) to show that

$$
\begin{aligned}
(\tilde{\mathbf{V}}_1, A(\tilde{\mathbf{V}}_2)) &= \pi \int \rho \tilde{\mathbf{V}}_1 \cdot A(\tilde{\mathbf{V}}_2) J \, \mathrm{d}\chi \\
&= \pi \int \left\{ \frac{1}{4\pi} [F(\xi_{y1})]^* [F(\xi_{y2})] + \left[\frac{\rho c^2}{c^2 + V_{\mathrm{A}}^2} N_\chi^2 + \right. \right. \\
&\quad \left. + \frac{\rho}{J B_\chi^2} \frac{\partial}{\partial \chi} \left(\frac{V_{\mathrm{A}}^2}{c^2 + V_{\mathrm{A}}^2} \frac{1}{J} \frac{\partial \Phi}{\partial \chi} \right) \right] \xi_{\chi 1}^* \xi_{\chi 2} + \\
&\quad \left. + \frac{\rho c^2 V_{\mathrm{A}}^2}{c^2 + V_{\mathrm{A}}^2} \left[F\left(\frac{\xi_{\chi 2}}{B_\chi} \right) \right] \left[F\left(\frac{\xi_{\chi 1}}{B_x} \right) \right]^* \right\} J \, \mathrm{d}\chi \\
&= \pi \int \rho A(\tilde{\mathbf{V}}_1)^* \cdot \tilde{\mathbf{V}}_2 J \, \mathrm{d}\chi \\
&= (A(\tilde{\mathbf{V}}_1), \tilde{\mathbf{V}}_2),
\end{aligned}
\tag{64}
$$

where $\tilde{\mathbf{V}}(\chi) = [\xi_y, \xi_\chi]^t$. The operator $A(\psi_0)$ on a given flux surface $\psi = \psi_0$ is Hermitian and this enables us to formulate a variational principle for the continuum frequencies:

$$
\begin{aligned}
\sigma^2(\psi) &= \frac{(\tilde{\mathbf{V}}, A\tilde{\mathbf{V}})}{(\tilde{\mathbf{V}}, \tilde{\mathbf{V}})} \\
&= \left[\int \left\{ \frac{1}{4\pi} |F(\xi_y)|^2 + \frac{\rho c^2 \mathcal{N}_\chi^2}{c^2 + V_{\mathrm{A}}^2} |\xi_\chi|^2 + \frac{\rho}{J B_\chi^2} \frac{\partial}{\partial \chi} \left(\frac{V_A^2}{c^2 + V_{\mathrm{A}}^2} \frac{1}{J} \frac{\partial \Phi}{\partial \chi} \right) |\xi_\chi|^2 + \right. \right. \\
&\quad \left. \left. + \frac{\rho c^2 V_{\mathrm{A}}^2}{c^2 + V_{\mathrm{A}}^2} \left| F\left(\frac{\xi_\chi}{B_\chi} \right) \right|^2 \right\} J \, \mathrm{d}\chi \right] \Big/ \int \rho \{ |\xi_y|^2 + |\xi_\chi|^2 \} J \, \mathrm{d}\chi.
\end{aligned}
\tag{65}
$$

The Alfvén continuum and the cusp continuum are uncoupled and we can derive a variational expression for each of them. For the frequencies in the Alfvén continuum we have, with $\xi_y \neq 0$ and $\xi_\chi = 0$,

$$
\sigma_{\mathrm{A}}^2(\psi) = \int \frac{1}{4\pi} |F(\xi_y)|^2 J \, \mathrm{d}\chi \Big/ \int \rho |\xi_y|^2 J \, \mathrm{d}\chi, \tag{66}
$$

and for the frequencies in the cusp continuum, we find, with $\xi_y = 0$, and $\xi_\chi \neq 0$,

$$
\begin{aligned}
\sigma_c^2(\psi) &= \int \left\{ \frac{\rho c^2}{c^2 + V_{\mathrm{A}}^2} \mathcal{N}_\chi^2 |\xi_\chi|^2 + \frac{\rho}{J B_\chi^2} \frac{\partial}{\partial \chi} \left(\frac{V_{\mathrm{A}}^2}{c^2 + V_{\mathrm{A}}^2} \frac{1}{J} \frac{\partial \Phi}{\partial \chi} \right) |\xi_\chi|^2 + \right. \\
&\quad \left. + \frac{\rho c^2 V_{\mathrm{A}}^2}{c^2 + V_{\mathrm{A}}^2} \left| F\left(\frac{\xi_\chi}{B_\chi} \right) \right|^2 \right\} J \, \mathrm{d}\chi \Big/ \int \rho |\xi_\chi|^2 J \, \mathrm{d}\chi.
\end{aligned}
\tag{67}
$$

Thus, the Alfvén continuum is always on the stable side of the spectrum, but the cusp continuum can have negative values when

$$\frac{c^2}{c^2 + V_A^2} \mathcal{N}_\chi^2 + \frac{1}{JB_\chi^2} \frac{\partial}{\partial \chi}\left(\frac{V_A^2}{c^2 + V_A^2} \frac{1}{J} \frac{\partial \Phi}{\partial \chi}\right) < 0. \tag{68}$$

Note also that $\mathcal{N}_\chi^2$ is not necessarily a non-negative number. An unstable stratification of density, pressure, gravity and magnetic field along magnetic field lines leads to an unstable cusp continuum. The present conclusion about the stability of the continuous spectrum is similar to the conclusion by Hellsten (1979) who also found that the Alfvén continuum is stable but that the cusp continuum can be unstable due to the unstable perpendicular and parallel pressure distributions along the field lines.

Let us consider the incompressible limit ($c^2 \to \infty$). The Equations (57) and (59) for the Alfvén continuum remain unchanged, but Equations (58) and (60) reduce to

$$\frac{B_\chi}{\rho V_A^2} B'_\chi + F^*\left[\frac{1}{\rho(\sigma^2 - N_\chi^2) B_\chi^2} F^*(B_\chi B'_\chi)\right] = 0, \tag{69}$$

$$\sigma^2 \xi_\chi = N_\chi^2 \xi_\chi - \frac{1}{\rho B_\chi} F^*\left[\frac{\rho c^2 V_A^2}{c^2 + V_A^2} F^*\left(\frac{\xi_\chi}{B_\chi}\right)\right], \tag{70}$$

where now

$$N_\chi^2 = -\frac{1}{J^2 B_\chi^2} \frac{1}{\rho} \frac{\partial \rho}{\partial \chi} \frac{\partial \Phi}{\partial \chi}.$$

Note that even in the incompressible limit the cusp continuum does not coincide with the Alfvén continuum. The cusp continuum in contrast to the Alfvén continuum which is stable, may become unstable. In the incompressible case, instability occurs when $\partial \Phi / \partial \chi$ and $\partial \rho / \partial \chi$ have the same sign so that along a magnetic field density increases in the direction of gravitational acceleration. This instability resembles the classical Rayleigh–Taylor instability of a heavier fluid on top of a lighter one.

Let us now turn back to Equations (50) and (51). Equations (50) and (51) are two uncoupled second-order differential equations and the solutions have no longer the classical polarization properties in the magnetic surfaces. The solutions have mixed properties and the continua are coupled. This coupling is caused by the component of the magnetic field in the ignorable direction, i.e. B_y. This can easily be understood if one compares the corresponding Equations (50)–(51) to (57)–(58) and notes that $B'_\parallel = B_\chi B'_\chi$ and $B'_\perp = B_\chi B'_y$ in the case of a purely poloidal magnetic field. If the solutions were to have the classical polarization properties and the continua to be uncoupled, then Equation (50) could only contain terms in $B'_\perp$ and Equation (51) only terms in $B'_\parallel$. But Equation (50) contains also terms in $B'_\parallel$ that are proportional to B_y, and conversely Equation (51) contains also terms in $B'_\perp$, that are proportional to B_y. Equations (50) and (51) are coupled through expressions proportional to B_y. The degree of coupling depends on the variation of the equilibrium quantities in the magnetic surfaces. The names 'Alfvén continuum' and 'cusp continuum' are here also meaningful when the coupling is weak so that the modes are indeed polarized almost purely perpendicular and almost purely parallel to the magnetic field lines. The classical Alfvén and cusp continua are coupled and this coupling persists even in the incompressible limit as can be seen by taking the limit $c^2 \to \infty$ in Equations (50) and (51). The continuous spectrum is affected by gravity and this implies that the continuous spectrum can be unstable.

4. Special Case

We now consider a uniform poloidal magnetic field of arbitrary direction,

$$\mathbf{B} = (B_x, 0, B_z), \tag{71}$$

in a plasma that is only stratified in the vertical direction, so that density, pressure and the component of gravitational acceleration are only functions of height z. Since B_x and B_z are constant, the magnetic flux function $\psi(x, z)$ is given by a linear expression in x and z:

$$\psi(x, z) = -xB_z + zB_x. \tag{72}$$

The magnetic surfaces $\psi(x, z) = C$ are parallel planes and their sections with the xz-plane are parallel straight lines as indicated in Figure 1. $\chi(x, z)$ is also a linear expression in x and z and a choice for $\chi(x, z)$ can be

$$\chi(x, z) = xB_x + zB_z. \tag{73}$$

The surfaces $\chi(x, z) = C$ are again parallel planes and their sections with the xz-plane are parallel straight lines as indicated in Figure 1.

From now on we confine our discussion to the xz-plane. The magnetic field lines are parallel straight lines. The equilibrium quantities only depend on height, and so at a given height an equilibrium quantity, like density, takes the same value on different field lines. The variation of the equilibrium quantities along a magnetic field line is independent of the magnetic field line considered. Since all equilibrium quantities are independent of x we can Fourier decompose the perturbed quantities also with respect to x and make them proportional to exp $(ik_x x)$. The operator F^* applied to a perturbed quantity takes the form

$$F^* = B_x ik_x + B_z \frac{\mathrm{d}}{\mathrm{d}z}, \tag{74}$$

so that the derivative along a field line is independent of the field line considered. In summary, the variation of the equilibrium quantities along a magnetic field line and the operator F^* are independent of the field line considered. The nonsingular eigenvalue problem (55) is the same for all magnetic field lines and there is no continuous spectrum.

The non-existence of the continuous spectrum for a uniform magnetic field with an arbitrary direction can also be made clear in the following way. Take as an example

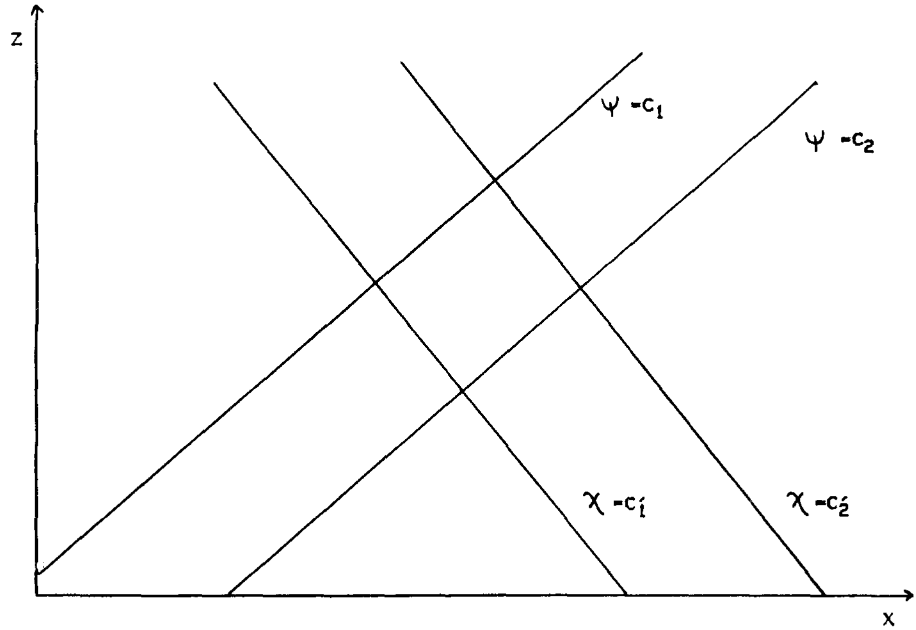

Fig. 1. Lines of constant ψ and constant χ for a uniform poloidal magnetic field.

Equation (59) for the Alfvén continuum and use Equation (57) to rewrite it as

$$\sigma^2 \xi_y = \frac{1}{4\pi\rho}\left(k_x^2 B_x^2 \xi_y - 2ik_x B_x B_z \frac{d\xi_y}{dz} - B_z^2 \frac{d^2\xi_y}{dz^2} \right). \tag{75}$$

Equation (75) is independent of the magnetic field line considered and the discrete eigenvalues are independent of the magnetic field line and do not spread out a continuous spectrum as the magnetic field line $\psi = \psi_0$ is varied.

In summary, there is no continuous spectrum for a vertically stratified plasma with uniform poloidal magnetic field because the variations of the equilibrium quantities along a magnetic field line are the same for all magnetic field lines. An equilibrium in which the variations of the equilibrium quantities along a magnetic field line, are identical for all magnetic field lines, is probably not often a good approximation of reality. The non-existence of the continuous spectrum for a uniform magnetic field with an arbitrary direction in a 1D equilibrium cannot be used to argue that the continuous spectrum has no physical relevance.

5. Conclusion

The continuous spectrum has been derived for a 2D magnetostatic equilibrium with y-invariance. The continuous spectrum arises because of the different behaviour of the equilibrium quantities in different magnetic surfaces. The continuous spectrum of a magnetostatic equilibrium with a purely poloidal magnetic field consists of two uncoupled parts which on the basis of the polarization properties of their solutions can be identified as an Alfvén continuum and a cusp continuum. The Alfvén continuum is stable, but the cusp continuum can be unstable and in the compressible limit the instability resembles the Rayleigh–Taylor instability of a heavier fluid on top of a lighter one. The continuous spectrum of a magnetostatic equilibrium with a mixed poloidal and toroidal equilibrium has solutions that are no longer polarized purely perpendicular or purely parallel to the magnetic field lines but that do show mixed properties. The coupling of the classical Alfvén and cusp continuum is due to the toroidal magnetic field and the degree of coupling depends on the variation of the equilibrium quantities in the magnetic surfaces.

The special case of a uniform poloidal magnetic field in a 1D magnetostatic equilibrium that is stratified with height, has been considered in detail. It is shown that this equilibrium has no continuous spectrum. This result cannot however be used to argue that the continuous spectrum has no physical relevance since it has not any general validity as it is completely due to the artificial property that all field lines are identical in the sense that the behaviour of the equilibrium variables along a field line is independent of the field line considered.

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