THE ORIENTATION OF THE SOLAR ROTATION AXIS FROM DOPPLER VELOCITY OBSERVATIONS

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Abstract. Mt. Wilson observations of solar velocity fields have been examined for evidence that the rotation axis of the nonmagnetic gas at the solar surface is oriented differently than the axis found by Carrington (1863) from sunspot observations. No difference is found with an accuracy of $0\overset{\circ}{.}15$ in the angle of inclination of the axis to the ecliptic.

1. Introduction

Many observations show differences between the rotation rates of the magnetized and unmagnetized gas on the surface of the Sun (Howard, 1978). This apparent absence of perfect coupling between the two systems raises the question of whether they should be considered independent and perhaps even have different rotation axes.

The rotation axis of the magnetic gas has been determined from the proper motions of sunspots by numerous observers (Table I of Wöhl, 1978; Clark *et al.*, 1979). All have found essential agreement with Carrington (1863), whose values we adopt as standards. The direction of the rotation axis is specified by i, the inclination of the Sun's equator to the ecliptic plane, and Ω, the longitude of the ascending node of the solar equator on the ecliptic. The observable effects of i are the tipping of the solar axis toward and away from the Earth-Sun line (B_0), and from side to side perpendicular to that line (P_s). B_0 and P_s are oscillatory with 1 year period and 90° out of phase. (The usual definition of the angle P includes the inclination of the Earth's equator to the ecliptic, an effect ignored here as it is known and removed with much higher precision than i.)

We measure the rotation axis of the nonmagnetic gas from the daily Mt. Wilson full disk observations of solar velocities. Each observation consists of 10 000 to 25 000 measures of the line of sight velocity, covering the disk with 17 to 12.5 arc sec resolution. From the interval January 1, 1971 to December 31, 1978, 1540 separate observations were available for this study.

We could solve for the angles i and Ω directly by fitting the original data with the velocity patterns which those angles produce. Such a massive rereduction would require several years of effort. We therefore choose an alternative but mathematically equivalent approach.

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Because the data clearly show that the rotation axis of the gas is very close to that of the sunspots, the observations have already been reduced using Carrington's elements, to derive the solar rotation, limbshift, and other large scale velocity fields (Howard *et al.*, 1980) and to form maps of the residual solar velocities. In this paper, we will try to measure any difference vector between the true rotation axis of the gas and that defined by Carrington, by examining the residual velocity patterns in space and time that would appear if the elements used in reducing the observations were incorrect. Since such a difference vector must be small, it is very nearly orthogonal to Carrington's axis. Therefore there is no problem of mathematical crosscorrelation, by which our use of a particular rotation axis direction for the data reduction would prevent the measurement of the error of that direction. The assumed axis and error in the assumed axis are orthogonal and independent, and we can measure the error vector with the same accuracy to which we could measure i and Ω directly.

The velocity data covers the full disk and thus includes the magnetic areas on the solar surface as well as the nonmagnetic areas. Perhaps this biases the results and invalidates the assumption that the velocity data refers essentially to the non-magnetic gas. In addition, the Mt. Wilson observations are made in the 5250 Å line of Fe I, which is sensitive to magnetic and thermal fields. This sensitivity may in some way distort the observed velocity in the magnetic areas and alter the large-scale velocity distribution. Neither of these factors is a problem for this study.

Magnetic fields cover only a small fraction of the surface. Direct measurement shows that even at the time of sunspot maximum the presence of magnetic fields affects large-scale velocity fields only at very low levels (Howard *et al.*, 1980). For example, the equatorial rotation rate measured on a single day changed by no more than 0.5% (10 m s^{-1}). This small effect is further decreased by the fact that the apparent solar axis direction varies with a 1 year period. The distribution of magnetic flux on the solar surface does not have systematic variation on this timescale. Characteristic timescales for the flux distribution are either much shorter (e.g. magnetic region lifetime, $\simeq$2 days to 2 months; solar rotation, $\simeq$27 days) or much longer (11 year sunspot cycle, 22 year polarity reversal) than 1 year. Thus the presence of magnetic fields on the surface does not systematically affect our results.

The Mt. Wilson observations show that large-scale velocity patterns such as solar rotation vary in amplitude from day to day (e.g. Howard and Harvey, 1970). It has been suggested by Scherrer *et al.* (1980) that these variations may be instrumental. Other Doppler machines show similar variations (Beckers, 1978) which have the character of an incorrect calibration factor, i.e. all velocities are uncertain by a multiplicative factor. This means that absolute measurements are uncertain, but relative measurements of the amplitudes of two different velocity patterns are correct since their ratio does not depend on the calibration. For this study, three of the four tests for the solar axis direction, including the most sensitive test, are relative measurements. They are not affected by the principal instrumental uncertainty.

2. Analysis

The component of the line of sight velocity due to solar rotation at a true heliographic longitude, latitude (L, B) is (Howard and Harvey, 1970)

$$V = \omega(B) \sin L \cos B \cos B_0 R_\odot. \tag{1}$$

The angular velocity profile is approximated by

$$\omega(B) = a + b \sin^2 B, \tag{2}$$

as higher order terms are not important to this analysis. The position (L, B) is inferred from an observed apparent position on the disk, through knowledge of the angles B_0, P_s. If incorrect values

$$\beta_0 = B_0 + E_B, \tag{3a}$$

$$\mathcal{P}_s = P_s + E_p \tag{3b}$$

are used, an erroneous position (λ, β) will be attributed to the true location (L, B). The true velocity pattern given by Equation (1) will then appear in a distorted form in the (λ, β) coordinates. The error angles E_B and E_P are orthogonal, and we will treat them separately to derive the expected velocity residuals.

2.1. B_0 ERROR

The improper value β_0 (Equation (3a)) is substituted in Equation (1) and into the formulas for computing (λ, β) (Howard and Harvey, 1970) to obtain

$$V = \omega(\beta) \sin \lambda \cos \beta \cos \beta_0 R_\odot + \tag{4a}$$

$$+ E_B \tan \beta_0 \omega(\beta) \sin \lambda \cos \beta_0 R_\odot - \tag{4b}$$

$$- 2 E_B b \sin \beta \cos^2 \beta \sin \lambda \cos \lambda \cos \beta_0 R_\odot. \tag{4c}$$

This form uses $E_B \ll 1$ to allow $\cos E_B = 1$, $\sin E_B = E_B$, and terms $0(E_B^2) = 0$.

The terms in Equation (4) are:

(a) The solar rotation measured in the (λ, β) system. The correct value of ω is found.

(b) A time variable increment to the measured rotation. Both E_B and $\tan \beta_0$ vary sinusoidally with 1 year periods; their product varies with 0.5 year period. We refer to this velocity pattern as the first B_0 error.

(c) A residual velocity pattern in which adjacent quadrants of the disk have average velocities of opposite sign. The pattern amplitude varies with time as E_B. This velocity pattern is the second B_0 error.

2.2. P_s ERROR

Substitution of the improper angle $\mathscr{P}_s$ (Equation (3b)) yields the P angle velocity pattern,

$$V = \omega(\beta) \sin\lambda \cos\beta \cos\beta_0 R_\odot - \quad (5a)$$

$$-E_p \omega(\beta) \sin\beta R_\odot + \quad (5b)$$

$$+2E_p b \sin^2\lambda \sin\beta \cos^2\beta \cos^2\beta_0 R_\odot . \quad (5c)$$

Again we have used $E_p \ll 1$ to simplify the algebra. It is more convenient to regroup the second and third terms and analyze the data as

$$V = \omega(\beta) \sin\lambda \cos\beta \cos\beta_0 R_\odot - \quad (6a)$$

$$-E_p a \sin\beta R_\odot + \quad (6b)$$

$$+E_p b \,(2 \sin^2\lambda \sin\beta \cos^2\beta \cos^2\beta_0 - \sin^3\beta)\, R_\odot . \quad (6c)$$

These terms are:

(a) The solar rotation measured in the (λ, β) coordinate system. Again, the correct value of ω is determined.

(b) A north-south velocity gradient, representing an apparent rotation of the disk about an east-west line. This velocity pattern is the first P_s error and varies in time as E_p.

(c) A scalloped pattern of residual velocities; in moving around the disk at a constant central distance, the velocity sign alternates in several sectors. The pattern amplitude and sign vary as E_p. This pattern is the second P_s error. In summary, there are two independent velocity patterns produced by each error angle (E_B and E_p). A total of four different tests can thus be made by searching for the presence of velocity patterns on the solar disk with the spatial and temporal variations specified by Equations (4b, c) and (6b, c).

The expected velocity amplitudes of the error patterns are small. Table I lists the peak observable line of sight velocities for 1°0 errors ($|E_B| = |E_p| = 1.7 \times 10^{-2}$ rad), derived by maximizing Equations (4b, c) and (6b, c) and substituting the values $|B_0| = i = 7°25$ and the measured values $a = 2.813\mu$ rad s^{-1} and $b = -0.335\mu$ rad s^{-1}. Clearly the most sensitive test, in terms of observable velocity per unit error, is the first P_s pattern.

TABLE I

Peak observable velocity for 1°0 error in solar axis direction

Error pattern	Equation	Peak velocity (m s^{-1})
1st B_0	(4b)	2.2
2nd B_0	(4c)	1.4
1st P_s	(6b)	34.0
2nd P_s	(6c)	4.1

Another consideration is that the second B_0 and both P_s error patterns (Equations (4c), (6b, c)) are orthogonal to the rotation velocity pattern (4a), (6a). This means these three patterns may be measured relative to the rotation. The daily measured values of a, b can be used to solve for E_B, E_p. This nets out any instrumental miscalibration. The first B_0 error pattern is, on the contrary, parallel to the rotation. It's amplitude must be measured absolutely, and any variations in ω on the Sun or caused by the instrument will increase the noise level of the measurement.

3. Results

The velocity patterns of the first B_0 and first P_s errors are measured in the standard data reduction (Howard *et al.*, 1980). To determine the second B_0 and P_s pattern amplitudes, a rereduction of the dataset was made, using the coarse (34 equal intervals in sine latitude and sine central longitude) velocity arrays generated as a part of the standard reduction. The time series of the measured daily pattern velocity amplitudes were Fourier analyzed to determine the pattern amplitude at the proper period.

As test of our ability to detect a weak signal in the presence of the observed noise, a sine wave of amplitude 17 m s^{-1} (corresponding to $E_p = 0\overset{\circ}{.}5$) was added to observed time series of the first P_s error. Our analysis retrieved this signal within 1% in amplitude and 3% in phase.

When the analysis is performed for each of the four expected terms, pure noise spectra are obtained, with no significant peaks corresponding to measurable error angles. To set an upper limit on the observed velocity, we choose the amplitude of the spectrum peak nearest to the correct period. These velocities are listed in Table II, along with the corresponding limits implied (from Table I) for the angular errors E_B, E_p.

TABLE II

Measured upper limits of the error in solar axis direction

Error pattern	Velocity limit (m s^{-1})	Angular limit
1st B_0	18	$\lvert E_B \rvert \le 8\overset{\circ}{.}1$
2nd B_0	7	$\lvert E_B \rvert \le 5\overset{\circ}{.}2$
1st P_s	5.2	$\lvert E_p \rvert \le 0\overset{\circ}{.}15$
2nd P_s	12	$\lvert E_p \rvert \le 3\overset{\circ}{.}0$

To interpret these values in terms of deviations E_i and E_Ω from the Carrington values, one can show that $|E_B|^2 + |E_p|^2 = |E_i|^2 \cos^2 i + |E_\Omega|^2 \sin^2 i$, in the case that sin $i = i$. The observed limits thus essentially apply directly to i, but the sensitivity to errors in Ω is reduced by tan i, in this limit. We conclude that the elements of the rotation axis of the nonmagnetic gas at the solar surface are

$$i = 7.25 \pm 0\overset{\circ}{.}15\,,$$

$$\Omega = 73.7 \pm 1\overset{\circ}{.}2 + 50\overset{\prime\prime}{.}25t\,,$$

where t is the time in years since 1850. These are just the Carrington values, with our best limit as errors.

4. Discussion

That the rotation axis is the same for both the magnetic and nonmagnetic gas suggests that any surface differences in their other rotation properties may not be evidence of deepseated differences between the two systems.

Our result conflicts with another determination of the rotation axis using Doppler velocity observations (Wöhl, 1978). He finds the inclination i to be $0\overset{\circ}{.}5$ different from Carrington's value. We believe our finding is correct for the following reasons:

(1) We have many more observations. This greatly reduces the effects of solar velocity noise (supergranulation, 5 min oscillations) which is a major contributor to the total noise amplitude.

(2) Our observations cover a larger number of cycles of the expected error signal. This reduces the likelihood that short term changes in the instrument or observing conditions will affect the measurement.

(3) Our test of the sensitivity of the analysis shows that an error signal corresponding to $0\overset{\circ}{.}5$ would be easily detected. This suggests that any difference from the Carrington elements cannot be this large.

The probable resolution of this discrepancy lies in a careful consideration of the actual errors of measurement. An alternative explanation is that shortlived solar velocities may imitate the B_0 and P_s error patterns. Such flows would not be caused by changes in the rotation axis, but might incorrectly be so interpreted. This is why we have derived a single solution from our entire dataset and not done separate reductions for subsets in space and time. Any shortlived velocities should be studied as such and not interpreted as rotation axis variations.

We note that there is no evidence for long-term changes in the rotation axis; all sunspot measurements in the last 300 years have agreed with Carrington's results within the observational error (Table 1 of Wöhl, 1978; Clark *et al.*, 1979).

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