MAGNETIC FLUX OF COLINEAR BIPOLAR SPOT PAIRS

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Abstract. It has been widely conjectured that solar flares are energized by the magnetic energy stored in complex active regions. Paradoxically, however, in attempting to show that magnetic changes cause or characterize flares, solar magnetic observations have produced equivocal results.

In previous attempts at resolving the paradox, it has been contended that magnetic measurements are simply imprecise or that magnetic theories of flares are incorrect. We present an alternative explanation: the present use of magnetograms to examine active region structure through numerical integration of miscellaneous field lines (under various force-free assumptions) provides qualitative information only and does not utilize the quantitative information available. Therefore, we propose a new approach to the analysis of magnetograms which is illustrated with a highly symmetrized example that permits integration in closed form. The proposed approach exploits the cellular structure of the flux of field lines present in a complex active region. The various topological connectivities distinguish parent and daughter flux cells. A function F is developed expressing the flux partitioned into the daughter cell of interconnected field lines in a potential field. This F is a function of the location, strength, and relative motions of the photospheric sources. Then dF/dt is used as an EMF in the direct calculation of the stored magnetic energy available for flare production. In carrying out this program the flux partitioning surface (separatrix) is calculated along with its line of self-intersection (separator). The separator is the location of the principal energy release site.

1. Introduction

Past attempts at correlating solar magnetic changes with solar flares have been only marginally successful. This is highly unfortunate in view of the fact that most flare theories tap magnetic energy as the driving mechanism for the flare. The indications would seem to be that the magnetic measurements are not sufficiently accurate or frequent or that flares are not derived from a dissipation of magnetic energy. Here we suggest that the magnetic data must be analyzed in a new way which accounts for the cellular nature of magnetic flux connectivity. Therefore, we will briefly review the relation of flares to magnetic fields, describe how solar magnetic fields have been interpreted, and discuss the relation between the topology of individual field lines and the cellular topology of flux surfaces. With these rudiments in mind, we shall proceed to calculate the cellular fluxes of a complex bipolar spot pair as a function of spotgroup separation. The calculation is done for the special limiting case where the spot pairs are colinear and of equal spot strength and will make use of the magnetic point charge model. Having demonstrated exact solutions for the cellular fluxes in this special case, it is suggested that solar magnetic field codes be further developed to compute the cellular fluxes in more general arrangements.

We shall conclude by showing a simple relationship between magnetic flux and energy changes which follows from electric circuit theory. The relationship will be used in discussing the energy requirements of flares.

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A. FLARES AND MAGNETIC ENERGY CHANGES

There are a number of observations indicating a decrease of magnetic energy during large flares (Evans, 1959; Howard and Severny, 1963; Gopasyuk *et al.*, 1963; Zvereva and Severny, 1970) and a later recovery to near its previous interflare value. Indeed, in the 3^+ flare of 16 July 1959 Howard and Severny (1963) report that the magnetic energy in the longitudinal magnetic field dropped almost 90%. While this is quite impressive, the same data show a 45% decrease in energy on 15–16 July 1959 which is not associated with a major flare.

The relation of flares to magnetic field changes have been reviewed by Rust (1976) who cites many examples where photospheric fields seem not to have changed when flares occurred. He cautions, however, that most sunspot field measurements are typically in error by 10–20% or 500 G which could easily mask expected changes. Rust (1976a) concludes that large flares are associated with the development of a strong transverse field $B_\perp$, parallel to the 'neutral line' where $B_\parallel = 0$. This conclusion is based on vector magnetogram measurements and photographs of penumbral fibrils. Since as noted the $B_\parallel$ measurements (which are simpler than $B_\perp$ measurements) are imprecise, it appears that some advances are required in observational procedures and/or interpretation before unequivocal flare related magnetic field changes can be firmly established.

B. ANALYSIS OF SOLAR MAGNETIC FIELDS

The study of magnetic field structure above bipolar sunspot pairs has been of interest for many years (e.g., Sweet, 1958a, b) because of a hypothesis that flare energy could be stored in a current system induced through the superposition or mutual intrusion of bipolar spot pair fields. Sweet (1958a) considered two bipolar spot pairs (4 spots) under the assumption of translational symmetry (infinitely long sunspots) making the problem two dimensional and obtaining estimates of the parameters describing the neutral current sheet assumed to form between spot pairs.

The analysis of solar fields was greatly advanced by Schmidt's (1964) introduction of a magnetic point charge method of computing potential solar fields and fluxes given the photospheric magnetic charge distribution or $B_\parallel$.

Skylab observations (Sheeley *et al.*, 1975) made it further apparent that the study of coronal potential magnetic fields above active regions was a valuable approach. In this approach one considers a pair of active regions rather than spot pairs as magnetic sources. The structures are essentially similar to spot pairs differing only in scale.

More recent progress in the structure of solar fields includes the mapping of the separatrix structure for coplanar but non-colinear bipolar spot pairs (Bratenahl and Baum, 1976), calculation of the field structure above three active regions in the point charge analogy (Sheeley *et al.*, 1975), and the insertion of current sheets into bipolar regions using complex variables (Tur and Priest, 1976). Sakurai and Uchida (1977) suggested an ingenious approach in which sunspots are treated as solenoids and individual field lines are computed for various arrangements of pairs of solenoids.

The lines thus obtained are quite similar to those which have been obtained from Schmidt codes but the program has a difference in that flux balance is assured in the region of computation since the total equivalent magnetic charge is set to zero. Sakurai and Uchida's method also allows the positioning of current sheets between pairs of magnetically non-connected bipolar spot pairs. This is equivalent to moving two bipolar pairs together under conditions of infinite plasma conductivity thereby excluding reconnection. The current sheet thus obtained corresponds to a calculated amount of energy ($\gtrsim 10^{31}$ ergs) above the potential energy. It may be noted that this energy is an overestimate or upper limit since no flux transfer is allowed during the buildup of sheet current.

C. MAGNETIC TOPOLOGY AND MAGNETIC FLUX

There exists considerable evidence that major flares tend to occur in magnetically complex regions (Švestka, 1976a) but recent X-ray observations show illuminated loop structures which prompted Spicer (1977) to propose the simple loop structure as the basic flare topology. Rust (1976b), however, views these loops as illuminations of selected parts of the general active region field. Ribes (1969) found that flares are correlated with an increase of one polarity flux and a decrease of the opposite polarity flux. This seems inconsistent with the concept of a simple flux loop where we expect flux conservation and implies that magnetically complex regions should be considered. Accordingly, we have studied the field of bipolar spot pairs not only from the point of view of discrete field lines, but have shown the cellular nature of flux in this arrangement. A perspective view of the three-dimensional flux cells were presented in Bratenahl and Baum (1976, Figure 2) for one general case. That case is redrawn here as Figure 1 to show the flux topology of two bipolar spot pairs in the plane of the photosphere. Cells 1 and 2 are termed parent cells while 3 and 3′ are the two parts of the daughter cell. The intersection of the separatrix with the photosphere is shown as the darkest pair of closed oval lines. The separatrix is the surface separating the parent and daughter flux. The separatrix intersects itself along an arched curve above the photosphere called the separator (not shown) which is the field line joining the points 'a' and 'b'. The points 'a' and 'b' are true x-type neutral points on the photosphere but there is a non-zero field component everywhere else along the separator. The separator is a generalization of what is a neutral line ($\mathbf{B} \equiv 0$) in two dimensions. The lighter lines are individual photospheric field lines linking the north (N) and south (S) spots.

From Faraday's law, the induced voltage V_S along the separator measures the rate of flux transfer from parent to daughter cells.

With the solar magnetic codes (e.g. Schmidt, 1964), it is common practice to compute the total flux and flux disbalance of a solar region and to develop contour maps of the line of sight magnetic field ($B_{\|}$). However, these analyses do not compute the flux within the various cells defined by the separatrix since the separatrix itself is not computed. It should be apparent from the separatrix shown in Figure 1 as well as from the pioneering work by Sweet (1958b, Figures 2, 3, and 4) that the separatrix

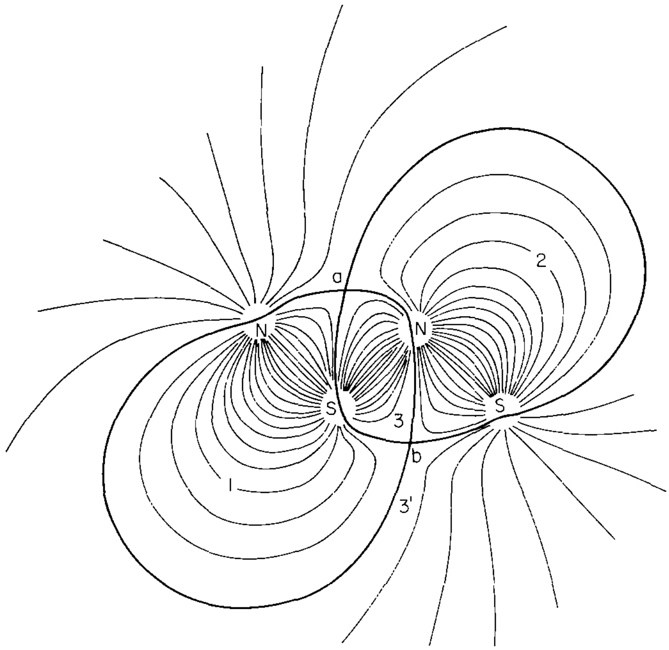

Fig. 1. The magnetic topology of two bipolar spot pairs arbitrarily arranged. The darkest curves are the separatrix at the photosphere and the lighter lines are discrete field lines. The separatrix divides the flux into cells 1, 2, 3, 3′ in which lines have differing connectivity. Points '*a*' and '*b*' are *x*-type neutral points ($\mathbf{H} \equiv 0$) and are the foot points of the separator field line.

passes through sunspots so that the flux of any given spot involves more than one connectivity. Hence, in determining flux transfers during flares, it is necessary to measure those fluxes which reside in parent and daughter cells. Further, it seems entirely reasonable as a first step to know the cell flux content in the potential case which represents the energy ground state of the spot fields. Therefore, in this paper we will derive fields, fluxes, and the separator height for colinear bipolar spot pairs using the point charge analogy. While spot pairs are rarely arranged in a colinear fashion, it has occurred, for example, during a flare of 7 September 1973 (Wu and Smith, 1977). Further, preliminary observations at this laboratory (Baum *et al.*, 1976) indicate that magnetoplasma processes above non-colinear bipolar solenoid pairs are quite similar to those above colinear pairs. Hence, it is appropriate to study the colinear problem as a guide for further interpretation and experimentation.

2. Approach

Figure 2 shows the magnetic topology of the system under study. We consider two bipolar pairs or four magnetic poles of which two are north and two are south polarity. Further, they are arranged in the colinear order NSNS along the Z-axis. Using the point charge analogy, the field is cylindrically symmetric (independent of

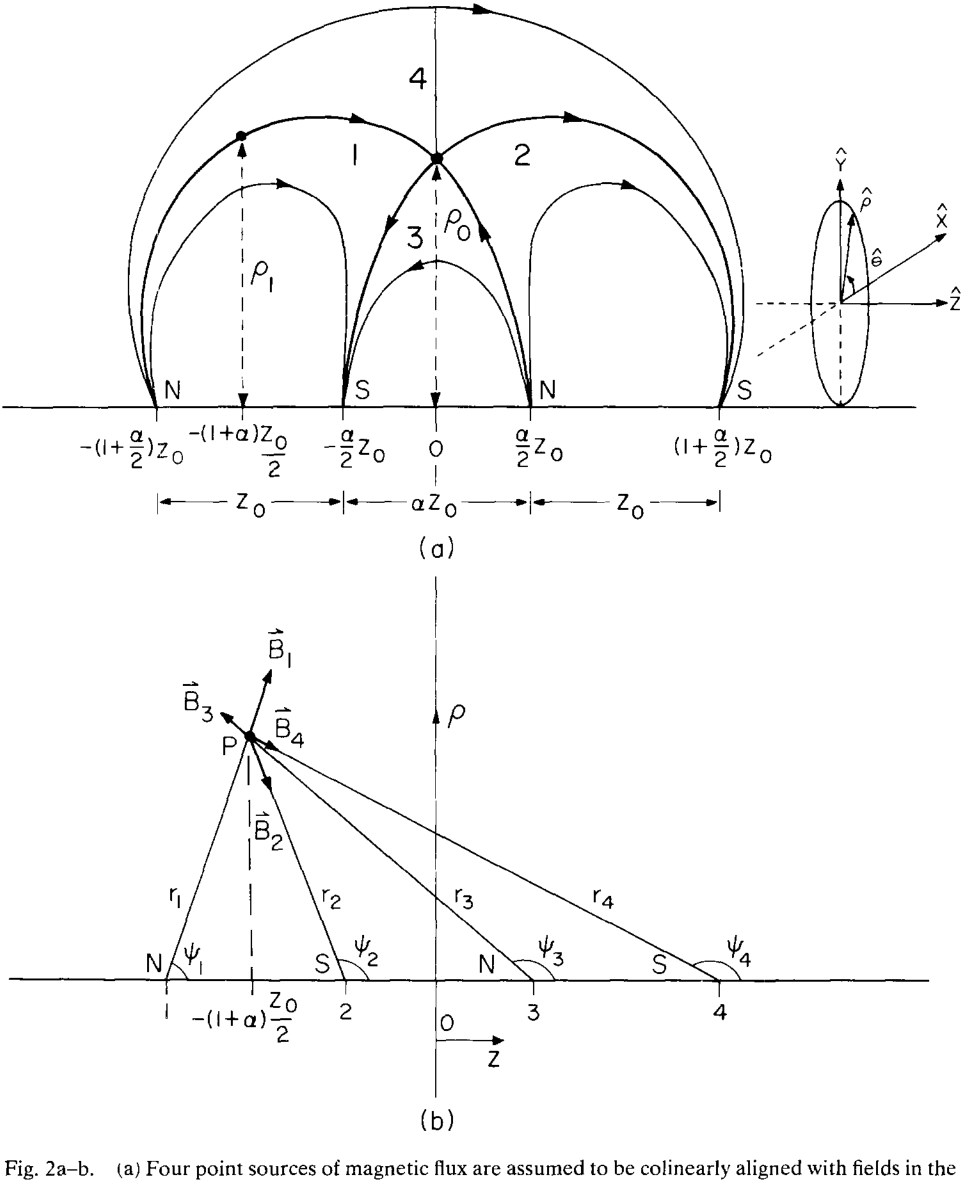

Fig. 2a–b. (a) Four point sources of magnetic flux are assumed to be colinearly aligned with fields in the order NSNS. The overall topology is schematically shown with the separatrix as the darker lines. The separatrix defines flux cells which are labelled 1 through 4, the configuration is cylindrically symmetric (in θ) about the z-axis with the separator (line of neutral points) located along $(\rho, \theta, Z) = (\rho_0, \theta, 0)$. The distance ρ_1 from $\rho = 0$ to the separatrix in cell 1 is used to compute the flux in cell 1. (b) The vector contribution of the magnetic fields at point P from spots 1 through 4. Point P is illustrated here in the plane in which ρ_1 is to be computed (i.e. midway between spots 1 and 2).

θ) and the radial coordinate is labelled ρ. The outermost spot pairs are separated by the distance Z_0 while the inner pair have a spacing αZ_0 where α is any real positive finite number. $\alpha = 1$ corresponds to equidistant spacing. The separatrix which is shown as the heavy solid curves in Figure 2a appears to divide the flux into four cells labelled 1–4. We shall find that cells 3 and 4 contain equal flux and may be considered as two parts of the same cell. This is true even when the fluxes in cells 1 and 2 are not equal. If the spots are labelled 1–4 as in Figure 2b, then cell 1 contains flux linking spots 1 and 2, in cell 2 flux links spots 3 and 4, in cell 3 flux links spots 2 and 3 and in cell 4 flux links spots 1 and 4. Note that the flux in cells 3 and 4 represent interconnected (perhaps reconnected) flux if spots 1–2 and 3–4 are 'parental pairs' (Bratenahl and Baum, 1976). In calculating these fluxes we will compute $\phi_n = \int \mathbf{B}_n \cdot \mathrm{d}\mathbf{S}_n$, where ϕ_n is the flux of cell n, $\mathbf{B}_n$ is the vector magnetic field in cell n and $\mathbf{S}_n$ is a vector normal to the appropriately chosen surface S_n. For example, in computing ϕ_3 the surface S_3 is chosen as a circular disk located in the plane $Z = 0$ and with radius ρ_0 which is the height of the separator. With this choice of S_3 the flux ϕ_3 can be calculated knowing only the horizontal (z) component of the magnetic field in cell 3, B_{3_z}. Similarly, ϕ_1 will be calculated through the surface S_1, which is a disk lying in the plane $z = -(1+\alpha)(z_0/2)$ and with radius ρ_1. The flux integrations will always begin or end at a point on the separatrix (e.g. ρ_0, ρ_1).

Using the point charge analogy, we will assume that each spot contains magnetic flux $\pm\phi_0$ and that in spherical coordinates centered on a spot,

$$\int \mathbf{B} \cdot \mathrm{d}\mathbf{A} = \pm\phi_0 , \tag{1}$$

where A is a hemispherical surface. Further, $\mathbf{B} = (K/r^2)\hat{r}$, where $\hat{r}$ is a radial unit vector and K is a constant so that the integration of (1) over a hemispherical surface results in

$$K = \pm\frac{\phi_0}{2\pi} . \tag{2}$$

Hence, at any radius r,

$$B_r = \pm\frac{\phi_0}{2\pi r^2} . \tag{3}$$

In Figure 2b we show the contributions at point P of $\mathbf{B}$ from all the sources. Since the flux can be calculated knowing only the horizontal field B_z, we construct

$$B_z = \sum_{i=1}^{4} B_i \cos \Psi_i = \frac{1}{2\pi} \sum_{i=1}^{4} \frac{\phi_{0_i}}{r_i^2} \cos \Psi_i . \tag{4}$$

In addition it is necessary to know B_z on only two planes which we choose midway between spots as $z = 0$ and $z = -(1+\alpha)(Z_0/2)$.

3. Solution and Results

In this section we will explicitly construct the horizontal component of the magnetic field, locate the neutral point, give a general expression for field lines and compute cellular flux partition functions called F and G.

Using Equation (4) and noting the symmetry about $z = 0$, we can construct the horizontal field on the plane $z = 0$:

$$B_z = \frac{1}{\pi}\left(\frac{\phi_0 \cos \Psi_1}{r_1^2} - \frac{\phi_0 \cos \Psi_2}{r_2^2}\right)$$

or

$$B_z = \frac{\phi_0 Z_0}{2\pi}\left\{\frac{(2+\alpha)}{[\rho^2 + Z_0^2(1+\alpha/2)^2]^{3/2}} - \frac{\alpha}{[\rho^2 + Z_0^2(\alpha/2)^2]^{3/2}}\right\}. \tag{5}$$

Equation (5) can be solved for the height of the neutral point by setting $B_z = 0$ with the general result

$$\frac{\rho_0}{Z_0} = \left\{\frac{\dfrac{(\alpha/2)^2}{\alpha^{2/3}} - \dfrac{(1+\alpha/2)^2}{(2+\alpha)^{2/3}}}{\dfrac{1}{(2+\alpha)^{2/3}} - \dfrac{1}{\alpha^{2/3}}}\right\}^{1/2} \tag{6}$$

which is plotted in Figure 3a for $0 < \alpha \leq 4$. Referring to the cylindrical coordinate system of Figure 2a, we now know that the points $(\rho, \theta, Z) = (\rho_0, \theta, 0)$ lie on the separatrix and in fact are the coordinates of the separator. In order to find the flux in cell 1, it is necessary to find an expression for the points $(\rho, \theta, Z) = (\rho_1, \theta, -[1+\alpha](Z_0/2))$ which can be done provided the equation for the separatrix can be found. The general expression for field lines in the colinear point charge model is known (Smythe, 1950; p. 13, Equation (1)). In the case under study here, the field lines are given by lines of constant C, where

$$C = \frac{\left(Z - \frac{\alpha Z_0}{2}\right)}{\left\{\left(Z - \frac{\alpha Z_0}{2}\right)^2 + \rho^2\right\}^{1/2}} - \frac{\left[Z - \left(1 + \frac{\alpha}{2}\right)Z_0\right]}{\left\{\left[Z - \left(1 + \frac{\alpha}{2}\right)Z_0\right]^2 + \rho^2\right\}^{1/2}} - \frac{\left(Z + \frac{\alpha Z_0}{2}\right)}{\left\{\left(Z + \frac{\alpha Z_0}{2}\right)^2 + \rho^2\right\}^{1/2}} + \frac{\left[Z + \left(1 + \frac{\alpha}{2}\right)Z_0\right]}{\left\{\left[Z + \left(1 + \frac{\alpha}{2}\right)Z_0\right]^2 + \rho^2\right\}^{1/2}}. \tag{7}$$

Now, knowing any point on the separatrix (e.g. the neutral point) we can evaluate C and subsequently, numerically solve Equation (7) to obtain the coordinates of any other point on the separatrix such as $(\rho_1, \theta, -[1+\alpha]Z_0/2)$. This was done and also appears in Figure 3a as ρ_1/Z_0. Note that $\rho_1 \geq \rho_0$ for the range $0 < \alpha \leq 4$ and that both ρ_1 and ρ_0 increase nearly linearly with α for $2 \lesssim \alpha \leq 4$.

For the calculation of fluxes, we further require the horizontal field on the plane $Z = -(1+\alpha)(Z_0/2)$. Symmetry now gives

$$B_z = \frac{\phi_0}{\pi r_1^2} \cos \Psi_1 - \frac{\phi_0}{2\pi r_3^2} \cos \Psi_3 + \frac{\phi_0}{2\pi r_4^2} \cos \Psi_4$$

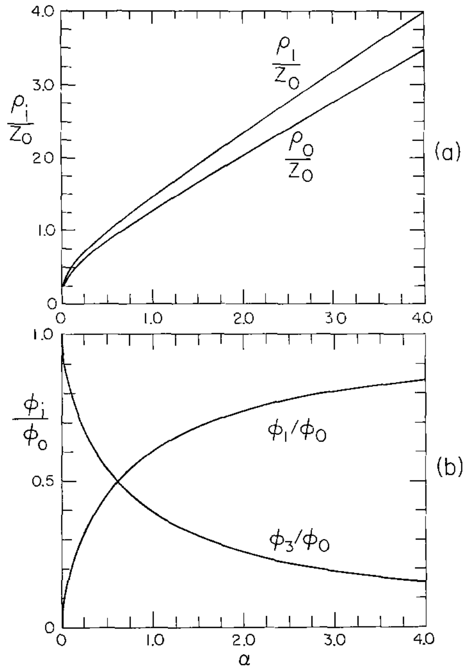

Fig. 3a–b. (a) Normalized separatrix heights calculated from the point charge model are plotted as a function of α. α is shown in Figure 2a. The lower curve, ρ_0, shows the height of the separator as a function of α whereas the upper curve, ρ_1, shows the height of the separatrix midway between spots 1 and 2. (b) The normalized calculated cellular fluxes are plotted as a function of α. ϕ_0 is the flux in the half space ($y \geq 0$) from a single spot. ϕ_1 is the flux in cell 1 and ϕ_3 is the flux in cell 3.

or

$$B_z = \frac{\phi_0 Z_0}{2\pi}\left\{\frac{1}{\left[\left(\frac{Z_0}{2}\right)^2+\rho^2\right]^{3/2}} - \frac{(\alpha+\frac{1}{2})}{\left[\left(\alpha Z_0+\frac{Z_0}{2}\right)^2+\rho^2\right]^{3/2}} + \right.$$
$$\left. + \frac{(\frac{3}{2}+\alpha)}{\left[\left(\frac{3Z_0}{2}+\alpha Z_0\right)^2+\rho^2\right]^{3/2}}\right\}. \tag{8}$$

We now form the fluxes (in the half space $y \geq 0$) above the photosphere:

$$\phi_1 = \phi_2 = \int_0^{\rho_1(\alpha)} B_{1_z}\rho \, \mathrm{d}\rho \int_0^{\pi} \mathrm{d}\theta ,$$
$$\phi_3 = \int_0^{\rho_0(\alpha)} B_{3_z}\rho \, \mathrm{d}\rho \int_0^{\pi} \mathrm{d}\theta , \tag{9}$$
$$\phi_4 = \int_{\rho_0(\alpha)}^{\infty} B_{4_z}\rho \, \mathrm{d}\rho \int_0^{\pi} \mathrm{d}\theta ,$$

where for B_{1_z} we use Equation (8) and for B_{3_z} and B_{4_z} we use Equation (5). The fluxes of Equation (9) are integrable giving

$$\phi_3 = \phi_4 = \frac{\phi_0}{2}\left\{\frac{(2+\alpha)}{\left[\left(1+\frac{\alpha}{2}\right)^2+\left(\frac{\rho_0}{Z_0}\right)^2\right]^{1/2}} - \frac{\alpha}{\left[\left(\frac{\alpha}{2}\right)^2+\left(\frac{\rho_0}{Z_0}\right)^2\right]^{1/2}}\right\}$$

$$\equiv F\left(\phi_0, \frac{\rho_0}{Z_0}, \alpha\right) \tag{10}$$

and

$$\phi_1 = \phi_2 = \frac{\phi_0}{2}\left\{\frac{-1}{\left[(\frac{1}{2})^2+\left(\frac{\rho_1}{Z_0}\right)^2\right]^{1/2}} + 2 + \frac{(\alpha+\frac{1}{2})}{\left[(\alpha+\frac{1}{2})^2+\left(\frac{\rho_1}{Z_0}\right)^2\right]^{1/2}} - \frac{(\frac{3}{2}+\alpha)}{\left[(\frac{3}{2}+\alpha)^2+\left(\frac{\rho_1}{Z_0}\right)^2\right]^{1/2}}\right\}$$

$$\equiv G\left(\phi_0, \frac{\rho_1}{Z_0}, \alpha\right). \tag{11}$$

Equations (10) and (11) are plotted in Figure 3b from which it is evident that $\phi_1 > \phi_3$ for $\alpha \gtrsim 0.67$. In this case the parental flux exceeds the interconnected or daughter flux (Bratenahl and Baum, 1976). The opposite is true for $\alpha \lesssim 0.67$. Since the separatrix partitions the spot flux ϕ_0 into two parts, we know that

$$\phi_4+\phi_1 \equiv \phi_1+\phi_3 \equiv \phi_3+\phi_2 \equiv \phi_2+\phi_4 \equiv \phi_0 \tag{12}$$

for the spots 1, 2, 3, 4 respectively, and therefore $F+G\equiv\phi_0$. This nontrivial result involving the use of (6), (7), (10), and (11) has been checked to computer accuracy for $0<\alpha\leq 4$. The component fluxes ϕ_i $(i=1\text{–}4)$ have now been calculated in the complex geometry of Figure 2a, but it remains a challenge to compute them in a more general configuration such as Figure 1. Finally in Figure 4 we present a numerical solution to Equation (7) which shows the computed field lines (equally spaced in flux) for the case $\alpha=1$.

Throughout the preceding sections we have treated only the special case of two bipolar spotgroups where all spots are of equal strength and increase at the same rate. There are, of course, other cases. For example, Heyvaerts *et al.* (1977) propose a case where one spot pair emerges into the flux of a pre-existing spot pair. While neither experimental nor computer simulations are yet complete, we presume that the mode of interaction will be quite similar to the symmetric case described here. The main difference we expect is that the separator inductance will initially be very small, later approaching the value of the symmetric case. The plasma physics and process of flux transfer should be quite similar to that described here.

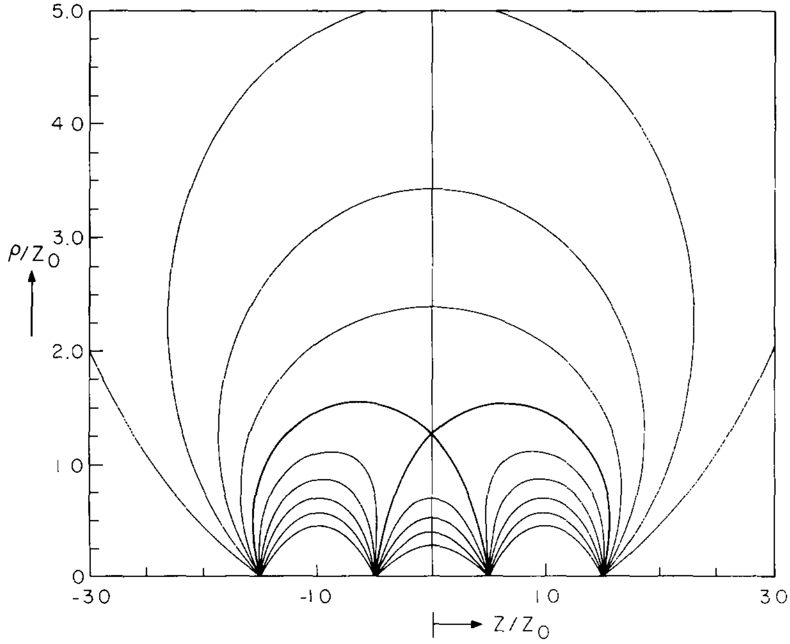

Fig. 4. Field lines calculated from the point charge analogy for the case $\alpha = 1$. The lines are equally spaced in flux.

4. Discussion and Conclusions

Let us compute the flux change needed to satisfy the solar flare energy requirement in reconnection models (Bratenahl and Baum, 1976a; Heyvaerts *et al.*, 1977). Consider that as current I flows along the arched separator its path defines an inductance $L \simeq 10H$ (Baum *et al.*, 1978) since from Figure 3a the separator height will be about equal to the sunspot spacing for $\alpha \sim 1$. Now on combining $\Delta U_m = \frac{1}{2}LI^2$ with the relation $\Delta\phi = LI$, the magnetic energy may be expressed as

$$\Delta U_m = \Delta\phi^2/2L\,, \tag{13}$$

a result which seems to have been introduced by Bratenahl *et al.* (1976). Equation (13) defines the available flare *energy stored* as the separator current I. $\Delta\phi$ is the stored flux corresponding to I. Previous energy analyses of solar flares, e.g. Schmidt (1964), have not studied the stored energy ΔU_m. As in Schmidt's case, solar measurements yield the (vector) sum $\mathbf{B} + \Delta\mathbf{B}$, where $\mathbf{B}$ is the 'background' magnetic field mainly emanating from the sunspots and $\Delta\mathbf{B}$ is the 'perturbation' magnetic field mainly due to (stored) currents in the solar atmosphere. Consequently, flux measurements yield the sum $\Phi = \phi + \Delta\phi$ of background and excess stored flux. Now forming the product Φ^2, as Schmidt did numerically, we find

$$\Phi^2 = \phi^2 + 2\phi\Delta\phi + \Delta\phi^2 \tag{14}$$

and the formal expression for magnetic energy

$$U_m = \Phi^2/2L \tag{15}$$

is not very meaningful since (15) contains the stored energy available for flare production

$$\Delta U_m = \Delta\phi^2/2L \tag{16}$$

as well as a cross product term and a background term $\phi^2/2L$ which is only dimensionally the spot flux energy since the appropriate subphotospheric inductance associated with the parental spot pair is unknown (L is the inductance of the superphotospheric current system). This is equivalent to a lack of detailed knowledge of the subphotospheric current distribution. Therefore since (15) contains relatively large background terms, it is not surprising that (15) has not been used for meaningful flare correlated energy changes. What is needed is a measurement of $\Delta\mathbf{B}$ or $\Delta\phi$ rather than the previously mentioned sum measurements. Shortly we shall see that $\Delta\phi$ and ΔU_m can be rather directly inferred from a knowledge of the time history of cellular flux. This, of course, implies a knowledge of the magnetic topology defining the flux cells, and especially a knowledge of the separatrix.

We now estimate the stored flux $\Delta\phi$ required for a large flare. Inverting Equation (16) we obtain the stored flux

$$\Delta\phi = [2L\Delta U_m]^{1/2}$$

which gives $\Delta\phi = 1.4\times10^{13}\ \text{V s}^{-1}$ (1.4×10^{21} Mx) for $L = 10H$ and $\Delta U_m = 10^{25}$ J(10^{32} ergs). On the other hand, a characteristic sunspot flux may be $\phi = 8\times10^{13}\ \text{V s}^{-1}$ (Bray and Loughhead, 1964) so that

$$\Delta\phi/\phi \approx 0.175\,.$$

Therefore, in a large flare the stored flux corresponds to only 17.5% of the flux linking one parental spot pair. If two equal bipolar pairs are involved, the dissipated flux is only ~9% of the flux of one spot pair. The preceding analysis is geometry independent and, therefore, applies to field aligned current models as well as flux transfer flare models.

Storage of flux $\Delta\phi$ and free magnetic energy ΔU_m to drive the flare process can come about through (i) a relative displacement of the spots (temporal change in α and/or Z_0), (ii) by a change in spot flux (temporal change in ϕ_0), or (iii) spot group rotations (not analyzed here). In the absence of a conducting plasma, changes (i) and (ii) simply change the flux partition functions, F and G, in the manner shown in Figure 3b. However, in the presence of plasma, as we shall presently see, these changes in F and G cannot take place *pari pasu* but are delayed. Flux $\Delta\phi$ and free energy $\Delta U_m = (\Delta\phi)^2/2L$ are therefore stored, and we can use Equations (10) and (11) (plotted in Figure 3b) as a basis for determining this flux and energy storage. In Case (i) examination of Figure 3b shows that a 9% change in ϕ_1 or ϕ_3 requires a change $\Delta\alpha \gtrsim 1.0$ for $\alpha \gtrsim 1$. If the bipolar regions are close ($\alpha < 1$), the displacement can be small ($\Delta\alpha < 0.25$). Pursuing Case (ii) for any constant α, ϕ_1 and ϕ_3 are directly proportional to the change in spot flux ϕ_0 (e.g. a 9% change in ϕ_0 produces a 9% change in ϕ_1 and ϕ_3). For constant spot area (or field), the spot field (or area) must change by 9% for a flux change of 9%. For a spot field of 2 kG we calculate a 9% spot field change of 180 G or a flux change of 7×10^{12} V s^{-1} (7×10^{20} Mx). It may be noted that usual magnetographs measure flux changes only *outside of sunspots* (because of saturation) and, therefore, do not produce evidence relevant to Case (ii). There is now a need for these measurements.

While the observations are ambiguous as noted in Section 1A, Tanaka (1977) has observed field changes up to 60 G and flux changes of 2×10^{20} Mx *during* a Class 2B flare of 10 September 1974. Earlier, Rust (1972) observed flux changes up to 10^{21} Mx although *not during* a flare. The solar observations, the present analysis, and laboratory observations (Bratenahl and Baum, 1976) emphasize the difficulty in correlating magnetic field changes with even fairly large flares.

We hypothesize that energy is continually supplied to the solar atmosphere by Poynting flux (evidenced by hydromagnetic waves), and that in the presence of a conducting plasma a portion of this energy may be stored for extended periods in current circuits or magnetic fields which need differ only slightly from potential fields. In extreme cases the structure may depart significantly from potential. The flare is more directly associated with the interruption of the current circuit than with the magnetic field itself (Baum *et al.*, 1978). Unfortunately, it is not possible to observe the current ($\bar{\nabla} \times \bar{\mathrm{B}}$) directly or even indirectly with good accuracy (Švestka, 1976b; p. 384). Further, if it were possible to observe the current, it would be necessary to know its spatial-temporal behavior since laboratory observations (Bratenahl and Baum, 1976) show that the current *decrease* along the separator at 'flare'-time is accomplished by its transport out to the frontier flux surfaces of the daughter flux cells by large amplitude waves. In this way currents are expected to *increase* at the frontier surfaces.

By default we must rely on measurements of magnetic field rather than current, but laboratory observations suggest that field changes at flare time are also complex. The field changes would be different for the two cases of spot separation and spot flux change. If energy is stored by spot motion (e.g. a temporal decrease in α), Figure 3b shows that the parental flux will decrease while the daughter flux will increase and the *net* spot flux will be unchanged. (Figures 1 and 2a show that both parental and daughter flux thread a spot and in the lowest energy state of the system (the ground state), the flux is partitioned according to (10) and (11).) However, the flux changes only need be of the order of 9% as indicated. If energy is stored by an increase in spot flux, both parental and daughter flux will increase by 9%. The small magnitude of the changes and the spatial resolution make observations difficult and we have still to consider plasma effects.

On the basis of laboratory studies (Bratenahl and Baum, 1976), we have acquired some knowledge of plasma effects in reconnecting systems. The presence of plasma controls the separator voltage $V_S = -\mathrm{d}\phi_3/\mathrm{d}t$ (Section 1C) which is the reconnection rate or rate of flux transfer into (or out of) the daughter cell. Thus with plasma present, Faraday's law, Ohm's law, and the circuit relation $\Delta\phi = LI$ give

$$-\frac{d\phi_3}{dt} = IR = \frac{R}{L}\Delta\phi = V_S. \tag{17}$$

Here I is the current induced along the separator, R is the effective resistance, L is the inductance associated with the current path, and $\Delta\phi$ is the stored flux associated with I. On the other hand, with no plasma present the voltage corresponding to the rate of change of flux in cell 3 is $-dF/dt$ which is greater than $-d\phi_3/dt$ by the amount $d(\Delta\phi)/dt$:

$$-\frac{dF}{dt} + \frac{d\phi_3}{dt} = \frac{d(\Delta\phi)}{dt}. \tag{18}$$

Hence, the equation for flux storage $\Delta\phi$ is

$$-\frac{dF}{dt} = \frac{d(\Delta\phi)}{dt} + \frac{R}{L}\Delta\phi \tag{19}$$

in which

$$\frac{dF}{dt} = \frac{\partial F}{\partial\phi_0}\frac{\partial\phi_0}{\partial t} + \frac{\partial F}{\partial\alpha}\frac{\partial\alpha}{\partial t} + \frac{\partial F}{\partial Z_0}\frac{\partial Z_0}{\partial t}. \tag{20}$$

Therefore, $-dF/dt$ acts as an EMF. From (19) the flux storage for times $t \geq t_1$ is (for $d^2F/dt^2 = 0$)

$$\Delta\phi = \frac{-L}{R}\frac{dF}{dt}\left\{1 - \left[1 - \frac{R}{L}\frac{\Delta\phi(t_1)}{-dF/dt}\right]\exp\left[-\frac{R}{L}(t - t_1)\right]\right\} \tag{21}$$

and the stored energy is

$$\Delta U_m = \frac{(\Delta\phi)^2}{2L}. \tag{22}$$

The details of the time release of this energy are presented elsewhere (Baum *et al.*, 1978). One way of estimating R is to measure the time $\mathfrak{T}$ between repeating flares and to assume that $\mathfrak{T} \sim L/R$. We note that (21) and (22) have the asymptotic flux and energy storage limits

$$\Delta\phi_\infty = \frac{-L}{R}\left(\frac{dF}{dt}\right) \tag{23}$$

and

$$\Delta U_{m\infty} = \frac{L}{2R^2}\left(\frac{dF}{dt}\right)^2. \tag{24}$$

If we now identify $\mathfrak{T} = L/R$ as an energy storage time then (24) can be rewritten

$$\Delta U_{m_\infty} = \frac{1}{2L}\left(\frac{\mathrm{d}F}{\mathrm{d}t}\right)^2 \mathfrak{T}^2 = \frac{(\Delta\phi)^2}{2L}\,.$$

From (17) and (23) we note that since $V_S = (R/L)\,\Delta\phi$, $V_S \to -\mathrm{d}F/\mathrm{d}t$ as $t \to \infty$. This is true for all times t in the no plasma case. Hence, the plasma reconnection rate asymptotically approaches the vacuum rate. This confirmation of a conjecture by Yeh and Axford (1970) and Yeh (1976) has been referred to as the Yeh–Axford theorem (Bratenahl and Baum, 1976). Note that many elements of the Yeh–Axford theorem are supported by the numerical MHD solution of Ugai and Tsuda (1979).

In (24) we note that the energy storage capacity is proportional to the square of both the no-plasma reconnection rate $\mathrm{d}F/\mathrm{d}t$ and the plasma conductivity. This latter condition makes it evident that the integrity of the energy storage reservoir depends on the stability of the conduction mechanism. Finally, we note that the foregoing no-plasma cellular flux analysis provides the formalism by which we can compute the energy available for flares in terms of changes in the parameters describing spot strengths and their geometrical arrangement.

In analogy to laboratory experiments (Bratenahl and Baum, 1976) we expect that during the intraflare phase the parent cells might decrease in flux for only a short time of the order of a few fast mode transit times (from the spots to points in the parent cells with a characteristic distance equal to the separator distance). Thereafter, the fields and fluxes in the parent cells may be restored to their previous interflare values. Therefore, we expect that considerable caution must be used in interpreting flare related magnetic field changes.

Based on a point charge model we have calculated the height of the separator, finding it to be proportional to the separation of the bipolar pairs for large separations. We have further calculated the magnetic flux in all cells of an interconnected colinear bipolar pair. For large pair separations the parental flux exceeds the interconnected flux.

In the potential limit by Faraday's law the vacuum separator voltage may be calculated from the time rate of change of flux. This was used as a driving EMF in calculating the separator electric field of reconnecting plasma systems and in computing the stored flare energy. The results will also be of use in testing modified numerical solar magnetic flux codes capable of computing cellular flux $\Delta\phi_i$ and their time changes $\Delta\dot{\phi}_i$ which are specific flare variables rather than active region variables as in the Schmidt method. Further, the results will be valuable in comparing with further laboratory data on bipolar pairs.

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