THEORY OF SOLAR BURSTS

(Invited Review Paper)

TATSUO TAKAKURA

Tokyo Astronomical Observatory, University of Tokyo, Mitaka, Tokyo, Japan

(Received 20 January, 1967)

1. Introduction

It is now the beginning of the third solar cycle since the birth of the solar radio astronomy. During the last two solar cycles, we have been able to observe enormous manifestations of solar radio bursts in a wide frequency range observable from the ground. Many theories have been proposed for the interpretation of the nature of the various types of bursts.

In the present review, an attempt is made to review the current theories and models of solar bursts in order to show the extent to which the theories contribute towards the interpretation of the nature of the bursts.

For observational background, a comprehensive text-book (KUNDU, 1965) and many good reviews have been available so far (KUNDU, 1963; FOKKER, 1963; WILD, 1964; MAXWELL, 1965) and a theoretical (and observational) review on the solar bursts has been given by WILD, SMERD, and WEISS (1963). Recently, a review on the implications of solar bursts for the study of the corona was given by TAKAKURA (1966). In order to avoid duplication of these reviews, emphasis is put in the present review on the theory of gyro-synchrotron emission and on the interpretation of bursts on microwaves.

In the solar atmosphere, radio waves are emitted directly from the electrons in acceleration. An acceleration of electrons is caused by the Coulomb force at the collisions with ions (free-free emission). Another acceleration is due to centrifugal force in the course of rotation of electrons in the magnetic field (gyro-synchrotron emission). The theory of gyro-synchrotron emission is given in Section 2, and the theory is applied in Section 3 to the interpretation of microwave bursts and Type IV bursts. Free-free emission is not so important as the origin of intense solar bursts, so that it is excluded in the present review.

Indirect generation of the radio waves is possible in the solar atmosphere through plasma electron-waves, which are generally longitudinal waves and can be excited by a fast stream of charged particles. In this case, it is necessary for the plasma waves to convert their energies to the electromagnetic waves. Theoretical background of the plasma waves and electromagnetic waves in the plasma is shown in Section 4, and it is applied in Section 5 to the interpretation of solar bursts mainly on meter waves. In these sections you will find some duplications of the previous reviews by WILD *et al.* (1963) and by TAKAKURA (1966).

The intensity spectrum and polarization state of the electromagnetic waves thus emitted in the solar atmosphere are modified during the propagation through the outer solar atmosphere. In this respect, the theory of propagation of electromagnetic waves in a plasma penetrated by a magnetic field (Magneto-Ionic theory) is important for the interpretation of observed characteristics of solar bursts. However, the M.I. theory is well established for the study of the propagation of waves in the terrestrial ionosphere, and many text-books have been published so far on this subject, so that the M.I. theory is excluded in the present review.

Finally, in Section 6 acceleration of electrons in the solar atmosphere is briefly reviewed.

2. Gyro-Synchrotron Emission

The charged particles, electrons in our case, gyrating in a magnetic field emit electromagnetic waves, which is generally called gyro-synchrotron emission. If the electrons are non-relativistic ($K \ll m_0c^2$), the emission is mainly confined to the gyro-frequency. This emission is generally called gyro-emission. If the electrons are extremely relativistic ($K \gg m_0c^2$), the emission predominates at higher frequencies than the gyro-frequency. This emission is called synchrotron emission.

The gyro-synchrotron emission is an efficient process to emit radio waves if the magnetic field is comparatively strong. In the solar atmosphere above sunspot regions, the magnetic field of the sunspots penetrates into the corona with decreasing intensity with height: typically 10^2 gauss at about 5×10^4 km and 1 gauss at about 10^6 km above the sunspot regions (TAKAKURA, 1964b, 1966). Thermal electrons are always gyrating in this magnetic field with r.m.s. speed V_T of 6.7×10^3 km/s for 10^6 °K in the

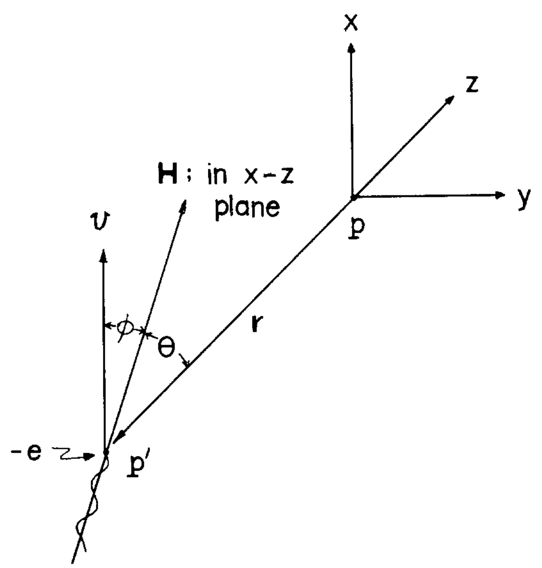

Fig. 1. Coordinate system for the gyro-synchrotron emission.

corona, thus they emit the gyro-emission or absorb incident waves as a reverse process of emission. When a flare or a similar activity occurs, a part of the thermal electrons is accelerated up to relativistic velocities and then emits gyro-synchrotron emission.

It should be remarked that the observed circular component of those solar bursts which are attributed to the gyro-synchrotron emission implies that an effective energy range of electrons is generally not extremely relativistic. It should be remarked also that the energetic electrons emit the radio waves in a plasma, whose density and magnetic field decrease outwards, so that the whole emission is not always observed from the earth. Low-frequency part of the radio spectrum cannot propagate, due to the over-dense plasma, or generally suffers absorptions, both gyro-absorption and free-free absorption, due to thermal electrons in the outer layers.

2.1. Gyro-synchrotron emission from an electron

A distant electric field due to an electron in motion is given by (e.g. Heitler, 1954)

$$\mathbf{E} = \frac{-e}{s^3 c^2}\left[\mathbf{r}\left[\mathbf{r} + \frac{r}{c}\mathbf{v}, \dot{\mathbf{v}}\right]\right], \tag{2.1}$$

with

$$s = r + \frac{(\mathbf{vr})}{c},$$

where the radius vector $\mathbf{r}$ is from the observer to the electron, and e is the charge of an electron (positive value).

In a general case in which the electron moves along a helical path making an angle φ with the magnetic field direction, the orthogonal components of the electric field at a distant observer situated on a line which passes through the electron and makes an angle θ with the magnetic field (cf. Figure 1) is given by (Takakura, 1960c)

$$E_x(t) = \frac{e\dot{v}}{c^2 r}\frac{\cos\theta\sin\zeta - \beta\cos\varphi\sin\zeta}{(1-\beta\cos\varphi\cos\theta - \beta\sin\varphi\sin\theta\cos\zeta)^3}, \tag{2.2}$$

$$E_y(t) = \frac{e\dot{v}}{c^2 r}\frac{\beta\sin\varphi\sin\theta - \cos\zeta + \beta\cos\varphi\cos\theta\cos\zeta}{(1-\beta\cos\varphi\cos\theta - \beta\sin\varphi\sin\theta\cos\zeta)^3}, \tag{2.3}$$

where

$$\zeta - \frac{\beta\sin\varphi\sin\theta}{1-\beta\cos\varphi\cos\theta}\sin\zeta = \omega_1 t.$$

ω_1 indicates a fundamental angular frequency at the observer, t is the time referred to the observer, and $\beta = v/c$. The fundamental angular frequency is given by

$$\omega_1 = \omega_H(1-\beta^2)^{1/2}(1-\beta\cos\varphi\cos\theta)^{-1}, \tag{2.4}$$

where $\omega_H = 2\pi f_H = eH/m_0 c$, which is angular gyro-frequency. The first bracket of

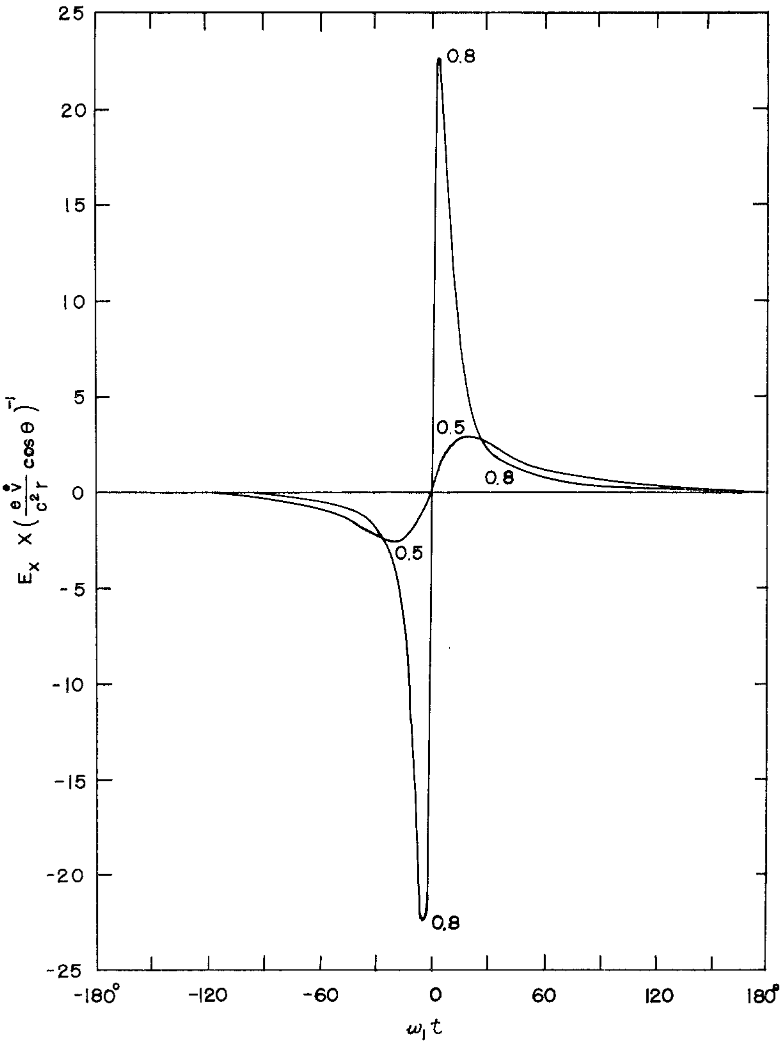

Fig. 2. Wave form of x-component of electric field emitted from an electron in a circular orbit ($\varphi = \pi/2$). Numerals on the curves indicate $\beta \sin\theta$. (After TAKAKURA, 1960b.)

Equation (2.4), $(1-\beta^2)^{1/2}$, represents a relativistic correction for the mass of electron, and the second bracket represents a Doppler shift.

As an example, the wave forms of the distant electric field are illustrated in Figures 2 and 3, for which $\varphi = \pi/2$, i.e., a trajectory of the electron is a circle. In the figures $t=0$, which is referred to the observer, corresponds to a retarded time referred to the electron when an angle between $\mathbf{v}$ and $(-\mathbf{r})$ is smallest (cf. Figure 1). As is shown in Figures 2 and 3, the peaks of the wave forms become sharper and concentrate near $t=0$ as β approaches to unity, showing that the harmonic components increase with β.

The nth harmonic components are derived from the Fourier transform of Equa-

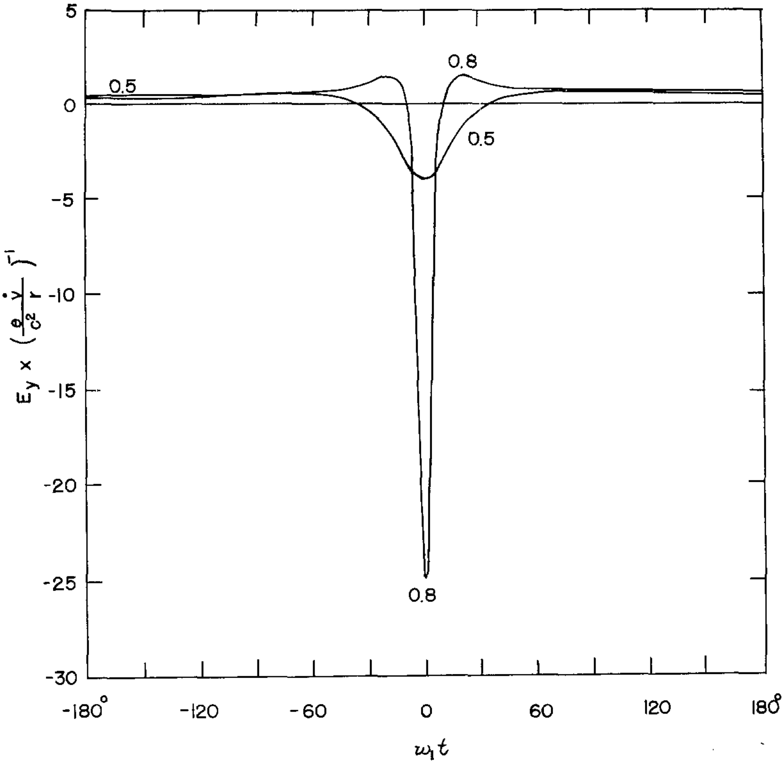

Fig. 3. Wave form of y-component of electric field (cf. Figure 2).

tions (2.2) and (2.3),

$$A_{x,n} = \frac{2e\dot{v}}{c^2 r} \frac{\cos\theta - \beta\cos\varphi}{\beta\sin\varphi\sin\theta(1-\beta\cos\varphi\cos\theta)^2} nJ_n(n\alpha), \tag{2.5}$$

$$A_{y,n} = \frac{-2e\dot{v}}{c^2 r} \frac{1}{(1-\beta\cos\varphi\cos\theta)^2} = nJ_n'(n\alpha), \tag{2.6}$$

where $\alpha = (\beta\sin\varphi\sin\theta)/(1-\beta\cos\varphi\cos\theta)$, $J_n(z)$ is the Bessel function of the first kind of argument z and order n, and $J_n'(z)$ is its derivative with respect to z.

The energy $P_n(\beta, \varphi, \theta)$ emitted by an electron per unit time per unit solid angle at θ in the nth harmonic is reduced from Equations (2.5) and (2.6):

$$P_n(\beta, \varphi, \theta) = \frac{cr^2}{4\pi}\left(\frac{A_{y,n}^2}{2} + \frac{A_{x,n}^2}{2}\right) = \frac{2\pi e^2 n^2 f_H^2 (\beta\sin\varphi)^2 (1-\beta^2)}{c(1-\beta\cos\varphi\cos\theta)^4} \times \\ \times \left[\{J_n'(n\alpha)\}^2 + \left(\frac{\cos\theta - \beta\cos\varphi}{\beta\sin\varphi\sin\theta}\right)^2 \{J_n(n\alpha)\}^2\right]. \tag{2.7}$$

The first term in the square bracket represents a component polarized normal to the magnetic field (y-component), and the second term is its orthogonal component (x-component). The frequency of the nth harmonic is (cf. Equation 2.4)

$$f = n\omega_1/2\pi = nf_H(1-\beta^2)^{1/2}(1-\beta\cos\varphi\cos\theta)^{-1}. \tag{2.8}$$

A general tendency of $P_n(\beta, \varphi, \theta)$ for $0<\varphi\leqslant\pi/2$ is as follows. For small values of β, the P_n is maximum at $n=1$ and $\theta=0$. With increasing n, the P_n decreases and $\theta_{\max}$ (which represents θ at which P_n is maximum for a given n) increases up to $\varphi<\theta_{\max}<\pi/2$. On the other hand, for large values of β, $(1-\beta^2\ll 1)$, P_n is maximum around a critical harmonic number $n_c\approx\frac{3}{2}(1-\beta^2)^{-3/2}\times\sin^3\varphi$ (cf. Equations 2.18 and 2.8) and $\theta_{\max}\approx\varphi$ with an effective width of $\delta\approx(1-\beta^2)^{1/2}$, except for small values of φ. With decreasing n, P_n decreases and $\theta_{\max}$ decreases: only in a special case of $n=1$, $\theta_{\max}$ is zero and $P_1(\beta, \varphi\ 0)$ is comparatively large even for the large values of β.

The shape of polarization ellipse R_n for the nth harmonic is given by

$$R_n \equiv A_{x,n}/A_{y,n} = -\left(\frac{\cos\theta-\beta\cos\varphi}{\beta\sin\varphi\sin\theta}\right)\frac{J_n(n\alpha)}{J_n'(n\alpha)}. \tag{2.9}$$

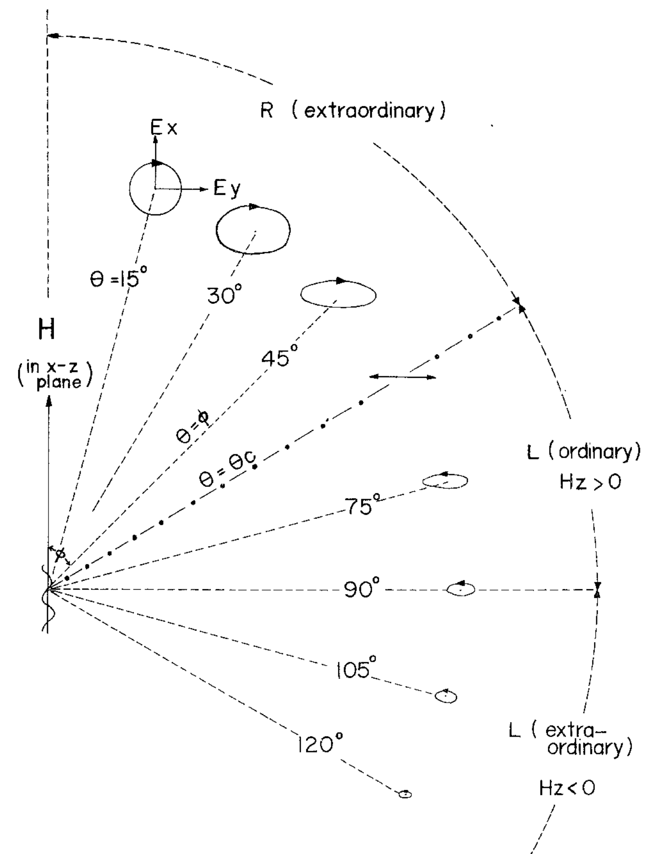

Fig. 4. Emission polar diagram of intensity and polarization at the third harmonic ($n=3$) of gyro-synchrotron emission from an electron with $\beta=0.7$ and a pitch angle $\varphi=45°$. The polarization is linear at $\theta=\theta_c$ which satisfies $\cos\theta=\beta\cos\varphi$.

If R_n is negative in a range of $\theta<\pi/2$, the sense of rotation of electric vector is the same as that of the extraordinary made in a tenuous plasma, and this is also the same as the sense of rotation of electrons in the magnetic field. In a range $\pi/2<\theta<\pi$, the positive sign of R_n corresponds to the sense of extraordinary mode, since the sign of the magnetic field is opposite to each other at $\theta<\pi/2$ and $\theta>\pi/2$. Equation (2.9) shows that the wave is polarized linearly ($R_n=0$) in y-direction at a critical angle θ_c given by,

$$\cos\theta_c = \beta \cos\varphi\,, \tag{2.10}$$

and at smaller and larger values of θ, the sense of rotation of the elliptical polarization is opposite to each other. For the extremely relativistic case $(1-\beta^2 \ll 1)$ this critical direction of emission θ_c is nearly equal to the pitch angle φ of the electron, while for the non-relativistic case $(\beta^2 \ll 1)$ θ_c is close to $\pi/2$.

The polarization state and the directivity of emission are diagrammatically illustrated in Figure 4 for an electron with a medium energy ($\beta=0.7$) and for $n=3$. It should be remarked that the sense of rotation of polarization ellipse is that of *ordinary mode* in a range of θ from θ_c to $\pi/2$. This range is wider for smaller values of pitch angle φ.

In an extremely relativistic case, the polarization ellipse at $\theta=\varphi+\Delta\theta$ is reduced from Equation (2.9) to

$$R_n \approx -\frac{(1-\beta)\cos\varphi - \Delta\theta\sin\varphi}{\sin^2\varphi}\,n^{1/3} \tag{2.11}$$

for high-order harmonics at which the emission is predominant; in this case $J_n(n\alpha)/J_n'(n\alpha)\approx J_n(n)/J_n'(n)\approx n^{1/3}$. For example, if $\varphi=\pi/2$, $R_n\approx\Delta\theta n^{1/3}$, so that at the critical harmonics $n_c\approx\frac{3}{2}(1-\beta^2)^{-3/2}=\frac{3}{2}\delta^{-3}$, around which the emission is maximum, R_n is about $3\Delta\theta/\delta$. Accordingly, the polarization is nearly circular at the half power width $\Delta\theta\approx\delta$; the sense of rotation is opposite to each other for $+\Delta\theta$ and $-\Delta\theta$.

In a non-relativistic case, Equation (2.9) reduces to

$$R_n \approx -\cos\theta + \beta\cos\varphi\sin^2\theta \tag{2.12}$$

for low-order harmonics, in which the emission is confined: $J_n(n\alpha)/J'(n\alpha)\approx\alpha$ for $(n\alpha)^2\ll 1$.

2.2. Volume Emissivity of an Ensemble of Electrons

a. *General Case*

Though the spectrum of emission from an electron is discrete lines, the emission from an ensemble of those electrons which have a distribution of both momenutm p ($p\equiv mv/m_0c=\beta(1-\beta^2)^{-1/2}$) and pitch angle φ shows a continuous spectrum. Kawabata (1964) has given a general formula for the volume emissivity. If we indicate the distribution function of electrons as $N(p,\varphi)$, the number density of those electrons which have the momentums of p to $p+\mathrm{d}p$ and pitch angles of φ to $\varphi+\mathrm{d}\varphi$ is $2\pi N(p,\varphi)$ $p^2\sin\varphi\,\mathrm{d}p\mathrm{d}\varphi$. Then we have a volume emissivity $\eta(f,\theta)$, that is, the emission energy per second from a unit volume in a unit bandwidth at f into a unit steradian at θ,

$$\eta(f, \theta)\,\mathrm{d}f = \sum_n \int\int P_n(p, \varphi, \theta)\, N(p, \varphi)\, 2\pi p^2 \sin\varphi\, \mathrm{d}p\, \mathrm{d}\varphi\,, \tag{2.13}$$

where

$$P_n(p, \varphi, \theta) = \frac{2\pi e^2 f^4}{c n^2 f_H^2}\left[p^2 \sin^2\varphi\, J_n'^2(n\alpha) + \frac{(\sqrt{1+p^2}\cos\theta - p\cos\varphi)^2}{\sin^2\theta} \times J_n^2(n\alpha)\right]$$

and $\alpha = fp\sin\varphi\sin\theta/nf_H$, which are a mere transformation of Equation (2.7) using $\beta = p(1+p^2)^{-1/2}$, and the integration must be carried out over a $p-\varphi$ domain, the electrons of which contribute to the emission in a frequency range from f to $f+\mathrm{d}f$. Since f is a function of p and φ, the integration domain is determined by (cf. Equation 2.8),

$$f = nf_H(\sqrt{1+p^2} - p\cos\varphi\cos\theta)^{-1}. \tag{2.14}$$

By using this equation, we can reduce Equation (2.13) to a single integral with respect to p or φ.

When $\cos\theta \neq 0$,

$$\eta(f, \theta) = \frac{(2\pi)^2 e^2}{c}\,\frac{1}{\cos\theta}\,\frac{f^2}{f_H}\sum_n \frac{1}{n}\int_{p_1}^{p_2}\left[p^2\sin^2\varphi\, J_n'^2(n\alpha) + \frac{(\sqrt{1+p^2}\cos\theta - p\cos\varphi)^2}{\sin^2\theta} J_n^2(n\alpha)\right] N(p, \varphi)\, p\, \mathrm{d}p \tag{2.15}$$

where p_1 and p_2 are positive roots of Equation (2.14) setting $\cos\varphi = \pm 1$, and in the integrand

$$\cos\varphi = \frac{\sqrt{1+p^2} - \dfrac{nf_H}{f}}{p\cos\theta}.$$

When $\theta = \pi/2$,

$$\eta(f, \pi/2) = \frac{(2\pi)^2 e^2}{c} f \sum_n \left(\frac{n^2 f_H^2}{f^2} - 1\right)^{3/2} \int_0^\pi [\sin^2\varphi\, J_n'^2(n\alpha) + \cos^2\varphi\, J_n^2(n\alpha)] \times N(p, \varphi)\sin\varphi\, \mathrm{d}\varphi \tag{2.16}$$

and in the integrand

$$p = \left(\frac{n^2 f_H^2}{f^2} - 1\right)^{1/2}.$$

The summation over n is to be carried out for $n > f\sin\theta/f_H$, which contribute to the emission at f, since the frequency range of the nth harmonic is from 0 for $p = \infty$ to a maximum frequency $nf_H/\sin\theta$ for $p = \cot\theta$ (cf. Equation 2.14).

As an example, the volume emissivity is shown in Figures 5 and 6 setting as $\theta = 60°$ or $\theta = 30°$ and $N(p, \varphi) = p^{-4}$. Figure 5 shows the contribution of each harmonic

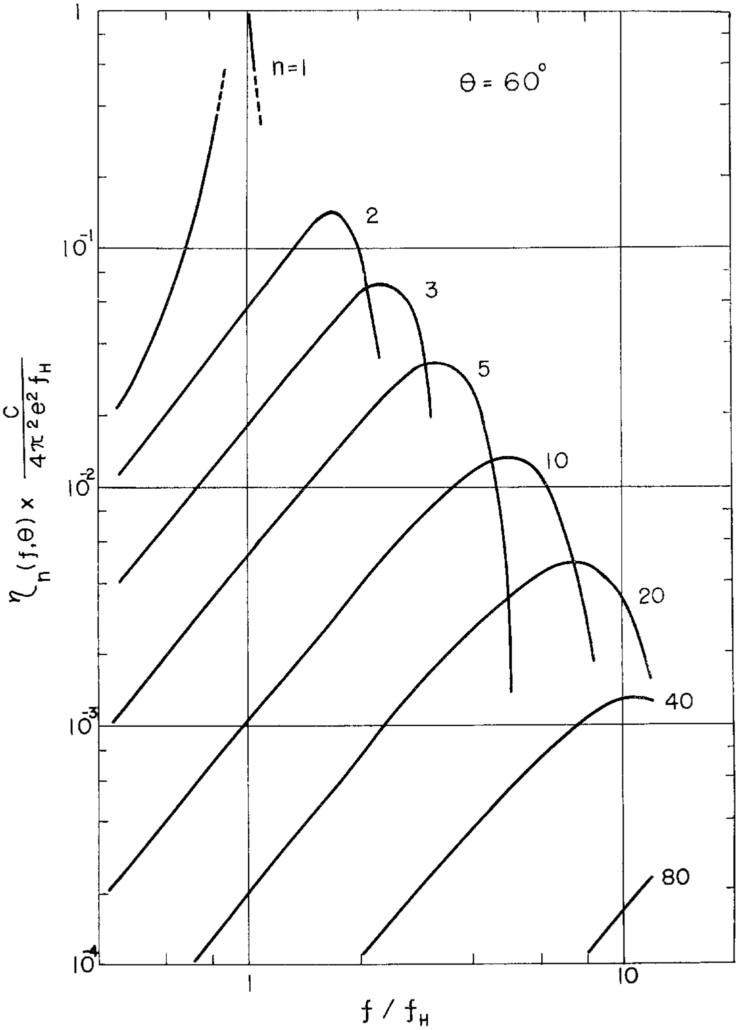

Fig. 5. Each harmonic component of volume emissivity of gyro-synchrotron emission from an ensemble of electrons is plotted against the frequency; $N(p, \varphi) = p^{-4}$, $\theta = 60°$. (After TAKAKURA, UCHIDA, and KAI.) Numerals on the curves indicate harmonic number n. The volume emissivity $\eta(f, \theta)$ is given by a summation over n from 1 to infinity (cf. Figure 6a).

separately for some values of n, and Figure 6a and 6b show the volume emissivity which is a summation over all harmonics as given by Equation (2.15). It should be remarked that the low frequency part of the spectrum ($f/f_H < 2$–3) suffers strong modification during the propagation due to the reflection or absorption (Section 2.4). The emissivity itself also suffers a depression due to an influence of the medium at low frequencies (Section 2.3).

b. *Extremely Relativistic Case* ($p^2 \gg 1$ or $1 - \beta^2 \ll 1$)

After SCHWINGER (1949), the power emitted per second per unit frequency interval

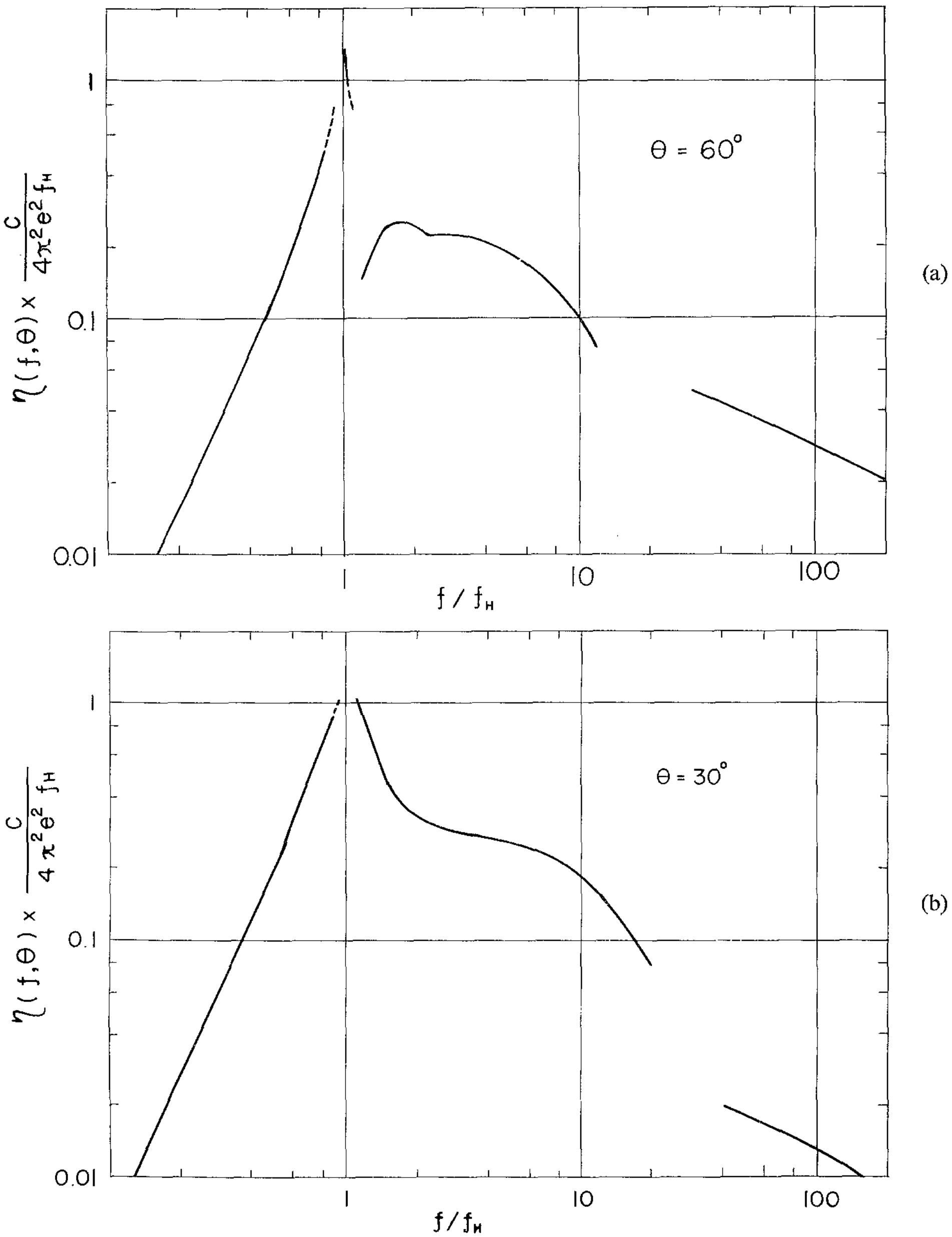

Fig. 6. Volume emissivity $\eta(f, \theta)$ of gyro-synchrotron emission from an ensemble of electrons with $N(p, \varphi) = p^{-4}$ is plotted against frequency. (After TAKAKURA, UCHIDA, and KAI.) – (a) $\theta = 60°$. At the frequencies above $f/f_H = 30$, extremely relativistic approximation is used (cf. Equation 2.20). With decreasing frequencies, the approximation of Equation (2.20) becomes poorer, since the contribution of low-order harmonics increases. – (b) $\theta = 30°$. At the frequencies above $f/f_H = 40$, extremely relativistic approximation is used. With decreasing frequencies, the approximation becomes poorer.

at f by an extremely relativistic electron in all directions is given by

$$P_f(p, \varphi) = \frac{2\sqrt{3}\,\pi e^2}{c} f_H \sin\varphi \left[u \int_u^\infty K_{5/3}(z)\,\mathrm{d}z \right], \tag{2.17}$$

where

$$u = f/f_c \quad \text{and} \quad f_c = \tfrac{3}{2} p^2 f_H \sin\varphi\,. \tag{2.18}$$

This emission is emitted into an effective solid angle contained between two cones of semi-angle $\varphi+\delta$ and $\varphi-\delta$. The effective solid angle is therefore $4\pi\delta\sin\varphi$, so that the emission into a unit solid angle around $\theta=\varphi$ is

$$P_f(p, \theta, \varphi) \approx \frac{1}{4\pi\delta \sin\varphi} P_f(p, \varphi), \tag{2.19}$$

where φ is to be set to θ. It should be noted that this approximation is poor at small values of θ even if $p^2 \gg 1$, since the contribution of low-order harmonics is comparatively large ($n_c \approx \frac{3}{2} p^3 \sin^3\varphi$, $\varphi \approx \theta$) and also the approximation $\varphi \approx \theta$ is not valid (cf., 'general tendency of $P_n(\beta, \varphi, \theta)$' mentioned previously).

The volume emissivity of an ensemble of electrons is given by (cf. Equation 2.13)

$$\eta(f, \theta) \approx \int_{\theta+\delta}^{\theta-\delta} \int_{p_1}^{\infty} P_f(p, \theta, \varphi) N(p, \varphi) 2\pi p^2 \sin\varphi\,\mathrm{d}p\,\mathrm{d}\varphi$$

$$\approx 2 \int_{p_1}^{\infty} \delta P_f(p, \theta, \varphi=\theta) N(p, \varphi=\theta) 2\pi p^2 \sin\theta\,\mathrm{d}p\,.$$

Substituting Equations (2.19) and (2.17), we have

$$\eta(f, \theta) = \frac{2\sqrt{3}\,\pi e^2}{c} f_H \sin\theta \int_{p_1}^{\infty} N(p, \varphi=\theta) p^2 \left[u \int_u^\infty K_{5/3}(z)\,\mathrm{d}z \right] \mathrm{d}p\,, \tag{2.20}$$

where p_1, lower limit to p, is set for the assumed extremely relativistic treatment ($p^2 \gg 1$), but may be set to 0 at $f > 40 f_H \sin\theta$, at which the contribution of electrons with p from p_1 to 0 is small. The function of u in the square bracket of Equation (2.20) is given by OORT *et al.* (1956) and also by others (e.g., GINZBURG *et al.*, 1965). Equation (2.20) is not valid for small values of θ as mentioned for Equation (2.19); e.g. at $\theta=0$, Equation (2.20) gives $\eta=0$, but Equation (2.15) gives $\eta\neq 0$ for $n=1$.

For the isotropic distribution of p, the distribution function of the momentum $N(p, \varphi)$ and that of the kinetic energy $N(\varepsilon)$ are equated by

$$4\pi p^2 N(p, \varphi)\,\mathrm{d}p = N(\varepsilon)\mathrm{d}\varepsilon\,, \tag{2.21}$$

where $\varepsilon = K/m_0c^2$; $(1+\varepsilon)^2 = 1+p^2$.

If $N(\varepsilon)=F\varepsilon^{-\gamma}$ in an extremely relativistic range of ε_1 to ε_2, it is well known that the volume emissivity is proportional to $f^{(1-\gamma)/2}$ in a frequency range $\frac{3}{2}\varepsilon_2^2 \sin\theta \gg f/f_H \gg \frac{3}{2}\varepsilon_1^2 \sin\theta$ (e.g., WESTFOLD, 1959); this is not valid, however, for small values of θ, as mentioned above. The extremely relativistic treatment, however, seems to be rare in the application to the solar bursts, except for the microwave bursts on millimeter wavelengths or shorter.

c. *Non-Relativistic Case* ($p^2 \approx \beta^2 \ll 1$)

The gyro-emission from a single electron in the nth harmonic is given by the approximation to Equation (2.7) for $(n\alpha)^2 \ll 1$,

$$P_n(p, \varphi, \theta) \approx \frac{2\pi e^2}{c} f_H^2 \frac{(np/2)^{2n}}{[(n-1)!]^2} (\sin\varphi)^{2n} (\sin\theta)^{2n-2} \times [1 + \cos^2\theta + 2\cos\theta\cos\varphi\{n + (n+2)\cos^2\theta\} p], \tag{2.22}$$

and the frequency of emission is

$$f \approx n f_H \left[1 - p\cos\theta\cos\varphi - \frac{p^2}{2}\right]^{-1}. \tag{2.23}$$

The volume emissivity of an ensemble of non-relativistic electrons is reduced from Equation (2.13) to (KAI, 1965b),

$$\eta(f, \theta) \approx \frac{4\pi^2 e^2}{c} f_H \frac{n^{2n+1}}{(n!)^2 2^{2n}} \frac{(\sin\theta)^{2n-2}}{\cos\theta} \{1 + \cos^2\theta + O(\Delta)\} \int_{p_1}^{p_2} (p^2 - p_1^2)^n \times N(p, \varphi)\, p\, \mathrm{d}p \tag{2.24}$$

at the frequency of $f = n f_H(1+\Delta)$, where

$$O(\Delta) = \left(n - 2 - \frac{n}{\cos^2\theta} + 2(n+1)\cos^2\theta\right)\frac{\Delta}{1+\Delta},$$

and the lower limit of integration p_1 is given by a positive root of

$$\frac{\Delta}{1+\Delta} = \pm\, p_1 \cos\theta - \frac{p_1^2}{2}. \tag{2.25}$$

The upper limit of integration p_2 is set as $p_2 \ll 1$ for the present assumption of non-relativistic treatment.

The gyro-emission seems to be unimportant as the origin of intense solar bursts, but it is more important as the absorption of the solar bursts as an inverse process of the gyro-emission from thermal electrons at the low-order harmonics of gyro-frequencies in the radio source and also in the outer layers through which the bursts propagate. More detail of the thermal gyro-absorption will be mentioned in Section (2.4).

2.3. Influence of the Ambient Plasma on the Gyro-Synchrotron Emission

In the previous sections, we have neglected the influence of ambient thermal plasma on the emissivity of gyro-synchrotron emission. In the solar atmosphere, energetic electrons which emit gyro-synchrotron emission gyrate in the ionized medium whose plasma frequency f_p is in some cases comparable to the gyro-frequency f_H. Even if $f_H > f_p$, the emissivity and polarization of the low-order harmonics are affected by the ambient medium. It is mainly attributed to the refractive index μ_f which is not unity in the ionized medium. Suppose that an electron is gyrating in a circular orbit and $\theta \neq 0$. A path length from the electron to the observer changes as a function of the location of the electron on the orbit. A difference of the path lengths for two points on the orbit, for example, results in a difference of travel times of the emission from the two points. The difference of the travel times is a function of the refractive index. Therefore, the radiation field given by Equations (2.2) and (2.3) (cf. Figures 2 and 3) is not valid if the refractive index of the medium *around the orbit* of the electron is not the unity. Moreover, the refractive index is a function of f, f_H, f_p and θ so that the general treatment of the gyro-synchrotron emission in the ionized medium is very complicated.

Twiss and Roberts (1958) have treated this problem and shown some numerical results. For $\beta^2 < 0.15$ and $(f_p/f_H)^2 = 0.8$, the total emission power decreases compared with the emission in a vacuum by a factor of about 0.02 for the fundamental, 0.8 for the 2nd harmonic, and 0.7 for the 3rd harmonic. Besides changing the total emission power, the ionized medium also modifies the emission polar diagram $P_n(\theta)$, especially at the fundamental frequency. Thus, the influence of the ionized medium is very strong for the fundamental frequency but may not be too serious for the harmonics even if f_p is comparable to f_H.

In the extremely relativistic case and $1 - \mu_f \ll 1$, where μ_f indicates refractive index, the emission which corresponds to Equation (2.17) reduces to (e.g., Ginzburg *et al.*, 1964, 1965),

$$P_f^*(p, \varphi) = \frac{2\sqrt{3}\,\pi e^2}{c} f_H \sin\varphi \left[1 + (1 - \mu_f^2)\, p^2\right]^{-1/2} \left[u^* \int_{u^*}^{\infty} K_{5/3}(z)\, \mathrm{d}z\right], \tag{2.26}$$

where

$$u^* = \frac{f}{f_c}\left[1 + (1 - \mu_f^2)\, p^2\right]^{3/2}.$$

The influence of the medium is therefore to be neglected if

$$(1 - \mu_f^2)\, p^2 \ll 1 \quad \text{for} \quad p^2 \gg 1. \tag{2.27}$$

Since $\mu_f^2 \approx 1 - (f_p/f)^2$ for $1 - \mu_f \ll 1$, Equation (2.27) reduces to

$$f^2 \gg (p f_p)^2, \tag{2.28}$$

or

$$(f/f_c)^2 \gg \left(\frac{f_p}{f_H \sin \varphi}\right)^2 \left(\frac{2}{3p}\right)^2 \quad \text{(cf. Equation 2.18.)} \tag{2.29}$$

2.4. Gyro-synchrotron absorption

The absorption of the wave is an inverse process of the emission, that is, an efficient emitter is an efficient absorber. Therefore the absorption coefficient κ is intimately related to the volume emissivity η. Since the gyro-synchrotron emission is an efficient emission process in the solar atmosphere, the gyro-synchrotron absorption is also important for the solar bursts. The absorption plays two roles for the bursts: One is self-absorption due to both non-thermal and thermal electrons in the radio source itself. If the number of electrons is sufficient, the source can be optically thick at certain frequencies, in other words, the emission is saturated at the corresponding frequencies. Another is thermal gyro-absorption due to thermal electrons in the outer layers during the propagation of the bursts. This absorption is generally strong, especially for extraordinary mode, at the low-order harmonics of gyro-frequencies in the outer layers.

By an analogy to the optical absorption, Twiss (1958) has treated the absorption coefficient at radio frequencies, and has shown a possibility of negative absorption for the gyro-emission, and for some other emission processes. The negative absorption means that the wave grows as the wave propagates in the medium similarly to the operations of the Maser and Laser. A comprehensive review on the absorption coefficient has been given by Wild *et al.* (1963) and also by Smerd (1965). The absorption coefficient is intimately related to

$$\frac{\partial N(p, \varphi)}{\partial W} + \left(\frac{m_0 c^2}{hf}\right) \Delta\varphi \frac{\partial N(p, \varphi)}{\partial \varphi} \tag{2.30}$$

where h is Planck's constant, W indicates total energy, $W = mc^2/m_0c^2 = (1+p^2)^{1/2}$, and $\Delta\varphi$ denotes a change in direction of the electron motion due to the emission or absorption of a photon. If the sign of (2.30) is positive, there is a possibility of negative absorption; but this is not sufficient to be negative absorption (Wild *et al.*, 1963; Smerd, 1965).

The general formulae of the absorption coefficients for the gyro-synchrotron emission in the tenuous plasma ($f_p \ll f$) have been given by Kawabata (1964). The general treatment of the absorption is, however, very complicated by the following reasons. The absorption due to an electron with a given p and φ is such that the polarization ellipse and the frequency of the absorption are the same as those of the emission from this electron. Accordingly the absorption coefficient due to an ensemble of the electrons is a complicated function of the polarization ellipse of the incident wave. On the other hand, the polarization ellipse of the incident wave generally changes as the wave propagates through the medium due to the absorption selective to the polarization. Therefore, the absorption rate changes according to the propagation of the wave, even if the medium is homogeneous. In order to know the attenuation of the incident wave of a given initial polarization ellipse, we have to solve the transfer equations (KAWABATA, 1964). Another more complicated effect arises, if the Faraday rotation is appreciable in the absorbing medium. The Faraday rotation is not necessarily negligible when the polarization is concerned, even if the refractive index of the medium is close to unity. Accordingly, the polarization ellipse generally changes as the wave propagates through the medium due to both the Faraday rotation and the above-mentioned absorption selective to the polarization.

Thermal gyro-absorption, that is the absorption due to the thermal electrons with Maxwellian distribution in the presence of the magnetic field, is a very important absorption for the solar bursts. This was first pointed out by GINZBURG and ZHELEZNYAKOV (1959b). The absorption coefficient of the thermal gyro-absorption has been calculated by several workers (SITENKO *et al.*, 1957; KAKINUMA *et al.*, 1962; ZHELEZNYAKOV, 1962; KAI, 1965b). The formulae of the absorption coefficients reduced from the general formulae of KAWABATA (1964) by KAI (1965b) are most convenient for the computation. The absorption coefficients at the nth harmonics of the gyro-frequency, for the extraordinary wave $\kappa_e^{(n)}$ and for the ordinary wave $\kappa_o^{(n)}$, are

$$\kappa_e^{(n)}(\omega, \theta) \approx \frac{(2\pi)^3 e^2}{m_0 c} \frac{1}{\omega_H} \frac{n^{2n-1}}{n!\,2^{2n}} \frac{\xi^{n-(3/2)}}{\pi^{3/2}} (\sin\theta)^{2n-2} \times$$
$$\times \frac{(c_1 + c_2 \cos\theta)^2}{\cos\theta} \exp\left\{ -\frac{\left(1 - \dfrac{n\omega_H}{\omega}\right)^2}{\xi \cos^2\theta} \right\}, \tag{2.31}$$

$$\kappa_0^{(n)}(\omega, \theta) \approx \frac{(2\pi)^3 e^2}{m_0 c \omega_H} \frac{n^{2n-1}}{n!\,2^{2n}} \frac{\xi^{n-(3/2)}}{\pi^{3/2}} (\sin\theta)^{2n-2} \frac{(c_2 - c_1 \cos\theta)^2}{\cos\theta}$$
$$\times \exp\left\{ -\frac{\left(1 - \dfrac{n\omega_H}{\omega}\right)^2}{\xi \cos^2\theta} \right\} \tag{2.32}$$

where $\xi = \kappa T / m_0 c^2$,

$$c_1 = \frac{\dfrac{Y_T^2}{2(1-X)} + \left\{ \dfrac{Y_T^4}{4(1-X)^2} + Y_L^2 \right\}^{1/2}}{\left[2Y_L^2 + \dfrac{Y_T^4}{2(1-X)^2} + \dfrac{Y_T^2}{1-X} \left\{ \dfrac{Y_T^4}{4(1-X)^2} + Y_L^2 \right\}^{1/2} \right]}$$

$$c_2 = \frac{Y_L}{\left[2Y_L^2 + \dfrac{Y_T^4}{2(1-X)^2} + \dfrac{Y_T^2}{1-X} \left\{ \dfrac{Y_T^4}{4(1-X)^2} + Y_L^2 \right\}^{1/2} \right]^{1/2}}$$

$$X = (\omega_p/\omega)^2, \quad Y_L = Y\cos\theta, \quad Y_T = Y\sin\theta \quad \text{and} \quad Y = \omega_H/\omega.$$

For the derivation of these formulae, it is assumed that the Faraday rotation is large enough, so that a phase relation between ordinary and extraordinary modes is randomized and the two modes propagate independently as if no coupling exists between them. These absorption coefficients thus derived coïncide numerically with those given by Sitenko and Stepanov (1957) on a strong field approximation, if the refractive indices of the waves are nearly equal to unity. An example of these absorption coefficients computed by Kakinuma *et al.* (1962) is shown in Figure 7.

Kai (1965b) has given a condition of negative absorption for the gyro-synchrotron emission. In the case of isotropic distribution of pitch angle φ, the absorption is always positive, even if $\partial N(p, \varphi)/\partial p$ is positive in a range of p. The only possible condition for the negative absorption is that the distribution of φ is confined around $\pi/2$ (nearly circular orbits) and the energies of electrons are not extremely relativistic. It is remarked that the absorption can be negative in this case, even if $\partial N(p, \varphi)/\partial p$ is negative.

2.5. The change of energy distribution of electrons with time

The radio spectrum of emission is a function of energy distribution $N(\varepsilon)$ (or momentum distribution $N(p, \varphi)$) as shown in Section 2.2. In order to account for the time variation of radio spectrum of radio bursts caused by the gyro-synchrotron emission, a time variation of the energy distribution $N(\varepsilon, t)$ must be computed. Though the decay rate of an electron due to the emission loss is simply given by the following Equation (2.34), the decay curve of a radio burst caused by an ensemble of electrons is not so simple, but it is a function of $N(\varepsilon, t)$. The decay curve has been calculated by Takakura and Kai (1966) for the application to the microwave impulsive bursts. It is assumed that an appreciable acceleration ends off at the time of maximum

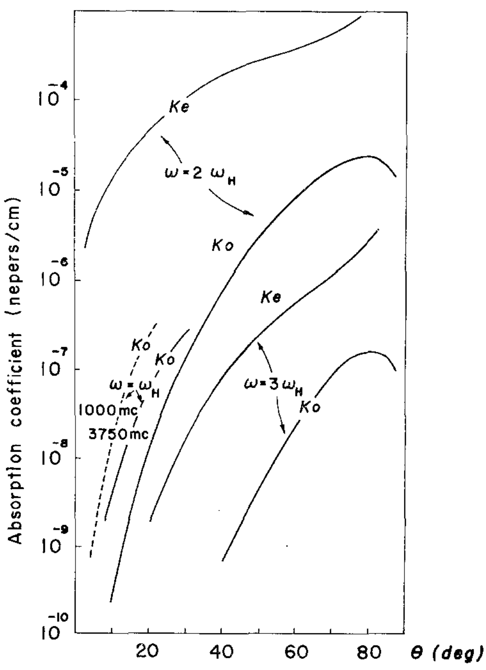

Fig. 7. Gyro-absorption coefficient at $\lambda = 8$ cm for an ionized gas with $N_0 = 10^9$ cm^{-3} and $T = 2 \times 10^6$ °K. (After Kakinuma and Swarup, 1962.)

intensity of the burst giving an initial energy distribution of the energetic electrons trapped in a magnetic tube situated in the lower corona between bipolar sunspots.

The total emission power per second from an electron is

$$P = \frac{8\pi^2}{3} \frac{e^2}{c} f_H^2 \langle \sin^2 \varphi \rangle \{\varepsilon^2 + 2\varepsilon\}, \tag{2.33}$$

and

$$- \mathrm{d}\varepsilon/\mathrm{d}t = P/m_0 c^2 . \tag{2.34}$$

The pitch angle φ changes with time for the electron trapped in the magnetic tube, but the period with $\varphi \approx \pi/2$ is generally longer than $\varphi \approx 0$. Therefore, $\langle \sin^2 \varphi \rangle$ is set to unity in the following treatment, for simplicity, but this factor may be included in f_H as a smaller effective value of the magnetic field-strength.

The change in the kinetic energy of an energetic electron due to both the emission and the collisions with ambient thermal electrons of number density N_0 is given by

$$- \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \nu_H \left(\frac{\varepsilon^2}{2} + \varepsilon \right) + \nu_c \varepsilon^{-3/2} , \tag{2.35}$$

where $\nu_H = 3.8 \times 10^{-9} H^2 \langle \sin^2 \varphi \rangle$ sec^{-1} (H in gauss), and $\nu_c \approx 1.5 \times 10^{-16} N_0$ sec^{-1} (N_0 in cm^{-3}) in the corona. Two terms in the bracket represent the synchrotron loss and gyro-emission loss respectively, and the last term represents the collision loss. $\varepsilon = 1$ at about 500 keV ($\beta = \sqrt{3}/2$ and $p = \sqrt{3}$), so that a dominant term in Equation (2.35) can easily be found for given values of the kinetic energy of an electron. For example, if H is several hundred gauss and N_0 is of the order of 10^9 cm^{-3}, as is a typical case of the bursts on microwaves, the collision term becomes smaller than emission terms for the electron above about 15 keV ($\varepsilon \approx 0.03$), while the ε^2 term is negligible at the energies below about 200 keV ($\varepsilon \approx 0.4$). As in that case of type IV bursts on meter waves, in which H is several gauss and N_0 is of the order of 10^6 cm^{-3}, the collision term becomes smaller for the electrons above about 40 keV ($\varepsilon \approx 0.08$). In a range $\varepsilon > 0.04$, however, another collision term $\nu_c' \varepsilon^{-1/2}$, where $\nu_c' \approx 2 \times 10^{-14} N_0$ sec^{-1} (N_0 in cm^{-3}), is more adequate than the collision term ($\nu_c \varepsilon^{-3/2}$) in Equation (2.35).

The change of the energy distribution $N(\varepsilon, t)$ of electrons with an initial distribution $N(\varepsilon_0)$ is given by

$$N(\varepsilon, t)\, \mathrm{d}\varepsilon = N(\varepsilon_0) \exp\left(- \int_0^t \mathrm{d}t/t_D(\varepsilon) \right) \mathrm{d}\varepsilon_0 , \tag{2.36}$$

where ε_0 is the initial value of ε at $t = 0$ and t_D is the deflection time (Spitzer, 1962). The exponential term in the above equation indicates a rate of decrease of number of electrons due to the escape of the electrons from the magnetic tube, since the electrons which have attained small pitch angles due to the collisions in the magnetic tube can penetrate into the chromosphere and lose energies by collisions there. In Equation (2.36), ε_0 and $\mathrm{d}\varepsilon_0/\mathrm{d}\varepsilon$ are the function of ε and t as a solution of Equation (2.35).

For example, if $N(\varepsilon_0)=F\varepsilon_0^{-\gamma}$, Equation (2.36) reduces to (TAKAKURA and KAI, 1966),

$$N(\varepsilon, t) = F\varepsilon^{-\gamma}\left[\exp\left\{(1-\gamma)\,\nu_H t - \frac{1}{\tau_D}\int_0^t \varepsilon^{-3/2}\,dt\right\}\right] \times \left[1 - \frac{\varepsilon}{2}\left\{\exp(\nu_H t) - 1\right\}\right]^{\gamma-2} \tag{2.37}$$

for $\varepsilon > 0.04$ but ε smaller than $2[\exp(\nu_H t)-1]^{-1}$, which corresponds to $\varepsilon_0 = \infty$ at $t=0$. Where $\tau_D \approx 2.4\times10^{12}\,N_0^{-1}$ sec (N_0 in cm^{-3}) in the corona.

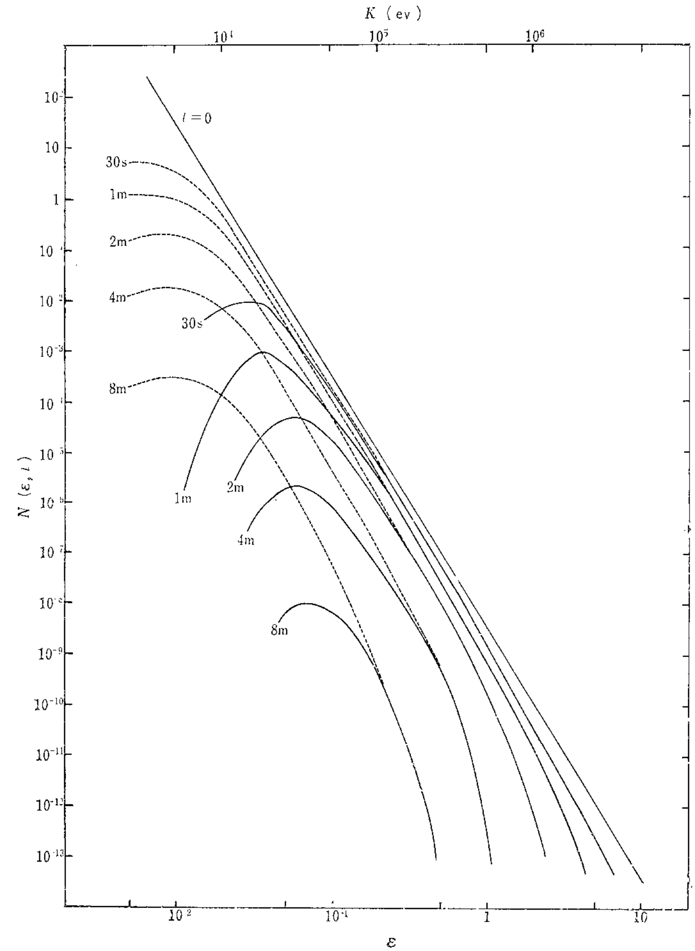

Fig. 8. Time variation of energy distribution function of energetic electrons, $N(\varepsilon, t)$. Initial distribution is assumed as $N(\varepsilon, 0) \sim \varepsilon^{-5}$ normalizing at $\varepsilon = 0.02$ ($K \approx 10^4$ eV); $\varepsilon = K/m_0c^2$. (After TAKAKURA and KAI, 1966.) Numerals on the curves indicate time. An effect of escaping electrons is taken into account for the solid curves, while dashed curves correspond to the case in which their escape is neglected. $H = 1000$ gauss and $N_0 = 10^9$ cm^{-3}.

For $\varepsilon > \frac{1}{700}$ but ε is smaller than a value which corresponds to $\varepsilon_0 \approx \frac{1}{3}$ at $t=0$,

$$N(\varepsilon, t) = F\varepsilon^{-\gamma}\left[\exp\left\{(1-\gamma)\,\nu_H t - \frac{1}{\tau_D}\int_0^t \varepsilon^{-3/2}\,\mathrm{d}t\right\}\right] \times \\ \times\left[\frac{\nu_c}{\nu_H}\varepsilon^{-5/2}\{1-\exp(-\tfrac{5}{2}\nu_H t)\} + 1\right]^{-(3+2\gamma)/5} . \quad (2.38)$$

It may be noted again that $N(\varepsilon)\,\mathrm{d}\varepsilon = 4\pi p^2 N(p, \varphi)\,\mathrm{d}p$ if the distribution of the pitch angle φ is isotropic; $(1+\varepsilon)^2 = 1+p^2$.

Examples of $N(\varepsilon, t)$ are shown in Figures 8 and 9, in which $\gamma=5$ and 3, respectively, $H=1000$ gauss and $N_0=10^9$ cm^{-3}. The gyro-synchrotron emission from these electrons decreases faster for larger values of γ, as is evident from these diagrams and also from a term $\exp(1-\gamma)\,\nu_H t$ in Equations (2.37) and (2.38). Therefore, the decay time of the solar bursts due to the gyro-synchrotron emission is not only the function of the magnetic field-strength and collision frequency, but also a function of the initial energy distribution function; for steeper slope of the distribution function the decay time is faster. The similar result is derived also for the initial distribution function of the exponential law, $\exp(-\varepsilon_0/\varepsilon_c)$, in which the decay time is faster for smaller value of ε_c (Takakura *et al.*, 1966).

Stein and Ney (1963) computed a time variation of the electron energy distribution for $\exp(-\varepsilon_0/400)$, but their computation is incorrect. Their result corresponds to set $\mathrm{d}\varepsilon_0/\mathrm{d}\varepsilon = 1$ in Equation (2.36); the exponential term in this equation can be neglected

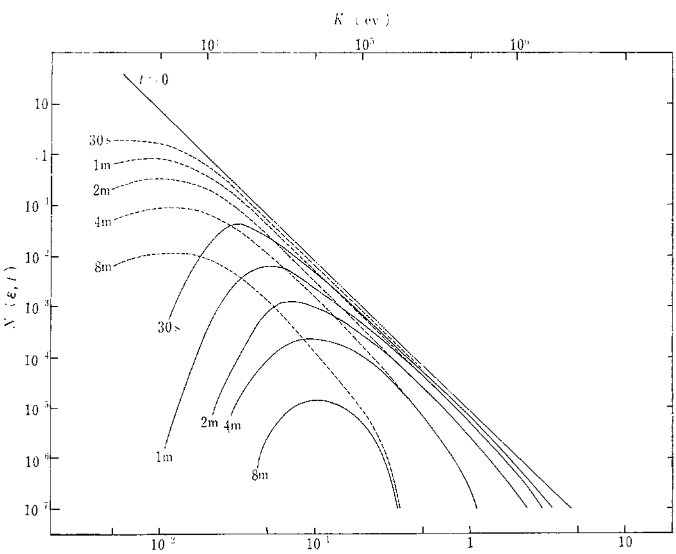

Fig. 9. $N(\varepsilon, t)$ for $N(\varepsilon, 0) \sim \varepsilon^{-3}$. (After Takakura and Kai, 1966.) The other conditions are the same as in Figure 8.

in their case. However, ε_0 and $d\varepsilon_0/d\varepsilon$ are the function of ε and t as a solution of Equation (2.35). If the computation is made correctly, $N(\varepsilon, t)$, dN/dE in their notation, slightly *increases* with time at low energies (TAKAKURA *et al.*, 1966) due to an accumulation of higher-energy electrons into the low and *narrower* energy range according to their emission loss.

3. Gyro-synchrotron Emission and Solar Bursts

In the various types of solar bursts, type IV bursts and microwave impulsive bursts have been attributed to the gyro-synchrotron emission. With the Nancay interferometer at 169 Mc/s, BOISCHOT (1957, 1958) was able to distinguish between ordinary noise storms related to active centers and an enhanced continuum emission following major outbursts associated with important flares. This prolonged continuum emission was designated as type IV.

BOISCHOT and DENISSE (1957) have explained this emission as due to synchrotron emission from relativistic electrons of 3 MeV assuming a coronal magnetic field of 1 gauss.

TAKAKURA (1959, 1960) has applied the gyro-synchrotron emission from mildly relativistic electrons (1 MeV–10 keV) in sunspot magnetic field of about 10^3 gauss to the interpretation of microwave bursts, designating also as type IV burst. Subsequently the designation of 'Type IV' has been extended with some confusions to any prolonged continuum bursts following big solar flares (Section 3.3).

3.1. MICROWAVE BURSTS (OBSERVATIONAL)

Some of bursts on microwaves (>1000 Mc/s) are gradual type, i.e., 'gradual rise and fall' and 'post-burst increase'. They are weak and are attributed to the thermal emission, both free-free emission and thermal gyro-emission, from sporadic coronal condensations (e.g., KAWABATA, 1960, 1966; KUNDU, 1963). These bursts will be excluded in the following description.

A great majority of bursts on microwaves are impulsive, that is, the intensity rises to a single maximum and then decreases more slowly. The duration of such simple bursts, at 2800 Mc/s for example, ranges from 1 minute to 10 minutes, but about 10% of them have longer durations of up to 1 hour (post-burst increases are excluded). Sometimes several bursts occur in succession to form a complex group of bursts. Their durations at 2800 Mc/s are mainly a few minutes to 30 minutes, but about 10% of them have longer duration up to 5 hours (post-burst increases are excluded).

The intensities of these microwave bursts cover a wide range from one flux unit (10^{-22} wm^{-2}(c/s)$^{-1}$) to 10^4 units; the lower limit may be determined by the sensitivity of radiometers.

It is controversial whether these microwave bursts can be divided into subgroups or not. The bursts of short duration, less than about 10 minutes, are generally called 'microwave impulsive bursts' (M-type or microwave early burst, cf. Figure 16), and prolonged bursts (above 20–30 minutes) associated with intense flares are designated as 'microwave type IV bursts (type IVμ)', which are generally complex groups of strong bursts but some of which are prolonged simple bursts. Nevertheless, if we plot the histograms of durations (τ), intensities (I) and $\tau \times I$ for all microwave bursts or for each simple and complex burst, we cannot find any significant subgroup from these histograms; significant maximum is only one for each histogram. Also the other characteristics of the bursts, i.e., spectrum, polarization, position and motion of the source and size of the source, cannot divide the microwave bursts (excluding gradual type) into *distinctly different* subgroups. It may imply that the mechanism of emission is the same for all microwave bursts but physical parameters involved in this mechanism vary from bursts to bursts in a wide range.

Even though the distinction between the impulsive bursts and type IVμ bursts is not clear, these are distinguished for convenience, roughly, as follows: The impulsive bursts are minor events which frequently occur associating with minor flares, and are not followed generally by type IV bursts on meter and decimeter waves. Type IVμ bursts are exceptional events associated with important flares, and are generally followed by type IV bursts on meter and/or decimeter waves.

The microwave bursts as a whole are generally associated with optical flares, both large and small. The starting time of the microwave bursts coïncides with the 'explosive phase' of the flares (Covington *et al.*, 1961).

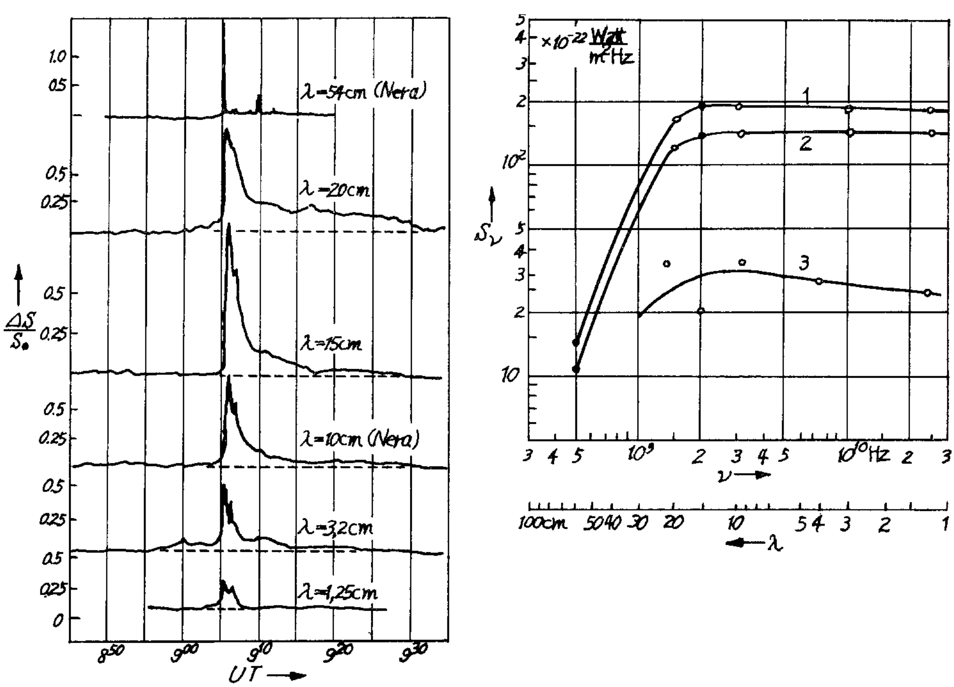

Fig. 10. Time profile and time variation of the spectrum of a microwave impulsive burst on July 12, 1959. (After Hachenberg and Wallis, 1961.) (1) Maximum at 0906 UT. (2) Maximum at 0907.5 UT. (3) Postburst increase at 0910 UT. The spectrum is flat at high frequencies.

The spectrum of the microwave bursts is a broad-band continuum with a maximum intensity generally at about $10^4 - 3 \times 10^3$ Mc/s. Towards the lower frequencies the spectrum decreases steeply, while towards the higher frequencies the decrease is generally more gradual (TAKAKURA, 1960c; HACHENBERG *et al.*, 1961). Some examples are shown in Figures 10 to 12. Figures 10 and 11 show two similar impulsive bursts but with different spectra. Figure 12 shows a type IVμ burst.

The microwave bursts are partially polarized circularly and, for about 60% of them, the sense of circular polarization reverses somewhere between 4000 Mc/s and 2000 Mc/s, at which the polarization degree is smallest (KAKINUMA, 1958; TANAKA *et al.*, 1959). The polarization degree at the higher frequencies is less than 50%. The sense of polarization, at least at high frequencies, seems to have a correlation with the occurring location of the burst on the solar disk (KAKINUMA, 1958) as is observed for the slowly varying components (PIDDINGTON *et al.*, 1951; KAKINUMA, 1956; TANAKA *et al.*, 1958; TANAKA *et al.*, 1960). That is, when the bursts occur on the northern hemisphere and the longitude is more than 40°, the senses of polarization are opposite to each other for the bursts occurring on the east side and on the west side of the northern hemisphere: right-handed for the sources on the east side in the last solar cycle. The bursts which occur in the southern hemisphere also have the similar tendency but the sense is opposite, i.e., the sense is left-handed for the sources

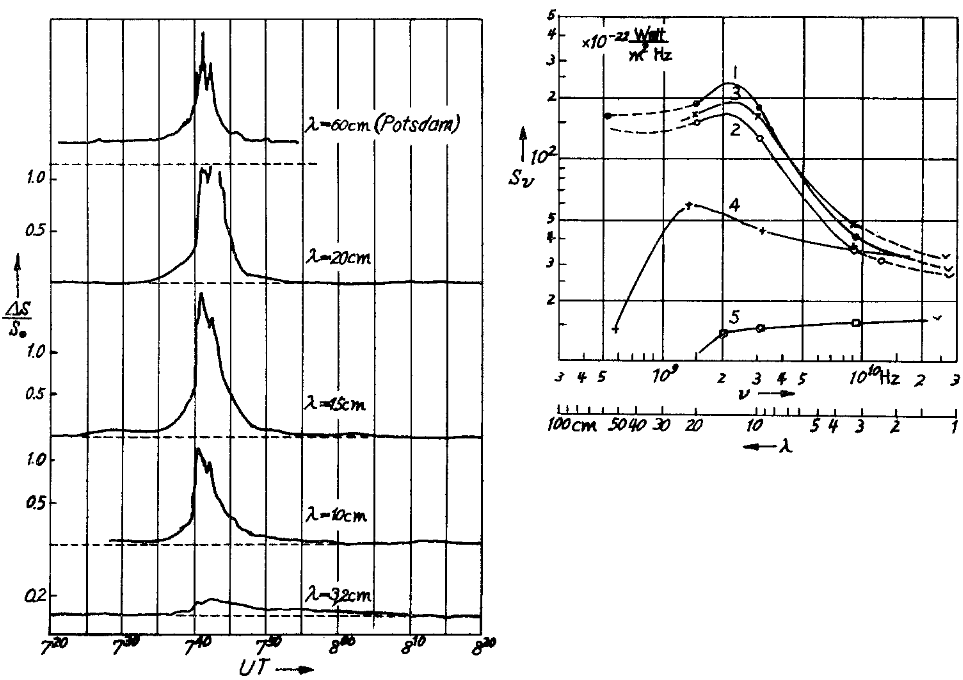

Fig. 11. Time profile and time variation of the spectrum of a microwave impulsive burst on September 2, 1959. (After HACHENBERG and WALLIS, 1961.) (1) Maximum at 0740.5 UT. (2) Minimum at 0741.5 UT. (3) Maximum at 0742.3 UT. (4) Decay part at 0745 UT. (5) Postburst increase at 0750 UT. The spectrum decreases at high frequencies, showing a non-thermal spectrum.

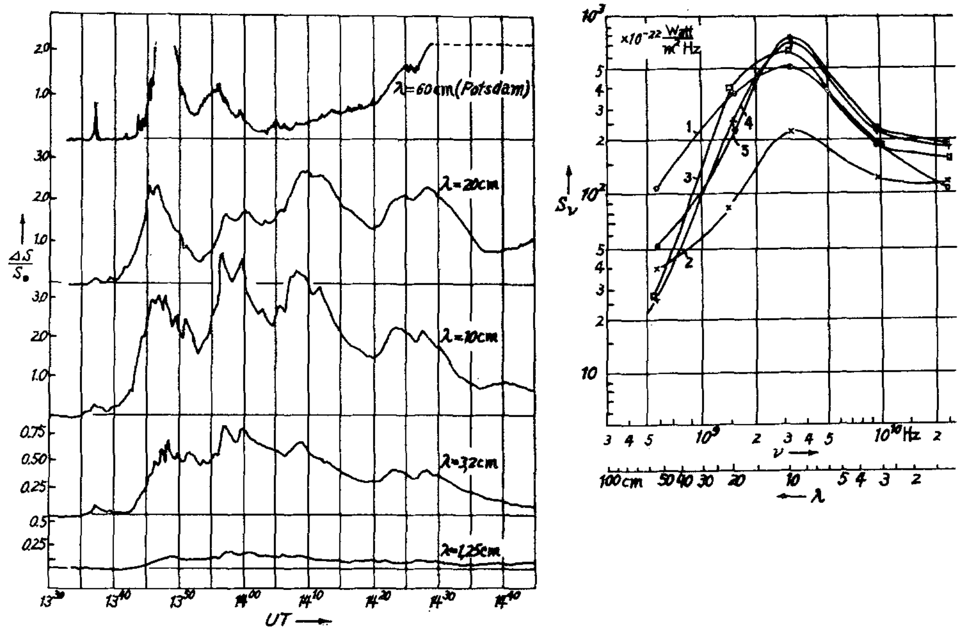

Fig. 12. Time profile and time variation of the spectrum of a microwave type IV burst on January 15, 1960. (After HACHENBERG and WALLIS, 1961.) (1) Maximum at 1348.5 UT. (2) Minimum at 1355 UT. (3) Maximum at 1357.5 UT. (4) Maximum at 1405.5 UT. (5) Maximum at 1408 UT.

on the east side. The bursts occurring in a central zone have both senses of polarization irrespective of the northern and southern hemispheres. Referring to the sense of polarization of slowly varying components, it has been concluded that the bursts at 9400 Mc/s is extraordinary mode (KAKINUMA, 1958).

The position of microwave burst coïncides fairly well with both an associated flare and a persistent source of slowly varying component. No significant motion of burst sources has been observed (KUNDU, 1959; KAKINUMA *et al.*, 1961).

Recently a brightness distribution of an impulsive burst with non-thermal spectrum was observed by chance at Toyokawa with the 9400 Mc/s interferometer with a fan beam of 1.1 minutes of arc (TANAKA *et al.*, 1967). The brightness distribution had two sharp maxima separated by 2 minutes of arc, each of which was above a sunspot of a bipolar group. The two maxima had opposite senses of circular polarization, and the polarization degree was 70% for the stronger preceding maximum. This fine structure is just as predicted by TAKAKURA and KAI (1966); cf. Figure 15.

Microwave bursts are intimately connected with bursts of X-rays in an energy range of 1 to several hundred keV. Of these X-rays, hard X-rays (>20 keV) show remarkably similar profiles to their coïncident impulsive bursts (e.g., KUNDU, 1961, 1963; ANDERSON *et al.*, 1962; FROST, 1964). Even in a lower-energy range of X-rays (1–2.4 keV), 'bursts' with non-thermal spectra were at times superposed on slow enhancements (POUNDS, 1965). These X-ray bursts were also coïncident with microwave impulsive bursts.

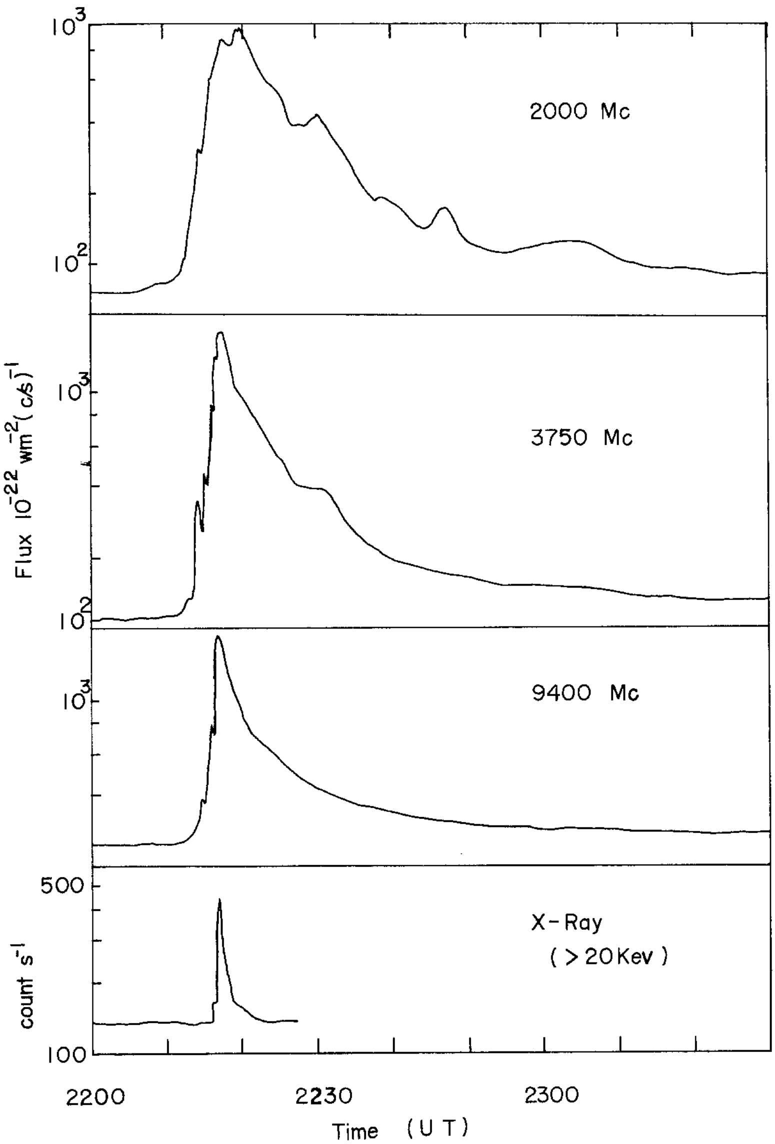

Fig. 13. Hard X-ray burst (> 20 keV) associated with a microwave type IV burst on September 28, 1961. The X-ray record is after Anderson *et al.* (1962). The microwave bursts were observed at Toyokawa, Japan (courtesy of H. Tanaka and T. Kakinuma). Decimeter and meter wave components of the type IV burst were also observed at Fort Davis (Maxwell, 1963).

At a type IV burst, however, the hard X-ray burst was associated only with the initial phase of the radio bursts, as shown in Figure 13: the duration of the X-ray burst was about 2 minutes, while that of the microwave burst was about 40 minutes. An intimate association of X-ray bursts (and microwave impulsive bursts) with 'explosive' flares has been shown by Moreton (1964).

Energetic electrons (>40 keV) ejected from the sun were detected in the interplanetary space by VAN ALLEN and KRIMIGIS (1965) and ANDERSON and LIN (1966). These electrons were generally preceded by the microwave bursts.

3.2. APPLICATION OF GYRO-SYNCHROTRON EMISSION TO MICROWAVE IMPULSIVE BURSTS

In this section, the gyro-synchrotron mechanism will be applied to the interpretation of the observed characteristics of the microwave impulsive bursts, but the same interpretation can be applied also to type IVμ bursts under some modification of physical parameters such as magnetic field, energy distribution of electrons, and density of ambient thermal electrons, since the impulsive bursts and type IVμ bursts seem to be one family, as mentioned in the previous section.

On the other hand, HACHENBERG and WALLIS (1961) have proposed, basing on the radio spectrum, that the impulsive burst is due to free-free emission (Bremsstrahlung) from a hot sporadic condensation with $T \approx 10^7$ °K and $N_0 \approx 10^{10}$ cm^{-3}. Even though many of the impulsive bursts have the spectra similar to the free-free emission, the others have non-thermal spectra. For example, Figure 10 shows a spectrum similar to the free-free emission, but Figure 11 shows non-thermal spectrum: the free-free emission never decreases with increasing frequency in this frequency range. Furthermore, their other characteristics are similar to each other, so that there is no basis to divide these bursts into two subgroups if these spectra can be accounted for by one emission mechanism. WILD (1964) pointed out that the *time variation* of the spectrum is difficult to be accounted for by the free-free emission, even when the spectrum is similar to that of free-free.

In the total flux of the burst, a component of the free-free emission may be included, e.g., as a part of post-burst increase. However, the flux value attributable to the free-free emission may be less than 50 units in 10^{-22}wm^{-2}(c/s)$^{-1}$ as the post-burst increases generally are: even for the post-burst increases, thermal gyro-emission may be predominant compared with the free-free emission at the frequencies below 10^4 Mc/s or so. Therefore, a major part of the impulsive burst which can have the flux of several thousands units is to be attributed to another process, such as gyro-synchrotron emission.

a. *Spectrum and Polarization*

Suppose that the magnetic field and energetic electrons are homogeneous throughout a radio source. The expected radio spectrum from this source can be deduced from Figure 6, though the spectrum depends on the energy distribution of the energetic electrons. At low frequencies, say $f/f_H < 3$, an absorption of the spectrum due to thermal gyro-absorption at low-order harmonics of local gyro-frequencies in the outer layers must be taken into account (cf. Section 2.4). The absorption is serious for both extraordinary and ordinary modes at $n=1$ and 2, and only extraordinary mode suffers some absorption at $n=3$ in a plausible model of a coronal condensation around which the microwave bursts originate (cf. Figure 7). Now, the local gyro-frequency $f_H(h)$ decreases with increasing height according to a decrease in the magnetic field-strength with height. Therefore, the gyro-synchrotron emission from the radio source at the frequencies $f/f_H<2$ (cf. Figure 6) must pass though the outer layers in which the second harmonics of the *local* gyro-frequencies $f_H(h)$ become equal to the frequencies $(f/f_H<2)$ of the emission. In the same manner, the emission from the radio source at the frequencies of $2f_H$ to $3f_H$ must pass through the outer layers in which the third harmonics of the local gyrofrequencies become equal to the frequencies of the emission. Consequently, the emission is almost absorbed in a frequency range of $f/f_H<2$. In the frequency range of $f/f_H=2$ to 3, an extraordinary component of the emission suffers some selective absorption. Thus, an expected spectrum to be observed on the earth is that the spectrum has a maximum at about $(3\sim4)f_H$ and a decreasing slope of the spectrum towards lower frequencies is steep. The decreasing slope at the higher frequencies depends on the energy-distribution function of electrons. If the energy distribution is given by a power law $N(\varepsilon)=F\varepsilon^{-\gamma}$, the decreasing slope at high frequencies, say $f/f_H>$ 50–100 at which extremely relativistic treatment is approximately valid, is nearly proportional to $f^{(1-\gamma)/2}$; for small values of θ, the approximation is valid only at higher frequencies (cf. Section 2.2.b).

An integral number density of energetic electrons necessary to account for the observed intensity is $1-10^3$ cm^{-3}.

The influence of ambient plasma on the emissivity itself, as mentioned in Section 2.3, depresses the spectrum at low frequencies. Even though quantitative study of the depression is not yet made for the gyro-synchrotron emission from mildly relativistic electrons, this influence would be important at low frequencies, say f/f_H is below 2–5 (cf. Section 3.4). Furthermore, the radio source may be optically thick at low frequencies due to self-absorption. Therefore, quantitative interpretation of the spectrum at low frequencies is left for the further study.

If a microwave burst originates from a double source above a bipolar spot group as was recentl yobserved (Section 3.1), the spectrum of the burst is a sum of two spectra from the double source. The two spectra have a similar profile but both intensity scale and frequency scale are to be shifted in proportion to f_H of the corresponding source, as shown diagrammatically in Figure 14. The sense of circular polarization of the gyro-synchrotron emission from electrons with isotropic distribution of pitch angles is generally that of the extraordinary mode. Therefore, the senses of polarization of emission from two sources with opposite senses of magnetic field (bipolar spots) are opposite to each other. Accordingly, the polarization state of the total spectrum is that the sense of polarization reverses *with frequency*, as shown in Figure 14. This polarization state may change during the propagation due to the mode coupling (COHEN, 1960) in the same manner as the slowly varying component (TAKAKURA, 1961b). That is, the two sources may have the same sense of polarization, for example at 9400 Mc/s, for the bursts occurring near the limbs of the solar disk, and the sense is opposite for the bursts occurring near the north-east limb and those of the north-west limb, as is actually observed (Section 3.1).

On an assumption that the source of a microwave burst is single, the sense reversal

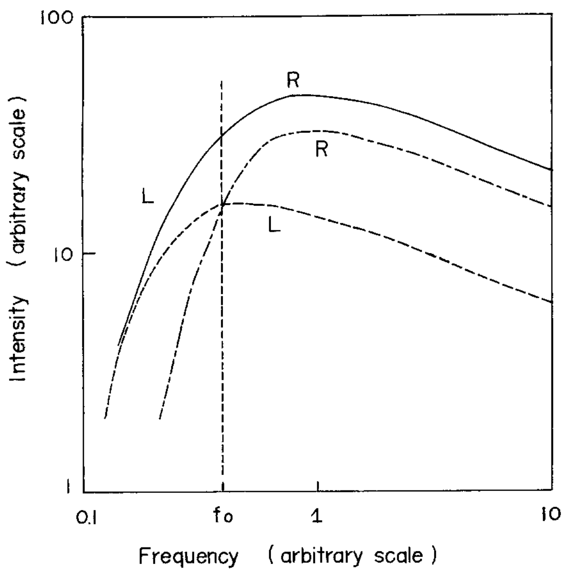

Fig. 14. Diagramatic representation of expected radio spectrum originating from a double source of a microwave burst (cf. Figure 15). Chain curve: spectrum due to the preceding (predominant) p-spot. Dashed curve: spectrum due to the following (weaker) f-spot, setting f_H (f-spot) $= \frac{1}{2} f_H$ (p-spot). Solid curve: Sum of above two spectra. Sense reversal of circular polarization from R to L with frequency is expected to occur at about f_0.

with frequency was attributed to other causes. KAKINUMA (1958) attributed this to a difference of reflection levels for extraordinary mode and ordinary mode; the latter is deeper. TAKAKURA (1960b,c) attributed it to a selective absorption of extraordinary mode due to thermal gyro-absorption at $f = f_H$, and later to its 2nd and 3rd harmonics (TAKAKURA, 1964b, 1966). COHEN (1961a) attributed it to the mode coupling.

b. *Time Variation of the Spectrum of Impulsive Burst*

The radio spectrum mentioned above changes with time according to a time variation of energy distribution $N(\varepsilon, t)$ of energetic electrons. On the other hand, $N(\varepsilon, t)$ can be computed as given in Section 2.5. Therefore, the time variation of the radio spectrum can be computed in principle and is to be compared with the observation. However, it is laborious so that the systematic computation has not yet been made. TAKAKURA and KAI (1966) have computed the time variation of that total emission, which is an integration of the spectrum with respect to the frequency, and compared with the observed decay curves of microwave impulsive bursts with a spectral maximum at about 4000 Mc/s. The observed decay curves fit with the initial energy distribution of electrons with a power law $N(\varepsilon_0) = F\varepsilon_0^{-\gamma}$ for γ of 3 to 5 in an assumed magnetic field of 500 gauss. If the power law is assumed in a range $0.01 < \varepsilon_0 < 40$ (5 keV–20 MeV), an effective energy range of electrons, from which 80% of the total emission are emitted, are 15 keV–1.5 MeV for $\gamma = 3$, 8 keV–125 keV for $\gamma = 4$, and 6 keV–30 keV for $\gamma = 5$.

c. *Microwave Bursts and X-Ray Bursts*

Hard X-ray (>20 keV) has been attributed to the Bremsstrahlung due to collisions of energetic electrons (>20 keV) with thermal particles including neutral hydrogen (e.g., PETERSON *et al.*, 1959). The total number of the energetic electrons required to account for the X-ray intensities is 10^3–10^4 times greater than that available to emit coïncident microwave impulsive bursts as the gyro-synchrotron emission. In order to account for this discrepancy, TAKAKURA and KAI (1966) have proposed a model of flare region emitting both a microwave impulsive burst and an X-ray burst (Figure 15). In this model, a major part of the energetic electrons is trapped in a region X in which the magnetic field is weak, say 10^2 gauss ($f_H = 280$ Mc/s), and the density of thermal electron is comparatively high, say 5×10^{10} cm^{-3} ($f_p = 2000$ Mc/s). Accordingly, gyro-synchrotron emission from the major part of the energetic electrons cannot escape, and only a small fraction of the energetic electrons trapped in an outer region contributes to the radio burst. This model also predicts such double-source structure of the radio burst as actually observed later (cf. Section 3.1).

In order that a part of energetic electrons can escape into the outer corona emitting type III bursts (Section 5.1) and further out to the interplanetary space, *a part* of the magnetic field lines from the sunspot region should be extended out to the interplanetary space. This radially outward magnetic field is also required for the interpretation of the limiting polarization of type I noise storms (TAKAKURA, 1961b).

At a type IVμ burst, hard X-ray was associated only at the initial phase of the radio burst (Figure 13). This implies that at type IVμ bursts a major part of energetic electrons, accelerated at an initial phase of a flare, are lost in a short time emitting an X-ray burst in region X (Figure 15), while smaller numbers of electrons are

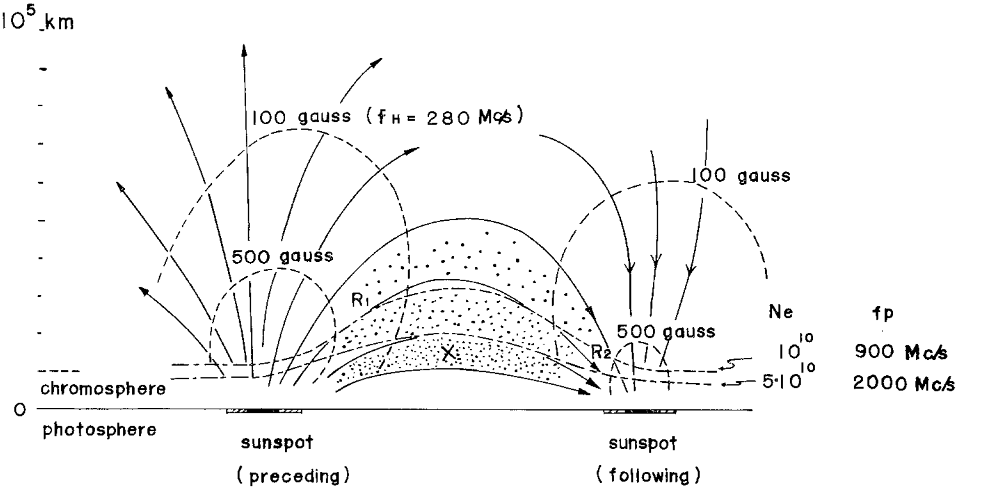

Fig. 15. A compound model of radio source and hard X-ray source. (After TAKAKURA and KAI, 1966.) X: Source for hard X-ray burst. R_1, R_2: Sources for microwave impulsive burst. R_2 may or may not appear as the radio source depending on the magnetic field strength of the following spot. A part of the field lines from the preceding spot may be extended into the interplanetary space.

accelerated again or accelerated for longer time only in the outer layer emitting a type IVμ burst. This may also imply a qualitative difference between type IVμ bursts and microwave bursts; the acceleration mechanism of electrons could be different at the initial phase and later, and the impulsive bursts correspond to the initial phase.

3.3. Type IV bursts (observational)

Type IV bursts are prolonged continuum bursts following big flares. A review on this type of burst has been given by Fokker (1963).

The spectrum of a fully developed type IV burst as a whole covers almost all frequency range observable on the earth, and it is complex and full of variety. Therefore, the nomenclature of type IV bursts was confused at the early stage of research. The wide-band spectrum does not mean that the whole emission originates from a single source at a time. A broad outline of the spectra of type IV bursts as a whole was to be seen from a spectral diagram derived from single frequency records at several different frequencies throughout the radio spectrum (Takakura *et al.*, 1961; Kakinuma *et al.*, 1961). Subsequently, such spectral diagrams have been obtained for the later type IV bursts. The question of how to define subclasses of type IV burst was treated also in a number of papers and was discussed at the conferences in Cloudcroft (I.A.U. Symposium No. 16, August 1961) and at Kyoto (Conference on Cosmic Rays and the Earth Storm, September 1961). The structure of a fully developed type IV burst, as envisaged during discussions at the Kyoto Conference is shown in Figure

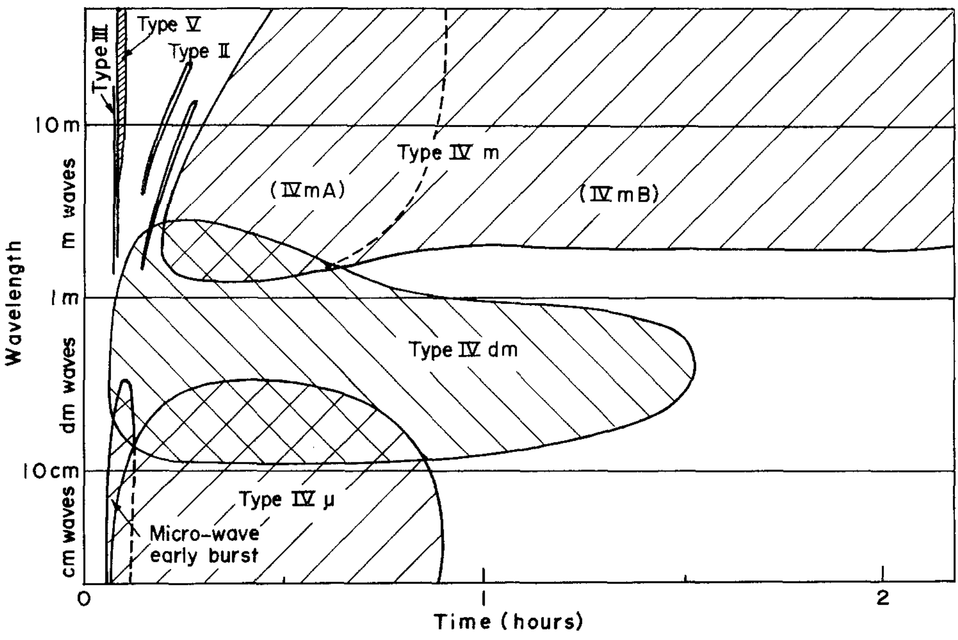

Fig. 16. The structure of a fully developed outburst showing the meter, decimeter and microwave components of the type IV burst, as envisaged during discussions at the Kyoto Symposium (after Wild, 1962).

16 (WILD, 1962). The original type IV designated by BOISCHOT (1957, 1958) is type IVmA and IVmB in Figure 16.

Type IVmA is also called 'moving type IV'. This burst lasts for tens of minutes following type III bursts and/or a type II burst, and the source of type IVmA moves outwards with speeds of 10^3–10^4 km/s to great heights in the corona, sometimes exceeding 10 solar radii, and then stops or moves slowly downwards with decreasing intensity (BOISCHOT, 1958; WILD *et al.*, 1959a; KRISHNAN *et al.*, 1961; KUNDU *et al.*, 1961; PHILIP, 1964; MALITSON *et al.*, 1966). The moving type IV seems to be rare at frequencies above 200 Mc/s (KUNDU *et al.*, 1961; MORIMOTO, 1961) but is common at lower frequencies. The emission is weakly circularly polarized and the sense of polarization seems to be that of extraordinary mode (WEISS, 1963b), but this may not be conclusive (KAI, 1965a). The source size is large ($>10'$) and an emission cone is broad.

Type IVmB is also called 'stationary type IV'. This burst may follow type IVmA or occurs without type IVmA. The duration is an hour or two, and the stationary type IV may further be followed by a 'continuum storm', which lasts for a few hours or more and may develop into type I noise storms (PICK-GUTMANN, 1961; BOISCHOT *et al.*, 1962). There is, however, still considerable doubt as to whether the continuum storm is a distinct component or merely a mixture of prolonged stationary type IV and type I noise storm. The stationary type IV burst is characterized by a source

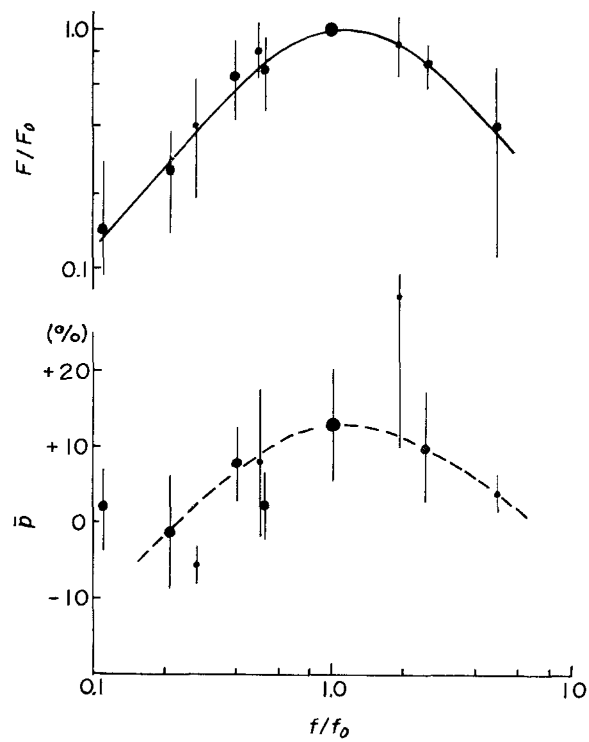

Fig. 17. Normalized spectrum (F/F_0) and circular polarization degree ($\bar{p}$) of microwave type IV bursts. (After KAI, 1965a.)

whose position is fixed close to the level from which type I noise storms, type III and type II bursts originate at a given frequency. The source size is small (3′–5′) and the emission is directive. The emission is usually strongly circularly polarized in ordinary mode (WEISS, 1963b; KAI, 1965a). Almost all of type IV at 200 Mc/s seem to have linearly polarized component of about 10% (KAI, 1963); the bandwidth of the receiver is 100 kc/s.

On microwaves, a microwave burst starts almost coïncidentally with a flare, ten to tens of minutes before the start of type IV burst on meter waves. This mïcrowave burst, which is now designated as type IVμ, is generally a complex group of bursts and lasts tens of minutes (cf. Section 3.1). The characteristics of type IVμ are almost identical with those of impulsive bursts, as mentioned in Section 3.1. An average spectrum and circular polarization degree are shown in Figure 17 (KAI, 1965a).

On decimeter waves, the start of type IVdm is obscure, since the spectrum of the early phase is very complex and full of variety; the low-frequency tail of microwave burst, amorphous patches and/or fast drift bursts, and sometimes high frequency part of a type II burst, are generally mixed together at the early phase. Whether the type IVdm is a distinct component or lower extension of type IVμ, or high frequency part

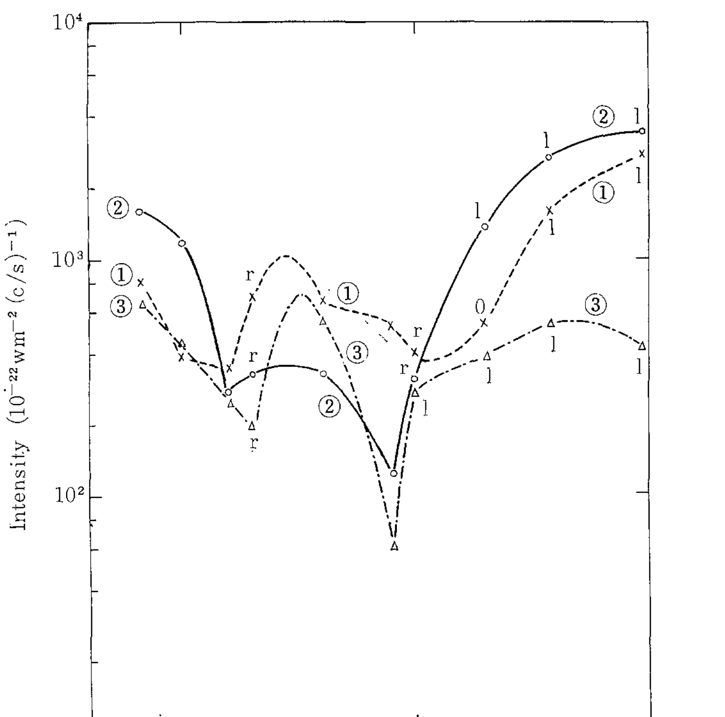

Fig. 18. Wide-range spectrum of a fully developed type IV burst on November 14, 1960, showing three maxima corresponding to meter, decimeter, and microwave components. (After TAKAKURA, 1963a.) Swept-frequency record of this burst was also obtained (TAKAKURA, 1962).

of type IVm, is still controversial. However, the wide-range spectrum of fully developed type IV bursts shows generally three maxima *at the same time* after 10 to 30 minutes from the start of the burst, as shown in Figure 18. Such a spectrum cannot be attributed to a single source, but we need at least three sources with different physical parameters such as magnetic field and/or energy distribution of electrons.

The duration of type IVdm is one to a few hours. The source is fixed close to the flare region (Kundu *et al.*, 1961), and the emission is sometimes characterized by a large fluctuation of intensity. But this may be attributed to an impulsive component superposed on a continuum. The impulses are generally highly polarized circularly (up to about 100%) as ordinary mode and have a bandwidth narrower than the continuum. The duration of each impulse is of the order of 1 minute. The continuum is also partially polarized circularly, in the same sense as the impulses.

3.4. Application of gyro-synchrotron emission to type IV bursts

Synchrotron emission was first applied by Boischot and Denisse (1957) to the interpretation of type IV on meter waves. Assuming a magnetic field of 1 gauss, they showed that 10^{33} electrons of 3 MeV were required to account for the observed intensity at 169 Mc/s. However, without knowing the intensity and polarization over the whole spectrum, we cannot uniquely determine both the energy of the electrons and the magnetic field. Here, it should be remarked again that the observed evidence of circular polarization implies that the electrons that contribute effectively to the emission at radio frequencies are not extremely relativistic; in the extremely relativistic

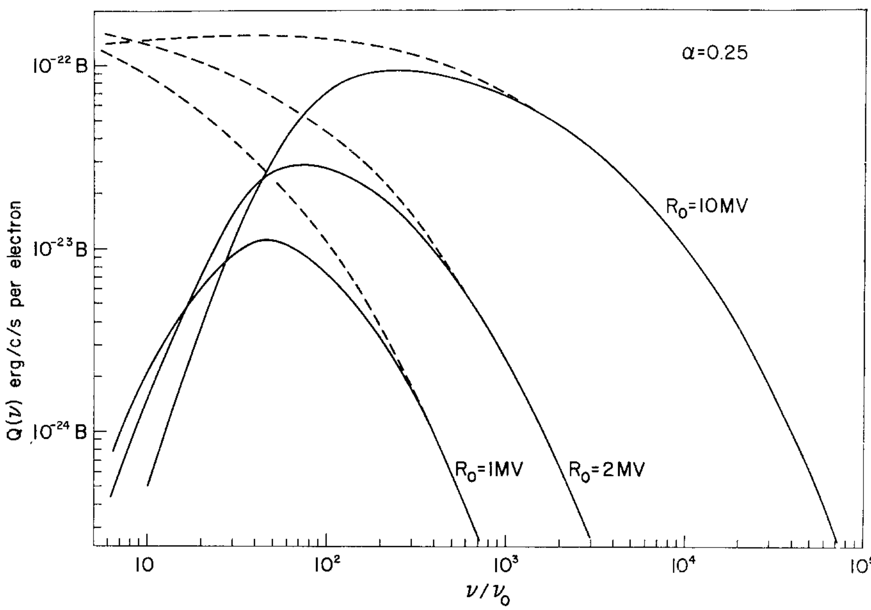

Fig. 19. Synchrotron emission spectra of electrons having an exponential rigidity distribution $\exp(-R/R_0)$ in a vacuum (dashed curves) and in a medium characterized by $\alpha \equiv 3f_H/2f_p = 0.25$ (solid curves). (After Ramaty and Lingenfelter, 1967.) $\nu = f$, $\nu_0 = f_H$ and $B = H$ in our symbols.

case, the polarization is linear at the source but becomes unpolarized due to Faraday rotations in the solar atmosphere.

TAKAKURA and KAI (1961) made rough estimates of effective energy of electrons and the magnetic field from the observed bandwidth and spectral maximum of each type, type IVμ, type IVdm and type IVm. These values were about 4×10^5 eV and 700–2000 gauss for type IVμ, 5×10^4 eV and 80–200 gauss for type IVdm, and 6×10^4 eV and 20–45 gauss for type IVm. The magnetic field-strengths are, however, to be reduced approximately by a factor $\frac{1}{2}$ if we take the thermal gyro-absorptions at low-order harmonics (cf. Section 2.4) into consideration (TAKAKURA, 1964b, 1966): the absorption shifts the maximum of emission spectrum towards higher frequencies (Section 3.2a).

Recently RAMATY and LINGENFELTER (1967) showed that the influence of ambient plasma described in Section (2.3) is very important for the interpretation of the spectra of type IV bursts. The ambient plasma suppresses the synchrotron emissivity appreciably at low frequencies. The suppression results in a shift of the maximum of emission spectrum towards higher frequencies (Figure 19). Thus, they have derived the magnetic field of 1–2 gauss for type IVm and the rigidity spectrum R $(= mvc/e = (m_0c^2/e)\, p)$ of electrons to be $\exp(-R/1\text{ MV})$. Even if their calculation is valid only for extremely relativistic cases so that the above result may include some errors at the low frequencies, the suppression of synchrotron emission at low frequencies, e.g. $f < 10 f_p = 60 f_H$ in their case, due to the influence of coronal plasma, seems to be very important for the interpretation of the spectra of type IV bursts on meter and decimeter waves. The influence is smaller for type IVμ since f_p is smaller than or comparable to f_H, but the influence would be worth consideration at low frequencies, probably at $f < 10 f_p = (2\text{–}5) f_H$, though the approximation used by Ramaty *et al.* cannot be applied in this case and we must use rigorous treatment as did TWISS and ROBERTS (1958) (cf. Section 2.3). Referring to their numerical estimate of the influence of medium described in Section 2.3, the influence may be smaller compared with the gyro-absorption at $f < 3 f_H$.

The interpretation of the spectrum and polarization can be made in more detail for type IVμ in the same manner as is made for microwave impulsive bursts in Section 3.2, since general characteristics of type IVμ and impulsive bursts are similar. Longer duration and complex time variation of type IVμ imply a successive supply of energetic electrons, and the acceleration of the electrons is to be made in an upper region for a limited number of electrons (cf. Section 3.2.c). Longer decay time and broader spectrum compared with short impulsive bursts of a minute or so imply that the slope of the energy distribution of electrons for type IVμ may be more gentle, i.e., γ is smaller if the distribution is power law.

The interpretation of stationary type IV (IVmB), continuum storm, and type IVdm by gyro-synchrotron emission meets with a difficulty. These bursts seem to be circularly polarized as ordinary mode, while the gyro-synchrotron emission is generally emitted predominantly in the extraordinary mode. In this respect, plasma waves have been proposed as the origin of these bursts (Section 5.4).

The gyro-synchrotron mechanism for the type IVmB and type IVdm cannot, however, be completely discarded. TAKAKURA (1962) proposed that a source of type IVdm burst could be above the following (weaker) sunspot; the emission is extraordinary mode referring to the *following* (weaker) sunspot but the polarization sense looks as that of ordinary mode if we refer to the preceding (stronger) sunspot. Another possibility is that a sense reversal occurs during the propagation of emission due to a mode coupling treated by COHEN (1960), though the reversal may or may not occur, depending on the magnetic configuration in the outer layer.

4. Plasma Waves and Electromagnetic Waves in a Plasma

In a plasma penetrated by a magnetic field, there are generally three wave-modes ascribed to the motion of electrons if we disregard the motion of ions. They are the plasma (electron) wave, the ordinary and the extraordinary waves. In the absence of a transverse component of the magnetic field, the plasma wave is a longitudinal space-charge wave and the other two are transverse electromagnetic waves. A transverse component of the magnetic field provides a coupling between the space charge wave and the electromagnetic waves as will be shown below. Such coupled waves have both transverse and longitudinal electric fields.

Low frequency modes ascribed to the motion of ions are important as VLF emission in the terrestrial magnetosphere, but have no direct relation to the solar radio bursts, since such low frequency waves cannot propagate up to the earth, even if they are excited in the solar corona.

4.1. DISPERSION EQUATION

In order to describe the characteristics of waves in the plasma, dispersion equation is usually used. The dispersion equation, which gives wave number k as a function of the frequency, is obtained by the use of Maxwell's field equations and Boltzmann's equation. This kinetic treatment has been made by SITENKO and STEPANOV (1957) and the detail has been published by GINZBURG (1961). More simple approximation has been given by a transport treatment, which uses Maxwell's field equations and Maxwell's momentum transfer equation. This transport treatment has been made by PIDDINGTON (1955) and a comprehensive treatise has been published by DENISSE and DELCROIX (1963).

The transport treatment gives,

$$\left.\begin{aligned} \left(1 - X - \left(\frac{V_T}{c}\right)^2 \mu^2\right) E_x - i(Y\sin\theta)(1-\mu^2)E_y &= 0 \\ i(Y\sin\theta)\ E_x + (1 - X - \mu^2)\ E_y - iY\cos\theta(1-\mu^2)E_z &= 0 \\ i(Y\cos\theta)(1-\mu^2)\ E_y + (1 - X - \mu^2)E_z &= 0 \end{aligned}\right\} \quad (4.1)$$

where $\mu = ck/\omega$ indicates refractive index, $X = (\omega_p/\omega)^2$, $Y = (\omega_H/\omega)$, V_T is r.m.s. thermal velocity of electrons, and a plane wave of the form $\exp\{i(\omega t - kx)\}$ propagates to the x-direction making an angle θ with the magnetic field. The magnetic field is in a x–z plane. $E_{x,y,z}$ are electric field components of the wave. The dispersion equation is given by setting the determinant of Equations (4.1) to zero.

The polarization state, e.g. E_y/E_z and E_x/E_y, for a given wave which satisfies the dispersion equation can be obtained by Equations (4.1).

If $(V_T/c)^2\mu^2 \ll 1-X$, Equations (4.1) are identical to those of magneto-ionic theory, which gives characteristics of ordinary and extraordinary modes. This approximation is not valid, if the refractive index μ is large: $(V_T/c)^2 \ll 1$ even in the corona in which $(V_T/c)^2 \approx 5\times 10^{-4}$ at a million degree. The characteristics of the ordinary and extraordinary modes are important for the radio bursts in order to know the propagation effect of the bursts. However, the magneto-ionic theory will not be described further in the present review, since it is well known and is shown in many text-books (e.g., Ratcliffe, 1959; Budden, 1961; Denisse *et al.*, 1963; Spitzer, 1962).

If $\mu^2 \gg 1$, Equations (4.1) give

$$\mu^2 \approx (c/V_T)^2 \{1 - X - (Y\sin\theta)^2 (1 - Y^2\cos^2\theta)^{-1}\}. \tag{4.2}$$

This is the dispersion equation for the electron plasma waves. For this mode, Equations (4.1) show that the longitudinal component (E_x) is predominant.

4.2. Excitation and Damping of the Plasma Waves

The refractive index of the electron plasma waves is large (Equation 4.2), i.e., the phase velocity $V_\varphi = \omega/k = c/\mu$ is small. Therefore, the plasma waves can be excited by a passage of a fast charged particle through the plasma at a speed V_0 greater than V_φ in a process analogous to the Cerenkov emission of electromagnetic waves.

Suppose that a single charged particle, either electron or proton, is passing through a plasma along the magnetic field with a speed V_0. The particle excites plasma waves which satisfy the dispersion Equation (4.2). Those waves which travel along with the electron satisfy the Cerenkov condition,

$$V_\varphi = \frac{\omega}{k} = V_0\cos\theta, \tag{4.3}$$

where θ is an angle between the wave and the velocity of the particle. From Equations (4.2) and (4.3), we have for $Y^4 = (\omega_H/\omega)^4 \ll 1$

$$\omega^2 \approx \frac{\omega_p^2 + \omega_H^2\sin^2\theta}{1 - \left(\dfrac{V_T}{V_0\cos\theta}\right)^2}. \tag{4.4}$$

Accordingly, the frequency of plasma waves excited by a charge particle is the function of θ, and may have very high frequencies if $V_0\cos\theta = V_\varphi \approx V_T$, but the damping of the wave increases with decreasing the phase velocity due to the Landau damping, and at $V_\varphi < V_T$ the waves are evanescent, as will be shown later. Therefore, the frequencies of plasma waves are mainly confined to near the plasma frequency f_p if $f_p \gg f_H$. The intensity of the waves in the absence of the magnetic field has been calculated by COHEN (1961b). The self-absorption of the plasma waves which gives an upper limit to the intensity of the plasma waves, has been given by GINZBURG and ZELEZNYAKOV (1958).

The above treatment is for a single electron (or proton). Now we shall treat an ensemble of electrons which have a velocity distribution $N(V_x)$. The distribution $N(V_x)$ includes both electron streams in x-direction and thermal electrons which form the plasma. Suppose that a plasma wave is propagating in this medium with a phase velocity V_φ in x-direction. The electrons are accelerated or decelerated by the longitudinal electric field of the plasma wave and the interaction is most efficient for electrons near the wave velocity, $V_x \approx V_\varphi$, because they stay in a force of the same phase for a long time. The electrons faster than the wave tend, on the average, to give energy to the wave, while the electrons slower than the wave gain energy. If a gradient of $N(V_x)$ is positive at $V_x = V_\varphi$, there are more faster electrons than slower electrons, so that the wave tends to be built up at the expense of the kinetic energy of the electrons. On the other hand, if the gradient is negative as in Maxwellian-like distribution, the wave tends to be damped. The damping of the plasma wave in a *Maxwellian distribution* is designated as Landau damping (LANDAU, 1946; BOHM *et al.*, 1949).

BOHM and GROSS (1949) have given the damping (or growing) factor of the plasma waves, with $\exp\{ik(x - V_\varphi t) + \Gamma t\}$,

$$\Gamma = -\frac{1}{2\tau} + \frac{\omega_p^3}{k^2}\frac{\pi}{2}\left[\frac{\partial N(V_x)}{\partial V_x}\right]_{V_x = V_\varphi} \tag{4.5}$$

for $V_\varphi \gg V_T$, where τ indicates collision time of thermal electrons. The first term leads to collision damping. The second term, however, can lead either to excitation if $\partial N/\partial V_x$ is positive at $V_x = V_\varphi$ or to further damping if $\partial N/\partial V_x$ is negative. In a Maxwellian distribution, $N(V_x) = (m_0/2\pi\kappa T)^{1/2} \exp(-m_0 V_x^2/2\kappa T)$, the second term reduces for $V_\varphi \gg V_T$ to the Landau damping given by

$$-\left(\frac{\pi}{8}\right)^{1/2} \{\omega_p/(kD)^3\} \exp\{-\tfrac{1}{2}(kD)^{-2}\}, \quad \text{for} \quad kD \ll 1, \tag{4.6}$$

where D indicates Debye length,

$$D \equiv (\kappa T/4\pi e^2 N_0)^{1/2} = \frac{1}{\sqrt{3}}(V_T/\omega_p). \tag{4.7}$$

The Debye length is a measure of a length that the thermal electrons with speed V_T travel during one period of plasma oscillation. Therefore, if $1/k \lesssim D$, i.e., the wavelength $\lambda \lesssim D$ or $V_\varphi \lesssim V_T$, the electrons which have contained within a wavelength of the plasma wave are almost renewed due to the thermal motion in one period of the oscillation, so that the wave becomes evanescent.

If a beam of fast electrons with velocities around $V_{0,x} (\gg V_T)$ is streaming in a plasma, the distribution function of electrons $N(V_x)$ which includes thermal electrons

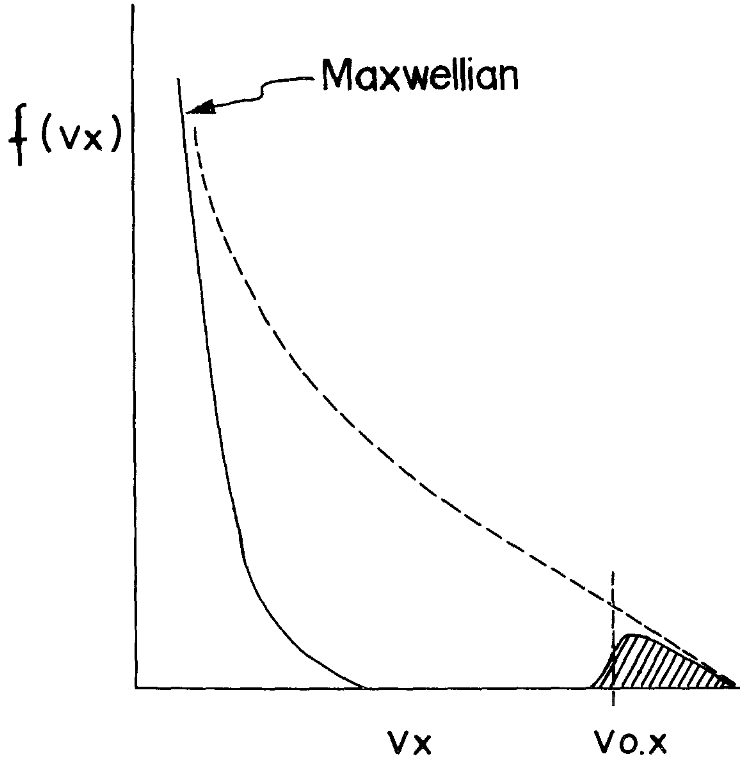

Fig. 20. Velocity distribution of electrons. Hatched part: a hump of fast electrons which can excite plasma waves. Dashed curve: at the origin in which the acceleration of electrons was made (Section 5.1).

may have positive gradient at about $V_{0,x}$ as shown by a hatched hump in Figure 20. In this case the plasma waves with $V_\varphi \approx V_{0,x}$ grow with time as shown by Equation (4.5), but we cannot obtain a finite amplitude of the plasma waves from this equation. Bohm and Gross (1949) have calculated the amplitude of electric field strength E_0 of the plasma waves in the absence of the magnetic field at the steady state, basing on a balance of an energy gain of the plasma waves from the beam of electrons and an energy loss of plasma waves due to the collision damping.

$$E_0 = \frac{m_0}{2e}\,\omega V_{0,x}^5 \left\{ \frac{16}{3}\left(\frac{\omega_p}{\omega}\right)^2 \frac{\tau}{\tau_0}\left[\frac{\partial N(V_x)}{\partial V_x}\right]_{V_x = V_{0,x}} \right\}^2, \tag{4.8}$$

where τ_0 and τ are collision time of the streaming electrons and that of thermal electrons, respectively. In Equation (4.8), ω is given by Equation (4.4), setting $\omega_H = 0$ and $V_0 \cos\theta = V_{0,x}$, but ω is nearly equal to ω_p if $V_{0,x} \gg V_\varphi$.

An upper limit of E_0 is given by,

$$E_0^2/8\pi = aN_0\kappa T, \tag{4.9}$$

where $a \approx 1$, but as an extreme case a can be appreciably greater than unity (Ginzburg *et al.*, 1966).

4.3. Conversion of energy from plasma waves to electromagnetic waves

The plasma waves are longitudinal mode and cannot propagate outside the plasma. In order that the electromagnetic waves originate from the plasma waves, conversion of energy from the longitudinal mode to the transverse mode is necessary. Two conversion mechanisms have been proposed by GINZBURG and ZHELEZNYAKOV (1958 and 1959a, b).

One is the mode coupling of the two waves due to large-scale inhomogeneities of the density of the plasma and/or the magnetic field. The coupling is greatest in the regions in which the phase velocities of the both modes are close to each other at a given frequency and also where the gradient of the density and/or magnetic field is steep. It is analogous to the mode coupling of two magneto-ionic modes treated by COHEN (1960). The coupling coefficient has been estimated to be of the order of 10^{-7} in the normal corona (GINZBURG and ZHELEZNYAKOV, 1958, 1959b). However, the coupling may be more efficient, if hydromagnetic waves are propagating in the coupling region; the waves have steep gradients of the density and the magnetic field.

Another mechanism of the conversion is scattering of the plasma waves on small-scale fluctuations of plasma density (GINZBURG and ZHELEZNYAKOV, 1958, 1959a). There are two kinds of fluctuations in the plasma: one is due to the motion of ions and another is due to the electrons. The former is density fluctuations with low frequencies and is quasi-neutral, while the latter is space-charge fluctuations at about the plasma frequency. Even if there exists no disturbance in the plasma, the plasma has inherent thermal fluctuations due to thermal motions of ions and electrons.

The power P' of radio waves caused by the scattering of plasma waves due to thermal density fluctuations has been calculated by GINZBURG and ZHELEZNYAKOV (1958, 1959a) on an analogy of the optical Rayleigh scattering,

$$P' \approx \frac{e^4 N_0 V}{6 m_0^2 c^3} E_0^2 \left(\frac{V_T}{V_\varphi}\right), \tag{4.10}$$

where V indicates a volume of the scattering region, E_0 is given by Equation (4.8), and the phase velocity of the plasma waves excited by a beam of electrons is given by $V_\varphi \approx V_{0,x}$. The scattered radio waves have about the same frequency as the plasma waves and the emission is mainly in the forward direction.

On the other hand, scattering of plasma waves by the space charge fluctuations shows quite different characteristics, since these fluctuations are also electron plasma waves. This wave-wave interaction is called combination scattering. The intensity of scattered radio waves P'' is given by (GINZBURG and ZELEZNYAKOV, 1958, 1959a),

$$P'' \approx \frac{e^4 N_0 V}{\sqrt{3} m_0^2 c^3} E_0^2 \left(\frac{V_T}{V_\varphi}\right)^2. \tag{4.11}$$

More rigorous treatment has been made by TERASHIMA and YAJIMA (1963). They have given smaller intensity for the combination scattering by a factor $(V_\varphi/c)^2$, but for the Rayleigh scattering they had the same result as Equation (4.10).

The radio waves caused by the combination scattering is approximately twice the plasma frequency and the emission is mainly in the transverse direction but slightly predominates in the backward direction (SMERD *et al.*, 1962). More efficient scattering may be possible if non-thermal fluctuations exist, as discussed by STURROCK (1964).

5. Plasma Waves and Solar Bursts

5.1. TYPE III BURSTS

This type of bursts is characterized by a short duration ($1 \sim 10$ sec) and a rapid frequency drift with time from high to low frequencies. The starting frequencies are distributed mainly from 350 Mc/s to 50 Mc/s (MALVILLE, 1962) and the lower extension of the bursts was observed down to 1.5 Mc/s (HARTZ, 1964). Harmonic structure (frequency radio of 1 to 2) is observed, but not easily recognized in most cases, because the rapid frequency drift and broad bandwidth tend to merge the fundamental and the second harmonic.

The existence of harmonics and systematic frequency drift led WILD *et al.* (1954) to the plasma hypothesis as the origin of the type III bursts, i.e., the plasma oscillations are excited successively at each layer of the corona by an agent moving rapidly outwards through the corona. A systematic variation of position with frequency, which was predicted by the hypothesis, has been confirmed by WILD *et al.* (1959b) by measurements with an interferometer. The velocities of the sources of type III bursts are 0.2 to 0.5 times the velocity of light. DE JAGER (1960, 1962) has suggested that the fast moving agent is a group of *electrons*.

The polarization of type III bursts is that they are almost unpolarized or only weakly polarized circularly at 200 Mc/s, but about 50% of the bursts at lower frequencies are fairly strongly polarized (KOMESAROFF, 1958; RAO, 1965). The sense of the circular polarization at 200 Mc/s seems to be that of the ordinary mode (ENOME, 1964). Linearly polarized component was detected with a narrow-band polarimeter (10 kc/s bandwidth) at 200 Mc/s (AKABANE *et al.*, 1961) and also at 74 Mc/s (BHONSLE *et al.*, 1964).

The plasma theory has been applied for the interpretation of the above characteristics of the type III bursts. Suppose that an acceleration of electrons for the order of 1 sec occurred at the flares or at more minor activities around the base of the corona. The velocity distribution $N(V_x)$ of the accelerated electrons may have negative gradient at the origin (Figure 20, dashed curve). The electrons with lower energies have a shorter mean free path, so that such electrons are lost by collisions during a passage through the corona. Therefore, after a certain time and at a certain distance from the origin, the velocity distribution may have a hump (positive gradient) at a high velocity range, as shown by a hatched hump in Figure 20. Thus, the plasma waves can be excited by the stream of the fast electrons. If a predominant magnetic field exists, the stream is guided along the lines of force.

Observed lower limit of the speed of type III sources of $0.2c$ (10 keV) is consistently explained by the above-mentioned propagation effect (DE JAGER, 1960; MALVILLE, 1962; STURROCK, 1964; TAKAKURA, 1966) if the stream starts from about the base of the corona. More energetic electrons (> 10 keV) can escape into the interplanetary space, unless the magnetic field prevents them (TAKAKURA, 1966). Actually the solar electrons of such energies were detected in the interplanetary space (VAN ALLEN *et al.*, 1965; ANDERSON *et al.*, 1966).

In order to make quantitative study of the time variation of spectrum of type III bursts, we have to solve the spacial and time variations of $N(V_x)$, but this problem is not yet solved. An estimate of the order of magnitude of intensity has been made by GINZBURG and ZHELEZNYAKOV (1958, 1959a) by using Equations (4.8), (4.10), and (4.11). The number density of the fast electrons of the order of 10^6 cm^{-3} is required for the burst at 100 Mc/s on an assumption that a half width of the hump in $N(V_x)$ is equal to V_T, though there is no theoretical basis on this assumption and furthermore $N'(V_x)$ is the most essential parameter for the intensity. If non-thermal fluctuations exist, they scatter the plasma waves more efficiently to emit the radio waves, so that smaller number of fast electrons is enough to explain the observed intensity.

5.2. TYPE II BURSTS

Type II bursts are generally associated with important flares and are characterized by a slow frequency drift. Harmonic structure is observed in about 60% of them. The starting frequency of the fundamental usually lies below 175 Mc/s, but can start at frequencies as high as 240 Mc/s (HADDOCK, 1959; MAXWELL *et al.*, 1962), and the lower extension is observed down to 10 Mc/s. The motion of type II sources is not always outward but tangential movement is sometimes observed (WEISS, 1963a). The speed of the source is of the order of 10^3 km/s.

In many type II bursts the individual harmonic bands are themselves double. The splitting is about 10 Mc/s in the fundamental band and is twice as great at the 2nd harmonic (ROBERTS, 1959). The polarization of type II bursts is that they are almost unpolarized, even if the bands are split. Some type II bursts exhibit a form of fine structure which appears to diverge from a type II burst. This structure was designated as herring-bone (ROBERTS, 1959), which at times exhibits strong circular polarization (STEWART, 1966).

Prior evidence suggesting the occurrence of frequency drift in type II bursts was given by PAYNE-SCOTT *et al.* (1947). They suggested that the frequency drift was related to a physical agency moving outwards through the solar atmosphere. This explanation was based on an idea put forward by MARTYN (1947) that the emission at any frequency originates near the level where the coronal electron density reduces the refractive index to zero (plasma hypothesis). WILD (1950) and WILD *et al.* (1953) confirmed this hypothesis, basing on the dynamic spectrum observations.

The question may arise as to how the disturbance of the order of 10^3 km/s can excite plasma waves, since the speed of the disturbance is below the thermal velocity of electrons in the corona: $V_T \approx 7 \times 10^3$ km/s at 10^6 °K.

The disturbance has been attributed to the hydromagnetic shock waves (UCHIDA, 1960; PIKEL'NER *et al.*, 1964). At the shock front of the wave propagating in x-direction across the magnetic field (z-direction) an electron current $i = -eNV_y$ flows in y-direction due to

$$\frac{\partial H}{\partial x} = \frac{4\pi}{c} i \quad \text{(in the shock system)}, \tag{5.1}$$

where ion current is neglected. The drift velocity of electrons V_y is then proportional to $\Delta H/\Delta l$, where both the thickness Δl of the shock front and an increment of the magnetic field ΔH at the shock front are the function of the magnetic mach number $\mathcal{M}$ of the shock. At the front of a weak shock (e.g., UCHIDA, 1960)

$$\Delta H/H \approx \tfrac{4}{3}(\mathcal{M} - 1). \tag{5.2}$$

A fine structure of the shock front has been calculated by SAGDEEV (1962a). The thickness Δl of the shock front is approximately given by

$$\Delta l \approx \frac{c}{\omega_p (1 - \mathcal{M}^{-2})^{1/2}}. \tag{5.3}$$

Putting $V_y = V_T$ at which $\mathcal{M} = \mathcal{M}_c$, we have

$$\mathcal{M}_c \approx 1 + \frac{3}{4}\left(\frac{8\pi N_0 \kappa T}{H^2}\right)^{1/3}, \tag{5.4}$$

for $(8\pi N_0 \kappa T/H^2) \ll 1$ (SAGDEEV, 1962b; PIKEL'NER *et al.*, 1964).

If the Mach number is greater than the critical value given by Equation (5.4), the drift velocity of electrons V_y at the shock front is faster than V_T, so that the plasma waves can be excited due to a relative motion of the electrons and ions; in a system fixed to the electrons ions are moving with $-V_y$. This critical condition (Equation 5.4) may determine that height range in the corona which corresponds to the observed frequency range of type II bursts (TAKAKURA, 1966).

The explanation of band-splitting was attempted without much success (ROBERTS, 1959; STURROCK, 1961; TAKAKURA, 1964a). Recently, ZAITSEV (1966) attempted to explain the band-splitting by means of the oscillatory structure of the magnetic field behind the shock front. The oscillatory magnetic field indicates electron drift currents of $+y$ and $-y$ directions. The plasma waves excited by these two opposite currents have band-splitting of $\Delta f/f \approx 2V_y/c$ due to the Doppler effect.

TIDMAN (1965) and TIDMAN *et al.* (1965) have proposed that the type II bursts are due to plasma waves excited by the suprathermal electrons coexisting with the thermal plasma in a turbulent collisionless bow shock wave. The suprathermal electrons have a Maxwellian distribution with high temperature. TIDMAN *et al.* (1966) have attributed the band splitting of the type II bursts to the magnetic field. If the suprathermal electrons have some anisotropic distributions, such as a pancake in velocity space drifting along the magnetic field, the magnetic field gives rise to a splitting of the plasma lines by an amount $\omega_H^2/2\omega_p$ for the fundamental and the amount is twice for the second harmonic.

5.3. Type I bursts

This type of bursts is characterized by a short duration and a narrow bandwidth. The bursts occur in succession to form a long enduring noise storm with a continuum. The frequency range of the noise storm is below about 300 Mc/s. In decametric range, noise storm is very rare (J. Warwick, private communication), but similar activities as the noise storms were observed in a range of 1.5–10 Mc/s from a satellite (Hartz, 1964). Type I bursts occur at times as a group forming a narrow chain with durations of 1 minute or several minutes (Wild, 1957; Wild *et al.*, 1964; Hanasz, 1966). The chain drifts systematically in frequency at a rate which corresponds to the speed of about 200 km/s. Wild *et al.* (1964) suggested that the chain may be produced in hydromagnetic waves or shock waves.

Most of the type I bursts and noise storms are strongly circularly polarized, but unpolarized noise storms are also frequent (Tsuchiya, 1963). Wide band type I bursts are observed at times (Vitkevich *et al.*, 1959, 1960). On the other hand, type III bursts also occur sometimes in groups for fairly long periods, and narrow-band type III bursts and polarized type III bursts are observed. Therefore, there is a question as to whether the type I bursts and type III bursts are distinctly different types of bursts.

On the basis of the general similarity between type I and type III, Takakura (1963b) has proposed to explain the type I bursts due to plasma waves excited by electron stream with rather slow speed V_0 as several times V_T. The slow speed is required to explain the narrow bandwidth and short duration of the type I bursts. The bandwidth and the duration are attributed to the lifetime of the electron beam. Owing to the short lifetime of the electron beam, the acceleration of the electrons must be carried out in the radio source. One possible mechanism of the acceleration is a collision of two wave packets of hydromagnetic waves. It is shown that a group of electrons has the speed $V_0 \approx 3V_A$ after the collision, where $V_A = H/(4\pi\rho)^{1/2}$ is the velocity of the hydromagnetic waves (cf. Section 6.2). Chertoprud (1963) has also proposed the plasma waves as the origin of type I bursts.

Recently, Trakhtengerts (1966) proposed that the countercurrent of electrons due to electric field produced by a charge separation at the pulse front of hydromagnetic waves is responsible for the excitation of plasma waves. He also suggested that the charge separation may inhibit the excitation of plasma waves proposed by Takakura (1963b). The model of Trakhtengerts seems to be favorable to the explanation of the rather slow drift speed of the chain of type I bursts. However, the charge separation is reduced by a large amount if we take into account the reflection of thermal electrons at the *back* of the pulse of the hydromagnetic waves; if the speed of the pulse is slow compared with the speed of the thermal electrons, many thermal electrons make overtaking collisions with the *back* of the pulse.

5.4. Other bursts

As mentioned in Section 3.4, some components of type IV burst could be due to plasma waves. Denisse (1960) has attributed the stationary type IV (IVmB) and the continuum storm (cf. Section 3.3) to the plasma waves excited by energetic electrons, which are accelerated during a flare, remain trapped in the corona and slowly diffuse downwards. Kai (1965b) has applied the plasma waves for the interpretation of both type IVdm and type IVmB.

Type V burst is a broad-band continuum appearing on meterwaves for a minute or so after a type III burst (Wild *et al.*, 1959a, b). Wild *et al.* (1959b) attributed the type V to the synchrotron emission from a part of that electron stream responsible for the type III burst. However, a lack of circular polarization and general similarity of characteristics between type V and type III led Weiss *et al.* (1965) to the suggestion that the type V is due to plasma waves excited by fast electrons trapped in a magnetic bottle in the corona.

6. Acceleration of Electrons in the Solar Atmosphere

A primary origin of solar bursts is an acceleration of electrons in the solar atmosphere. However, quantitative study of the acceleration of electrons from a viewpoint of the radio bursts has not been well established. General treatment of acceleration of charged particles has been given, for example, by Hayakawa *et al.* (1964) and Sakurai (1965).

Effective energy range and spectrum of the energetic electrons have been indirectly estimated from the characteristics of radio bursts and X-rays, as shown in Table I. Recent direct observations of solar electrons ($10^4 \sim 10^5$ eV) in the interplanetary space are included in the Table (Van Allen *et al.*, 1965; Anderson *et al.*, 1966). More energetic electrons (>100 MeV) also were detected by Meyer *et al.* (1962). If the white light from flares is caused by synchrotron emission as suggested by Stein and Ney (1963), 10^{32} electrons of 10^8 eV with a spectrum $\exp(-E/200\ \text{MeV})$, must be created at such a flare. However, the white-light flares could be accounted for also by the free-free emission from low energy electrons of the order of 10 eV; thermal emission from dense hot gas with 10^5 °K, or even slightly less, is enough to appear as the white-light flare.

Several mechanisms have been proposed for the acceleration of electrons. The differences between the acceleration conditions for electrons and protons are as follows: Electrons suffer more energy losses due to both collisions and gyro-synchrotron emission as compared with ions. Gyroradius of electron is smaller for given energy than proton in the non-relativistic range. Initial velocities of electrons are higher than protons if the acceleration starts from a thermal distribution. Observational evidence shows that the final velocities, rather than energies, of electrons and protons seem to be of the same order after the acceleration at the flares.

TABLE I

Estimates of Solar Electrons associated with Flares

Type of phenomenon	Effective energy (eV) of electrons	Energy spectrum (differential)	Total number (NV)	Acceleration time	References (not comprehensive)
μ-wave gradual X-rays gradual (10–1 Å)	10^3	Maxwell $\approx 10^7$ °K	$10^{38} - 10^{39}$ ($N^2V = 10^{48-49}$ cm^3)	several min.	KAWABATA, 1960, 1966. KUNDU, 1963. WHITE, 1964. FRIEDMAN, 1964. ELWERT, 1964.
μ-wave impulsive	$10^4 - 10^6$	$K^{-3} - K^{-5}$	$10^{30} - 10^{32}$	1 min.	TAKAKURA, 1966.
X-rays impulsive ($\lambda < 0.6$ Å), (10–1 Å)	$10^4 - 10^6$ 10^3	$\{K^{-1.5} - K^{-4.5}$ $(10^4 < K < 10^5)\}$ non-thermal?	$\{10^{34} - 10^{36}$ $(N_i NV = 10^{45}$ cm$^{-3})\}$ $(N_i NV = 10^{50}$ cm$^{-3})$	1 min. 1 min.	$\{$VETTE *et al.*, 1961. WINCKLER, 1964. KUNDU, 1963.$\}$ POUNDS, 1965.
IVμ	$10^5 - 10^6$	K^{-3} or more flat?	$10^{31} - 10^{33}$	10 min.	
IVdm	10^5 $10^5 - 10^6$	monoenergy $\exp(-R/1\text{MV})$ or steeper	$10^{32} - 10^{33}$ $10^{32} - 10^{33}$	10 min.	TAKAKURA, 1962. RAMATY *et al.*, 1966
IVm	3×10^6 10^5 $10^5 - 10^6$	monoenergy monoenergy $\exp(-R/1\text{MV})$	10^{33} $10^{31} - 10^{33}$ $10^{31} - 10^{33}$	10–30 min.	BOISCHOT *et al.*, 1957. TAKAKURA, 1962. RAMATY *et al.*, 1966.
III	$10^4 - 10^5$	$\frac{dN}{dK} > 0$, in a range	$10^{36} - 10^{37}$	1 sec.	
II	10^3	$\frac{dN}{dK} > 0$, in a range	$10^{36} - 10^{37}$ sec^{-1}	continuous at shock front	
White light flare	10^8 (synchrotron) 10 (free-free)	$\exp(-E/200\text{MeV})$ Maxwell, 10^5 °K	10^{32} $\{10^{42}$ $(N^2V = 10^{53}$ cm$^{-3})\}$	several min.	STEIN *et al.*, 1963.
Electrons detected in interplanetary space (at ≈ 1 AU)	$10^8 - 10^9$ $4 \times 10^4 - 10^4$ 4×10^4	E^{-2} $\exp(-K/10$ keV$)$	(cm^{-2}s^{-1}ster^{-1}) 0.04 $\{$5–80 $(10^{33} - 10^{34}$ at the sun$)\}$ 10–3000		MEYER *et al.*, 1962. VAN ALLEN *et al.*, 1965. ANDERSON *et al.*, 1966.

6.1. Betatron acceleration

Electrons are accelerated if the magnetic field changes with time. The energy gain ΔK of an electron during one period T of gyration is given by

$$\Delta K = e\oint E\,\mathrm{d}l = \frac{e}{c}\pi r_g^2 \frac{\mathrm{d}H}{\mathrm{d}t}, \tag{6.1}$$

where $r_g = (T/2\pi)\, V_\perp = (mc/eH)\, V_\perp$ indicates the gyroradius of the electron. Equation (6.1) reduces to

$$\frac{\mathrm{d}K}{\mathrm{d}t} = \frac{\Delta K}{T} = \frac{m_0 V_\perp^2}{2(1-\beta^2)^{1/2}}\left(\frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t}\right). \tag{6.2}$$

In a non-relativistic range,

$$\frac{\mathrm{d}K_\perp}{\mathrm{d}t} = K_\perp\left(\frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t}\right), \tag{6.3}$$

or

$$\frac{\mathrm{d}\varepsilon_\perp}{\mathrm{d}t} = \varepsilon_\perp\left(\frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t}\right),$$

which reduces to

$$\varepsilon_\perp/\varepsilon_{\perp,0} = H/H_0, \tag{6.4}$$

where the suffix 0 indicates initial value. Only velocity component $V_\perp$ perpendicular to the magnetic field increases due to the betatron acceleration in the non-relativistic range.

In an extremely relativistic case,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{\sin^2\varphi}{2}\varepsilon\left(\frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t}\right), \tag{6.5}$$

where φ indicates an angle between the velocity of electron and the magnetic field. The maximum energy attainable $(\varepsilon/\varepsilon_0)$ is limited by H/H_0 as shown by Equation (6.4). A probable ratio of H/H_0 may be 10^2, e.g., $H_0 = 1$ gauss and $H = 100$ gauss, and at most 10^3. The minimum energy of those electrons which can gain energy is determined by equating the energy loss (Equation 2.35) and the energy gain (Equation 6.3 or 6.5).

6.2. Fermi acceleration

Electrons are accelerated statistically by the reflections due to hydromagnetic shocks. This acceleration mechanism is a kind of Fermi acceleration, which is famous as the acceleration process for the cosmic rays.

A momentum gain of an electron at a head-on collision with a shock front is

$$\Delta p = 2\left(\frac{u}{c}\right)(1+p^2)^{1/2} \quad \text{for} \quad u^2 \ll c^2, \tag{6.6}$$

where u indicates a speed of the shock which is of the order of the speed of Alfven wave $V_A = H/\sqrt{4\pi\rho}$ and $p = mV/m_0c$. Δp is positive for a head-on collision and is negative for an overtaking collision, so that a statistical net gain of the momentum is proportional to a difference between the collision frequencies of the head-on and overtaking collisions. The mean frequency of the collision is $(V+u)/l$ for the head-on collisions and $(V-u)/l$ for the overtaking collisions, where l indicates a mean distance between the shock fronts; hence, the difference is $2u/l$. Therefore, the rate of momentum gain or energy gain is statistically,

$$\frac{\mathrm{d}p}{\mathrm{d}t} = 4\left(\frac{u}{c}\right)^2 \frac{c}{l}(1 + p^2)^{1/2},$$

or

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = 4\left(\frac{u}{c}\right)^2 \frac{c}{l}(\varepsilon^2 + 2\varepsilon)^{1/2}. \tag{6.7}$$

In this process, only momentum parallel to the magnetic field increases, so that the pitch angle of electron φ decreases with increasing momentum. A reflection condition at the hydromagnetic shock is given by the conservation of magnetic moment of the electron $\frac{1}{2}m_0 V^2 \sin^2\varphi/H$, such as

$$\frac{H_0}{H_s} < \sin^2\varphi_0, \tag{6.8}$$

where H_s indicates the magnetic field at the shock front and φ_0 is the pitch angle at the static field H_0. Therefore, some mechanism is necessary to redistribute the pitch angle for the electron to have repeated accelerations. TAKAKURA (1961a, 1962) proposed that the redistribution could be attributed to the Coulomb collisions with ambient thermal electrons, since the deflection time t_D (SPITZER, 1962) is one order shorter than the energy exchange time t_E (SPITZER, 1962).

For the above statistical acceleration, the rate of energy gain is proportional to $(u/c)^2$. More efficient Fermi type acceleration of electrons is possible, for which the rate of energy gain is proportional to u/c. WENTZEL (1963) has presented a successive acceleration of this type due to a hydromagnetic shock which moves to a stronger magnetic field. While the electrons are accelerated ahead of the shock and able to penetrate into stronger field, the shock field may become sufficiently stronger, so that the electrons are unable to escape the trap to be accelerated for a long time. The maximum energy attainable is limited by a condition that

$$\text{gyroradius of the electron } r_g \approx \text{shock thickness } \Delta l$$

where $\Delta l \approx c/\omega_p$ (cf. Equation 5.3), since for $r_g \gg \Delta l$ the electron cannot be reflected at the shock front.

TAKAKURA (1963b) has calculated a time variation of velocity-distribution function of electrons in front of a wave packet of Alfvèn waves after a collision of two wave packets propagating in opposite directions to each other. As an example, the distribution function may have a hump at about $V=(3.1\text{–}3.2)V_A$ in a layer several thousand kilometers ahead of the wave with an effective thickness of 10^2 km for 0.2–0.4 seconds after the collision. This acceleration process has been applied to the origin of type I bursts. However, as suggested by TRAKHENGERTS (1966), if the electric field would be caused at the pulse front of hydromagnetic wave and it would have an essential effect on the distribution of electrons (Section 5.3), the above estimate is not valid.

Fermi type acceleration of electrons due to reflections by the electric field at the wave packets of plasma waves may be possible. This is an inverse process of the excitation of plasma waves by the fast electrons. However upper limit of this acceleration seems to be low, as follows:

In order that the electrons are reflected due to the plasma waves, the electric potential φ_p must be greater than K/e, where K is kinetic energy of the electron. The upper limit of the kinetic energy $K_{\max}$ is then

$$K_{\max} = e\varphi_p .$$

On the other hand, an upper limit of φ_p is given by Equation (4.9) setting $E_0 = k\varphi_p$. The upper limit reduces to

$$K_{\max} = \sqrt{\tfrac{2}{3}} m_0 V_\varphi V_T , \tag{6.9}$$

where we have set $a=1$ and $k=\omega_p/V_\varphi$. Therefore the maximum speed of the electron attainable by this process is below V_φ; since $V_\varphi > V_T$. On the other hand, if the plasma waves are excited by electron beams, we have assumed that the electrons are already accelerated due to a certain process up to $V \geqslant V_\varphi$, so that the acceleration of other electrons up to $V < V_\varphi$ may be less important.

6.3. Cyclotron acceleration

In laboratory experiments, X-rays with energies greater than 10^5 eV were observed coming from a plasma in a magnetic field of 2000 gauss penetrated by an electron beam of 10^4 eV. Stix (1964) has attributed this acceleration to the cyclotron acceleration of electrons due to electric field of those plasma waves which are excited by the electron beam and propagating slightly obliquely to the magnetic field.

In order that this acceleration occurs in the solar atmosphere, another primary acceleration of electrons up to 10^3–10^4 eV is necessary for the excitation of the plasma waves; the excitation of the plasma waves may also occur at the hydromagnetic shock front (Section 5.2). Then a part of these electrons and/or thermal electrons could be accelerated due to the cyclotron process to higher energies.

7. Concluding Remarks

In concluding this review it should be remarked that part of the present theories of solar bursts are not firmly established. Improved measurements of solar bursts during the coming active period will possibly upset some theories or force us to modify the present theories. Quantitative studies of the bursts on decimeter waves and longer waves are especially delayed because of the poorness of both observational and theoretical backgrounds. Quantitative study of the acceleration of electrons is also highly desirable.

References

Akabane, K. and Cohen, M. H.: 1961, *Astrophys. J.* **133**, 258.
Anderson, K. A. and Lin, R. P.: 1966, *Phys. Rev. Letters* **16**, 1121.
Anderson, K. A. and Winckler, J. R.: 1962, *J. Geophys. Res.* **67**, 4103.
Bhonsle, R. V. and McNarry, L. R.: 1964, *Astrophys. J.* **139**, 1312.
Bohm, D. and Gross, E. P.: 1949, *Phys. Rev.* **75**, 1864.
Boischot, A.: 1957, *Compt. Rend.* **244**, 1326.
Boischot, A.: 1958, *Ann. Astrophys.* **21**, 273.
Boischot, A. and Denisse, J. F.: 1957, *Compt. Rend.* **245**, 2194.
Boischot, A. and Pick, M.: 1962, *J. Phys. Soc. Japan* **17**, Suppl. A-II, 203.
Budden, K. G.: 1961, *Radio Waves in the Ionosphere*. Cambridge Univ. Press, Cambridge.
Chertoprud, V. E.: 1963, *Soviet Astron. – A.J.* **7**, 35.
Cohen, M. H.: 1960, *Astrophys. J.* **131**, 664.
Cohen, M. H.: 1961a, *Astrophys. J.* **133**, 978.
Cohen, M. H.: 1961b, *Phys. Rev.* **123**, 711.
Covington, A. E. and Harvey, G. A.: 1961, *Nature* **192**, 152.
Denisse, J. F.: 1960, XIIIth U.R.S.I. General Assembly, London.
Denisse, J. F. and Delcroix, J. L.: 1963, *Plasma Waves* (translated by M. Weinrich and D. J. Bendaniel) Interscience Publishers, New York.
Elwert, G.: 1964, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. H. Hess). U.S. Government Printing Office, Washington, p. 365.
Enome, S.: 1964, *Publ. Astron. Soc. Japan* **16**, 135.
Fokker, A. D.: 1963, *Space Sci. Rev.* **2**, 70.
Friedman, H.: 1964, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. H. Hess). U.S. Government Printing Office, Washington, p. 147.
Frost, K. J.: 1964, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. H. Hess). U.S. Government Printing Office, Washington, p. 139. (Time designation on the data was inaccurate and has been revised by private communication.)
Ginzburg, V. L.: 1961, *Propagation of Electromagnetic Waves in Plasma* (translated by Roger and Roger, Inc.). Gordon and Breach Science Publishers, New York.
Ginzburg, V. L. and Ozernoy, L. M.: 1966, *Astrophy, J.* **144**, 599.
Ginzburg, V. L. and Syrovatskii, S. I.: 1964, *The Origin of Cosmic Rays* (translated by H. Massey). Pergamon Press, New York.
Ginzburg, V. L. and Syrovatskii, S. I.: 1965, *Ann. Rev. Astron. Astrophy.* **3**, 297.
Ginzburg, V. L. and Zheleznyakov, V. V.: 1958, *Soviet Astron. – A.J.* **2**, 653.
Ginzburg, V. L. and Zheleznyakov, V. V.: 1959a, *Paris Symposium on Radio Astronomy* (ed. by R. Bracewell). Stanford Univ. Press, p. 574.
Ginzburg, V. L. and Zheleznyakov, V. V.: 1959b, *Soviet Astron. – A.J.* **3**, 235.
Hachenberg, O. and Wallis, G.: 1961, *Z. Astrophys.* **52**, 42.
Haddock, F. T.: 1959, *Paris Symposium on Radio Astronomy* (ed. by R. N. Bracewell). Stanford Univ. Press, p. 188.
Hanasz, J.: 1966, *Australian J. Phys.* **19**, 635.
Hartz, T. R.: 1964, *Ann. Astrophy.* **27**, 831.
Hayakawa, S., Nishimura, T., Obayashi, H., and Sato, H.: 1964, *Prog. Theor. Phys.*, Suppl. **30**, p. 86.
Heitler, J. W.: 1954, *The Quantum Theory of Radiation.* 2nd ed., Clarendon Press, Oxford, p. 21.
Jager, C. de: 1960, *Space Research* **1**, 628.
Jager, C. de: 1962, *Space Sci. Rev.* **1**, 487.
Kai, K.: 1963, *Publ. Astron. Soc. Japan* **15**, 195.
Kai, K.: 1965a, *Publ. Astro. Soc. Japan* **17**, 294.
Kai, K.: 1965b, *Publ. Stron. Soc. Japan* **17**, 309.
Kakinuma, T.: 1956, *Proc. Res. Inst. Atmospherics Nagoya Univ.* **4**, 78.
Kakinuma, T.: 1958, *Proc. Res. Inst. Atmospherics Nagoya Univ.* **5**, 71.
Kakinuma, T. and Swarup, G.: 1962, *Astrophys. J.* **136**, 975.
Kakinuma, T. and Tanaka, H.: 1961, *Proc. Res. Inst. Atmospherics Nagoya Univ.* **8**, 39.
Kawabate, K.: 1960, *Rep. Ionosphere Space Res. Japan* **14**, 405.
Kawabata, K.: 1964, *Publ. Astron. Soc. Japan* **16**, 30.

KAWABATA, K.: 1966, *Rep. Ionosphere Space Res. Japan* **20,** 118.
KOMESAROFF, M.: 1958, *Australian J. Phys.* **11,** 201.
KRISHNAN, T. and MULLALY, R. F.: 1961, *Nature* **192,** 58.
KUNDU, M. R.: 1959, *Ann. Astrophys.* **22,** 1.
KUNDU, M. R.: 1961, *J. Geophys. Res.* **66,** 4308.
KUNDU, M. R.: 1963, *Space Sci. Rev.* **2,** 438.
KUNDU, M. R.: 1965, *Solar Radio Astronomy*. Interscience Publishers, New York.
KUNDU, M. R. and FIROR, J. W.: 1961, *Astrophys. J.* **134,** 389.
LANDAU, L.: 1946, *J. Phys. USSR* **10,** 25.
MALITSON, H. H. and ERICKSON, W. C.: 1966, *Astrophys. J.* **144,** 337.
MALVILLE, J. M.: 1962, *Astrophys. J.* **136,** 266.
MARTYNE, D. F.: 1947, *Nature* **159,** 26.
MAXWELL, A.: 1963, *Planet. Space Sci.* **11,** 897.
MAXWELL, A.: 1965, *The Solar Spectrum* (ed. by C. de Jager). D. Reidel Publ. Co., Dordrecht, Holland, p. 342.
MAXWELL, A. and THOMPSON, A. R.: 1962, *Astrophys. J.* **135,** 138.
MEYER, P. and VOGT, R.: 1962, *Phys. Rev. Letter* **8,** 387.
MORETON, G. E.: 1964, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. H. Hess). U.S. Government Printing Office, Washington, p. 209.
MORIMOTO, M.: 1961, *Publ. Astron. Soc. Japan* **13,** 285.
OORT, J. H. and WALRAVEN, T.: 1956, *Bull. Astr. Inst. Netherlands* **12,** 285.
PAUNDS, K. A.: 1965, *Ann. Astrophys.* **28,** 132.
PAYNE-SCOTT, R., YABSLEY, D. E., and BOLTON, J. G.: 1947, *Nature* **160,** 256.
PETERSON, L. E. and WINCKLER, J. R.: 1959, *J. Geophys. Res.* **64,** 697.
PHILIP, K. W.: 1964, *Astrophys. J.* **139,** 723.
PICK-GUTMANN, M.: 1961, *Ann. Astrophys.* **24,** 183.
PIDDINGTON, J. H.: 1955, *Phil. Mag.* **46,** 1037.
PIDDINGTON, J. H. and MINNETT, H. C.: 1951, *Australian J. Sci. Res.* **A4,** 131.
PIKEL'NER, S. B. and GINTSBURG, M. A.: 1964, *Soviet Astron. – A.J.* **7,** 639.
RAMATY, R. and LINGENFELTER, R. E.: 1967, *J. Geophys. Res.* **72,** 879.
RAO, U. V. G.: 1965, *Australian J. Phys.* **18,** 283.
RATCLIFFE, J. A.: 1959, *Magneto-Ionic Theory and Its Applications to the Ionosphere.* (Cambridge Univ. Press, Cambridge.
ROBERTS, J. A.: 1959, *Australian J. Phys.* **12,** 327.
SAGDEEV, R. Z.: 1962a, *Soviet Phys. Technical Phys.* **6,** 867.
SAGDEEV, R. Z.: 1962b, *Proc. Symp. Electromagnetic and Fluid Dynamics of Gaseous Plasma.* Polytechnic Press, Brooklyn, N. Y., p. 443.
SAKURAI, K.: 1965, *Rep. Ionosphere Space Res. Japan* **19,** 408; *Publ. Astron. Soc. Japan* **17,** 403.
SCHWINGER, J.: 1949, *Phys. Rev.* **75,** 1912. (See p. 1918.)
SITENKO, A. G. and STEPANOV, K. N.: 1957, *Soviet Phys. J. E. T. P.* **4,** 512.
SMERD, S. F.: 1965, *The Solar Spectrum* (ed. by C. de Jager). D. Reidel Publ. Co., Dordrecht, p. 398.
SMERD, S. F., WILD, J. P., and SHERIDAN, K. V.: 1962, *Australian J. Phys.* **15,** 180.
SPITZER, L.: 1962, *Physics of Fully Ionized Gases.* 2nd ed., Interscience Publishers, New York.
STEIN, W. A. and NEY, E. P.: 1963, *J. Geophys. Res.* **68,** 65.
STEWART, R. T.: 1966, *Australian J. Phys.* **19,** 209.
STIX, T. H.: 1964, *Phys. Fluids* **7,** 1960.
STURROCK, P.: 1961, *Nature* **192,** 58.
STURROCK, P.: 1964, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. N. Hess). U.S. Government Printing Office, Washington, p. 356.
TAKAKURA, T.: 1959, *Paris Symposium on Radio Astronomy* (ed. R. N. Bracewell). Stanford University Press, p. 562.
TAKAKURA, T.: 1960a, *Publ. Astron. Soc. Japan* **12,** 55.
TAKAKURA, T.: 1960b, *Publ. Astron. Soc. Japan* **12,** 325.
TAKAKURA, T.: 1960c, *Publ. Astron. Soc. Japan* **12,** 352.
TAKAKURA, T.: 1961a, *Publ. Astron. Soc. Japan* **13,** 166.
TAKAKURA, T.: 1961b, *Publ. Astron. Soc. Japan* **13,** 312.
TAKAKURA, T.: 1962, *J. Phys. Soc. Japan* **17,** Suppl. A-II, 243.

TAKAKURA, T.: 1963a, *Publ. Astron. Soc. Japan* **15,** 327.
TAKAKURA, T.: 1963b, *Publ. Astron. Soc. Japan* **15,** 462.
TAKAKURA, T.: 1964a, *AAS-NASA Symposium on Physics of Solar Flares* (ed. by W. Hess). U.S. Government Printing Office, Washington, p. 383.
TAKAKURA, T.: 1964b, *Publ. Astron. Soc. Japan* **16,** 230.
TAKAKURA, T.: 1966, *Space Sci. Rev.* **5,** 80.
TAKAKURA, T. and KAI, K.: 1961, *Publ. Astron. Soc. Japan* **13,** 94.
TAKAKURA, T. and KAI, K.: 1966, *Publ. Astron. Soc. Japan* **18,** 57.
TANAKA, H. and KAKINUMA, T.: 1958, *Rep. Ionosphere Res. Japan* **12,** 273.
TANAKA, H. and KAKINUMA, T.: 1959, *Paris Symposium on Radio Astronomy* (ed. by R. N. Bracewell). Stanford Univ. Press, p. 215.
TANAKA, H. and KAKINUMA, T.: 1960, *Proc. Res. Inst. Atmospherics Nagoya Univ.* **7,** 79.
TANAKA, H., KAKINUMA, T., and ENOME, S.: 1967, *Proc. Res. Inst. Atmospherics Nagoya Univ.* **14,** 23.
TERASHIMA, Y. and YAJIMA, N.: 1963, *Prog. Theoretical Phys.* **30,** 443.
TIDMAN, D. A.: 1965, *Planet. Space Sci.* **13,** 781.
TIDMAN, D. A. and DUPREE, T. H.: 1965, *Phys. Fluids.* **8,** 1860.
TIDMAN, D. A., BIRMINGHAM, T. J., and STAINER, H. M.: 1966, *Astrophys. J.* **146,** 207.
TRAKHTENGERTS, V. YU.: 1966, *Soviet Astron. – AJ.* **10,** 281.
TSUCHIYA, A.: 1963, *Publ. Astron. Soc. Japan* **15,** 368.
TWISS, R. Q.: 1958, *Australian J. Phys.* **11,** 564.
TWISS, R. Q. and ROBERTS, J. A.: 1958, *Australian J. Phys.* **11,** 424.
UCHIDA, Y.: 1960, *Publ. Astron. Soc. Japan* **12,** 376.
VAN ALLEN, J. A. and KRIMIGIS, S. M.: 1965, *J. Geophys. Res.* **70,** 5737.
VETTE, J. I. and CASAL, F. G.: 1961, *Phys. Rev. Letters* **6,** 334.
VITKEVICH, V. V. and GORELOVA, M. V.: 1960, *Soviet Astron. – AJ* **4,** 595.
VITKEVICH, V. V., GORELOVA, M. V., and LOZINSKAYA, T. A.: 1959, *Soviet Astron. – AJ* **3,** 626.
WEISS, A. A.: 1963a, *Australian J. Phys.* **16,** 240.
WEISS, A. A.: 1963b, *Australian J. Phys.* **16,** 526.
WEISS, A. A. and STEWART, R. T.: 1965, *Australian J. Phys.* **18,** 143.
WENTZEL, D. G.: 1963, *Astrophys. J.* **137,** 135.
WESTFOLD, K. C.: 1959, *Astrophys. J.* **130,** 241.
WHITE, W. A.: 1964, *AAS-NASA Symposium on Physics of Solar Flares.* U.S. Government Printing Office, Washington, p. 131.
WILD, J. P.: 1950, *Australian J. Sci. Res.* **A3,** 399.
WILD, J. P.: 1957, *Symp. IAU* **4,** 321.
WILD, J. P.: 1962, *J. Phys. Soc. Japan* **17,** Suppl. A-II, 249.
WILD, J. P.: 1964, *AAS-NASA Symposium on Physics of Solar Flares.* U.S. Government Printing Office, Washington, p. 161.
WILD, J. P. and TLAMICHA, A.: 1964, *Nature* **203,** 1128.
WILD, J. P., MURRAY, J. D., and ROWE, W. C.: 1953, *Nature* **172,** 533.
WILD, J. P., ROBERTS, J. A., and MURRAY, J. D.: 1954, *Nature* **173,** 532.
WILD, J. P., SHERIDAN, K. V., and TRENT, G. H.: 1959a, *Paris Symposium on Radio Astronomy* (ed. by R. N. Bracewell). Stanford University Press, p. 176.
WILD, J. P., SHERIDAN, K. V., and NEYLAN, A. A.: 1959b, *Australian, J. Phys.* **12,** 369.
WILD, J. P., SMERD, S. F., and WEISS, A. A.: 1963, *Ann. Rev. Astron. Astrophys.* **1,** 291.
WINCKLER, J. R.: 1964, *AAS-NASA Symposium on Physics of Solar Flares.* U.S. Government Printing Office, Washington, p. 177.
ZAITSEV, V. V.: 1966, *Soviet Astron. – AJ* **9,** 572.
ZHELEZNYAKOV, V. V.: 1962, *Soviet Astron. – AJ* **6,** 3.