ON THE IONISATION OF HYDROGEN IN OPTICAL FLARES

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Abstract. Non-steady state and non-LTE effects on the ionisation equilibrium of hydrogen in optical flares are considered in terms of a two-level hydrogen atom. It is shown that, just as in the quiet low chromosphere, the ionisation equation is controlled by spontaneous recombination to the second level and by photoionisation from this level by photospheric radiation, and is independent of the nature of the flare energy input mechanism.

Adjustment of the ionisation then occurs on the recombination time scale which is short compared to the flare heating time scale. Consequently the ionisation is given by a simple LTE-modification of Saha's equation at the instantaneous electron temperature obtained from the energy equation. These conclusions are contrasted with those of previous authors who, by using one-level atoms only, have omitted the optically thin Balmer continuum which dominates the optically thick Lyman transitions.

1. Introduction

An important test of any flare heating model is the correct prediction of the radiative emission, especially at optical and UV wavelengths which contain the bulk of the thermal emission energy (Bruzek, 1967). The intermediate optical thickness of the flare at these wavelengths (see Equations (3) below and Kandel *et al.*, 1971) demands, however, that any such model include a solution of the transfer equation and a non-LTE treatment of the hydrogen ionisation. It is possible, furthermore, that the ionisation equilibrium of hydrogen may not even satisfy the steady state approximation, (i.e. be determined by the instantaneous electron temperature – Cox and Tucker, 1969), if the time scale of the heating is short compared to that for adjustment of the ionisation to the rising temperature (cf. the analysis for heavy elements in X-ray flares – Kafotos and Tucker, 1972). Considering flare heating models, both Kandel *et al.* (1971) and Hudson (1972) claim that the hydrogen ionisation in optical flares is indeed far from a quasi-steady approximation and, in addition, that it is directly affected by the non-thermal electrons present and which may contribute to the flare heating.

In this paper it is shown that these conclusions of Kandel *et al.* (1971) and of Hudson (1972) arise spuriously from the use of a hydrogen atom model with only one discrete level plus continuum. When a two-level atom is adopted, the ionisation equilibrium is found to be given accurately by the steady state approximation and to depend on the non-thermal particle input only via the local electron temperature, determined by the energy equation. The ionisation is in fact then given by the same 'modified Saha equation' as originally derived by Pottasch and Thomas (1959) for the quiet low chromosphere.

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2. The Ionisation Equation

Pottasch and Thomas (1959) first demonstrated the need to use a hydrogen atom model with at least two discrete levels for an adequate description of hydrogen ionisation in chromospheric conditions. Inclusion of the second level alters the essential character of the ionisation equilibrium by introducing transitions in the optically thin Balmer continuum (BC) which dominate the optically thick Lyman continuum (LC) transitions near detailed balance. Studies of multi-level atom models show that the two-level atom gives a good first approximation to the hydrogen ionisation at densities and temperatures encountered in the low chromosphere (e.g. Thomas and Athay, 1961; Mihalas, 1970; Jefferies, 1968; de Feiter, 1966) and it will therefore be adopted here. The ionisation equation is then

$$n\frac{\partial x}{\partial t} = \sum_{j=1,2} (C_j + P_j + C_j^* + P_j^*), \tag{1}$$

where n and x are respectively the local values of the total number density, and the ionisation, of hydrogen. C_j and P_j are *nett* ionisation rates ($\text{cm}^{-3}\,\text{s}^{-1}$) from Level j due, respectively, to *thermal* collisions and to the *local* radiation field, while C_j^* and P_j^* are the corresponding *nett* rates of ionisation by *non-thermal* particles and by an *external* radiation field. Suffices + and − will refer respectively to *gross* ionisation and recombination rates so that, for example, $C_j = C_{j+} - C_{j-}$.

As previous analysis of the two-level atom Equation (1) (i.e. Pottasch and Thomas, 1959; and subsequent authors) has been in terms of the steady conditions of the quiet chromosphere, it is necessary here to reconsider this analysis with reference to transient flare conditions. In the optical flare the temperature is of order 10^4 K (Fritzová-Švestková and Švestka, 1967) with a corresponding density scale height of about 5×10^7 cm and consequent optical thicknesses of

$$\tau_{\text{LC}} \simeq 3\times10^{-10}\, n(1-x) \tag{2}$$

and

$$\tau_{\text{BC}} \simeq 6\times10^{-10}\, nq_2(1-x)$$

where absorption coefficients have been taken from Allen (1963) and q_2 is the proportion of neutral atoms in Level 2. Electron densities in optical flares (de Feiter, 1966; Fritzová-Švestková and Švestka, 1967) show that n can be higher than $5\times10^{13}\ \text{cm}^{-3}$ and is never less than about $3\times10^{12}\ \text{cm}^{-3}$ so that

$$1.5\times10^4(1-x) \gtrsim \tau_{\text{LC}} \gtrsim 1.0\times10^3(1-x), \tag{3}$$

and

$$3\times10^4\, q_2(1-x) \gtrsim \tau_{\text{BC}} \gtrsim 2\times10^3\, q_2(1-x).$$

Since typical preflare values of x lie in the range 4×10^{-2} to 2×10^{-3} for the above n (Gingerich *et al.*, 1971), Equation (3) shows that τ_{LC} will remain high throughout optical flares except perhaps in their uppermost layers when these are near their peak temperature and $(1-x)\lesssim10^{-3}$. On the other hand, τ_{BC} is small even in preflare conditions since q_2, being near LTE (Pottasch and Thomas, 1959; see also Equation (10)), is small. As originally shown by Pottasch and Thomas (1959), the regime of large τ_{LC} and small τ_{BC} simplifies the ionisation equation by eliminating the need for explicit solution of the transfer equation. Thus large τ_{LC} ensures detailed balance in LC transitions (i.e. $P_1 = P_{1+} - P_{1-} \simeq 0$) as well as excluding external Lyman fields ($P_1^* = 0$). Small τ_{BC}, on the other hand, results in a local Balmer field that is very weak compared to the *external photospheric field*, which thus controls photoionisation from Level 2, spontaneous recombination providing a return route. Equation (1) thereby simplifies to

$$n \frac{\partial x}{\partial t} \simeq P_{2+}^* - P_{2-} + \sum_{j=1,2} (C_j + C_j^*). \tag{4}$$

It is well known that thermal collisional ionisation is, under chromospheric conditions, minor in comparison to photoionisation (e.g., Ambartsumyan, 1958; Pottasch and Thomas, 1959) so that terms C_1 and C_2 may be neglected in a first approximation to Equation (4). Non-thermal collisions might, however, be large under flare conditions (e.g. Hudson, 1972) – a possibility which can only be tested by comparison of C_{1+}^* with P_{2+}^*. The photoionisation term is due to the photospheric Planckian field $B_\nu(T_R)$ at radiation temperature $T_R(\simeq 6000\,\mathrm{K})$ diluted in the chromosphere by a factor $W(\gtrsim 0.5)$. If the Balmer frequency limit is ν_2 and the photo-absorption cross-section is $a_2(\nu)$, the result is

$$P_{2+}^* = nq_2(1-x) \times 4\pi W \int_{\nu_2}^{\infty} \frac{B_\nu(T_R)\, a_2(\nu)}{h\nu}\, \mathrm{d}\nu \tag{5a}$$

where h is the Planck constant. Substituting numerical values gives

$$P_{2+}^* \gtrsim 2.5 \times 10^4 nq_2(1-x). \tag{5b}$$

Non-thermal electrons penetrating the atmosphere deeply enough to be a heating agent for the optical flare region must have energies $E^* \gtrsim 35$ keV (Brown, 1973). Such electrons are observed, from X-ray burst data, to be injected at a rate $\gtrsim 10^{36}\ \mathrm{s}^{-1}$ even in the largest events (Brown, 1971). These must, furthermore, be distributed over the optical flare area which exceeds $5 \times 10^{18}\ \mathrm{cm}^2$ even in small events and can be as great as $5 \times 10^{19}\ \mathrm{cm}^2$ in large flares – de Feiter (1966), (unless the electron stream were highly filamented within these areas). Thus the electron flux in a large flare cannot exceed $10^{17}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ which, with the electron/H-atom collisional ionisation cross-section from Mott and Massey (1965), yields

$$C_{1+}^* < 10^{-1} n(1-x \ . \tag{6}$$

Finally, by (5b) and (6)

$$\frac{C_{1+}^*}{P_{2+}^*} < 10^6 \times 10^{-49\,600/T} \tag{7}$$

where the population ratio $q_2 \simeq n_2/n_1$ has its Boltzmann value due to the high opacity, and consequent detailed balance, in $L\alpha$ transitions throughout the flare. De Feiter (1966) found that flare Balmer emission was only consistent with T near 10^4 K for which (7) shows that the photospheric field is at least ten times more important than non-thermal electrons. With a more reasonable area of over 10^{19} cm^2 for the large event considered, the electron flux is proportionately less and the photoionisation even more dominant.

In disk flares the optical emission may emerge from a region at as low a temperature as 8000 K (Švestka, 1972). This does increase the exponential factor in (7) by a factor of 10 but will be offset by a comparable diminution of the electron flux at this greater depth in the atmosphere (Brown, 1973). At the lesser depth from which EUV flare emission emerges, on the other hand, the electron flux is considerably greater – perhaps a factor of 10 (Brown, 1973) – but, since $T \gtrsim 2 \times 10^4$ K there, the increased exponential factor in (7) more than cancels this, leaving radiative processes entirely dominant. (In fact, in this region, the now optically thin LC transitions would have to be considered.) Just as in the quiet atmosphere (Thomas and Athay, 1961, p. 134) the influence of the mechanical energy input (i.e., non-thermal electrons) on ionisation in the flare is then only through the local electron temperature (determined by the energy equation), entirely contrary to the conclusions of Hudson (1972) and of Kandel *et al.* (1971).

To a good approximation, therefore Equation (6) may finally be written

$$n \frac{\partial x}{\partial t} \simeq P_{2+}^{*} - P_{2-}. \tag{8}$$

In the *steady state* solution of Equation (8), applicable to the quiet low chromosphere, ionisation from Level 2 by the photospheric Balmer field is balanced by spontaneous recombination (to Level 2) of electrons at kinetic temperature T. Substituting the appropriate rate coefficients, (Allen, 1963), this steady state is such that

$$\frac{n_2}{n^2 x^2} = \frac{\alpha_2(T)}{\dfrac{4\pi W}{h} \displaystyle\int_{\nu_2}^{\infty} B_\nu(T_R)\, a_2(\nu)\, \mathrm{d}\nu/\nu}, \tag{9}$$

and α_2, a_2 is the recombination coefficient for the second level. Solution of Equation (9) for x is obtained by use of the other condition that Levels 1 and 2 are tied by detailed balance of Lα line transitions (large $\tau_{L\alpha}$) resulting in an LTE value for q_2 (Pottasch and Thomas, 1959). In terms of the usual LTE departure coefficient b (e.g. Thomas and Athay, 1961) the solution is, in obvious notation

$$\frac{nx^2}{1-x} = \frac{f(T)}{b_1(T)},$$

where

$$f(T) = (2\pi mkT/h^2)^{3/2}\, e^{-\chi_H/kT} \tag{10}$$

and

$$b_1(T) \simeq b_2(T) = \frac{T}{WT_R} \exp\left[\frac{\chi_H}{4kT}\left(\frac{T}{T_R} - 1\right)\right]$$

which is the Saha equation modified by the factor $b_1(T)$.

Deviation of the ionisation from this steady state may arise in transient conditions of varying T, such as in flare heating. Considering Equation (8), it is clear that an increase of T first reduces the recombination rate $P_{2-} \sim n^2 x^2 T^{-1/2}$ followed by increase of x, due to the unbalanced photospheric radiative ionisation P^*_{2+} (which varies only with n_2), until a new steady state at higher T obtains. The characteristic time t_I for adjustment of the ionisation to changing temperature in optical flares is thus, by Equation (8):

$$t_I \simeq \frac{n}{P^*_{2+}} \tag{11}$$

i.e. by Equation (9):

$$t_I \simeq \frac{n}{P_{2-}}$$

which is just the recombination time t_R to Level 2, given approximately by Allen (1963):

$$t_R(s) \simeq \frac{10^{11}\, T^{1/2}}{nx} \tag{12}$$

(This contrasts with the discussion of the steady state approximation to ionisation of heavy elements in X-ray flares by Kafotos and Tucker (1972). In these low density, high temperature regions the ionisation time scale t_I is determined by collisional, and not radiative, processes (Cox and Tucker, 1969)).

In the quiet chromosphere (Gingerich *et al.*, 1971) the function $nx/T^{1/2}$ (for $10^{12} \lesssim n \lesssim 10^{14}$ cm^{-3} and x given by Equation (10)) is approximately constant at 80×10^{-11} cm^{-3} so that $t_R \simeq 80$ s by Equation (12). This is less than the typical time scale of about 100 s for non-thermal electron input in large flares (Cline *et al.*, 1968) which is comparable to observed heating times t_H for the flash phase of optical flares (Ellison, 1943). Thus even with initial conditions appropriate to the quiet atmosphere, heating of a large flare occurs slowly enough for the ionisation to follow the quasi-steady Equation (10). At the higher temperatures present in a (preflare) active region and after onset of the flare heating, increase of x from the quiet Sun values (10^{-2}–10^{-3}) greatly reduces t_R and the steady state approximation is even better. (e.g. for $n = 10^{13}$ cm^{-3}, and $T = 10^4$ K then Equation (10) gives $x \simeq 0.5$ and $t_R \simeq 2.1$ s). Optical flare ionisation is therefore amply described by the steady-state non-LTE expression (10) entirely analogous to that in the quiet chromosphere (Pottasch and Thomas, 1959) but with a density n and an instantaneous temperature

T appropriate to flare conditions. In particular it may be noted that this implies a degree of ionisation far below that in LTE at the same T. The detailed behaviour of $b_1(T)$ is discussed fully elsewhere (e.g. Thomas and Athay, 1961, p. 105) but a typical value of $b_1 \simeq 46$ at $T = 10^4$ K suffices here to emphasise the large deviation from LTE. For $n \simeq 10^{13}$ cm^{-3} this implies $x \simeq 0.56$ as against $x \simeq 0.95$ in LTE (by Equation (10)). If T were, however, somewhat lower (as it may be in disk flares – Švestka, 1972) and n correspondingly higher, then deviations from LTE are not so great ($b_1 \simeq 4$). Nevertheless the condition of large τ_{LC} is again amply met and the argument for use of a steady state approximation to the ionisation equation still valid.

3. Discussion of the Discrepancy with Previous Authors

Considering a model of flare heating by high energy particles, Kandel *et al.* (1971) used only a one-level hydrogen atom. Their neglect of the second level from Equation (1) does not merely eliminate the Balmer terms but, due to τ_{LC} being large and consequent detailed balance in LC transitions, also ensures that the ionisation is initially extremely close to LTE (cf. Pottasch and Thomas, 1959). In this state of affairs *nett* ionisation (C_1) by thermal collisions is also very small. In their model, therefore, Kandel *et al.* (1971) have effectively a time dependent ionisation equation.

$$n \frac{\partial x}{\partial t} \simeq C_1^* \tag{13}$$

which clearly describes a steady increase of x (over its initial LTE value) due entirely to non-thermal electron collisions and continuing until τ_{LC} is sufficiently reduced to break down the detailed balance of thermal processes. (This is indeed the behaviour computed in detail by Kandel *et al.* (1971)). Comparison of Equations (13) and (8) shows clearly, however, that this conclusion of dominance of non-thermal collisions C_1^* in the ionisation equation is a spurious result of the omission of much larger terms from *optically thin* Balmer transitions. Thus, far from complicating the problem of optical flare ionisation inclusion of the second level reduces the coupled time-dependent ionisation and transfer equations of Kandel *et al.* (1971) to the single time independent Equation (10).

Hudson (1972) has also considered the ionisation in optical flares heated by non-thermal electrons and assumed an ionisation equation of the form

$$n \frac{\partial x}{\partial t} = \sum_j (C_j^* - P_{j-}) \simeq C_1^* - \sum_j P_{j-}. \tag{14}$$

This model yields a quasi-steady state in which non-thermal collisional ionisations are balanced by radiative recombination. No account is taken of photoionisation terms, neither from Level 1 (by the local Lyman field) nor from higher levels by the photospheric Balmer field, which results in a great underestimate of the ionisation since these latter terms are dominant ($C_1^* \ll P_{2+}^*$ – Section 2). The unsuitability of Equation (14) is clear when it is observed that, by neglecting ionisation by photospheric radiation, it fails to express even the steady state of the chromosphere prior to heating by the electron stream – i.e. when $C_1^* = 0$ in Equation (14). Thus, the conclusion (Hudson, 1972) that C_1^* enhances flare ionisation over thermal collisional equilibrium is both incorrect and inappropriate since the ionisation is radiatively and not collisionally, controlled.

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