CURRENTS IN THE SOLAR ATMOSPHERE AND A THEORY OF SOLAR FLARES

H. ALFVÉN and P. CARLQVIST
Royal Institute of Technology, Stockholm 70, Sweden

(Received 19 October, 1966)

1. Definition of the Electro-Magnetic State

The electro-magnetic state in a certain volume is defined if we know the electric field E and the magnetic field H as functions of space and time. However, because of the first Maxwell equation

$$\operatorname{curl} H = \frac{1}{c}\left[4\pi i + \frac{\partial D}{\partial t}\right] \tag{1}$$

the magnetic field variations are defined by the electric current (comprising the current density i and the displacement current $\partial D/\partial t$). Hence a description in terms of currents is possible, and often physically more interesting than a description in terms of magnetic fields. The displacement current is of importance only for frequencies of the order of the plasma frequency or higher. As in this paper we describe stationary or slowly varying phenomena, we shall neglect it.

2. Measurements of Magnetic Fields and Currents

In astrophysics no direct measurements of electric fields have yet been made. All conclusions about electric fields have been reached in an indirect way. The magnetic fields are much better known, because it is possible to measure them by means of the Zeeman effect or the Faraday rotation. However, up to quite recently it was essentially the longitudinal effects which were measured. Of the six electro-magnetic vector components only one, viz. the magnetic component along the line of sight, was measured, so no real understanding of the electro-magnetic state has been possible.

Important progress in the knowledge of solar electro-magnetic conditions was recently made by Severny (1964, 1965) with systematic measurements of the transverse Zeeman effect. In active regions, including sunspots, he has sometimes found a remarkable rotation of the transverse component $H_\perp$ and a change of its magnitude from one point to another.

If the z-axis of an orthogonal coordinate system points along the line of sight we can calculate the current component i_z from

$$i_z = \frac{c}{4\pi}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \tag{2}$$

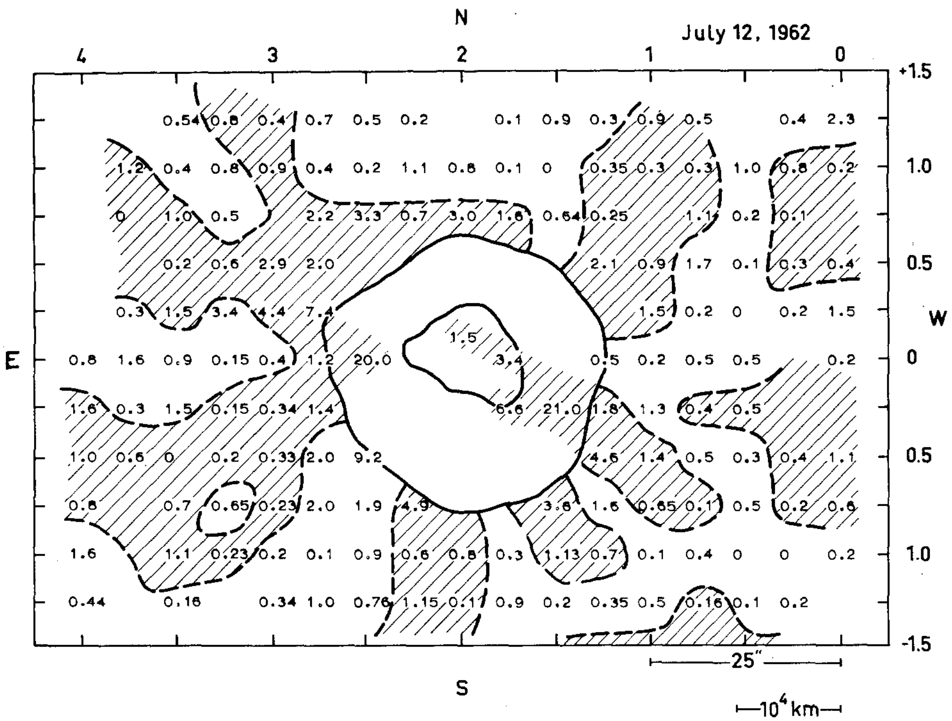

Fig. 1. Vertical electric currents in the neighbourhood of a sunspot. White areas and shaded areas represent regions with currents of opposite directions. The numbers give i_z in units of $10^4/4\pi$ amp/km^2.

Hence, if the space variation of H_x and H_y, measured by the transverse Zeeman effect, is calculated, we can find the electric current along the line of sight. Combined measurements of the longitudinal and the transverse Zeeman effect yield the current system in three dimensions.

By using methods of this kind Severny has mapped vertical currents near the centre of the solar disk. Especially in the neighbourhood of sunspots he has found small regions ($\lesssim 10^9$ cm in diameter) in which vertical currents of the order of 10^{11} amps flow, sometimes upwards, sometimes downwards. Figure 1 shows such a map, constructed by Severny, where many regions of opposite current directions are located around a sunspot. The value $(4\pi/c)i_z$ is given in units of 10^{-2} gauss/km.

3. Description of the Electro-Magnetic State by the Current System

As the advance in observational technique has given us a possibility to describe the electro-magnetic state by currents instead of magnetic fields it would be of interest to give a survey of the advantage of the current description in different respects. However, in this paper we shall confine ourselves to one important problem, viz. the theory of the solar flares and further develop ideas which earlier have been discussed by

JACOBSEN and CARLQVIST (1964). As we shall see, the current system picture may help us to clarify the basic mechanism of a flare.

4. Circuit Interruption

An inductive circuit in which a current exists has a general tendency to "explode". This means that if we try to interrupt the current at a certain point, the whole magnetic energy of the circuit tends to be dissipated at that point. A few examples will illustrate this.

In a high-power transmission network the switches must be constructed in such a way that they can take up most of the magnetic energy of the whole network, because this is released in them when they break the current.

The modern technique of producing very strong magnetic fields by superconducting coils has to meet the difficulty that, if at one point the conductor ceases to be superconducting, the whole magnetic energy of the circuit is released at that point with the result that the coil may explode.

The most interesting case from our point of view is when a plasma is introduced into a high power transmission system. This is the case, for example, when a mercury rectifier is used for the transformation of alternating current to direct current. If then the current through the mercury plasma exceeds a certain limit which is essentially given by the density of the plasma and the geometry of the rectifier, a sort of explosion occurs. Under certain conditions most of the magnetic energy of the circuit is suddenly released in a high impedance region.

5. Maximum Current Through a Plasma

The current limitation in low pressure discharges has been studied by a number of investigators (LANGMUIR and MOTT-SMITH Jr, 1924; TONKS, 1937; HULL and ELDER, 1942). It has been investigated in simple laboratory experiments of the type shown in Figure 2. An arc discharge in mercury vapour between the mercury cathode C and the anode A is produced by a voltage source V. An inductance L is inserted in series with the discharge and the current can be regulated by a resistor R. A spark gap G is connected in parallel with the discharge. The pressure of the mercury vapour is regulated by the temperature of the mercury pool serving as the cathode. The voltage drop over the discharge is V_D. Normal values are:

length of discharge tube 100 cm	$I = 10$ amps
diameter of tube 3 cm	$L = 0.1\ H$
$p = 10^{-3}$ torr	$R = 16\ \Omega$
$V = 200$ volts	$V_D = 40$ volts

If we increase the current by diminishing R or increasing V, a discharge over the spark gap suddenly occurs. Usually the discharge in the tube goes out at the same time and must be re-ignited. The spark occurs even if the distance between the gap

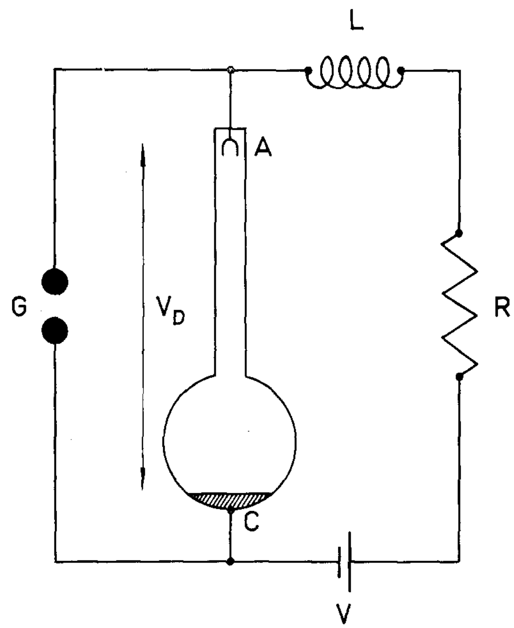

Fig. 2. Laboratory experiment on current limitation. A is the anode and C is the mercury cathode of a tube in which a mercury vapour discharge is produced by a voltage source V in series with an inductance L and a resistor R. A spark gap G is inserted in parallel with the tube.

electrodes is so large that a voltage of some ten or even some hundred kilovolts is needed for a flash-over. This shows that the mercury plasma in the tube has suddenly become essentially non-conducting and can support a voltage drop of 10^5 volts or more. Because of the inductance the current cannot stop immediately, and the voltage drop over the rectifier increases until the spark gap ignites. Part of the magnetic energy of the inductance is consumed by the spark.

If the spark gap distance is increased very much and the tube is insulated so well that an external spark-over cannot take place, the whole energy of the inductance is consumed inside the tube. The voltage drop over the tube may then be 10^5 volts or more.

The disruption of the current can be produced either by increasing the current (diminishing R or increasing V) or by decreasing the vapour pressure by decreasing the temperature of the mercury pool. The sudden change of the low impedance plasma to a very high impedance medium occurs when the neutral gas pressure becomes lower than a certain limit given by POLETAEV (1951). Below this limit the production of ions is smaller than the losses of ions to the walls. The discharge cannot be upheld any more.

The spark gap may be ignited also if it is connected to two probes which are inserted into the plasma at different distances from the anode. This shows that the disruption is not a cathode (or anode) phenomenon but occurs in the plasma itself. The transition of the low impedance plasma to a high impedance region occurs very rapidly, often in a time τ of the order of a few microseconds. Since $V = L dI/dt \approx LI/\tau$, we find with $V = 10^5$ volts, $I = 10$ amps, $L = 0.1$ H, a time constant $\tau = 10^{-5}$ sec.

6. The Disruption of the Current

The experiments show that in a plasma there is a maximum current which cannot be exceeded. If the maximum current is reached, the discharge suddenly increases its impedance by many orders of magnitude. The current through the plasma is mainly carried by electrons, and their space charge is normally neutralized by positive ions. At the disruption this space charge neutralization is destroyed and the electrons move under conditions which more closely resemble a high vacuum diode. According to LANGMUIR (1913) the electron current density in a high vacuum diode is given by

$$i = \left[2e/m_e\right]^{1/2} \cdot \frac{V^{3/2}}{9\pi d^2} \qquad \text{(esu)} \quad (3)$$

where $-e$ is the electron charge, m_e the electron mass, V the potential drop over the diode, and d the distance between the anode and the cathode.

Hence, if in the experiment we have $V = 10^5$ volts $i = 3$ amps/cm^2 we find that high vacuum conditions should prevail in a high impedance region with the thickness $d \approx 5$ cm. In reality this region must be somewhat thicker, because a partial neutralization by positive ions is always present.

7. Instability of a Current in a Completely Ionized Gas

We shall now treat a simple model of a completely ionized current-carrying plasma and discuss a mechanism by which it becomes unstable so that the current is disrupted. It is possible that this disruption mechanism is active in the mercury vapour discharge we have described. However, this problem needs further clarification.

In order to study the instability we shall treat a simplified stationary model of a fully ionized plasma, in which both the electron temperature and the ion temperature is zero. The electron and ion density $n_e = n_i = n$ of the plasma is uniform and the current $i = -env_e$ is carried by electrons moving along the s-axis with constant velocity v_e in relation to the ions, which are assumed to be at rest.

Suppose that in this plasma there is a region where the density n becomes somewhat smaller than the normal value (see Figure 3). Because of the definition of a plasma both n_i and n_e must be smaller. As the current i_e must be the same everywhere, the electrons move with a slightly higher velocity

$$v_e = -\frac{i}{en} \qquad (4)$$

through this region. In a stationary model this can occur if they are accelerated by an electric field

$$E_s = -\frac{m_e}{e}\frac{dv_e}{dt} \approx -\frac{m_e i v_e}{e^2 n^2}\frac{\partial n}{\partial s} \qquad (5)$$

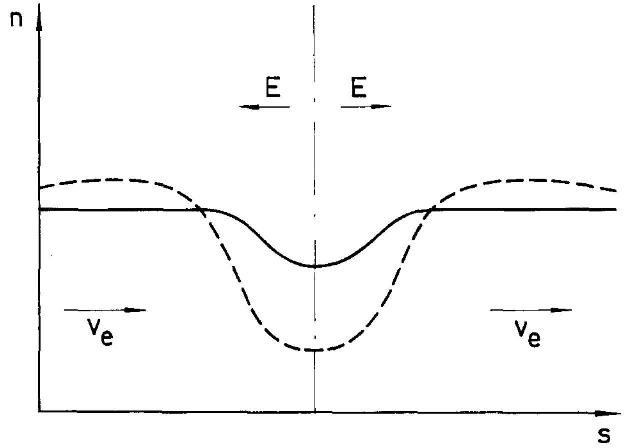

Fig. 3. Particle density $n = n_e = n_i$ given as a function of the distance s. The solid line shows the initial state indicating a small density dip, and the dashed curve shows the configuration at a later time. The electrons move to the right with the velocity v_e. Acceleration and retardation of the electrons are produced by an electric field E.

when entering the region of lower density, and decelerated by a field of opposite direction when leaving the region. However, such an electric field will also act on the ions, which so far have been supposed to be at rest. It will accelerate the ions away from the region of low density, and hence lower the density still more.

This shows that a stationary state is unstable. The simplest time development of a model would be a continuous decrease in density in the region of initially low density leading to a high vacuum region. In reality the process is probably much more complicated.

The instability does not occur if the temperature is sufficiently high. In fact, if the ion temperature is T_i, the ions diffuse with a velocity equal to the drift velocity in an electric field (see ALFVÉN and FÄLTHAMMAR, 1963, p. 158).

$$E_D = -\frac{kT_i}{ne}\frac{\partial n}{\partial s} \tag{6}$$

Comparing this with (5), we find the condition for instability:

$$|i| > en\sqrt{\frac{kT_i}{m_e}}; \tag{7}$$

or

$$v_e^2 > \frac{kT_i}{m_e}. \tag{8}$$

Hence, when the systematic velocity exceeds the thermal velocity the plasma becomes unstable.

A detailed calculation of the conditions necessary for a stationary state has been made by CARLQVIST (1967). Instead of the simple inequality (8) he finds

$$\frac{m_i v_i^2}{2} + \frac{m_e v_e^2}{2} > kT \tag{9}$$

where the temperature T is supposed to be the same for electrons and ions.

8. Applications to Solar Conditions

As mentioned in section 2, there is observational evidence for the existence of large vertical currents in the solar atmosphere. As all currents they must form closed loops. In a typical example they may follow a field line which twice intersects the photosphere. The circuit is closed by currents in the deep layers of the sun (see Figure 4).

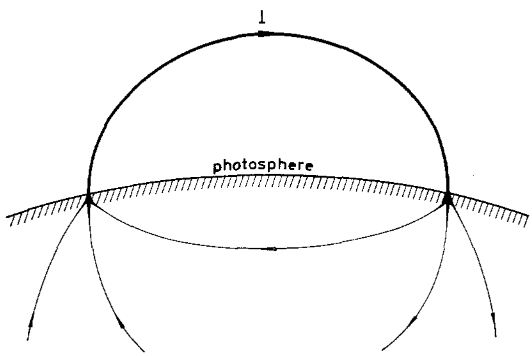

Fig. 4. General pattern of electric currents in the solar atmosphere. The current exists in narrow channels passing through the solar atmosphere and is closed in the photosphere or in deeper layers.

In the layers below the photosphere the density is so high that the current behaves in the same way as in an ordinary conductor. In the high layers of the solar atmosphere the current exists in a plasma of rather low density, and we cannot exclude the possibility of a disruption if the maximum plasma current is exceeded. By analogy with ordinary circuits such a disruption would lead to a concentrated and sudden release of most of the magnetic energy of the circuit.

The disruption we have discussed earlier is fundamentally independent of the magnetic field. In principle we should not expect that the disruption should be influenced by a longitudinal magnetic field. However, the walls of a discharge tube, which limit the discharge sideways, have no direct correspondence in the solar atmosphere. Their place may be taken by the magnetic field tubes which limit the discharge channel sideways.

Moreover, magnetic effects are likely to be important for the constriction of the discharge in the solar atmosphere. The Zeeman effect measurements indicate currents of the order of 10^{11} amps over an area with a diameter of 10^9 cm or sometimes less. However, these values obviously give only a lower limit to the current density. It is well known that the solar atmosphere has a complicated fine structure, which may lead to much higher current densities in some regions. In fact, the theory of line currents in the solar atmosphere shows that we can expect phenomena similar to the pinch effect, by which the current is constricted to very narrow channels. In fact, with $n=10^8$ cm^{-3} and $T=10^6$ °K, a local constriction to a radius of the order of 10^6 cm is necessary to reach the disruption current given by Equation (7).

9. Qualitative Picture of Solar Flare

What has been said leads to the following qualitative picture of a solar flare:

Hydromagnetic phenomena produce currents which exist in the solar atmosphere. They are especially strong in active regions. At least in the higher layers these currents follow the magnetic field lines. Local constrictions due to the pinch effect and similar phenomena increase the current density in some small regions to much higher values than the average over large regions. If at a certain point the current density in a flux tube exceeds the maximum value which the plasma can carry in a stable way, a local decrease in plasma density is produced. The current which earlier could exist as a plasma current, driven by a very low electric field, has now to pass a high vacuum region, where a very high voltage is required because the region resembles a high vacuum diode. A short-circuit by the adjacent plasma is inefficient because the explosion spreads to the neighbourhood. Hence a major part of the magnetic energy of the whole circuit should be released near the point where the instability started. An essential part of the energy is transformed into kinetic energy of electrons and ions, accelerated by the excessive voltage of the high impedance region.

10. Quantitative Considerations

The inductance L of a circular current loop with the radius R consisting of a wire with the radius r is

$$L = 4\pi R\left[ln\,\frac{8R}{r} - \frac{7}{4}\right] \tag{10}$$

when the current is homogeneously distributed within the wire. If we approximate the circuit of Figure 4 to a circular loop, the value of R/r may be between 10 and 10^3, which gives $L \approx 50R$. The real inductance is probably not so high because the magnetic field from the current is screened by the conducting plasma in the surroundings. In any case loops with a radius of 10^9–10^{10} cm could often have inductances of $L=10^{10}$ cm $=10\,H$. If the time needed to produce a solar flare is of the order of 10^3 sec, we have:

current $I=10^{11}$ amps
inductance $L=10\ H$
time constant $\tau=10^3$ sec
voltage drop over the interruption $V=L\,dI/dt \approx LI/\tau=10^9$ volts
magnetic energy of the circuit $W=\frac{1}{2}LI^2=0.5\cdot10^{23}$ joules$=0.5\cdot10^{30}$ ergs

The above values seem to be acceptable from an observational point of view (see SMITH and SMITH, 1963). In individual flares the values may vary by one order of magnitude or more.

Of special interest is that the voltage drop in the flare region should be of the order of 10^9 volts. Solar flares often produce cosmic rays of corresponding energies. According to our model the acceleration should be produced in a high impedance region in the solar atmosphere with a voltage difference of about 10^9 volts. In this region ions and electrons are accelerated to the same energy.

There are many astrophysicists, who are accustomed to consider a plasma as a very good conductor (some always put $\sigma = \infty$). They will probably object to the idea that a very large voltage difference should be established in the solar atmosphere. A careful study of what happens at a current disruption in a mercury discharge will make such objections less convincing. For a quarter of a century the electrical engineers have been familiar with high voltage surges in plasmas, and the time seems now ripe for astrophysical applications of this phenomenon.

References

ALFVÉN, H. and FÄLTHAMMAR, C-G: 1963, *Cosmical Electrodynamics*. 2nd ed., Clarendon Press, Oxford.
CARLQVIST, P.: 1967 (in the press).
HULL, A. W. and ELDER, F. R.: 1942, *J. Appl. Phys.* **13**, 372.
JACOBSEN, C. and CARLQVIST, P.: 1964, *Icarus* **3**, 270.
LANGMUIR, I.: 1913, *Phys. Rev.*, ser. 2, **2**, 450.
LANGMUIR, I. and MOTT-SMITH, H. Jr.: 1924, *Gen. Electric Rev.* **27**, 762.
POLETAEV, I. A.: 1951, *Zhurnal Tekhnicheskoi Fiziki* **21**, 1021.
SEVERNY, A.: 1964, *Space Sci. Rev.* **3**, 451.
SEVERNY, A.: 1965, *Astron. Zh.* **42**, 217.
SMITH, H. J. and SMITH, E. V. P.: 1963, *Solar Flares*. The Macmillan Company, New York.
TONKS, L.: 1937, *Trans. Electrochemical Soc.* **72**, 167.