LARGE-SCALE STRUCTURE OF THE INTERPLANETARY MEDIUM

I: *High Coronal Source Longitude of the Quiet-Time Solar Wind*

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Abstract. We propose that the coronal source longitude and latitude of solar wind plasma can be estimated within $\sim 10°$. Previous writers have argued that the solar wind in the ecliptic should originate near the equator and that a quasi-radial hypervelocity (QRH) approximation (constant radial flow) is valid beyond the magnetohydrodynamic critical points. We demonstrate that an extension of the QRH approximation (as if the solar wind flowed radially with constant velocity from the center of the Sun) yields a proper estimate of the high coronal source location at the 'release zone' where the solar wind makes its transition to radial interplanetary flow. This 'extrapolated' QRH (or EQRH) approximation succeeds because the two main corrections to this source estimate, coronal corotation and interplanetary acceleration, tend to cancel (the former correcting the source location eastward, the latter westward). Although this 'ideal spiral' approximation was first suggested by Snyder and Neugebauer (1966), only recently has it been demonstrated that it relates a wide range of interplanetary plasma, magnetic field and energetic particle data to observed coronal magnetic structure. We estimate quantitatively the error in the EQRH approximation by comparison with steady-state streamlines predicted by azimuthally independent and dependent theoretical solutions to the steady-state plasma equations. We find the error in both cases $\leqslant 10°$ in longitude and therefore suggest that the EQRH approximation offers the means to relate observed solar 'initial conditions' in the 'release zone' directly to interplanetary measurements. If, in addition, the EQRH approximation also leads to agreement with *low* coronal structure, then there should be a straightforward correspondence to otherwise unobservable *high* coronal structure.

1. Introduction

To the plasma physicist, the solar wind and interplanetary magnetic field appear as a medium of inexhaustible complexity. To the solar cosmic ray physicist, that medium appears mainly as the matrix material in which the interplanetary magnetic field is embedded, and he is primarily interested in only the configuration of the field on a scale ~ 1 AU since the guiding centers of non-relativistic cosmic rays rather faithfully follow the large-scale field lines (Lin *et al.*, 1968; Fan *et al.*, 1968; Kavanagh *et al.*, 1970; McCracken and Rao, 1970; Roelof and Krimigis, 1973). The cosmic ray physicist with an interest in solar physics therefore is always asking the question "where does this field line on which I see particles at 1 AU connect back to the Sun?" If he is successful in obtaining an estimate of that connection point, he also gains an additional piece of information useful to the solar and interplanetary plasma physicist: to the extent that the approximation of 'frozen-in' field lines holds, the coronal 'source' longitude of the field line is also that of the solar wind plasma through which it threads. At the current state of the art, it seems that all interested parties would be pleased to be able to identify coronal field and/or plasma source regions with an accuracy of $\sim 10°$ in latitude and longitude.

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In this paper, we advocate a simple technique for estimating the high coronal quiet-time solar wind source locations. This technique, which we have called the extrapolated quasi-radial hyperverlocity (EQRH) approximation (Section 2), consists of extrapolating instantaneous ideal spirals to the center of the Sun to determine the source longitude of the solar wind. Although this technique obviously does *not* produce an exact representation of solar wind *streamlines* even at quiet times (due to the effects of coronal corotation, interplanetary acceleration and stream interactions), we shall show that the high coronal *source* longitude is given correctly within $\sim 10°$. As we discuss below, it then requires direct comparison with solar data to establish the correspondence (if any) between the high and low coronal source longitude. We offer mathematical justification for the approximation (based on recent theoretical analyses of the solar wind), showing that its accuracy results from the tendency toward cancellation of the two most important deviations from the simple estimate.

In a companion paper (Nolte and Roelof, 1973a, hereafter called Paper 2) we shall extend the approximation to the general condition in which the plasma velocity from a given source longitude is varying with time (so that the solar wind/interplanetary magnetic field pattern is not one that simply 'corotates' with the Sun's angular velocity). We shall also make a comparison of the predictions with plasma and particle data from three spacecraft spread over nearly a radian in longitude near 1 AU.

Observational justification of our method for estimating source longitude and latitude has come from direct comparison with solar data. If structure in a wide range of interplanetary particle, plasma and field data, when mapped back to the estimated connection points, is well-ordered by observed *low* coronal magnetic features, then we believe that the approximation is validated because in such cases (1) there must be a simple relationship between low and high coronal structure and (2) the EQRH high coronal connection longitudes must be accurate. Furthermore, even when the agreement with low coronal structure is not present, it is plausible that high coronal EQRH longitudes are just as accurate but the lack of agreement is due to non-radial magnetic structure in the i nner corona since our mathematical analysis of interplanetary flow is independent of conditions in the inner corona.

Working with P. S. McIntosh of NOAA/ERL, we and our colleagues have compared interplanetary data with equatorial chromospheric magnetic neutral-line structure using his method of inferring the neutral lines from Hα filtergrams of the Sun. He has given a recent review of the technique (McIntosh, 1972). We present briefly the results of several analyses below. The interested reader is referred to these papers for detailed results of the studies.

The EQRH approximation has been used recently by Krieger *et al.* (1973) to identify a magnetically open structure (a coronal 'hole' observed at the solar equator in X-ray images of the lower corona) as the coronal source of a recurrent high velocity solar wind stream. This identification is consistent with the suggestions (Parker, 1963; Billings and Roberts, 1964; Wilcox, 1968) that the interplanetary solar wind tends to come from weak field regions on the Sun, and supports the hypothesis that there is no close correlation between solar wind streams and solar active regions. Krieger *et al.*

(1973) hypothesize that the occurrence of open magnetic structures has been a dominant factor in the evolution of solar wind structure during the present cycle.

The EQRH approximation has also been used recently by Roelof and Krimigis (1973), again assuming that solar wind in the ecliptic originates near the solar equator, to determine the connection longitudes of the interplanetary field. These connection points have been used to correlate low energy ($\geqslant 0.3$ MeV) solar proton observations with chromospheric magnetic features photographed in the Hα line and mapped onto synoptic charts by P. S. McIntosh. The time histories of these events, as observed by three spacecraft (Mariners 4 and 5 and Explorer 35) separated by 0.1 AU (Krimigis *et al.*, 1971) can be understood in terms of an equatorial coronal injection profile, organized by the large-scale coronal magnetic field. In this picture the particle flux seen by a particular detector at any time is determined by the coronal injection at the connection longitude of the detector. An extended discussion of this work, oriented more toward solar physics, may be found in Roelof (1973).

McIntosh and Roelof (1972) have also used the EQRH approximation to compare the interplanetary magnetic field polarity with the large-scale chromospheric polarity, as inferred from structures photographed in the Hα line. This comparison yielded a correlation (see Figure 3 of Roelof, 1973) strongly peaked near the solar equator (0°–N10°), supporting the hypothesis of similar chromospheric and high coronal magnetic structure during July–September 1967 when equatorial polarities were well-defined.

In a similar study, Nolte and Roelof (1973b) have compared the interplanetary magnetic field polarity measurements from Mariner 4 (Coleman *et al.*, 1967) with those in the chromosphere during early 1965 (near solar minimum). Based on data from twelve consecutive solar rotations, the equatorial correlation in 1965 is low (consistent with the weak equatorial neutral line structure at that time), while significant correlation ($\chi^2 > 20$) is obtained with the more clearly defined polarity structure at $\sim \pm 25°$ which therefore must have dominated the equatorial field of the outer corona at this time since the observed wind beyond the magnetohydrodynamic critical points must originate near the equator. However, in a further study of this period when only times in which 0.5 MeV solar protons were present were considered, the correlation coefficient peaked significantly only at the *equator*, implying that particles were released preferentially when the interplanetary field was well-connected to the inner corona.

The ordering of interplanetary data by the EQRH mapping demonstrates the desirability of further theoretical examination of this approximation in order to: (i) understand how this simplified approximation can give connection points which produce consistent association of chromospheric and interplanetary structures with an accuracy $\sim 10°$ in solar longitude, even though the approximation gives neither an exact representation of solar wind streamlines nor the magnetic structure inside the magnetohydrodynamic critical points; and (ii) generalize the approximation to disturbed (time-dependent) solar wind conditions.

In this paper we shall show that this approximation provides a good estimate for the connection points of the quasi-steady solar wind. We discuss deviations of the quasi-steady solar wind flow from the constant-speed radial-velocity ideal spirals in Section 2. In Section 3 we shall show that the effects of the two major quiet-time deviations of the solar wind streamlines from the EQRH approximation streamlines (corotation of the solar wind with the Sun within the magnetosonic critical points, and acceleration beyond them) tend to cancel, so that the connection points determined by the EQRH approximation are accurate within $\sim 10°$ in solar longitude during quiet times. We also show how the observational agreement of high and low coronal source longitudes is not necessarily inconsistent with observed low coronal non-radial structure. In Section 4 we shall discuss the applicability of the EQRH approximation to quasi-stationary stream-stream interaction periods. We generalize the approximation to the time-dependent solar wind in Paper 2.

2. The EQRH Approximation

The connection-point estimate we use was first proposed in the analysis of the first extended solar wind plasma data that was obtained by spacecraft. Snyder and Neugebauer (1966) attempted to locate sources of high speed solar wind streams, using the assumption that the solar wind speed is constant and the velocity is radial from the Sun to the point of observation ('ideal spirals'). Their failure to identify the sources consistently with calcium plage regions on the Sun led them to conclude:

> The data seem to indicate that there is no close correlation between the plage regions and the solar streams, unless either (1) the velocity is not constant, or (2) the high-velocity plasma is not shot out from the Sun in a radial direction, so that the simple Archimedes-spiral model is incorrect.

We can equally logically draw the converse conclusion: if the assumption of radial solar wind velocity at constant speed *is* a good approximation for solar wind propagation, then there is no close correlation between solar wind streams and solar active regions. Indeed, this latter interpretation has been borne out by recent observations (Krieger *et al.*, 1973) that locate a stream source *between* active regions.

The question of the validity of the constant speed radial velocity approximation can be clarified by consideration of the results of two recent theoretical studies. Sakurai (1971) and Matsuda and Sakurai (1972) have shown that in the quasi-radial hypervelocity (QRH) approximation, the solar wind propagates radially beyond the critical points, at constant speed. The QRH-approximation consists of the assumption that the sonic and Alfvén Mach numbers are large and that the effects of gravitational potential and azimuthal convection are negligible. Based on the estimates of Weber and Davis (1967) for the critical points, and also on the extrapolation of observations near the orbit of the Earth, Matsuda and Sakurai (1972) conclude that the QRH-approximation assumptions are valid only for radial distances larger than $30\,R_{\odot}$. Thus, solar wind streamlines within $30\,R_{\odot}$ are not described by the constant radial velocity approximation.

The method for determining the source longitude of solar wind plasma which we are advocating here consists of extrapolating the QRH approximation to $r=0$. This 'extrapolated' QRH (EQRH) approximation does *not* describe solar wind streamlines near the Sun. The two major quiet-time deviations of the solar wind streamlines from the EQRH approximation streamlines are first, the transfer of angular momentum to the solar wind by the magnetic field near the Sun, so that the solar wind maintains an angular momentum comparable to rigid rotation inside the Alfvénic critical point (Parker, 1969), and second, radial acceleration of the solar wind beyond the critical radius. We shall show in Section 3 that these two effects tend to cancel, so that the EQRH approximation provides a reasonably accurate estimate of the solar source longitude of the quiet-time solar wind measured near 1 AU, even though it does not accurately describe the streamlines near the Sun even during periods without interplanetary stream-stream interactions.

The EQRH approximation may also provide a first estimate for the source longitude of plasma near a quasi-stationary (corotating) stream-stream interaction front, since Hundhausen (1973b) has shown that the acceleration of the slow stream and deceleration of the fast stream are localized within the interaction region. The accuracy of the EQRH approximation source longitudes near stream interactions will be investigated in Section 4.

Implicit in the approximation of radial solar wind velocity is the assumption that solar wind observed by a spacecraft originated beyond the MHD critical points at the sub-satellite latitude. This assumption is validated by the work of Siscoe and Finley (1969) who have investigated the latitude dependence of the solar wind using a linearized perturbation technique. They conclude that solar wind in the ecliptic originates within 3 to 12° of the subterral latitude. In another study, including azimuthal dependence also (Siscoe and Finley, 1970), they conclude that the effect of latitudinal divergence is very small. On the basis of their work, we shall assume that quiet-time solar wind measured by a satellite originated within 10° of the sub-satellite solar latitude. Thus, the EQRH approximation for determining solar source locations consists of extrapolating a solar wind velocity measurement to the sub-satellite solar latitude, assuming constant radial velocity for the propagation from $r=0$.

A final introductory point requiring discussion is the 'random walk' of interplanetary field lines based on the 'random walk' of photospheric fields (Leighton, 1964) proposed some years ago (Michel, 1967) and invoked by Jokipii and Parker (1968) to explain the wide longitude spread of low energy (<1 MeV) solar protons associated with active regions (Fan *et al.*, 1968). If such a random walk were extensive (a rms displacement >10°), then clearly even if the EQRH approximation gives the correct source longitude for the plasma, it is not necessarily the source longitude for the large-scale field. However, Kavanagh *et al.* (1970) pointed out that Jokipii and Parker (1968) over-estimated the effect (from Leighton's analysis) by at least a factor of four. The *observational* support for the effect is limited and subject to interpretation. The power at low frequencies in the spectrum of the magnetic irregularities (to which such a random walk would contribute) has recently been revised downward (Fisk and Sari, 1973).

Direct observational evidence *contrary* to extensive random walk of interplanetary field lines has been found by several workers. Lin *et al.* (1968) established that there was spatial structure in low-energy solar particle (LESP; protons $\lesssim 1$ MeV, electrons ~ 50 keV) events on the scale of $\leqslant 6°$ in longitude. Anderson (1969) pointed out that meandering of field lines from one active region to another was inconsistent with LESP data. McCracken and Rao (1970) in their extensive review of LESP phenomena agree with Anderson and base their analysis of long-lived low-energy events on the discussion of Lin *et al.* (1968) which assumed LESP quasi-stationary longitude profiles are mapped out from the corona along interplanetary field lines. McCracken and Rao state:

> It is difficult to use exactly the same model of localized injection, near-Sun diffusion and Sun–Earth propagation along meandering field lines to fit observations (of prompt and delayed solar particle events).

They then propose a two-region coronal propagation model which obviates the need for interplanetary random walk of field lines by introducing a low-coronal transverse diffusing region.

Evidence against random walk of field lines from LESP observations from well-separated spacecraft has steadily increased. Krimigis *et al.* (1971) point to specific observations in 1967 (see pp. 5931, 5934, and 5944 in particular), and Roelof and Krimigis (1973), after examining 80 days of continuous multispacecraft data make the strong introductory statement:

> *Low energy solar particles cannot cross interplanetary field lines to any measurable extent and there is no indication of 'random walking' of field lines; hence particle flux populations should be traceable back along field lines to the solar longitude of [their interplanetary injection].* (Italics in the original.)

In the same paper, these authors show that when the particle fluxes are mapped back to the Sun using the EQRH approximation, well-defined spatial structures in the LESP histories consistently agree with chromospheric Hα structures within 10° over three consecutive solar rotations. Therefore their study presents direct observational evidence against significant random walking of interplanetary field lines and for the validity of the EQRH approximation.

3. The Quiet-Time EQRH-Approximation

The first step in our analysis is the justification of the EQRH-approximation for the determination of quiet-time solar connection points of the interplanetary magnetic field. In this section we are not including effects due to azimuthal variations since Urch (1972) has argued that the mean flow parameters are well represented by a spherically symmetric model. We shall therefore consider (for the discussion in this section) a spherically symmetric model for average flow conditions.

We assume that the large-scale configuration of a quiet-time interplanetary field line originating at a given equatorial longitude is the locus of all elemental volumes of plasma from the same equatorial solar longitude. In the 'frozen-in' approximation, each field line is then uniquely specified by its connection longitude. Superimposed upon this large-scale configuration will be smaller-scale (<0.1 AU) irregularities due to interplanetary plasma dynamics, but we assume that the small-scale local structures deform but do not obliterate the large-scale structure determined by the plasma locus. The justification of this assumption must ultimately come from agreement between interplanetary and solar data, and we have cited in the Introduction several studies demonstrating agreement.

In Figure 1 we present the situation schematically, plotting radius vs longitude in Cartesian coordinates as done by Snyder and Neugebauer (1966). Solar plasma starts

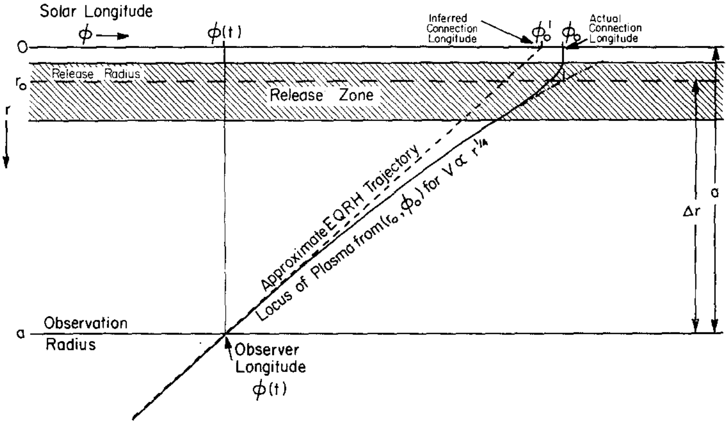

Fig. 1. Comparison of the steady-state EQRH approximation with a model plasma streamline with a radial velocity $V(r) \sim r^{1/4}$ for $r > r_0$. The solar longitude coordinate frame rotates with the Sun (e.g., Carrington) and the radial coordinate r is measured outward from the center of the Sun. The 'release radius' r_0 gives the order of magnitude extent of the 'release zone', the region in which the plasma undergoes the transition from corotation to radial expansion. The plasma is essentially corotating within r_0, and has nearly radial velocity beyond r_0. For plasma observed at radius a and solar longitude $\phi(t)$, the EQRH approximation gives an inferred connection longitude ϕ'_0. Interplanetary acceleration alone would put the actual connection longitude far to the west of ϕ'_0, as shown by the broken extension of the interplanetary curve for $V \sim r^{1/4}$. The coronal corotation of the plasma places the actual connection longitude ϕ_0 back to the east and thus closer to the EQRH estimate ϕ'_0.

expanding outward from the base of the corona, where it is corotating with the Sun and dominated by the (possibly non-radial) inner coronal magnetic fields. As the plasma moves out, it passes through a zone of transition from corotation to nearly radial expansion. We have called the region where this transition takes place the 'release zone'. For convenience in the discussion to follow, we shall characterize the 'release zone' by a 'release radius' r_0, a parameter that gives the order of magnitude extent of the 'release zone'. The interplanetary field far from the Sun should be little changed when the true, more gradual transition is replaced by the assumption of a completely corotating solar plasma within r_0, and radial expansion outside r_0. We emphasize that r_0 is not a radial magnetohydrodynamic critical point in the solution of the hydrodynamic equation for coronal expansion; it is a parameter, characteristic of a particular solar wind streamline that marks the radius beyond which the solar wind is essentially radial. However, as indicated by Parker (1969), r_0 should be of the order of the Alfvénic critical radius, estimated by Weber and Davis (1967) to be $\sim 25\ R_\odot$. A large r_0 is also consistent with the observation of a mean azimuthal velocity of ~ 10 km s^{-1} near 1 AU (Brandt, 1967; Hundhausen *et al.*, 1967; Lazarus and Goldstein, 1971). We shall proceed to show that for r_0 in the range of 0.1 AU to 0.25 AU the EQRH-approximation gives solar connection longitudes within $\sim 10°$.

At this point we must consider the effects of *non-radial* magnetic structure inside the release radius r_0. Such structures certainly exist low in the corona since closed structures are directly observable several tenths of a solar radius beyond the solar surface and may be estimated by potential (current-free) calculations based on photospheric field strength. At heights of several solar radii, the non-radial alignments of coronal streamers over enhancements observed in eclipse photographs directly imply non-radial magnetic structure. Thus even if the EQRH *high* coronal connection longitude at r_0 is accurate, the association of that longitude with *low coronal* features at the *same* longitude might be questioned.

However, it may well be that there is no contradiction between the non-radial magnetic structure of coronal streamers or potential field calculations and the documented successes (listed in Section 1 above) of the EQRH approximation in ordering a wide range of interplanetary and solar data. As we mentioned in Section 1, there is a growing acceptance that solar wind streams in the ecliptic tend to originate in *weak* field regions near the equator, while streamers often originate over coronal enhancements and over a wide range of latitudes. Consequently, the occasional equatorial streamer may come from a distinctly different region than solar wind sources (where the coronal field can be open and nearly radial), and hence the probability of non-radial magnetic structure inside r_0 distorting the longitude associations within a solar wind stream is much reduced. Since solar wind streams are present during the major portion of each solar rotation, most interplanetary data will then correspond to stream sources and not streamer locations.

As for potential field-calculations, the assumption of a current-free corona becomes unduly restrictive beyond a few solar radii because of the acknowledged presence of current sheets in the outer corona which are implied by the sharply defined sector structure in the interplanetary magnetic field and the correlation of sector boundaries with polarity boundaries of the solar field. The effect of such current sheets is to reduce the transverse (non-radial) component of coronal fields that would be predicted by a potential calculation (Schatten, 1972).

Turning these arguments around, we can say that since it has already been established that there are extended periods in which the EQRH connection longitudes successfully correlate interplanetary and equatorial solar data (see Section 1), the simplest implication is that non-radial magnetic structure inside the release zone cannot be dominant (on the average) and there must usually be a straightforward correspondence between high and low coronal structure during those periods.

In order to estimate the effects of interplanetary acceleration on the connection longitude, we take Burlaga's (1967) approximation $V(r) = Kr^{1/4}$ as a fit to Parker's

(1963) theoretical curves for quiet-time coronal expansion as a first approximation to the radial dependence of a typical quiet solar wind stream. The locus of plasma from ϕ_0 has been drawn in Figure 1 beyond r_0 using this assumption.

We can now estimate (using this acceleration function) the Carrington longitude ϕ_0 to which a given interplanetary field line is connected at the release radius r_0 for a relatively undisturbed solar wind.

If a solar wind bulk velocity measurement $V(t)$ is made at time t by a detector at heliocentric radius a, the actual average bulk velocity for this element of plasma, released at time t_0 and radius r_0, from longitude ϕ_0 between r_0 and a is $\bar{V}(t_0) = (a-r_0)/(t-t_0)$. Then the transit time $\Delta t = t - t_0$ for this element of plasma from r_0 to a may be expressed in terms of $V(t)$ as:

$$\Delta t = \Delta r/\bar{V}(t_0) = \{(\Delta r/a)\,[V(t)/\bar{V}(t_0)]\}\,[a/V(t)]. \tag{1}$$

Thus, if the bracketed expression can be approximated, we may estimate the true transit time Δt from the measured quantities a and $V(t)$. Since $r_0 \gg R_\odot$, no consideration of the plasma transit time from the photosphere into the corona is necessary here for an estimate of the transit from r_0 to 1 AU. Unless there are strongly non-radial magnetic fields in the inner corona, the connection longitude at r_0 will then be essentially the coronal connection point relevant for energetic particles and solar wind, due to the corotation of the plasma inside the critical points.

Using the assumption $V = Kr^{1/4}$ we find:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = V\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{K^4}{4V^2}. \tag{2}$$

A first integration from (r_0, V_0) to $[r(t), V(t)]$ gives

$$V(t) = \frac{\mathrm{d}r}{\mathrm{d}t} = (\tfrac{3}{4}K^4 t + V_0^3)^{1/3}. \tag{3}$$

Integrating again:

$$r - r_0 = \frac{1}{K^4}\,[(\tfrac{3}{4}K^4 t + V_0^3)^{4/3} - V_0^4], \tag{4}$$

so the time for propagation from $r = r_0$ to $r = a$ is

$$\Delta t = \left[1 - \left(\frac{r_0}{a}\right)^{3/4}\right]\frac{4a}{3V}. \tag{5}$$

For $r_0 = 0$, this reduces to Burlaga's (1967) result, $\Delta t = 4a/3V$.

With $\phi(t)$ defined to be the Carrington longitude of the solar central meridian observed from the location of the detector at time t, the transit time is also given by:

$$\Delta t = [\phi_0 - \phi(t)]/\Omega, \tag{6}$$

where Ω is the sidereal angular velocity of the Sun.

The EQRH-approximation ϕ_0' (for ϕ_0) is written

$$\phi_0' = \phi(t) + \Omega a / V(t). \tag{7}$$

In this approximation, the bracketed quantity in Equation (1) is estimated to be unity. The error $\delta\phi_0$ introduced by using the approximation in Equation (7) is

$$\delta\phi_0 = \phi_0' - \phi_0 = \Omega\left[\frac{a}{V(t)} - \frac{\Delta r}{\bar{V}}\right] = \Omega(a/\bar{V})\left[\frac{\bar{V}}{V(t)} - \frac{\Delta r}{a}\right]. \tag{8}$$

Note that the two unknown factors in the brackets are both less than one and tend to cancel, since as $\Delta r/a$ increases, so does $\bar{V}/V(t)$ for a weak dependence of solar wind velocity on radius.

For $V(t) = 400\ \mathrm{km\ s^{-1}}$, and $a = 1$ AU, the two cases $(r_0/a) = 0.1$ and 0.25 give respectively $\delta\phi_0 \simeq -6°$ and $+9°$. Thus this simplified analysis leads us to expect that the EQRH-approximation of $\Delta t = a/V$, first used by Snyder and Neugebauer (1966), may give the correct connection longitude ϕ_0 at the release radius r_0 within 10° (during quiet-time coronal expansion). We see this approximation works because the two major deviations of solar wind propagation from the EQRH-approximation streamlines tend to cancel: corotation of the solar wind near the Sun tends to put the EQRH estimate of connection points westward, while interplanetary acceleration alone would put the EQRH estimate to the east of the actual source longitude. Since the accuracy of the EQRH estimate for ϕ_0 is relatively insensitive to the choice of r_0 for this simple model that includes both the effects of corotation within and acceleration beyond the MHD critical points, we conclude that the EQRH-approximation for quiet-time connection points is adequate for studies currently underway that require $|\delta\phi_0| < 10°$.

4. The EQRH-Approximation and the Azimuthally Dependent Solar Wind

In this section we discuss the applicability of the EQRH-approximation near the corotating stream-stream interface in the steady state (time independent) azimuthally varying solar wind. The extension to a time varying solar source, and mapping the large-scale interplanetary field (in the QRH-approximation) from plasma data from widely separated spacecraft will be considered in Paper 2.

The azimuthally varying solar wind has received considerable attention in the last ten years. Early qualitative studies by Sarabhai (1963) and Parker (1963) described various physical effects expected. Dessler (1967) points out that the slow and fast streams should be deflected in the interaction. Carovillano and Siscoe (1969) argued on the basis of a linearized perturbation calculation that these deflections should be less than 10° near 1 AU. Siscoe *et al.* (1969) and Siscoe (1972) report observation of local deflections, indicating that the magnitude of the deflection is up to about 5° within the interaction domain. Since Hundhausen (1973a) argues that the ratio of azimuthal to radial velocity is a maximum near 1 AU, we expect that these deflections will not have a large effect on the validity of the ERQH approximation. We shall return to this point later.

Another analysis of the steady state stream-stream interaction has been performed by Matsuda and Sakurai (1972). They point out that the infinite values of density, magnetic field and azimuthal velocity obtained at the intersection of fast and slow streams in the QRH-approximation are a result of neglecting azimuthal convection. They are thus led to the hypervelocity (H) approximation, which allows azimuthal convection and includes magnetic field terms. Mathematically, these additional terms couple the equations describing the solar wind flow; the high pressure in the compressed region is fed back to alter the shape of the characteristic streamlines of the flow. The physical interpretation is that the plasma is deflected, and the density, magnetic field and azimuthal velocity remain finite in the stream-stream interaction regions. Urch (1972) also concludes that the inclusion of additional terms is necessary to make the QRH-approximation valid in these regions. Since Matsuda and Sakurai concluded that the modified azimuthal profile of V_r was "essentially the same as the case without the magnetic field", it appears that the QRH-approximation is valid in interplanetary space, except in the compression regions in stream-stream interactions.

Thus the EQRH-approximation may be expected to be valid both before the compression region arrives and immediately afterwards (when it provides a good first estimate for the source location of the fast stream in the steady state situation).

For a further test of this conclusion, we have also compared the EQRH-approximation to the theoretical calculations of Goldstein (1971). Using a non-linear numerical calculation, he derived solar wind streamlines and velocity and density profiles as functions of heliographic longitude at various radial distances, assuming sinusoidal perturbations in velocity and density at a source surface at 10 $R_\odot$.

In Figure 2 we compare the EQRH-approximation with his calculated streamlines for the initial radial velocity varying as $V_r=(350+100\cos 4\phi_0)$ km s^{-1} with no initial density perturbation at the source surface. Figure 2a shows this input velocity perturbation at 10 $R_\odot$. We have redrawn Goldstein's calculated streamlines on a $r-\phi$ plot in Figure 2b. The straight lines on this plot are the EQRH-approximation streamlines, shown also in Figure 2c, derived from the values for V_r he calculated at 155 $R_\odot$ (Figure 2d).

As shown in Figure 2b, the solar 'source' longitudes inferred by the EQRH-approximation (at $r=0$) never differ by more than 10° from the actual source longitudes at 10 $R_\odot$. Only at the leading edge of the high velocity stream (the compression region) are the errors greater than 5°; but this is the region where the QRH-approximation has been shown to be somewhat inaccurate (Matsuda and Sakurai, 1972). We conclude that the EQRH-approximation gives a realistic first estimate for the solar connection longitude even in the compression region. Furthermore, Figure 2b suggests that an eastward correction to the EQRH estimate should be made in the connection longitudes for compression regions. We will consider this correction further in Paper 2.

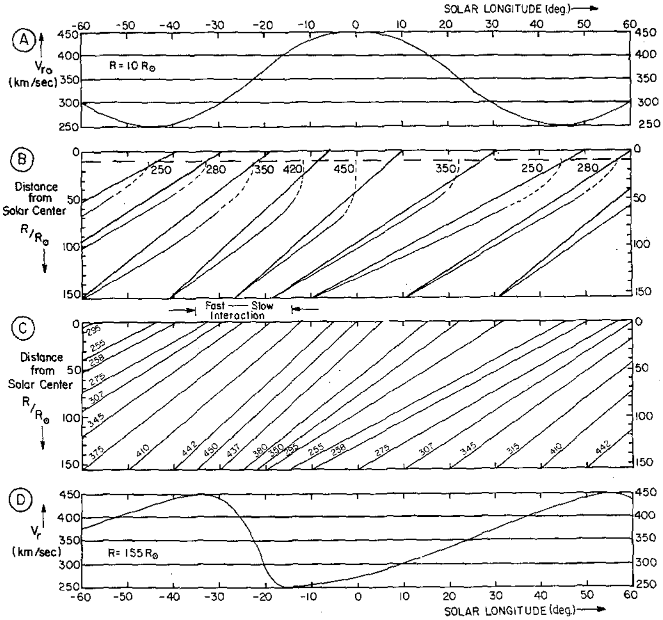

Fig. 2. Comparison of the EQRH approximation for solar plasma source longitude with Goldstein's (1971) calculation for an azimuthally-dependent source velocity: (A) The steady azimuthal profile of the radial component of the source velocity at 10 $R_\odot$. The azimuthal component is chosen to be zero in the corotating reference frame. (B) Goldstein's calculated streamlines, redrawn on a Cartesian plot of r vs ϕ (the heavy and dashed lines) and the EQRH approximation straight lines. (C) The EQRH lines constructed from and labeled by (D) Goldstein's calculated azimuthal profile of the radial velocity at 155 $R_\odot$, plotted vs solar longitude in the frame of reference corotating with the Sun. The EQRH and theoretical source locations agree within 10°.

A further theoretical test of the applicability of the EQRH-approximation is provided by the comparison with Goldstein's calculation which started from an azimuthally-varying density at 10 $R_\odot$. The variation in density as a function of solar longitude at the source surface is shown by the sinusoidal curve in Figure 3a. The source velocity is independent of longitude for this calculation.

We have considered Goldstein's calculated longitudinal profiles in density and radial velocity at 200 $R_\odot$, shown in Figure 3b, to be data, and have used these 'measured' velocities to determine solar connection longitudes in the EQRH-approximation. We then plotted the density ratio, 'measured' at 200 $R_\odot$, as a function of solar connection longitude (the dashed curve in Figure 3a) for comparison with the 'actual' source density. With the exception of the easily-identified (and commonly observed) density ridge preceding the maximum of the high velocity stream, the interplanetary density ratio, mapped back to the Sun using the EQRH-approximation, agrees quite well with the source density ratio. We therefore suggest that the EQRH-approximation can be used to map interplanetary density profiles back to their approximate source *locations* (if it can first be established that they are solar in origin).

Similarly, as shown above (Figure 2), an observed perturbation in the velocity

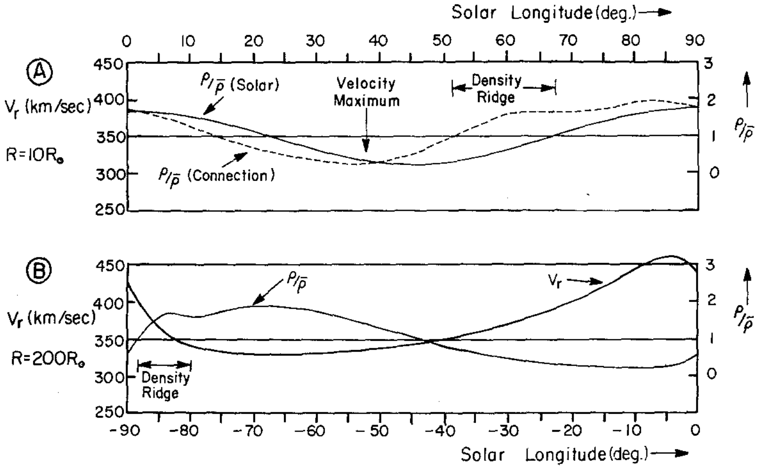

Fig. 3. Comparison of the EQRH approximation with Goldstein's (1971) calculation for an azimuthally-dependent source density. Read velocity on left-hand scale and density (expressed in a ratio to the mean density $\bar{\rho}$ averaged over longitude) on the right-hand scale. Solar longitude (ϕ) is measured in a frame rotating with the sun (e.g., Carrington). (A) The azimuthal profiles of the source density ratio $\rho/\bar{\rho} = 1 + 0.75 \cos 4\phi$ (solid curved line) and radial velocity (constant at 350 km s^{-1}) at 10 $R_{\odot}$. (B) The calculated azimuthal profiles of density (light curve) and radial velocity (heavy curve) at 200 $R_{\odot}$. These profiles were used as the 'data' for the comparison. The velocities were used to determine the source longitudes of the observed densities in the EQRH approximation. The 'observed' density ratios are plotted vs their EQRH source longitudes as $\rho/\bar{\rho}$ (Connection) in (A) (the dashed line). The maximum and minimum of $\rho/\bar{\rho}$ agree within 10°, even without removal of the 'density ridge' just prior to the velocity maximum (the result of stream-stream interaction rather than an indication of source density).

profile can be mapped back to its solar source. In either case, we do *not* claim that the mapped-back profile in density or velocity is the exact coronal profile in density or velocity. However, the EQRH-approximation can be used to identify the *solar source* location of the observed perturbations. It was in this sense that Krieger *et al.* (1973) identified the source of a recurrent high speed solar wind stream and independently deduced coronal conditions at the source from X-ray images of the solar corona. Since Krieger *et al.* (1973) demonstrated that the EQRH mapping of the stream back to its likely source in a coronal 'hole' was accurate to within 10°, this independent observational determination of the physical conditions at the source of the stream together with the observation of the stream near 1 AU provide specific boundary conditions both near the Sun and near 1 AU for any model of solar wind propagation.

We present the results of our final comparison of the EQRH approximation with a theoretical calculation in Figure 4. This calculation was also made by Goldstein (1973, private communication), who has extended his numerical technique to include the effects of the magnetic field on a two-component plasma. For this calculation, he has assumed a 'source surface' at 0.1 AU, and also has neglected the effects of latitudinal divergence. At this 'source' radius, the number density is chosen to be ~ 700 cm^{-3}, the magnetic field is ~ 300 γ, and radial, the azimuthal velocity is as-

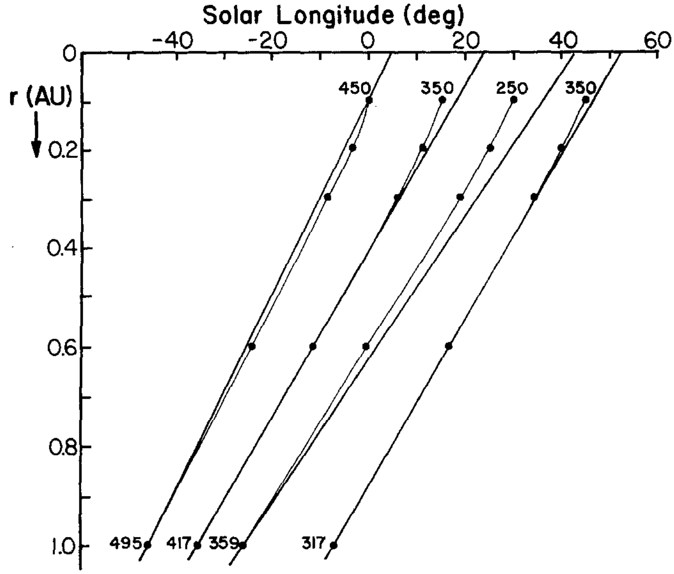

Fig. 4. Comparison of the EQRH approximation for solar wind plasma source longitude with Goldstein's (1973, private communication) calculated streamlines, including interplanetary effects of the magnetic field. The plotted points are calculated points on the streamlines, which have been drawn in as light curved lines from the 'source surface' (0.1 AU) to 1 AU. The heavier straight lines are the EQRH lines drawn using the 'observed' velocities indicated at 1 AU. Source velocities are marked near 0.1 AU. The differences between the EQRH source location estimates (at $r = 0$) and the 'actual' source locations (at $r = 0.1$ AU) are less than 10° except for 12° for the slowest stream, due to interplanetary deflection. This stream also shows the largest interplanetary acceleration.

sumed to be ~ 44 km s^{-1} (rigid corotation), radial velocity varies as $(350 + 100 \cos 6\phi)$ km s^{-1}, electron temperature is chosen to be $\sim 8 \times 10^5$ K and nearly uniform, and ion temperature varies from 5–8×10^5 K, in phase with the radial velocity variation. At 1 AU, the calculated number density ranges from 4 to 10 cm^{-3}, the radial and azimuthal components of the magnetic field range from 2 to 5 γ (in phase), azimuthal velocity from -20 to 40 km s^{-1}, radial velocity from 360 to 500 km s^{-1}, electron temperature from 1.3 to 2×10^5 K, and ion temperature ranges from 5 to 9×10^4 K. The slightly curved light lines in Figure 4 are Goldstein's calculated streamlines; the straight heavier lines are the EQRH lines drawn from the 'observed' radial velocities, shown at 1 AU. The source radial velocities are labeled at 0.1 AU.

As Matsuda and Sakurai (1972) also found, the inclusion of magnetic field terms in the calculation results in greater azimuthal convection of the solar wind plasma. In Goldstein's calculation, this is most noticeable in the slow stream where the azimuthal velocity remains near 40 km s^{-1} all the way from 0.1 AU to 1.0 AU. The result of this deflection is that the EQRH estimate for the source of this slow region is in error (to the west) by 12°. Note also that this slow stream is accelerated by the stream-stream interaction. This additional interplanetary acceleration causes the EQRH line drawn from the observed velocity to have a steeper slope relative to the actual average velocity of the stream from 0.1 AU than the lines for the faster streams. This shifts the EQRH estimate for the connection point to the east, partially cancelling the effects of the deflection. Similarly, also as anticipated by Siscoe *et al.* (1969), the fast stream is deflected toward the East and decelerated in the interaction. The deceleration implies that the EQRH line has too small a slope, so that the connection point estimate is moved back to the West, again in the opposite direction of the effects of the deflection. Thus, just as in the case of the quiet azimuthally symmetric solar wind, the EQRH approximation produces a better estimate for the connection longitudes of solar wind near a quasi-stationary stream-stream interaction than might be expected because two effects tend to cancel.

The errors in the EQRH estimates of the sources of all streams except the slowest are between 5° and 8°. Thus this comparison with Goldstein's calculation including interplanetary magnetic field also suggests that the EQRH approximation provides an estimate for the source location of the quiet-time solar wind accurate within 10°, and additionally a reasonable first estimate for the source of solar wind in a quasi-stationary (corotating) stream-stream interaction. For a discussion of the applicability of the EQRH approximation to non-steady stream interaction periods, the reader is referred to Paper 2.

5. Summary and Conclusions

We have shown that the constant-speed, radial-velocity, or EQRH-approximation, provides a reliable first estimate for the high coronal solar source location of solar wind plasma observed near 1 AU, for the quasi-stationary solar wind. This approximation produces a source location accurate within $\sim 10°$ in solar longitude because the two largest errors (caused by corotation near the Sun and radial interplanetary acceleration) tend to cancel. Thus, although the EQRH connection longitude is the same as that obtained by constructing an Archimedian spiral field line, the actual field line deviates significantly from the spiral near and within the 'release zone' of the plasma (i.e., the vicinity of the magnetohydrodynamic critical points). We have also assumed that observed solar wind came from the sub-satellite solar latitude. This assumption is supported by the theoretical calculations of Siscoe and Finley (1969 and 1970) which indicate that the source latitude is within $\sim 10°$ of the sub-satellite latitude for quiet times. In view of the present uncertainty in the location of the 'release zone' (as well as in the true radial dependence of the solar wind velocity), we feel that the EQRH-approximation provides an estimate for the solar source location of an observed solar wind stream that is sufficiently accurate for contemporary analysis. Therefore, whenever interplanetary data correlate well with low coronal features at the EQRH longitudes, we conclude that there is a straightforward correspondence between high and low coronal structures.

We have provided theoretical justification for the EQRH-approximation in this paper. This justification consists of a comparison of the EQRH connection longitudes to the calculated source longitudes of the solar wind both for Parker's (1963) spherically symmetric model (assuming $V \sim r^{1/4}$) in Section 3 and for Goldstein's quasi-stationary azimuthally varying model in Section 4.

The ultimate justification of this method, however, must come from direct comparison with solar data. The ordering of a wide range of interplanetary data (low energy solar proton data, interplanetary magnetic field polarity, and solar wind velocity and density) by the EQRH mapping reported in several recent studies (see references in Section 1) provides initial indication that the approximation is generally valid for quasi-stationary solar wind structures. Paper 2 contains the extension of the approximation to evolving structures and the use of solar wind data from well-separated spacecraft.

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