THEORY OF THE SOLAR CYCLE* **

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Abstract. The properties of kinematic $\alpha\omega$-dynamos are briefly reviewed. The mean field concept, including turbulent diffusivity, is defended against recent criticism. It is pointed out that although the Maunder minimum cannot be explained by kinematic dynamo theory alone, this does not invalidate dynamo theory in general. A special discussion is devoted to attempts to evaluate the coefficients of the mean field induction equation in the case of very large conductivity. The field then behaves intermittent, in the form of locally concentrated flux tubes, and the α-effect and the turbulent diffusivity may be determined by asymptotic techniques or with the help of an exact solution of the non-dissipative induction equation in Lagrangian co-ordinates.

Magnetic cycles of main sequence stars other than the Sun are briefly discussed. Besides rotation, the depth of the convection zone is probably the most influencial parameter for period and amplitude of the stellar cycle.

Observational programmes to advance the theory of the solar cycle must include the solar magnetic and velocity fields, over the entire Sun and on all scales. In particular the angular velocity as a function of depth should be studied further with the help of the p-eigenmodes. The knowledge of luminosity, radius and (or) temperature variations with the solar cycle would also stimulate the theoretical approach.

1. Introduction

During a discussion at the Symposium 'Basic Mechanisms of Solar Activity' in Prague 1975, one of the theoreticians made the prediction that solar activity would be rather low in the next cycle. However, as you know, we are presently witnessing a high maximum of activity; the yearly mean sunspot number may well reach its second highest value since 1610. I mention the failure of this prediction here because it reflects our inadequate knowledge of how the solar cycle works. It is true, there is a group of kinematic dynamo models, the so-called $\alpha\omega$-dynamos, which apparently explain much of the observed long-term and large-scale behaviour of solar magnetic fields. But these models are being criticized on various grounds. I shall try, in Section 2, to summarize the $\alpha\omega$-dynamos and their difficulties, as far as the kinematic theory is concerned; some of the non-linear problems, which arise when the Lorentz force acts back upon the fluid motion, are considered in Section 3.

One particular source of criticism is the use of 'first order smoothing', which, in the case of high electrical conductivity, depends on the condition $v \ll l/\tau$, where v, l, and τ are r.m.s. velocity, and correlation length and time of the turbulent convective motions. On the Sun we have rather $v \approx l/\tau$, with the consequence that 'first order smoothing' is inadequate to describe the concentration of magnetic flux into narrow tubes of large field strength. Attempts to treat this situation more realistically will be discussed in Section 4.

* Proceedings of the 14th ESLAB Symposium on *Physics of Solar Variations*, 16–19 September 1980, Scheveningen, The Netherlands.

** Mitteilung aus dem Kiepenheuer-Institut Nr. 194.

Solar Physics **74** (1981) 79–101. 0038–0938/81/0741–0079 $03.45.

For the understanding of the solar cycle it would be extremely useful to know how the basic *stellar* parameters influence the properties of the cycle. In Section 5 I therefore report on $\alpha\omega$-dynamo models for main sequence stars. In Section 6 a few observational problems are discussed.

This review is concerned exclusively with dynamo theory and its possible relevance to the solar cycle. As an alternative, oscillator theories, sometimes in combination with a primordial field, have been proposed (Richardson and Schwarzschild, 1952; Piddington, 1971; Layzer *et al.*, 1979; Dicke, 1979). I do not discuss these, mainly because they are mathematically not sufficiently developed to be susceptible to criticism.

2. Kinematic αω-Dynamos

Parker (1955) suggested that the solar magnetic cycle is maintained by the combined action of differential rotation ('ω') and cyclonic convection (or 'α-effect').

A theoretical deduction of the latter effect, and also of the effect of turbulent electromagnetic diffusion, was subsequently provided by Steenbeck *et al.* (1966) and Krause (1967). How these effects are used to construct models of oscillatory mean fields which, in both hemispheres, migrate towards the equator has been summarized in the reviews of Stix (1976a, 1978a) and in the books by Moffatt (1978), Parker (1979), and Krause and Rädler (1980).

The main characteristics of $\alpha\omega$-dynamos are the following:

(a) They describe the behaviour of *mean* fields. Only these are subject to turbulent diffusion; and the α-effect is a *mean* electric current, $\sigma\alpha\bar{\mathbf{B}}$, parallel to the *mean* field, $\bar{\mathbf{B}}$. It is difficult to identify the mean field on the Sun, where the magnetic flux is concentrated into narrow tubes of large field strength. In default of ensemble averages, averages over a time span of 1 to 2 years seem to serve best (e.g. Stix, 1976a, Figure 4).

(b) A mean toroidal field is generated from an initial poloidal mean field by differential rotation. The α-effect then causes a toroidal current, with a concomitant new poloidal field. The relative strength of the poloidal to the toroidal mean field componenents is given by $(\alpha/\Delta\omega r_\odot)^{1/2}$, where $\Delta\omega$ is a typical difference of angular velocities within the convection zone, and $r_\odot$ is the solar radius. For the Sun the ratio between the two field components has been estimated to $1:100$ (Steenbeck and Krause, 1969), although the magnitude of α is not well-known – see below.

(c) The mean field *propagates* along the surfaces of isorotation (Parker, 1955; Yoshimura, 1975b), in the direction $\alpha\nabla\omega\times\mathbf{e}_\phi$, where $\mathbf{e}_\phi$ is the unit vector in the azimuthal direction. With $\alpha>0$ in the northern, and $\alpha<0$ in the southern hemisphere this means that an inwards increasing angular velocity is required in order to explain the observed butterfly diagram. The same direction of field propagation is obtained when both α and $\partial\omega/\partial r$ reverse their signs, but the phase relation between the poloidal and toroidal field components indicates $\partial\omega/\partial r<0$ (Stix, 1976b). The

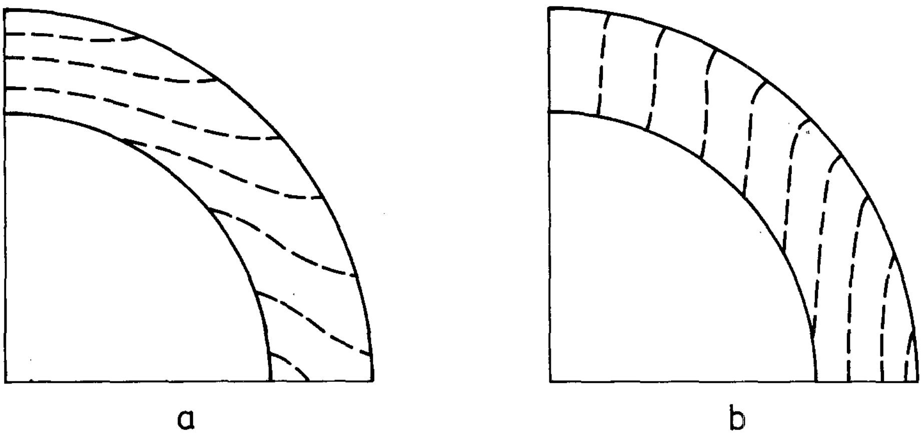

Fig. 1. Surfaces of constant angular velocity ω, in a meridional cross-section through the Sun: (a) With $\partial\omega/\partial r < 0$, as suggested by $\alpha\omega$-dynamo models, (b) with $\partial\omega/\partial r > 0$.

role of the angular velocity distribution in models of the solar cycle has been discussed further by Stix (1978b), in particular in the context of a tensorial α-effect.

(d) Two physical time scales determine the *period* of the oscillatory field: The time $r_\odot^2/\eta_t$ of turbulent electromagnetic diffusion (η_t being the turbulent diffusivity), and the period of the dynamo wave, $|\alpha\nabla\omega|^{-1/2}$. The condition of marginal dynamo instability expresses the equality of these two time scales. For the Sun estimates generally predict a period which is shorter than the observed 22 years by at least one order of magnitude, but again these estimates rest on a not well-known α; in addition, non-linear effects may lengthen the period (Stix, 1972; Kleeorin and Ruzmaikin, 1980). – Stationary solutions to the model equations of $\alpha\omega$-dynamos also exist (Levy, 1972; P. H. Roberts, 1972; Stix, 1973; Yoshimura, 1978b). However, in particular in a spherical shell such as the solar convection zone, the oscillatory $\alpha\omega$-dynamo is the normal case (Deinzer *et al.*, 1974).

(e) The *parity* of the mean field excited by an $\alpha\omega$-dynamo can be odd or even with respect to the equator. If $\alpha\ \partial\omega/\partial r < 0$ in the northern hemisphere, the odd parity is excited at smaller $|\alpha\ \partial\omega/\partial r|$ and is thus preferred. There is almost no such parity selection in models where the induction effects, $\Delta\omega$ and α, operate at middle and high latitudes (e.g. Belvedere *et al.*, 1980b). In order to avoid this degeneracy we must therefore assume, for a solar model, that the shear and the α-effect are concentrated at rather low latitudes. Of course, this does not only lead to the desired odd parity of the mean solar field, but also produces the toroidal field, and with it the east–west oriented bipolar groups, at low latitudes, where they are observed.

The above results (a) to (e) are obtained on the basis of a scalar α-effect. It is however known that in general α is a tensor, as in Equation (1) below (e.g. Moffat, 1978, Chap. 7). In the framework of kinematic theory, and using first order smoothing the α-tensor has been determined by Krause (1967) for turbulence influenced by slow rotation, and having one additional preferred direction. The case of fast rotation has been included by Rüdiger (1978). Wälder *et al.* (1980), employing the same approximations, and following earlier treatments of Moffat (1970a, b, 1972), determined the α-tensor for a sea of random waves in a stratified rotating fluid. The tensorial α-effect has been incorporated into spherical dynamo models by Weißhaar (1978), Busse (1979), and Busse and Miin (1979). They find oscillatory mean field solutions. These models however do not consider differential rotation, and thus are probably not applicable to the Sun.

Rädler (1980) has recently given a comprehensive formulation of the tensorial α-effect, as it occurs in a spherical geometry. In particular, the antisymmetric part of the α-tensor, which is equivalent to a mean fluid motion, can provide a 'magnetic pumping'. A similar effect occurs when the intensity of turbulence or r.m.s. velocity, varies in space. As noticed by Rädler (1968) and Vainshtein and Zel'dovich (1972), not only an effective electrical conductivity, but also an effective permeability, lowered by an equal amount, must then be considered. Ruzmaikin and Vainshtein (1978) pointed out that this 'diamagnetism' of turbulence contributes an effective mean flow. In the deeper part of the convection zone this flow, directed radially downward, could help to keep the buoyant magnetic field in the dynamo region. The diamagnetic effect also lengthens the period of $\alpha\omega$-dynamos (Ivanova and Ruzmaikin, 1976).

The use of $\alpha\omega$-dynamo models to explain the origin of the 22-year solar magnetic cycle has been criticized on various grounds. In particular, Piddington (1978, and further references therein) and Layzer *et al.* (1979) dispute the concept of turbulent diffusivity. To some extent, this criticism is based on a misunderstanding: Of course η_t must not be used in the equations governing the total magnetic field, $\mathbf{B}$, or its fluctuating part, $\mathbf{B}'$. Only the *mean* field is affected by turbulent diffusion. This has been deduced for isotropic and slightly anisotropic turbulence (e.g. Krause, 1967); there can be no doubt that the effect also occurs in strongly anisotropic turbulence such as in the Sun's convection zone. Another misunderstanding is that the mean field used in dynamo theory is often confused with a smooth or 'diffuse' background field. Dynamo theory would be inapplicable, since on the Sun all, or almost all, magnetic flux occurs in highly concentrated form. However, according to its definition the mean field is the *average* field, rather than the field in between the flux concentrations. Even without such background field there would be a mean field if there exists an disbalance of positive and negative flux over a certain area. Even in the framework of linear theory the equilibrium ratio of the r.m.s. field strength to the mean field can be estimated to $\sim R_m^{1/2}$, where $R_m = \mu\sigma v l$ is the magnetic Reynolds number (Bräuer and Krause, 1973); we have $R_m \gg 1$ in the solar convection zone. The 'curious' result of Layzer *et al.* (1979) that the r.m.s. field decays just as slowly as in the absence of turbulence is not new; this slow decay is the reason for the result of Bräuer and Krause.

The cornerstone of turbulent dynamo action is the mean electric field $\mathscr{E} = \overline{\mathbf{u}' \times \mathbf{B}'}$. Formally, we can always expand $\mathscr{E}$ in the following way:

$$\mathscr{E}_i = \alpha_{ij}\bar{B}_j + \beta_{ijk}\frac{\partial \bar{B}_j}{\partial x_k} + \cdots . \tag{1}$$

The coefficients α and β have been computed under a variety of assumptions. The only serious approximation, and the only one which really deserves criticism, is the 'Second Order Correlation Approximation' (or 'quasilinear approximation' in the terminology of Layzer *et al.*, 1979). This constitutes a rather crude method of closure, in particular in the case of magnetohydrodynamic turbulence in a highly conducting medium. In Section 4 below I shall further discuss this point, and also comment on two attempts to avoid this approximation.

Other assumptions which have been made to compute α and β include: Global non-axisymmetric convection in a *shallow* layer, in the limit of slow rotation (Yoshimura, 1972); *isotropic* turbulence *slightly* influenced by rotation and a gradient of density or turbulence intensity (Steenbeck *et al.*, 1966; Krause, 1967); or *small amplitude* random *waves* in a rotating medium (Moffatt, 1970a, b, 1972; Wälder *et al.*, 1980). Often such calculations yield too large values of α: An application of Krause's (1967) formula

$$\alpha = -\tfrac{16}{15}\tau^2 v^2 \boldsymbol{\omega} \cdot \boldsymbol{\nabla} \ln(\rho v) \tag{2}$$

to velocities and scales obtained from a mixing length model of the Sun's convection zone (Köhler, 1973) gives $\alpha \approx 100\ \mathrm{m\ s^{-1}}$. A similar value is found from the helicity of global non-axisymmetric convection as computed by Gilman and Miller (1980). In contrast to this, the observed ratio of poloidal and toroidal field components indicates $\alpha \approx 1$ to $10\ \mathrm{cm\ s^{-1}}$. In view of this discrepancy between required and computed α values it is little comfort that, at least, the sign of α seems to be the same in all cases: positive in the northern, and negative in the southern hemisphere, except possibly in the deepest part of the convection zone.

Polar field reversals are an essential feature of the oscillatory $\alpha\omega$-dynamo (as opposed to the torsional oscillator model). Such reversals have been observed in 1957/58 (Babcock, 1959) and in 1969/71 (Howard, 1974). Still, Piddington (1977) disputes this observational evidence. Another reversal is expected to take place sometime during the present maximum of activity. In fact Dr Howard tells me that it has already been observed. Of course it would be desirable to measure the polar fields as longitudinal fields from an out of ecliptic spacecraft. The two ISPM's will measure the interplanetary field at high ecliptic latitudes. A safe extrapolation as to the polarity of the solar polar cap fields and a comparison with ground measurements will then be possible.

A 'crucial aspect of turbulent dynamo theory' . . . is 'the spatial separability of the α and ω regenerative processes'. This statement by Layzer *et al.* (1979) is simply wrong. Many $\alpha\omega$-dynamos with co-spatial shear and α regions have been computed in the past, including the very first treatment of dynamo waves by Parker (1955). Contrary to the original view of Steenbeck and Krause (1969) and Krause and Rädler (1971) these waves do not travel back and forth between the α and ω regions, but instead along the shear surfaces (Yoshimura, 1975b). This result has nothing to do with the neglect of a term curl $(\bar{\mathbf{u}} \times \mathbf{B}')$ in the equation governing $\mathbf{B}'$, as Layzer *et al.* (1979) seem to believe. Incidentally Krause (1967) has included this term in the case where the vector gradient $\partial \bar{u}_i / \partial x_k$ is constant.

The *Maunder minimum*, i.e. the period from 1645 to 1715 when solar activity remained at a very low level, 'presents a serious difficulty for conventional dynamo theories, because if the surface fields are weak or absent for an extended period, the poloidal field cannot be regenerated, and the dynamo must be quenched'. In order to support this conclusion Layzer *et al.* (1979) quote Leighton's (1969) model of the solar cycle, which indeed depends on surface fields of finite magnitude. But although Leighton's model has much in common with $\alpha\omega$-dynamos (Stix, 1974), it differs in this particular point: $\alpha\omega$-dynamos are capable of maintaining a field at arbitrary low levels. In fact, in the kinematic approximation, where the Lorentz force is neglected, dynamo theory is strictly applicable only as long as the field amplitude is infinitesimal. Thus, we may simply adopt the view that during the Maunder minimum the solar dynamo was continously in opertion, but with a mean field so weak that very few toroidal flux tubes became strong enough to be buoyant and erupt through the photosphere. Of course, in its kinematic (linear) form, dynamo theory is altogether uncapable to predict a small field amplitude during the Maunder minimum, and a large amplitude during other periods of time. But this criticism does not invalidate dynamo theory in general. Attempts to include non-linear effects into the dynamo equations in order to simulate 'anomalous' periods of activity will be described in the following section.

3. Non-Linear Effects

Kinematic dynamo theory, turbulent or not, is based upon the induction equation alone, which is homogeneous and linear and yields no information about the field strength. The Lorentz force acting on the fluid motion is neglected. Piddington (1978) believes that 'this passive field concept is a basic feature of dynamo theory'. Quite wrong. Many examples of nonlinear, or hydromagnetic, dynamos do exist, and mean-field equations for the non-linear case have been formulated, e.g. by Rädler (1976).

In most non-linear models which have been applied to the Sun the induction equation alone has been used, and the feedback mechanism has been parametrized by a $\bar{\mathbf{B}}$-dependence of α or (and) ω (Stix, 1972; Rüdiger, 1973ab; Jepps, 1975; Yoshimura 1975a, 1978a, c, 1979; Ivanova and Ruzmaikin, 1977; Kleeorin and Ruzmaikin, 1981). Particular forms of the functional dependence $\alpha(B)$ are $\alpha \sim 1-\xi B^2$ for weak fields, where ξ can be negative or positive, and $\alpha \sim B^{-3}$ for strong fields (Moffatt, 1972; Rüdiger, 1974). The properties of linear $\alpha\omega$-dynamos are usually reproduced by such calculations. In addition, improvements such as a longer period of field oscillation can be obtained. Most extensive calculations of this kind have been carried out by Yoshimura. By a proper choice of the parameters describing the geometry and field-dependence of α and ω, and introducing other parameters describing the eruption of buoyant magnetic flux or a feedback mechanism with time-delay he was able to simulate many characteristics of observed solar magnetic fields: the division of the poloidal mean field into two branches, one travelling polewards, and the other equatorwards (Yoshimura uses the term 'quadrupole' for this field-structure; in view of the parity, however, 'octupole' would be more appropriate); the occurrence of long-term variations such as a 55-year period; the occurrence of periods of weak mean fields and so presumably low surface activity, such as the Maunder minimum; the possibility of a transition from the oscillatory dynamo mode to a steady mode; and sporadic field reversals of these steady modes, simulating geomagnetic reversals.

There is some observational evidence of a variation of the solar velocity field during the solar cycle. Howard (1976) reports a gradual increase of the equatorial rotation rate between 1967 and 1976, and recently a small torsional oscillation, of order 3 m s^{-1}, superimposed on the differential rotation, has been discovered (Howard and LaBonte, 1980a; Scherrer and Wilcox, 1981). Such an oscillation may be explained by a mean Lorentz force. Using one of his $\alpha\omega$-model calculations, Yoshimura (1980) has demonstrated that this Lorentz force travels like a wave from high to low latitude, with a period of 11 years (Figure 2). Such nice reproduction of the observed feature does however not necessarily mean that all details of this particular model are correct: Any dynamo model having the correct period of 22 years and the correct latitude migration of the field would give such a Lorentz force (which is of second order and thus must vary with half the period.) Also, we must

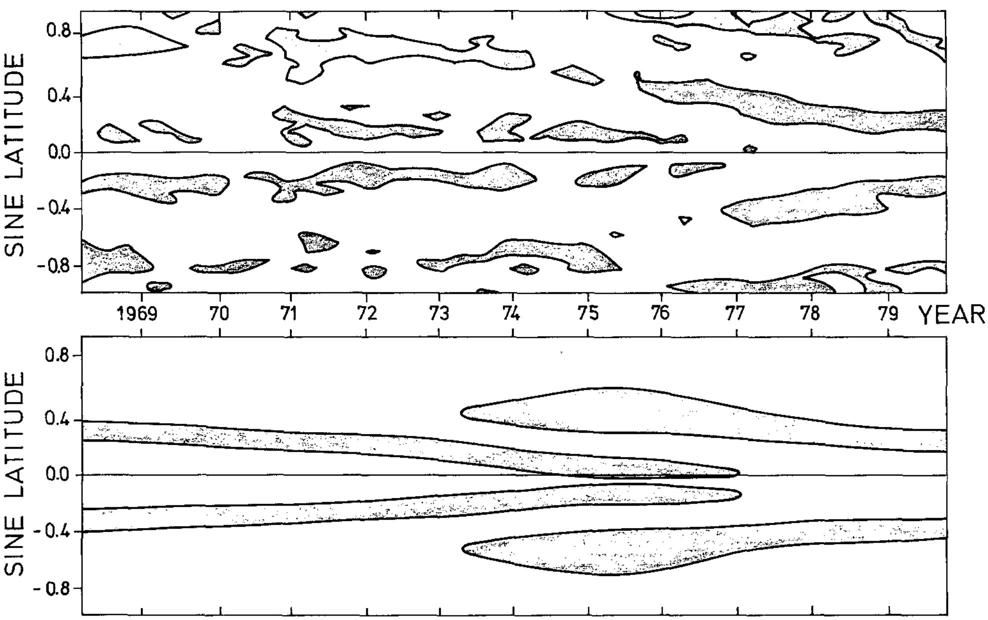

Fig. 2. The shaded areas in the upper diagram show where four-rotation averages of the azimuthal velocity exceed the mean differential rotation by more than 1.5 m s^{-1} (the corresponding negative deviations are not shown). The lower diagram shows areas of positive azimuthal mean Lorentz force in an $\alpha\omega$-dynamo model. Adapted from Howard and LaBonte (1980) and Yoshimura (1980).

keep in mind that the part $\overline{\mathbf{j}' \times \mathbf{B}'}$ of the mean Lorentz force should be taken into account, in addition to the term $\bar{\mathbf{j}} \times \bar{\mathbf{B}}$ considered by Yoshimura.

Gilman and Miller (1980) have criticized $\alpha\omega$-dynamos in that they 'ignore the hydrodynamics altogether' or 'base the chosen motions on solutions to hydrodynamic equations which are ... severely approximated'. It is difficult to assess to what extent this criticism is justified. For example, the hydrodynamic treatment by Gilman (1976, 1978, 1979) of non-axisymmetric Boussinesq convection in a spherical shell leads to an angular velocity which is approximately constant on cylinders parallel to the axis of rotation, as indicated in Figure 1b. It is true, none of the existing solar $\alpha\omega$-dynamos makes use of such a cylindrical rotation law. But then, we cannot yet be sure whether the Boussinesq model correctly describes the Sun's differential rotation: One obtains, in addition to the equatorial acceleration, flux and velocity variations which exceed the observational upper limits; the argument (Gilman, 1980, Figure 4) that a turbulent layer near the surface screens those effects from the observer has been studied by Stix (1981) who found that (at least for the velocity) this screening is probably not sufficiently effective.

Other attempts to explain the Sun's differential rotation employ anisotropic or space-dependent transport coefficients, and some of them lead to angular velocity profiles which qualitatively resemble those used in $\alpha\omega$-dynamos (Figure 1a; e.g. Belvedere *et al.*, 1980a; Durney, 1981). In one case (Belvedere *et al.*, 1980b) a solar $\alpha\omega$-dynamo has been directly computed together with the profile of ω. This model gives the correct latitude migration of the mean field, but on the other hand exhibits too-high-latitude fields and the concomitant degeneracy of parities (cf. Section 2). The fact that these results rest on an approximation of slow rotation (small Taylor number) seems to be not of grave consequence, as recent calculations (W. Schmidt, 1980, private communication) indicate. It appears that compressibility more severely affects the law of rotation than has previously been thought (P. A. Gilman, 1980, private communication).

In their own numerical model Gilman and Miller (1981) simultaneously solve the equations for the evolving magnetic field and velocity, including the feedback of the field on the motion. With the exception of isotropic transport co-efficients (the thermal, kinematic, and magnetic turbulent diffusivities) they do not parametrize small scale motions. In particular, their induction equation – although describing the evolution of a mean field, because it uses the turbulent magnetic diffusivity – does *not* employ an α-term. Although they have not yet been able to successfully simulate the solar cycle, their model is a consistent hydromagnetic dynamo, and exhibits a number of interesting results. One is that the *time-dependence* of the motion field, which is generally neglected in kinematic dynamos, substantially inhibits dynamo action, i.e. increases the effective ohmic dissipation. Another is a strong feedback of even *weak* magnetic fields, with magnetic energy several orders of magnitude smaller than kinetic energy: a slight change in the motion, caused magnetically, grows, due to unstable (diverging) hydrodynamic solutions, to a large deviation, and the flow may turn to a state which is less favourable to dynamo action.

The main problem with such numerical calculations (as with conventional $\alpha\omega$-dynamos) is that concentrations of magnetic flux cannot be adequately described. Such 'flux tubes' are observed; they should be described by the high wave number end of the magnetic spectrum. Figure 6 of Gilman and Miller (1980) shows that with decreasing magnetic Prandtl number, Q, i.e. increasing electrical conductivity, the spectrum becomes flatter; for $Q < 0.1$ difficulties with spatial resolution are encountered. I shall comment further on this problem in the following section.

A special feature of non-linear systems is bifurcation. Consider e.g. the following system of ordinary differential equations:

$$\begin{aligned} \dot{A} &= -A + DB - CB \,, \\ \dot{B} &= -\sigma B + \sigma A \,, \\ \dot{C} &= -\nu C + AB \end{aligned} \tag{3}$$

(Zel'dovich and Ruzmaikin, 1980), where A is the ϕ-component of the vector potential of the poloidal mean field, $B = \bar{B}_\phi$, and C is proportional to the deviation of the α-effect from its kinematic value; D is the dynamo number, i.e. a dimensionless product of shear and α-effect, and σ and ν are positive constants. The solutions of (3) may remain in the neighbourhood of one of the three steady solutions $(A, B, C) = (\pm\sqrt{\nu(D-1)}, \pm\sqrt{\nu(D-1)}, D-1)$ and $(0, 0, 0)$ for some time and then change over to another. Such behaviour is called 'strange attractor'. Ruzmaikin (1981) in particular suggests that the Maunder minimum of solar activity can be described by a strange attractor to a singular point similar to the point $(0, 0, 0)$ in the above example.

4. Large Electrical Conductivity and Magnetic Flux Tubes

Magnetic fields on the solar surface are observed as highly concentrated flux tubes. This is true not only for sunspots and pores, but also outside active regions where the field is confined to small areas, with diameters of order 200 km and field strengths of order 2000 G (e.g. Stenflo, 1976). The main reason for the concentration of magnetic flux is convection occurring in a fluid of large magnetic Reynolds number, i.e. large electrical conductivity. This effect has been numerically modelled for laminar (Galloway *et al.*, 1978) as well as turbulent flow; in the latter case, the field becomes 'intermittent' (e.g. Kraichnan, 1976; Orszag and Tang, 1979), just as seen in the solar photosphere. Thus it is only plausible to assume that concentrated fields pervade the entire convection zone since the magnetic Reynolds number becomes even larger with depth, increasing from $\approx 10^3$ to $\approx 10^9$ (e.g. Stix, 1976a, Figure 1). We should keep in mind that we discuss here the interaction of the magnetic field with individual convection cells and are, therefore, concerned with the 'local' magnetic Reynolds number, $R_m = \mu\sigma v l$ where σ is the 'molecular' conductivity (Spitzer, 1962), v and l are velocity and size of the cell, and μ is the permeability.

The properties of individual magnetic flux tubes have been reviewed recently by Parker (1979) and Spruit (1980). Concerning the solar dynamo, their existence does *not* force us to abandon the mean field concept. As already emphasized in Section 2 above, the mean field is an *average* and therefore includes the contributions of the flux tubes. If the magnetic field would vanish exactly in between the flux concentrations, we could consider the mean field as a distribution function for flux tubes.

The problem is how to reliably compute the coefficients α and β in the expansion (1). Before considering a distribution of many flux tubes, we may see what happens with one isolated tube. In the kinematic regime, where the Lorentz force is neglected, the radius of the central flux concentration formed in a cylindrical container of convecting fluid will be of order $R_m^{-1/2}r_0$, where r_0 is the radius of the cylinder (Galloway *et al.*, 1978). The maximum field strength is of order $R_m B_0$, and the profile is gaussian; B_0 is the total flux across the container divided by its cross-section, and nearly all of this flux is concentrated near the axis. Childress (1979) has recently studied the consequences of a toroidal velocity in the cell in addition to the poloidal one which produces the central tube. The situation is illustrated in Figure 3: The azimuthal velocity was chosen such that the flow $\mathbf{u}$ possesses helicity, i.e. $\int \mathbf{u}\,\mathrm{curl}\,\mathbf{u}\,\mathrm{d}V \neq 0$, where the integral is over the entire cell volume. In cylindrical co-ordinates (s, ϕ, z), the component α_{zz} of the α-tensor is of particular interest. No 'first order smoothing' (another name for the above-mentioned second order correlation approximation) was necessary in Childress' analysis. Instead he utilized the fact that – due to the large value of R_m – the field is confined to narrow zones at the boundaries of the cylinder, see Figure 3. Using boundary layer methods he describes the generation of a toroidal field, B_ϕ, of order

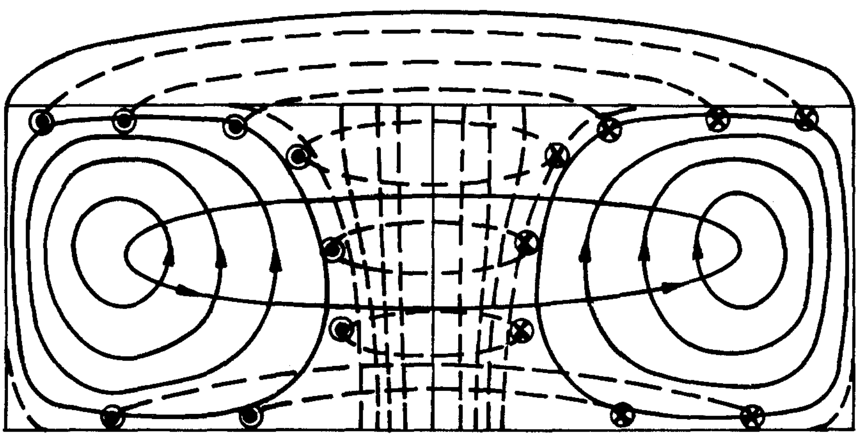

Fig. 3. Poloidal and toroidal laminar flow in a cylindrical container (solid lines), and the resulting magnetic flux distribution (broken): A concentrated poloidal tube at the axis, and boundary layers of toroidal flux around the central tube and along the surfaces of the container. Schematically composed from Galloway *et al.* (1978) and Childress (1979).

$R_m^{1/2} B_0$ around the central flux tube. This field is then advected by the poloidal flow into the remainder of the volume, but, again due to the large value of R_m, is essentially confined into boundary layers of thickness $r_0 R_m^{-1/2}$. The integral

$$\alpha_{zz} = \frac{2}{B_0 r_0^2 d} \int_0^d \int_0^{r_0} s(u_s B_\phi - u_\phi B_s)\, \mathrm{d}s\, \mathrm{d}z \tag{4}$$

has contributions only from the boundary layers. The dominant ones come from the top and bottom surfaces of the cylinder, and are of order U_0 where U_0 is essentially the velocity of the material rising along the axis. The contribution from the flux rope itself is only of order $R_m^{-1/2} U_0$.

Cylindrical containers with rigid top and bottom surfaces do not exist in the Sun's convection zone, but flux tubes *do* occur. We may therefore speculate that the $R_m^{-1/2} U_0$ contribution to the α-effect from the central flux tube in Childress's model is more significant than the much bigger contributions from the surfaces. An $\alpha \approx R_m^{-1/2} U_0$ is also obtained in a different example (Childress, 1979), a two-dimensional flow of a convection role of infinite length. The result is consistent with numerical calculations of G. O. Roberts (1972), who considered magnetic Reynolds numbers as large as $R_m = 64$.

Suppose the α-effect in the solar convection zone would be reduced everywhere by a factor $R_m^{-1/2}$ in comparison to the original result of Krause (1967). Would it be still sufficiently large in order to regenerate the poloidal field? I have computed (Figure 4) the modified α as a function of depth, using the results shown in Figures 1 and 7 of Stix (1976a). The maximum value is $\approx 4 \times 10^{-3}$ m s^{-1}. The critical value of the dynamo number, $P = \alpha \Delta\omega r_\odot^3 / \eta_t^2$, which must be exceeded for dynamo action to occur, is of the order 10^3 in numerical calculations. Using $\Delta\omega = 10^{-6}$ s^{-1} and $r_\odot = 7 \times 10^8$ m this means that the diffusivity, η_t, should be smaller than 4×10^7 m^2 s^{-1}. The value obtained for isotropic turbulence, using first order

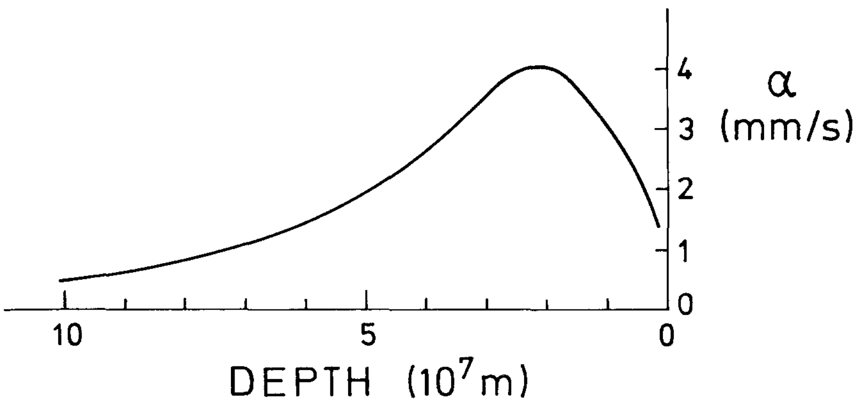

Fig. 4. The α-effect as a function of depth in the solar convection zone. The coefficient α is everywhere reduced by a factor $R_m^{-1/2}$, where R_m is the magnetic Reynolds number.

smoothing, and without any concern about flux tubes, is $\frac{1}{3}U_0 l \approx 10^9 \text{ m}^2 \text{ s}^{-1}$ (Stix, 1976a, Figure 7). Thus, α would be too small for dynamo action, unless the value of the turbulent magnetic diffusivity in the solar convection zone would *also* be reduced by the presence of magnetic flux tubes. Such a reduction would also favourably change the period, T, of the cycle models, which generally yield dimensionless frequencies $\Omega \approx 100$: we would obtain $T = 2\pi r_\odot^2/\Omega\eta_t \approx 25$ yr. We should however not be too keen to apply the results of Childress to the Sun: his flow is steady and incompressible, of special geometry, and is not subject to the feedback by the Lorentz force. The latter has been included into a boundary layer treatment by Galloway *et al.* (1978). Their result, that the flow is gradually quenched when B_0, i.e. the total flux across the cell, is increased, seems plausible; but they use the limit of small Reynolds number, $\text{Re} = U_0 d/\nu$, where the Lorentz force is balanced by the viscous force. This does probably not apply to the Sun; there an equilibrium must be established essentially between the Lorentz force, the pressure gradient and the inertia force (e.g. Vainsthein, 1979).

In the limit of infinite conductivity (or magnetic Reynolds number) the α-effect obtained in the model of Childress tends to zero or non-zero values, depending on whether or not we accept the R_m^0-contribution from the top and bottom surfaces of the cylinder. This question seems to be very critical for the solar dynamo, because the Sun comes close to the limit $R_m \to \infty$. It is therefore useful to note a few other results. In isotropic incompressible turbulence, with first order smoothing, we have $\alpha \to 0$ with $R_m \to \infty$, since the fluctuations $\mathbf{u}'$ and $\mathbf{B}'$ of flow and field are in phase, so that the average $\overline{\mathbf{u}' \times \mathbf{B}'} = 0$ (Moffatt, 1974). This is still true (Deinzer, 1976) when compressibility and one preferred direction (rotation) is admitted; but a second preferred direction, stratification of density or turbulent intensity, leads to finite non-zero α, as demonstrated by Krause (1967) for slightly anisotropic turbulence and by Wälder *et al.* (1980) for a sea of random waves. In order to avoid first order smoothing, 'Cauchy's integral' of the vorticity equation,

$$B_i(\mathbf{X}(\mathbf{a}, t), t) = B_j(\mathbf{a}, 0)\, \partial X_i/\partial a_j\,, \tag{5}$$

can be used to write down an exact solution of the induction equation in the case $R_m = \infty$ (Parker, 1971; Moffatt, 1974). To evaluate this solution the fluid-element trajectories $\mathbf{X}(\mathbf{a}, t)$ must be known as functions of time, t, and initial position, $\mathbf{a}$. Kraichnan (1976) computed such trajectories numerically. In the case of 'normal' turbulence, when the velocity covariance decays in time, he found that $\alpha(t)$, being zero initially because the initial field in Equation (5) contains no fluctuations, tends to a finite value of order v_0, the r.m.s. velcoity (see Figure 5). But again we cannot yet draw conclusions for the Sun: the treatment is for isotropic (although helical) and incompressible turbulence, and purely kinematic. As Kraichnan points out, the magnetic field in his calculations soon becomes intermittent. Thus it seems that the Lorentz force may become important locally due to the formation of flux tubes.

Kraichnan (1976) has also computed the coefficient of turbulent diffusivity; again in the 'normal case', he found that η_t approaches a finite value, of order v_0/k_0, where

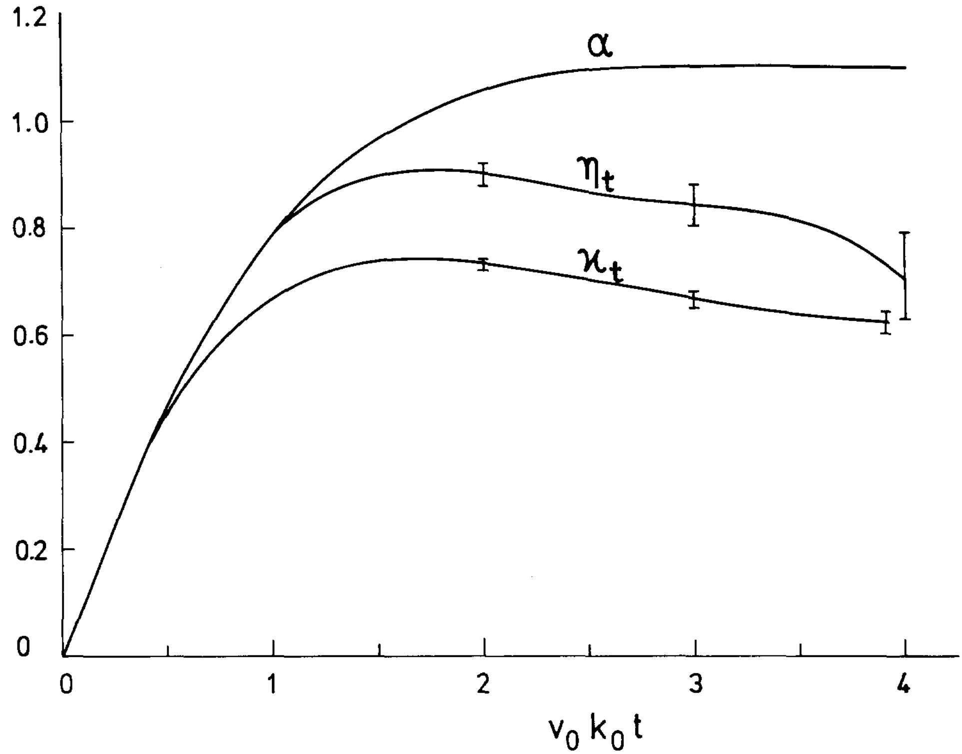

Fig. 5. Coefficients $\alpha(t)$, in units of v_0, and $\kappa_t(t)$ and $\eta_t(t)$, in units of v_0/k_0, for normal isotropic turbulence with maximal helicity, in the case of infinite conductivity. κ_t and η_t are the turbulent diffusivites for a passive scalar and for the mean magnetic field, respectively; v_0 and k_0 are a characteristic velocity and wave number of the turbulence. Adapted from Kraichnan (1976).

k_0 marks the peak of the energy spectrum. Figure 5 shows some of these results; η_t is some 40% smaller than the diffusivity, κ_t, of a scalar quantity. Only for non-normal turbulence, with strong helicity fluctuations, η_t can be reduced substantially or even become negative. The problem of turbulent electromagnetic diffusion of mean fields has been extensively treated by Parker (1979, Chap. 17).

Albregtsen and Maltby (1978) reported evidence that the intensity of sunspot umbrae, I_u, in particular in the infrared, increases during the sunspot cycle. The increase of I_u/I_{phot} is from 0.44 to 0.59, at 1.67 μm, from one minimum to the following. As sunspots are the most prominent examples of magnetic flux tubes, this result indicates that flux tubes 'know' the epoch at which they appear during the cycle, or that they have a lifetime comparable to the period of the cycle, or that the efficacy of sunspot cooling varies during the cycle (as suggested by Albregtsen and Maltby). All these explanations are difficult to understand physically. We know of course that the spots originate at decreasing latitudes as the cycle proceeds, but why should this lead to varying intensities if, at the same time, there is apparently no systematic variation of the total flux of a spot? Or how can individual flux tubes in the convection zone survive for years, with a memory of their initial conditions, when the turbulent convection continuously forms, reshapes, and shreds such tubes? On the Sun's surface, we observe that the age of flux tubes is at most a few months, and the decay is attributed to the action of turbulence (Meyer *et al.*, 1974). And how should we understand *variations* of the cooling efficacy when the cooling mechanism itself is hardly understood (cf. Parker, 1979, Chap. 10)?

The importance of Albregtsen and Maltby's observation for any theory of the solar cycle has been pointed out by Schmidt (1978), and Schüßler (1980) has proposed a simple model incorporating the spot intensity variation as an effect of aging flux tubes. The model is an $\alpha\omega$-dynamo, with a shear region near the bottom of the convection zone. There a toroidal field is generated from an original poloidal field, and is concentrated into a number of toroidal flux tubes. These become buoyant and leave the shear region sooner or later, depending on their field strength, and rise. The poloidal field also varies within a cycle. When it is stronger, the toroidal tubes are formed quickly, and emerge as 'young' flux at the surface. When it is weak, the process takes longer, and 'old' flux tubes appear. Heat exchange with the surrounding material, possibly by radiation, would make old tubes less dark. At the same time, convective motions separate smaller portions from the tube; these are distributed over a wider area and appear as ephemeral active regions. Some of the tubes, in particular at high latitudes, would thus be dissolved completely. Schüßler suggests this as an explanation for the cyclic variation of the appearance of these regions (Martin and Harvey, 1979), and the related X-ray bright points (Golub *et al.*, 1980, and further references therein). The occurrence of X-ray bright points has also been correlated directly to the sunspot intensity at 1.67 μm (Maltby and Albregtsen, 1979). Figure 6 shows, as a function of time, a computed activity curve, together with the age of the emerging flux tubes. There is a slow increase of age during the descending phase of activity maximum rather than at the beginning of the cycle.

A somehow related suggestion concerning the origin of magnetic flux outside active regions has been put forward by Golub *et al.* (1980). They also consider small-scale, intermittent magnetic flux in the convection zone as 'debris' produced by turbulent dissipation of a large-scale, dynamo-produced field. This dissipation would be greatest during the sunspot minimum, when oppositely directed toroidal fields annihilate each other. The observation that the number of X-ray bright points is largest around sunspot minimum, is thus explained.

5. Stellar Cycles

Main sequence stars later than F possess outer convection zones. Presumably they also rotate, although the rotation is generally too slow to be detected through spectral line broadening (e.g. Tassoul, 1978). The main ingredients of the solar $\alpha\omega$-dynamo are differential rotation and helical turbulent convection, and are both caused by the influence of rotation upon convection. In spite of the possibly very

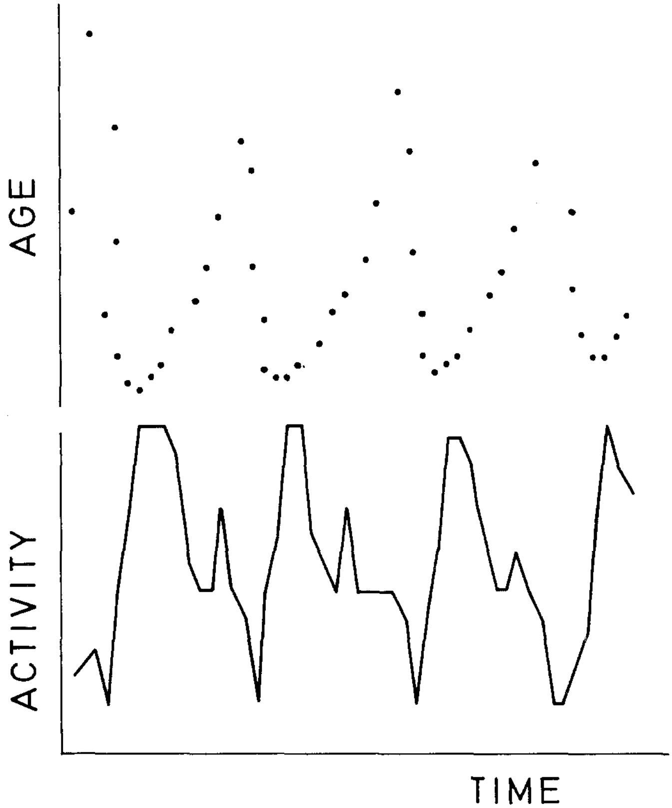

Fig. 6. The flux-tube dynamo of Schüßler (1980): The age (top) of the tubes varies in antiphase with the activity index (bottom), which represents the number and field strength of the erupting flux tubes.

slow rotation of most late main sequence stars the rotational influence is probably important for the following reason (Durney and Latour, 1978): Later stars have deeper convection zones, extending into regions of higher temperature, and therefore larger scale height, H. The turnover time, H/U of convection elements of velocity U therefore increases, presumably more rapidly than the possible increase of the rotational period, P. The ratio H/UP (the inverse Rossby number), indicating the strength of the Coriolis force in comparison to the inertia force, may therefore be of order unity or larger even for the slow rotators on the late main sequence.

Figure 7 shows this ratio, as a function of spectral type (Gilman, 1980), and also the depth of the convection zone. The angular velocity of all stars is assumed to be the same as for the Sun; two values l/H of the ratio mixing length/scale height are considered, and convection extends deeper and the rotational influence becomes stronger for larger l/H.

One possible cause of the solar differential rotation is a latitude-dependent convective heat transport coefficient (Durney and Roxburgh, 1971). Belvedere and Paternò (1977) and Belvedere *et al.* (1980a, c) have used this idea in order to

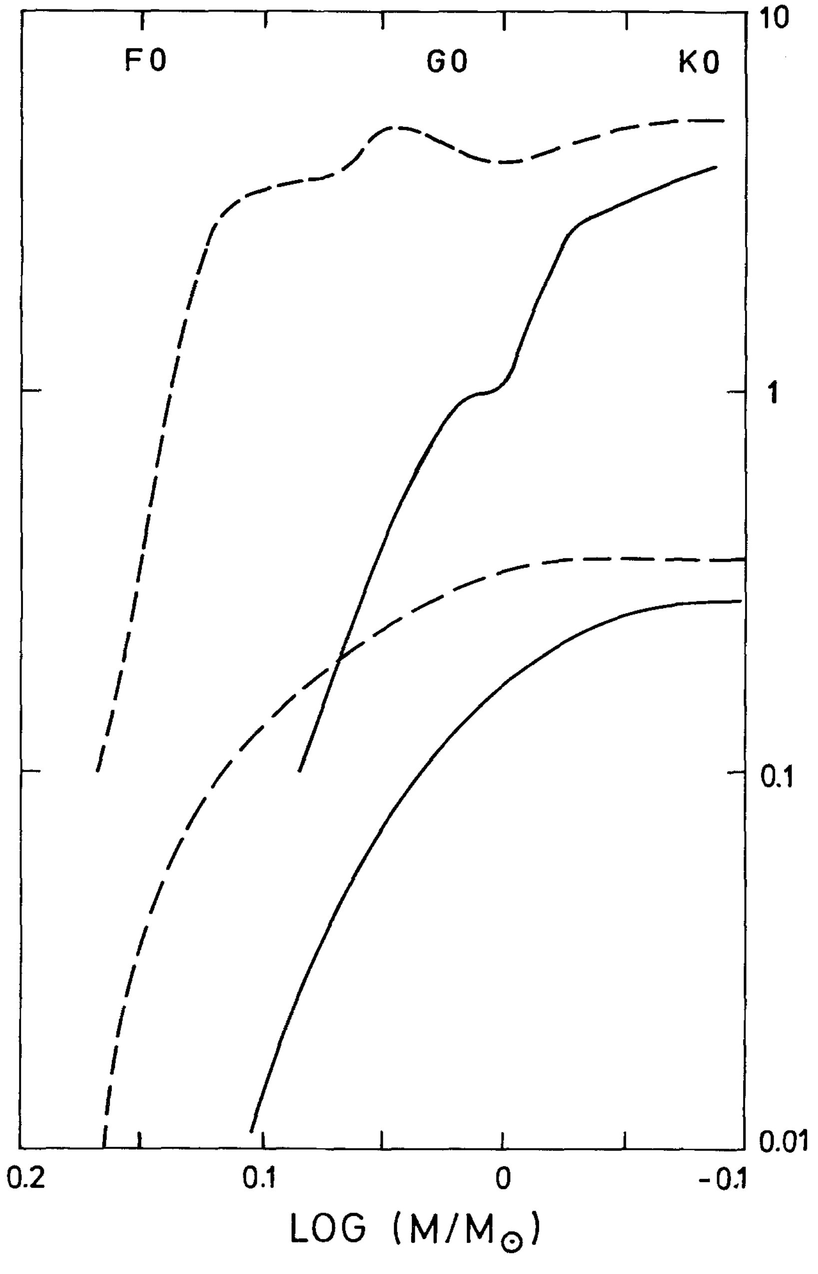

Fig. 7. Depth of the convection zone (lower curves) and the ratio $H\omega/U_0$ of turnover time to period of rotation (upper curves) for late main sequence stars, as a function of stellar mass, or spectral type. The mixing length to scale height ratio is 1 for the solid and 2 for the broken curves. From Gilman (1980).

compute models of differentially rotating solar and stellar convection zones. The unknown strength of rotational interaction with convection is obtained by calibrating the models for the solar case. The interaction coefficient is then assumed to be the same for all stars. Also, for stars later than the Sun, the angular velocity is essentially assumed to be constant. For these stars the differential rotation, shown in Figure 8, should therefore be considered as an upper limit. Using these results, and an α-effect as used in solar $\alpha\omega$-dynamos, Belvedere *et al.* (1980b, d) then computed models for stellar magnetic cycles. All these models generate oscillatory mean fields. Their period increases toward later stars, essentially due to the increase of the magnetic

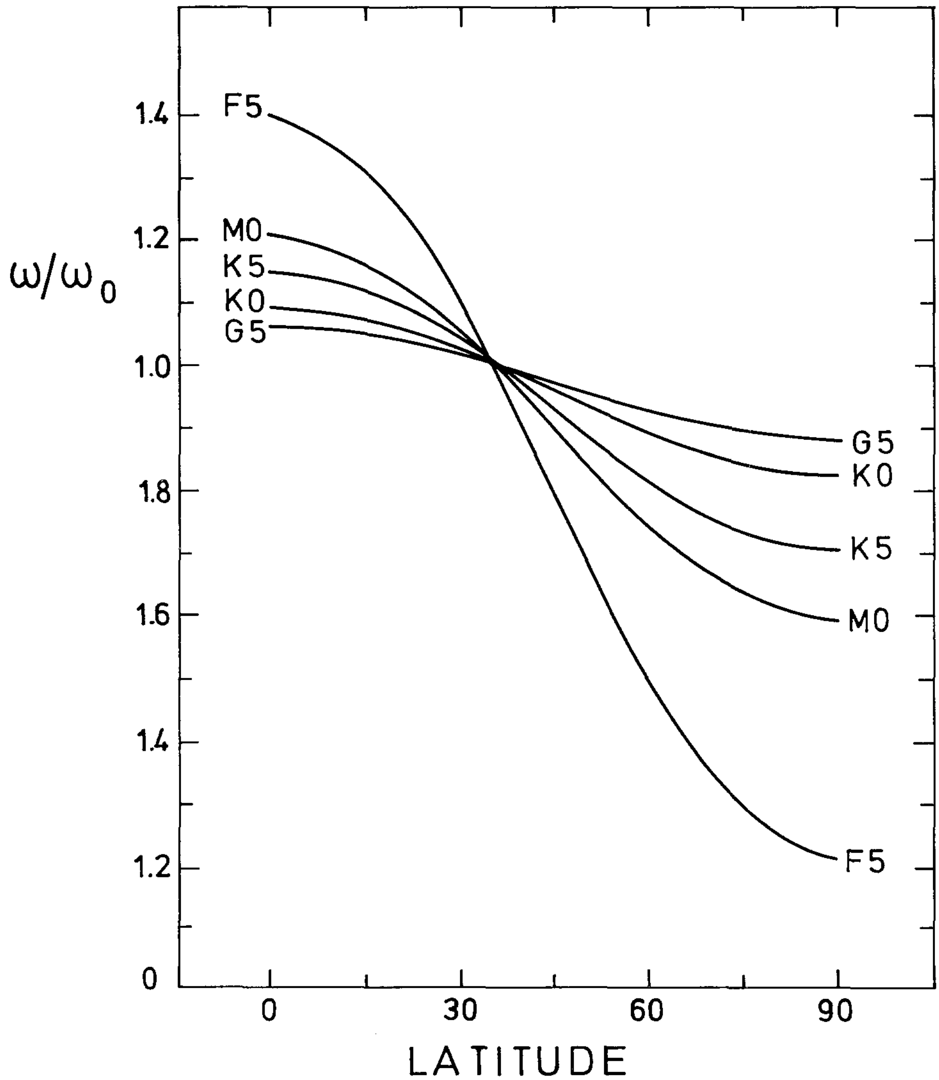

Fig. 8. Angular velocity distribution at the surface of 5 main sequence stars; ω_0 is the mean value. From Belvedere *et al.* (1980c).

diffusion time, d^2/η_t. We have suggested (Belvedere *et al.*, 1980d) that such a systematic period variation should be obtained in any stellar dynamo theory. It does, however, not transpire from the observations by Wilson (1978) of time-dependent Ca II-emission in stars; possible reasons are that the observations are too widely spaced in time (ca. 1 year) or of insufficient total duration (approx. 10 years). Theoretical oversimplifications such as neglect of non-linear effects may of course also be the reason for the disagreement. One particular influence which has not been discussed comes from stellar age. Younger stars have faster rotation and higher levels of Ca II-emission (Skumanich, 1972) than older stars of the same spectral type. It is thus conceivable that their dynamo has a shorter period, $|\alpha\nabla\omega|^{-1/2}$, due to stronger actions of differential rotation and α-effect (cf. Section 2).

The strength of stellar Ca II-emission (Wilson, 1978) and of stellar X-ray surface flux increases towards later main sequence stars (Vaiana, 1980). The reason could be that these stars generate cyclic fields with larger amplitudes. Thanks to their deep convection zone, the dynamo possibly is based on stronger α and $\nabla\omega$ effects. Belvedere *et al.* (1980e) have used this idea, and plausible relations between the equipartition field of the turbulent flow and the dynamo-generated field in order to estimate the surface X-ray flux of main sequence stars. Their result agrees, by order of magnitude, with the observations. Another important factor – probably the most important – determining the field strength is magnetic buoyancy. It removes the field from a *deep* dynamo zone more slowly than from a shallow one, and amplification to a higher level is possible (Parker, 1975).

6. Observational Problems

In the preceding sections I have already mentioned some of the observations which are relevant to the theory of the solar cycle. Here I list a few more, but the list is certainly not complete.

(a) Long time series of *magnetograms* of the entire Sun show how much, where, and on which scales, magnetic flux is generated by the solar dynamo. Yoshimura (1976a) has proposed a method how to extract information about the poloidal and toroidal mean field components from such magnetograms:

$$B_{\text{pol}} = \frac{1}{2\pi} \int_0^{2\pi} B(\vartheta, \phi)\, d\phi\,, \tag{6}$$

$$B_{\text{tor}} = \frac{1}{2\pi} \int_0^{2\pi} |B(\vartheta, \phi) - B_{\text{pol}}|\, d\phi\,, \tag{7}$$

where B is the observed field; the integrals are over the whole longitude of a synoptic solar chart, and B_{pol} and B_{tor} depend on latitude, ϑ. As pointed out by Golub *et al.* (1980), the spectrum of emerging flux covers the range from 10^{23} down to below

10^{19} Mx, and only the smaller fraction of the total flux seems to be associated with active regions. This appears to be in contrast to the results of Howard and Labonte (1980b) who find most of the flux in active regions. High resolution magnetograms, covering all latitudes, and frequent observations of ephemeral active regions are necessary to settle this question which, as illustrated by Schlüßler's dynamo (Section 4), is important for the theory.

(b) In addition to the umbral intensity variation (Albregtsen and Maltby, 1978) and a variation of the ratio penumbra radius/umbra radius (Jensen *et al.*, 1955; Antalovà, 1971), are there other *properties of single spots* which depend on the phase of the cycle? The answer to this question could help to clarify the role of concentrated flux tubes in the dynamo.

(c) In order to understand the Sun's magnetic field we must observe the *velocity field* which induces it. Is there non-axisymmetric global convection, causing differential rotation (Gilman, 1976, 1977, 1978) and α-effect (Yoshimura, 1972)? Even with very small amplitudes, of order 1 m s^{-1}, such velocity fields would be important because they would indicate larger velocities at greater depth, partly screened by turbulent diffusion in the upper layer (Stix, 1981). What are the upper limits of axisymmetric meridional circulation and of heat flux deviations from the spherical symmetric mean?

Does the angular velocity increase with depth, as indicated for the first 10^7 m by the rotational splitting of solar p-modes (Deubner *et al.*, 1979)? The extension in depth of such information requires high spectral resolution of the $k-\omega$-diagram at small wave number k and high frequency ω. Measurements should therefore cover periods substantially longer than a day. It has been emphasized in the recent report 'Study of the Solar Cycle from Space' (Newkirk, 1980) that observations from a station in a full sunlight orbit would be particularly suited. The oscillatory velocities of p-modes and non-oscillatory motions of large scale could be observed with the same equipment. And the spectral resolution necessary to identify and utilize the p-modes and their rotational splitting would not be distorted by the side lobes of a window transform (which occur at $\Delta\nu = 12$ μHz for a ground based observatory and at 180 μHz for a satellite in low latitude orbit). Uninterrupted observing runs of several days length may of course also be obtained from the South Pole, as the recent success of Grec *et al.* (1980) shows, but longer runs at any desired season can be measured from space.

I have already mentioned Howard and LaBonte's (1980a) discovery that there seem to be deviations from the mean differential rotation in form of a torsional wave. These observations must be continued, and should be accompanied by a search for axisymmetric meridional circulation.

(d) Does the Sun's luminosity, L, vary with the solar cycle, perhaps because the presence of magnetic flux tubes influences the efficiency of convective energy transport? The convection zone would thus become a temporal heat reservoir (Yoshimura, 1978a). Simulating such an effect through a variation of the ratio l/H of mixing length to scale height, Dearborn and Newman (1978) found $\delta L/L \approx$

$0.2\delta(l/H)$. The response is fast, and essentially is caused by the change of l/H in the outer, superadiabatic region of the convection zone (Dearborn and Blake, 1980). According to Sofia *et al.* (1979) the luminosity change should be accompanied by a *radius* change, with $\delta_{r\odot}/r_{\odot} \approx 0.075\delta L/L$. Existing radius determinations would thus limit the possible variation of the Sun's luminosity to less than, say, 0.3%; much lower limits could be obtained by use of the SCLERA instrument (Hill and Stebbins, 1975) for the radius measurements.

Thomas (1979) has pointed out that the Sun's radius may change during the cycle even at constant luminosity if the *temperature* changes. The apparent decrease of surface temperature during the ascent of the present activity cycle (Livingstone, 1978) is however obtained only from certain lines; other lines suggest a variation (in time) of the temperature variation with depth (Livingston and Holweger, 1981). Moreover, evaluation of photoelectric radius measurements from Mt. Wilson observatory indicates (R. Howard, 1980, private communication) that the solar radius oscillates with an amplitude of (0.07 ± 0.03) arc sec, in *antiphase* with the solar activity, in contrast to Thomas' suggestion that magnetic flux tubes would add to the net pressure and so cause the Sun's outer layers to expand. Clearly, simultaneous observations of the solar luminosity, its radius, and its mean surface temperature over at least one cycle are necessary. Made from space, the accuracy of such measurements could only win; in case of the luminosity (or solar 'constant') it is a necessity to go to space.

(e) Observation of *stellar activity cycles* could substantially improve the theory of the solar cycle. Imagine you undertake a theory of stellar evolution without an observed HR-diagram! We now have strong observational indications (Wilson, 1978) that stellar cycles do exist. Moreover, the Ca II and X-ray emissions indicate that the amplitudes of these cycles increase with rotation and toward later spectral types. As stated above in Section 5, this indicates that the stellar dynamos operate in the deep parts of the convection zones. Most urgently needed are however reliable observational values of the *periods* of stellar cycles. Periods can be theoretically determined from *linear* theory, and a comparison would show whether such theory is at all applicable. The stars observed by Wilson (1978) should therefore be monitored for at least another 10 years. Also, we should observe the X-ray emission for a number of stars repeatedly over many years in order to detect cyclic behaviour. In addition, other manifestations of activity of stars, like stellar flares, could be used to establish stellar cycles and to determine their periods.

Unfortunately for almost all programmes discussed in this section success will come only after many years of observation. But, as the examples of Wilson (1978) and Howard and LaBonte (1980a, b) show, the result will be worth the effort.

Acknowledgements

I thank the participants of the workshop 'Theory of Planetary and Stellar Magnetism' in Budapest (1980) for many stimulating discussions. For suggestions and for sending unpublished material I am, in addition, indepted to Drs B. R. Durney, E. Fossat, P. A. Gilman, R. Howard, G. Newkirk, Jr., L. Paternò, M. Schüßler, H. C. Spruit, S. I. Vainshtein, and H. Yoshimura.

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* Articles marked with an asterisk have been translated into English (Roberts and Stix, 1971).