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A multi-winner nested rent-seeking contest *

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Abstract. This paper considers a symmetric imperfectly discriminating rent-seeking contest in which there may be several winners. We first demonstrate a serious flaw in previous work and then go on to suggest an alternative method for analyzing the contest. In contrast to the previous work, we show that the value of the rent is fully dissipated in equilibrium as the number of players becomes large.

1. Introduction

The bulk of the literature on imperfectly discriminating rent-seeking contests has concentrated on the case in which a number of contestants compete to win a single prize.[1] An exception is the paper by Berry (1993) in which there are several identical prizes on offer, and each player may win no more than one prize.[2] In a completely symmetric model, Berry shows that the amount of rent-seeking which arises in the symmetric equilibrium with a large number of contestants is only a fraction of the value of the total rent. This is in stark contrast to the one prize case in which a large number of identical contestants fully dissipate the value of the rent. Thus, introducing this multi-prize framework seems to resolve Tullock's (1988) conundrum that theoretical models predict much more rent-seeking than is observed in practice.

In Section 2 we show that this result is directly due to an unreasonable selection process which underlies the probability of winning function in Berry's model. Specifically, we show that Berry implicitly assumes that only one of the prizes is awarded on the basis of the contestants' rent-seeking outlays; the probability that a player wins one of the remaining prizes is independent of these outlays.

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When we allow the imperfectly discriminating rent-seeking contest to have several winners, there is no unique method for selecting these winners. Retaining the assumption made by Berry that each player may win no more than one prize, Section 3 presents one method which we believe to be reasonable in that it describes an easily justifiable *process* for choosing the winners. The process we consider is one in which contenders make a rent-seeking outlay once and the winners are chosen sequentially where the winner of the previous round is eliminated from future participation. In this case a player decides his rent-seeking outlay based upon the total probability of winning one of the rounds. As the probability of winning one contest is nested in the probability of not having won the previous contests, we refer to this mechanism as a nested game.

To keep the analysis as simple and tractable as possible, and in order to compare our results directly with those of Berry (1993), we assume throughout the paper that all prizes are identical and that each player has an identical valuation of each prize.[3]

2. Berry's results

Assume that there are $k \geq 1$ identical indivisible rents for which $N > k$ players compete. All players are risk-neutral and have the same symmetric valuation, $V(k)$, of one of the rents; it appears reasonable to assume $V'(k) \leq 0$ where $V'(k)$ is the first derivative of the valuation function with respect to the number of prizes.[4] However, this assumption is not necessary for deriving the equilibrium, but will affect its comparative static properties. Thus the total rent to be distributed is $kV(k)$. Denoting by $q_i(k)$ the probability that player i=1,2,...,N wins one of the k prizes, and $x_i \geq 0$ the rent-seeking outlay of player i, the expected payoff for player i is:

$$q_i(k)V(k) - x_i. \tag{1}$$

In order to describe the probability function $q_i(k)$, Berry postulates that the probability that player i wins a prize is the sum of the rent-seeking expenditures of combinations of k from N players *which involve player i* divided by the sum of rent-seeking outlays in *all possible* combinations of k from N players. (This leads to a complicated expression given as equation (1) in Berry). The function which Berry thus obtains is indeed a valid probability function in that it is bounded between zero and one, the probability that i wins is increasing in x_i, and the sum of all players' probabilities of winning one of the prizes is equal to k. However, simple rearrangement of his complicated expression shows that the probability of winning function which Berry is using is actually (see the appendix for a derivation of this result):

$$q_i(k) = \frac{x_i}{X} + \left(1 - \frac{x_i}{X}\right)\frac{k-1}{N-1} \text{ where } X = \sum_{m=1}^{N} x_m \tag{2}$$

Whilst (2) describes a valid probability of winning function, our rearrangement shows the underlying process which Berry is using. The first prize is awarded according to the standard Tullock (1980) rent-seeking game where the probability of winning for player i is x_i/X; if player i does not win the first prize, then he is assigned an equal probability $(1/(N-1))$ of winning one of the remaining $k-1$ prizes. Thus, rent-seeking outlays affect only the distribution of the first prize; after this is awarded, rent-seeking outlays are forgotten and each player has an equal probability of winning. Notice that the probability function in (2) allows a player a chance of winning one of $k > 1$ prizes by setting $x_i = 0$.

Substituting (2) into (1) and maximizing the resulting expression with respect to x_i, yields the following first order condition for an interior optimum for player i:

$$\frac{X - x_i}{X^2}\left(\frac{N-k}{N-1}\right)V(k) = 1 \tag{3}$$

As in Berry, if we assume that $x_i = x \; \forall \; i$ then we find the total amount of rent-seeking $X(N,k)$ in a symmetric equilibrium as:

$$X(N,k) = Nx = \frac{N-k}{N}V(k) \tag{4}$$

Berry thus achieves the result that total rent-seeking outlays increase in the number of players and that, in the limit, $X(\infty,k)=V(k)$; in contrast to a one-winner contest where the whole rent is dissipated in this competitive case, Berry's game has rent underdissipation. From (4) we find also that $\partial X(N,k)/\partial k \leq 0$ so that increasing the number of winners weakly decreases total rent-seeking outlays (there is a zero effect if $N \to \infty$ and $V'(k) = 0$).

3. A nested contest with k rounds

In contrast to the process underlying the previous model, we believe the following to be one of many reasonable alternatives. The players make one rent-seeking contribution which is valid for all k rounds of a nested contest. The winner of the first prize is decided by using the probability distribution which arises after the rent-seeking outlays of all N players are collected. The winner of this round is then eliminated and the second prize is distributed by using the probability distribution which arises when we exclude the outlay of the first winner. This process continues until all k prizes have been distributed. Thus the probability distribution is updated after each round to reflect the fact that previous winners are eliminated from future rounds. The point is that the rent-seeking outlays are used to determine the winners of all k prizes; in Berry (1993) these outlays determine the winner of the first contest and then are forgotten.

We focus again on symmetric equilibrium.[5] To contrast with the results of the previous section, denote z_i as the rent-seeking outlay of player i. We use $\underline{z}=(z_1,\dots,z_N)$ to denote the vector of rent-seeking outlays. To get to the probability, $P_i(\underline{z};k)$, that player i wins in a k-round nested contest, let $p_i^s(\underline{z};k)$ represent the conditional probability that player i wins the s'*th* prize, where $p_i^s(\underline{z};k)$ is assumed to be homogeneous of degree zero in the rent-seeking outlays of the players remaining in stage s, increasing and strictly concave in z_i, and symmetric in all z_j, $j \neq i$.[6] The total probability that player i wins the first prize is given by $p_i^1(\underline{z};k)$. If $k > 1$, then the probability that i wins the s'*th* prize is given by the probability that i has not won any of the previous s–1 rounds of the contest multiplied by the conditional probability of winning the s'*th* round (in which there are N–s+1 players remaining). Thus the probability that player i wins one of the rounds of the contest, and hence a prize, can be written as:[7]

$$\begin{aligned} P_i(\underline{z};1) &= p_i^1 \text{ for } k=1 \\ P_i(\underline{z};k) &= p_i^1 + (1-p_i^1)p_i^2 + (1-p_i^1)(1-p_i^2)p_i^3 + \dots \\ &\quad + \Pi_{s=1}^{k-1}(1-p_i^s)p_i^k \\ &= p_i^1 + \sum_{j=1}^{k-1}\left[\Pi_{s=1}^{j}(1-p_i^s)p_i^{j+1}\right] \text{ for } k>1 \end{aligned} \tag{5}$$

Equation (5) indicates the nested structure of the contest. The expected payoff of player i is thus

$$P_i(\underline{z};k)V(k) - z_i. \tag{6}$$

Evaluating the first order conditions for this problem at an interior symmetric situation ($z_i = z \ \forall \ i$) gives the total level of rent-seeking, $Z(N,k)$, as[8]

$$\begin{aligned} Z(N,k) = Nz &= V(k)Np_{ii}^1(1;k) \text{ for } k=1 \\ &= V(k)Np_{ii}^1(1;k)\left[k - \tfrac{N}{N-1}\sum_{j=1}^{k-1}\tfrac{k-j}{N-j}\right] \text{ for } k>1 \end{aligned} \tag{7}$$

$$\text{where } p_{ii}^1(1;k) = \tfrac{\partial p_i^1}{\partial z_i} \mid z_1 = z_2 = .. = z_i = .. = z_N = 1$$

The assumption that $p_i^s(z;k)$ is homogeneous of degree zero allows the use of $p_{ii}(1;k)$ in (7).

Considering the simple Tullock probability function $p_i^1 = z_i/Z$, equation (7) can be written,

$$Z(N,k) = Nz = V(k)\left[\frac{N-1}{N}\right] \text{ for } k = 1$$
$$= V(k)\left[\frac{k(N-1)}{N} - \sum_{j=1}^{k-1} \frac{k-j}{N-j}\right] \text{ for } k > 1 \qquad (8)$$

Equation (8) is comparable to the result obtained by Berry which we have repeated in our equation (4). Common to Berry's results, we find that $\partial Z(N,k)/\partial N > 0$; however, our model yields $Z(\infty,k) = kV(k)$, i.e. the rent is *fully dissipated* in the case of competitive rent-seeking. This corresponds to the widely known result in the literature with one winner: a symmetric imperfectly discriminating rent-seeking game with constant returns to rent-seeking and one winner exhibits full rent dissipation in equilibrium when there is free entry to rent-seeking.

Consider now the effect of k on the total amount of rent-seeking. For any valuation function with $V'(k) \leq 0$, the amount of rent-seeking obtained by Berry is weakly decreasing in the number of winners. However, we cannot unambiguously determine the sign of $\partial Z(N,k)/\partial k$ in our model without specifying a functional form for $V(k)$. From (8), using the valuation function in Berry (1993), $V(k) = V/k$ where V is a positive constant, yields $\partial Z(N,k)/\partial k \leq 0$ (where we have equality when $N \rightarrow \infty$). Equation (4) yields $\partial X(N,k)/\partial k < 0$ in this special case (independent of N).

As rent-seeking outlays affect only the distribution of the first prize in Berry's setup, whereas they affect the distribution of all prizes in our model, our model exhibits a larger amount of rent-seeking. The apparent efficiency of the multi-winner contest in Berry arises due to the lack of attention to the micro-foundations in his model.

4. Conclusion

In studying multi-winner rent-seeking contests, we should take care to pay attention to the *process* which we use to describe the distribution of these rents. We have shown that defining a valid probability of winning function is not enough in this context; we must also consider the micro-foundations which underlie this function. There is no unique method for picking out several winners; we have presented one possibility which we consider reasonable and have shown that this model shares common features with the literature on imperfectly discriminating, symmetric, single-winner rent-seeking games.

The total amount of rent-seeking increases in the number of players; when entry to rent-seeking is free, the whole of the rent is dissipated. We cannot, however, be sure about the effect on rent-seeking of increasing the number of winners; this depends upon the valuation function used by the players.

Notes

1. For a recent survey of the rent-seeking literature, see Nitzan (1994). For various extensions of Tullock's (1980) contest see, for example, Leininger (1993), Hillman and Riley (1989), Michaels (1989), Paul and Wilhite (1990), Nitzan (1991a, 1991b) and Hillman and Katz (1984).
2. For a perfectly discriminating rent-seeking contest with several winners see Clark and Riis (1995).
3. Van Long and Vousden (1987) present a model in which players compete for a (possibly asymmetric) share of a divisible rent. Here, we assume that the potential winnable share of the total rent is fixed and identical for each player.
4. Berry in fact assumes that $V(k) = V/k$ where V is a positive constant.
5. It can be shown that the symmetric equilibrium is in fact the unique equilibrium of this nested game. The proof is available from the authors on request.
6. We have slightly abused notation here by writing the conditional probabilities as $p_i{}^s(\underline{z};k)$ as these conditional probabilities are dependent upon the identity of the winners of the previous rounds. This is of no consequence for the later analysis as the unique equilibrium is symmetric.
7. To ease notation in (5), we write $p_i^s(\underline{z};k)$ simply as p_i^s.
8. A pre-requisite for the calculation leading to (7) is that one can establish the relationship between $\partial p_i^s/\partial z_i$ for different numbers of players. One simple way of characterizing this is to assume that p_i^s obeys the axiom of independence of irrelevant alternatives; this underlies the expression in (7) and states simply that the probability that player i wins when player j is not included in the set of players is the same as if j is in the set of players but does not win. For more details, see Clark and Riis (1995a).

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Appendix

This appendix contains a derivation of equation (2) which is a rearrangement of the probability of winning function in Berry (1993). With N players and k prizes, Berry expresses the probability that player i wins a prize as:

$$q_i(k) = \frac{\text{sum of outlays of combinations of k from N players which include i}}{\text{sum of outlays of all combinations of k from N players}} \tag{A1}$$

Dealing with the denominator in (A1) first, the number of combinations of k from N which involve any arbitrarily chosen player is $\binom{N-1}{k-1}$. Thus, the expenditure of each player appears in the denominator this number of times so that the denominator is

$$\binom{N-1}{k-1}\sum_{i=1}^{N} x_i = \binom{N-1}{k-1} X.$$

In the numerator there are $\binom{N-1}{k-1}$ combinations which include player i; player $j \neq i$ appears in $\binom{N-2}{k-2}$ of these. Thus (A1) can be written

$$\begin{aligned} q_i(k) &= \frac{\binom{N-1}{k-1} x_i + \binom{N-2}{k-2}\sum_{j\neq i} x_j}{\binom{N-1}{k-1} X} \\ &= \frac{x_i}{X} + \frac{\binom{N-2}{k-2}}{\binom{N-1}{k-1}} \frac{\sum_{j\neq i} x_j}{X} \\ &= \frac{x_i}{X} + \frac{\frac{(N-2)!}{(k-2)!(N-k)!}}{\frac{(N-1)!}{(k-1)!(N-k)!}} \left(1 - \frac{x_i}{X}\right) \\ &= \frac{x_i}{X} + \frac{k-1}{N-1}\left(1 - \frac{x_i}{X}\right) \end{aligned}$$

which is equation (2) in the text.