SIMULATION OF SOLAR FLARE PARTICLE INTERACTION WITH INTERPLANETARY SHOCK WAVES

M. SCHOLER and G. MORFILL
Max-Planck-Institut für Physik und Astrophysik, Institut für extraterrestrische Physik, 8046 Garching b. München, F.R.G.

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Abstract. In order to study the propagation of solar cosmic rays in interplanetary space a computer program has been developed using a Monte-Carlo technique, which traces the histories of particles released impulsively at the Sun. The particle propagation model considers the adiabatic deceleration during the convective and diffusive transport of the particles, and the model of the interplanetary medium incorporates a radially expanding blast wave which exerts a sweeping action on the particles and accelerates them through the first-order Fermi process. It is shown that energetic storm particle events cannot be simulated by assuming a pure sweeping action of the interplanetary blast wave, but that energization of the particles while reflected at the shock can explain many observed features of such events.

1. Introduction

Large flux enhancements of low energy solar cosmic rays have sometimes been observed several hours before the arrival of interplanetary shock waves near the Earth. These energetic storm particles (ESP) are usually seen only when solar energetic particles from an earlier flare are present in interplanetary space and can greatly alter the intensity-time profiles of a solar particle event. A particular feature common to most ESP events is a steepening of the spectrum at energies $\gtrsim 1$ MeV and an abrupt flux decrease following the passage of the interplanetary shock wave (e.g. Bryant *et al.*, 1962, 1965; Rao *et al.*, 1967; McCracken *et al.*, 1967). In addition, short lasting (~ 10 min) particle enhancements at low energies associated with the shock front itself have been reported by Lanzerotti (1969), Singer (1970), Armstrong *et al.* (1970), Ogilvie and Arens (1971), and Palmeira *et al.* (1971).

Several explanations for the ESP events have been proposed: Modifying the original Axford and Reid (1963) hypothesis van Allen and Ness (1967) suggest Fermi acceleration of energetic particles by scattering between the shock and interplanetary field irregularities far upstream of the shock. Gold (1959) has suggested trapping of solar accelerated particles in a magnetic tongue and Parker (1965a) and Rao *et al.* (1967) propose that the ESP's are accelerated in the shock itself. Other interpretations of energetic storm particle enhancements involve the core-halo injection (Lin *et al.*, 1968) and co-rotation processes (Kahler, 1969); they have, however, been challenged by Lanzerotti (1970) and Datlowe (1972).

Palmer (1972) has proposed a model for ESP events based on the hypothesis that efficient sweeping of the particles by the shock waves results in a 'banking up' of particles ahead of the shock. He developed a Monte-Carlo model, where protons are released impulsively from the Sun and diffuse through the interplanetary medium.

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These particles are reflected or transmitted at the shock wave associated with the solar flare event. The shock wave is released from the Sun at the same time as the particles and reflection is obtained in Palmer's model with an arbitrarily chosen transmission coefficient. In this calculation Palmer (1972) neglects energy changes due to the moving solar wind (adiabatic deceleration) and due to particle reflection at the shock. Fisk (1971) also considered interaction of low energy cosmic rays with interplanetary shocks. His one-dimensional steady-state model considers the passage of the shock through a homogeneous background of particles, which is not the case for an impulsive injection, as in flare events.

In the present study we have developed a Monte-Carlo model following the method given by Jokipii and Owens (1975), which also traces the histories of particles released impulsively at the Sun. The particle propagation model considers the adiabatic deceleration during the convective and diffusive transport of the particles and the model of the interplanetary medium incorporates a radially expanding blast wave. The particles are reflected or transmitted at the blast wave under the assumption of conservation of the first adiabatic invariant (magnetostatic reflection). Thus the blast wave exerts a sweeping action on the particles and, in addition, accelerates them through the first order Fermi process. It is shown that Fermi acceleration is by far the most important process for producing ESP enhancements, completely dominating the simple sweeping action of the shock.

2. The Monte-Carlo Method and the Interplanetary Environment

In order to investigate the interaction of solar cosmic rays with interplanetary shock waves we assume that the cosmic ray omnidirectional intensity U is a spherically symmetric function of heliocentric radius r, particle kinetic energy per nucleon T, and time t. In the simplest case of pure radial diffusion the cosmic-ray diffusion equation is (Parker, 1965b; Gleeson and Axford, 1967; Jokipii, 1971):

$$\frac{\partial U}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2 \varkappa_{rr}\frac{\partial U}{\partial r} - r^2 v_{sw} U\right\} + \frac{4}{3}\frac{v_{sw}}{r}\frac{\partial}{\partial T}(TU), \tag{1}$$

where κ_{rr} is the radial cosmic-ray diffusion coefficient and v_{sw} is the solar wind velocity. Assuming κ_{rr} to be independent of r and introducing the new function

$$f = 4\,\pi r^2 U \tag{2}$$

one obtains from Equation (1)

$$\frac{\partial f}{\partial t} = \varkappa_{rr}\frac{\partial^2 f}{\partial r^2} - \frac{\partial}{\partial r}\left[\left\{v_{sw} + \frac{2\,\varkappa_{rr}}{r}\right\} f\right] + \frac{\partial}{\partial T}\left[\left\{\frac{4\,v_{sw}T}{3r}\right\} f\right], \tag{3}$$

where $f\,\mathrm{d}r$ is the number of particles in a spherical shell of thickness $\mathrm{d}r$.

As pointed out by Jokipii and Owens (1975) Equation (3) is identical to the equation obtained from one-dimensional random walk in radius with the random step size given by

$$\Delta r_{\mathrm{diff}} = \pm\, (2\, \kappa_{rr}\, \Delta t)^{1/2} \tag{4}$$

a convection term

$$\Delta r_{\mathrm{con}} = (v_{sw} + 2\, \kappa_{rr}/r)\, \Delta t \tag{5}$$

and an energy change term

$$\Delta T = -\left[\frac{4\, v_{sw} T}{3\, r}\right] \Delta t\,. \tag{6}$$

We therefore adapt the method of Jokipii and Owens (1975) and simulate the cosmic-ray diffusion by a Monte-Carlo process in which a particle's position and energy is followed explicitly in time according to Equations (4) to (6), i.e. a particle of kinetic energy T is released near the Sun (at $r = R_0$) and after each time step the radius and energy are recorded. Since we want to study the interaction of particles with interplanetary shock waves, we will follow the random motion of a 'real' particle, i.e. we choose Δt such that Δr_{diff} equals the mean free path $\lambda = 3\, \kappa_{rr}/v$:

$$\Delta t = 9\, \kappa_{rr}/2\, v^2\,,$$

where v is the particles' velocity. Since this procedure might introduce rather large Δt steps (if κ_{rr} is large), we have to average the convection and energy change term over the particles mean free path, e.g., if the particle moves from r to $r + \lambda$

$$\langle \Delta r_{\mathrm{con}} \rangle = \left\{ \frac{1}{\lambda} \int_{r}^{r+\lambda} (v_{sw} + 2\varkappa_{rr}/r)\, \mathrm{d}r \right\} \Delta t\,.$$

The particle is assumed to mirror without energy loss at the inner boundary and is assumed to have escaped when it reaches a certain outer boundary R_a. Another particle is then released near the Sun, and so on. The distribution function $f = 4\, \pi r^2 U$ is then given by the three-dimensional histogram of the corresponding values of radius, energy and time. Since we are interested in the flux vs time profile near the Earth, we register at what time the particles cross a plane at $r = 1$ AU and we also note the particle energy. The corresponding two-dimensional histogram is then proportional to the flux at 1 AU and can be directly compared with measurements. After having constructed a two-dimensional histogram in energy and time for a specific initial particle energy at the Sun, the whole process is repeated for another initial energy. Finally, by superposing all these histograms, it is possible to construct the flux vs energy and time profile at 1 AU for any injection energy spectrum at the Sun, by weighting the histograms accordingly. The model of the interplanetary medium contains a shock wave released from the Sun simultaneously with the impulsive particle emission. The shock wave is simulated by adapting Parker's (1963) blast wave solution for high Mach number of an explosion into a stationary interplanetary medium with a density ahead of the wave varying as r^{-2}. The blast wave solution is completely specified by the shock velocity V_0 at 1 AU and a parameter λ, which specifies the way in which the total energy ε of the blast wave is increasing with time t according to the relation $t^{3/\lambda-2}$. For simplicity, we have assumed that the solar wind velocity varies linearly between the shock and the following discontinuity or piston, at the rear of the blast wave, on which the enhanced solar corona is pushing. The quiet interplanetary magnetic field is assumed to have the usual Archimedian spiral direction. Magnetic field variations across the shock are the following: the radial component of the field does not change through the shock front and the azimuthal field component is increased by a factor 4, corresponding to the density jump of a factor 4 through a high Mach number shock. This immediately gives the ratio B_1/B_2 of the magnetic field strengths in front and behind of the shock front.

3. Particle Reflection at the Shock

The energy gain of a charged particle due to magnetostatic reflection at a shock is given by (e.g. Sonnerup, 1973):

$$\Delta T = mV_S^* \tilde{v}\,(\cos\tilde{\beta}_1 + \cos\tilde{\beta}_2)/\cos\alpha \tag{7}$$

where V_S^* is the speed with which the shock moves into the stationary plasma, $\tilde{v}$ is the speed of the incident particle, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are the particle's pitch angle before and after reflection and α is the angle between the shock normal and the magnetic field in front of the shock. $\tilde{v}$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are to be taken in a frame of reference $\tilde{S}$ in which the shock is stationary and the plasma flows along the magnetic field lines. The pitch angle after reflection, $\tilde{\beta}_2$, in general is a function of the phase angle of the particle at the moment of impact. Hudson (1965) has calculated $\tilde{\beta}_2$ for a given $\tilde{\beta}_1$ and various phase angles and α's. However, in our numerical calculations we will use $\tilde{\beta}_2 = \tilde{\beta}_1$ for all values of α, which represents approximately the average over all phase angles (Sonnerup, 1973). In the frame of reference $\tilde{S}$ the plasma flows with a velocity $V_S^*/\cos\alpha$ parallel to the magnetic field into the shock. Introducing the solar wind velocity v_{sw1} in front of the shock, the shock velocity V and the particles' velocity v and pitch angle β in the solar wind system we have the transformation

$$\begin{aligned}\tilde{v}_\perp &= v_\perp \\ \tilde{v}_\parallel &= v_\parallel + (V - v_{sw1})/\cos\alpha,\end{aligned} \tag{8}$$

where $_\perp$ and $_\parallel$ refer to the magnetic field direction. Introducing (8) in (7) gives finally

$$\Delta T = 2\,m\,(V - v_{sw1})\left[v\,\frac{\cos\beta}{\cos\alpha} + \frac{V - v_{sw1}}{\cos^2\alpha}\right]. \tag{9}$$

The pitch angle $\tilde{\beta}$ of the particle incident on the shock in the frame of reference $\tilde{S}$ can be found from

$$\tan\tilde{\beta} = \frac{\tilde{v}_\perp}{\tilde{v}_\parallel} = \frac{\sin\beta}{\cos\beta + (V - v_{sw1})/v\cos\alpha}. \tag{10}$$

This shows that only particles with pitch angles less than a maximum pitch angle $\beta_{\max}$ can reach the shock. Particles with pitch angles $\beta > \beta_{\max}$ have $\tilde{\beta} > 90°$ in the system $\tilde{S}$, which is given by

$$\cos\beta_{\max} = -(V - v_{sw1})/v \cos\alpha . \tag{11}$$

Whether a particle will be reflected or transmitted through the shock is decided according to the criterion for adiabatic reflection in the frame of reference $\tilde{S}$

$$\sin^2\tilde{\beta} \geqslant B_1/B_2 , \tag{12}$$

where B_1 and B_2 are the magnetic field magnitudes in front and behind the shock, respectively.

In order to determine the particle pitch angle β in the numerical calculation we proceed in the following way: After each random walk step the radial position of the particle before and after the step is compared with the shock wave position. If a particle has encountered the shock, a random number η between 0 and 1 is generated and the particle pitch angle is determined from

$$1 - \cos\beta = \eta(1 - \cos\beta_{\max}) . \tag{13}$$

This ensures that the possible pitch angles β are distributed within $0 \leqslant \beta \leqslant \beta_{\max}$ according to $\sin\beta$ or since the flux of an isotropic distribution within a solid angle element $d\Omega$ is proportional to $\sin\beta \, d\beta$, we have made certain that we choose the right pitch angle according to a probability law for an isotropic particle distribution in the solar wind frame. Of course, in reality (and this is also reflected in our Monte-Carlo calculation) the particle pitch angle distribution is not completely isotropic, so that the above criterion is not entirely self-consistent. However, anisotropy ratios between fluxes towards and away from the Sun of the order 0.5 or 2 should not affect the results significantly. After computation of $\tilde{\beta}$ from Equation (10) the adiabatic reflection condition (12) determines whether the particle is going to be reflected or transmitted.

4. Results

There are a number of parameters which can be varied in the numerical calculation. To keep the problem tractable, we have adapted a solar wind velocity of $v = 350$ km s^{-1}, an inner boundary R_i at 2 $R_\odot$ and an outer boundary at $R_a = 2$ AU for all computer runs. The blast wave parameter λ was assumed to be $\frac{4}{3}$, resulting in a time dependence of the total blast wave energy proportional to $t^{1/4}$ and a ratio of shock front distance to the distance of the driving sphere of ~ 1.4. The radial diffusion coefficient was assumed to be independent of r. The energy dependence of κ_{rr} in the case of an axisymmetric spectrum of irregularities with respect to the magnetic field, is given by

$$\kappa_{rr} \sim T^{(3-n)/2} , \tag{14}$$

where n is the exponent in the representation of the power in one perpendicular magnetic field component, $P(f) \sim f^{-n}$. Most runs were performed with $n = 2$, which

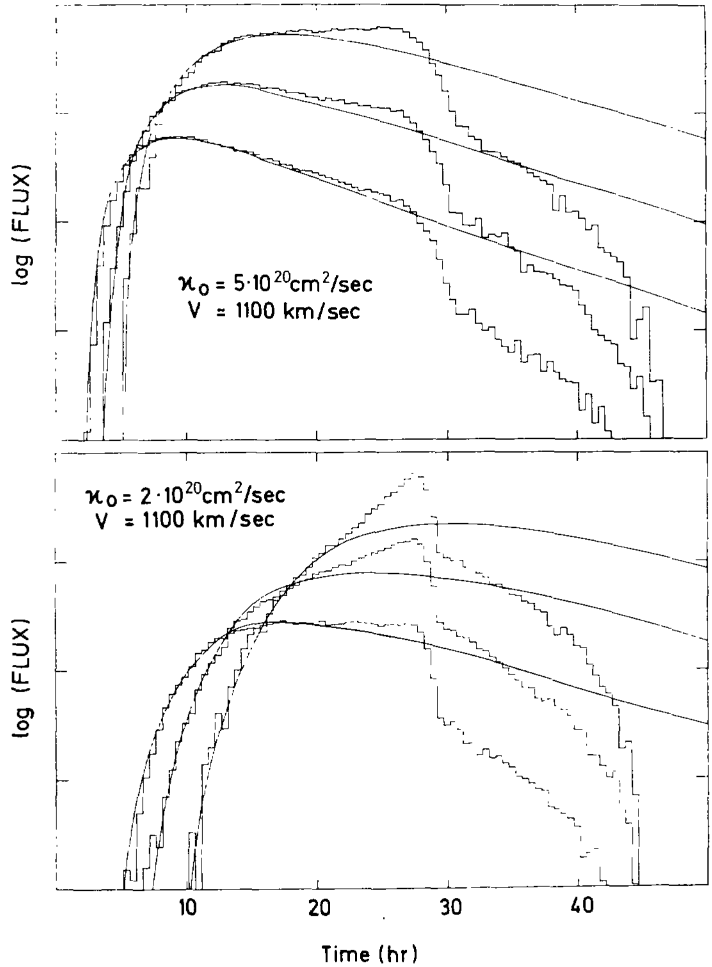

Fig. 1. Flux vs time at 1 AU in three energy channels (0.4–0.63 MeV, 1.0–1.58 MeV, 2.5–3.98 MeV) for the pure 'shock sweep' model taking a shock velocity of 1100 km s^{-1}. Solid line represents the diffusive-convective profile in the absence of a shock wave.

because of the relation

$$\kappa = \lambda v/3 \tag{15}$$

between the diffusion coefficient and the particles mean free path, is equivalent to a constant mean free path, independent of energy. Of course, in the quasilinear derivation of the diffusion coefficient in an axially symmetric wave field (e.g. Jokipii, 1966; Hasselmann and Wibberenz, 1968; Völk *et al.* 1974), we have a singularity for power spectral exponents of $n=2$, giving $\kappa \to \infty$. However, this singularity disappears when non-linear effects are considered (see e.g. review by Völk (1975), and references therein), without changing the energy dependence of κ. In interplanetary space, measurements yield power spectra with exponents lying somewhere between 1.3 and 2, so that the choice which we have made here is not unphysical.

The injection energy spectrum consisted of a series of monoenergetic spectra centered on different energies T_i. Ten different values of T_i were spaced logarithmically within each energy decade. A given injection energy spectrum was approximated by taking the flux f of the monoenergetic spectra to be equal to the flux of the continuous injection spectrum at the corresponding energies. The lowest energy, T_0, was assumed to 63 keV, i.e. each injection energy spectrum was arbitrarily cut off at 63 keV. It can be shown, 'a posteriori', that this cut off has little or no influence on the particle flux at the energies which we are primarily concerned with here, i.e. at around 1 MeV. As boundary condition at the discontinuity behind the blast wave, it was assumed that the particle is lost when encountering the discontinuity.

In Figure 1 we have repeated the shock sweeping model of Palmer (1972) with one modification; we have taken into account the effect of adiabatic deceleration. The shock velocity at 1 AU was assumed to be 1100 km s^{-1} and the flux vs time profile at 1 AU was calculated for two different values of κ_0 (the diffusion coefficient at 1 MeV): 5×10^{20} cm^2 s^{-1} and 2×10^{20} cm^2 s^{-1}, respectively. The injection particle energy spectrum was chosen to be of the form $j=AT^{-\gamma}$, where T is the kinetic energy and where the spectral exponent was taken as $\gamma=2$, typical for solar flare events (e.g. Lanzerotti and Maclennan, 1973). The histograms represent the flux calculated for the shock sweeping model in three different energy channels: 0.4–0.63 MeV, 1.0–1.58 MeV and 2.5–3.98 MeV, respectively. The solid curve represents a solar flare simulation with the same number of particles, without an interplanetary shock wave. As can be seen, a flux increase due to the sweeping of the particles ahead of the shock wave occurs only if the diffusion coefficient κ_0 is about 2×10^{20} cm^2 s^{-1} or less, diffusion coefficients of 5×10^{20} cm^2 s^{-1} or higher do not result in any banking up of particles. However, even for a diffusion coefficient as low as 10^{20} cm s^{-1} the flux increases only by a factor of two as compared to the reference profile (solid line). In addition, the energy spectrum does not steepen due to the sweeping process. Thus, the simple sweeping action cannot explain flux increases by more than an order of magnitude as observed during ESP events.

How does this compare with the work of Palmer (1972)? First of all, Palmer neglected the adiabatic deceleration, but a more important omission in Palmer's model was the fact that he did not use a physical model for the reflection and transmission properties at the shock. He could only produce an efficient banking up of particles ahead of the shock by assuming an almost impenetrable nature of the shock wave barrier. Even then, he obtained at best only particle increases of the order of three as compared to the reference profile (observe that Palmer shows flux vs time on a linear scale). In his comparison with actual measurements, Palmer (1972) presents only events with a small intensity increase at the time of the storm sudden commencement (SSC) and the more usual profiles, as shown schematically in Lanzerotti (1974; Figure 25) cannot be simulated by Palmer's model nor indeed by any model employing purely the sweeping effect of the shock wave. This situation is changed completely when the

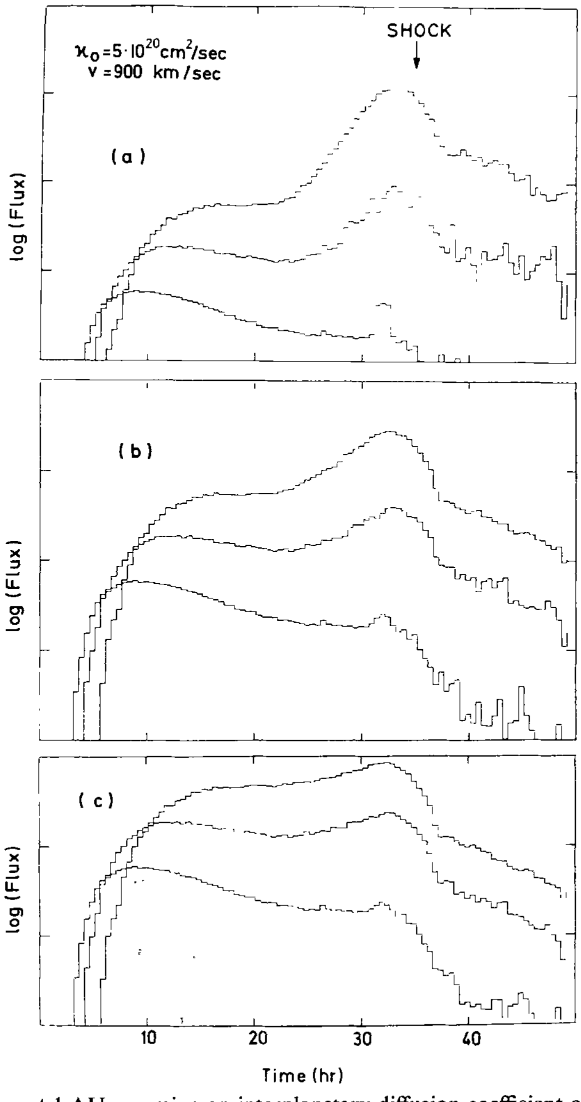

Fig. 2. Flux vs time at 1 AU assuming an interplanetary diffusion coefficient of 5×10^{20} cm^2 s^{-1} at 1 MeV and a shock velocity of 900 km s^{-1} at 1 AU. For (a), (b), and (c) different assumptions about the injection spectrum have been made (see text).

energy increase due to the reflection at the moving shock is taken into account. Figure 2 shows calculated flux profiles for a shock with a velocity of 900 km s^{-1} at 1 AU and a diffusion coefficient of $\kappa_0 = 5 \times 10^{20}$ cm^2 s^{-1}. In Figure 2a the injection spectrum was assumed to be of the form $j \sim T^{-2}$. Figure 2b is the result for an injection spectrum of the form $j \sim T^{-2}$ for $T > T^* = 0.4$ MeV and $j = \text{const}$ (and equal to the value at T^*) for $T \leqslant T^*$ and the injection spectrum used to obtain the results shown in Figure 2c is equivalent to that used for Figure 2b but with $T^* = 1$ MeV.

The model is in good agreement with actually observed ESP events: It results in a flux increase beginning several hours before the arrival of the interplanetary shock superimposed on an exponentially decaying profile. The spectrum as observed at 1 AU steepens above ~ 1 MeV with the arrival of the shock accelerated particles in accordance with the observations. At lower energies ($T \leqslant 500$ keV) the model predicts a spectrum which is much more flat than the injection energy spectrum and turns over at ~ 200 keV. Model calculations for other shock velocities have shown that the flattening of the spectrum is shifted towards higher energies with increasing shock velocity. The maximum intensity occurs ~ 1 to 1.5 hr before the shock arrival and the intensity decreases behind the shock within about one hour by an order of magnitude. From there on the intensity decreases gradually towards the driving discontinuity. In order to keep the flux increase at the shock within typical observation limits (about one order of magnitude above decaying reference profile), some levelling off of the injection energy spectrum below some hundreds of keV has been assumed. This leveling off is consistent with some theoretical deliberations, such as performed by e.g. Englade, 1971. When comparing the profiles in Figure 2 with the reference profile it is found, that the very first ESP particles (first deviation of the ESP profile the reference profile) appear $\tau \approx 12$ hr before the shock arrival. There is a strong, linear dependence of τ on the shock velocity, i.e. $\tau \sim 1/V_0$, and τ is approximately independent of energy.

The anisotropy A of the particle distribution is obtained from

$$A = \frac{j_+ - j_-}{j_+ + j_-}, \tag{16}$$

where j_+ and j_- are the outward and sunward flux, respectively. An example of the time variation of the anisotropy is shown in Figure 3, together with the anisotropy profile as obtained in the absence of a shock wave. With the appearance of the ESP particles the anisotropy increases and changes its direction abruptly with the passage of the shock wave. This behaviour is in very good agreement with the anisotropy measurements during ESP events as reported by, e.g. Rao *et al.* (1967).

Figure 4 shows the dependence of the ESP model on the diffusion coefficient, κ_0. The shock velocity V_0 at 1 AU was assumed to be 1100 km s^{-1} and the injection spectrum is given by $j \sim T^{-2}$ for $T \geqslant 0.4$ MeV and $j = \text{const}$ for $T \leqslant 0.4$ MeV.

The flux increase before the shock arrival strongly depends on the diffusion coefficient κ. Whereas a diffusion coefficient of 10^{21} cm^2 s^{-1} at 1 MeV results in only a minor flux increase due to multiple reflection of the particles at the shock, the case with a diffusion coefficient of 5×10^{20} cm^2 s^{-1} at 1 MeV seems quite representative of actually measured ESP events. Calculations with 2×10^{20} cm^2 s^{-1} at 1 MeV result already in unreasonably high flux increases and, in addition, to a very flat energy spectrum at around 1 MeV, which is in disagreement with the measurements.

Fisk (1971) also considers energy changes during reflection at the shock front. However, in contrast to our model, he considers a linear one-dimensional geometry, a homogeneous background of particles through which the shock passes and an

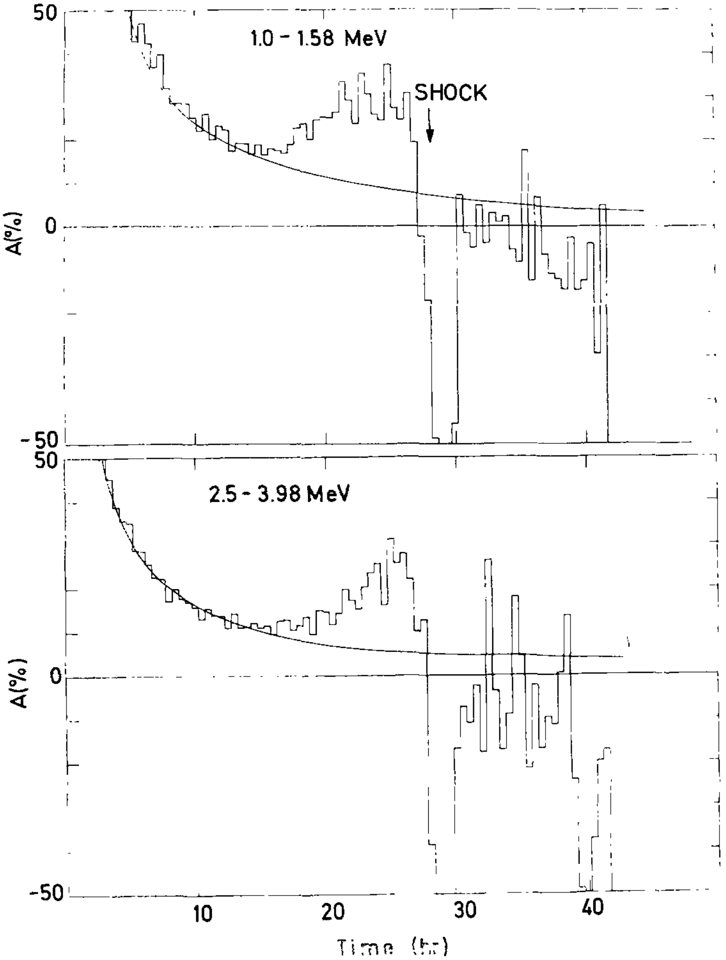

Fig. 3. Anisotropy variation with time for an interplanetary diffusion coefficient of 5×10^{20} cm^2 s^{-1} at 1 MeV and a shock velocity of 1100 km s^{-1} at 1 AU. Shown for comparison is the anisotropy profile calculated in the absence of a shock wave.

energy independent diffusion coefficient. In addition, he assumes 100% reflection independent of shock position and pitch angle of the incident particle. By choice of a very low diffusion coefficient, which does not increase with particle energy, he prevents the particles from leaving the vicinity of the shock and can thus generate shock spikes, but not ESP events. However, his result that the energy spectrum around ~1 MeV hardens at the shock is in agreement with our calculations for very low diffusion coefficients.

5. Discussion

At first sight, the enormous flux increase due to the energy increase of the reflected particles is the most startling feature of the Monte-Carlo simulation. The maximum

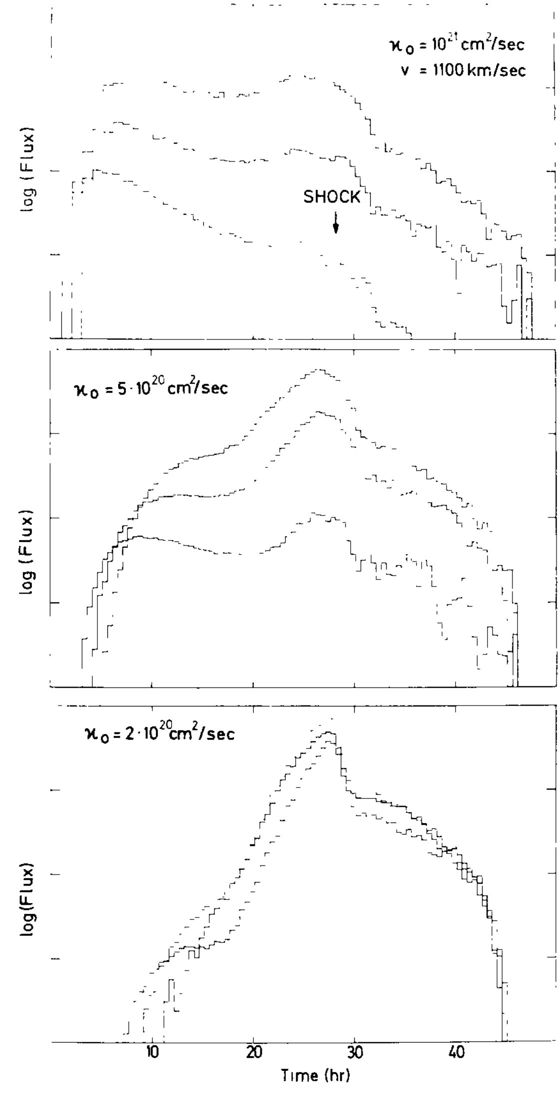

Fig. 4. Flux vs time at 1 AU for a shock velocity of 1100 km s^{-1} and three different values of the interplanetary diffusion coefficient.

energy gain per encounter with the approaching shock is $\Delta T = 2\,mvV$. Under the assumption that the particle makes $v/2\,L$ round trips per second, where L is the distance from the shock to the mirroring irregularity, the particle increases its energy at the rate

$$dT/dt = 2\,TV/L, \tag{17}$$

L is in the order of the mean free path λ. Using the relation (15) between the mean free path and the diffusion coefficient and assuming that the diffusion coefficient is proportional to the particle velocity $\kappa=\kappa_0 T^{1/2}$, ($n=2$ in Equation (14)), we find:

$$\mathrm{d}T/\mathrm{d}t \approx TVc/(\kappa_0\sqrt{m})$$

or

$$\frac{T}{T_0} = \exp\left\{\frac{Vc}{\varkappa_0\sqrt{m}}\, t\right\}, \tag{18}$$

where c is the velocity of light, and m is the particle rest mass in MeV. Taking $\kappa(T=1\text{ MeV})=5\times 10^{20}\text{ cm}^2\text{ s}^{-1}$, $V\sim 1000\text{ km s}^{-1}$ and $t=30$ h we find $Vtc/(\kappa_0\sqrt{m})$ of the order of 10 which gives an enormous energy increase. However, the particle does not stay with the shock all the time, as was assumed in deriving (17). Let us assume, instead, that the particle encounters the shock α times per second. We have then an energy increase at the rate

$$\mathrm{d}T/\mathrm{d}t = \sqrt{8mT}\, V\alpha$$

or

$$T/T_0 = [1 + \sqrt{2m}\, V\alpha t/T_0^{1/2}]^2 \tag{19}$$

for a praticle energy T at time t which started with an energy T_0. Let us now assume that we have an initial energy spectrum $j\sim T_0^{-2}$. In order to have a flux increase by a factor of ten, we require $T/T_0=\sqrt{10}$. Inserting typical values in (19) we find at $T\approx 1$ MeV a value for α of about $\frac{1}{4}$ encounters per hour or about 7 encounters till the shock has reached 1 AU, i.e. about 10 reflections at the shock can easily result in a flux increase by one order of magnitude at 1 MeV. Of course, the above calculations are only very rough and should be used as order of magnitude estimates.

Equation (18) also indicates a strong dependence of the energy increase on the interplanetary diffusion coefficient. To keep the flux increase within the observed range ($\sim$ one order of magnitude) we had to assume a diffusion coefficient $>2\times 10^{20}$ $\text{cm}^2\text{ s}^{-1}$ for 1 MeV protons. Diffusion coefficients of that order of magnitude are inferred from the strong anisotropy and the time profile during the early stages of solar flares (e.g. Webb and Quenby, 1973; Innanen and van Allen, 1973; Fisk and Axford, 1968). Models used to explain solar flare decay-phase profiles (Lupton and Stone, 1973) yield values of $\sim 3\times 10^{20}\text{ cm}^2\text{ s}^{-1}$ at 1 MeV which is just about acceptable as a lower limit from our model calculations. However, it should be pointed out that these relatively high diffusion coefficients are not supported by the theoretically calculated coefficient (from observed power spectra) at 1 AU (e.g. Jokipii and Coleman, 1968).

When comparing our model calculations with actual measurements, one important fact has to be kept in mind: Our model of the interplanetary diffusion process as well as the shock wave model is independent of the solar longitude. Thus our model really applies only to near Earth measurements of western hemisphere flares, or more generally to flares, where the observer (at 1 AU) is connected via an interplanetary field line to the flare site on the Sun. For other flare sites the combination of the shock accelerated particles within a flux tube and the corotation of the flux tube over an observer at 1 AU may considerably change the flux time profile.

In the present model we have not considered variations in the diffusion coefficient due to the different plasma-field topology between the shock and the discontinuity, e.g. increase in solar wind speed, magnetic field strength and wave amplitudes (Scholer and Belcher, 1971), nor the possibility of second order Fermi acceleration in the turbulent region behind the shock. These effects, which might be of some importance at low energies, will be considered in a subsequent publication.

In summary, we have developed a model, which predicts the intensity-time distribution of an idealized western-hemisphere impulsive injection event with an accompanying shock. Energisation of the flare particles due to multiple reflection at the moving shock front can easily result in flux increases at low energies (< 1 MeV) by more than an order of magnitude. The model is only consistent with measured flux increases if the diffusion coefficient for ~ 1 MeV protons is $\sim 5 \times 10^{20}$ cm^2 s^{-1} or larger. The model predicts correctly a steepening of the spectrum at energies $\gtrsim 1$ MeV and an anisotropy directed outward or inward in front or behind the shock wave, respectively.

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