THE DEDUCTION OF ENERGY SPECTRA OF NON-THERMAL ELECTRONS IN FLARES FROM THE OBSERVED DYNAMIC SPECTRA OF HARD X-RAY BURSTS

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(Received 6 January, 1971)

Abstract. The derivation of dynamic spectra of high energy electrons in flares from high resolution hard X-ray observations is considered. It is shown that the Bethe-Heitler formula for the electron-proton bremsstrahlung cross-section over the 20–100 keV range of energies admits of a general analytic solution for the electron spectrum in terms of the X-ray spectrum, in a form convenient for computation. The bearing of this analysis on different models of flare conditions is considered. In examining the hypothesis that the X-rays are produced in regions of high ambient density, the duration of the burst being governed by modulation of the electron source rather than by the decay of trapped electrons injected impulsively, it is emphasised that the energy spectrum of the electrons at their source is different from their effective spectrum in the X-ray emitting region. This spectrum, at the source, is found to be much steeper than that in the X-ray region which means that the entire energy of the flare could reside in the injected electrons.

1. Introduction

Since the first balloon-borne observations of hard X-rays from solar flares (Peterson and Winckler, 1959), great advances have been made in the temporal and spectral resolution of the bursts. Continuous monitoring by satellites, such as those in the OSO series, is now providing dynamic X-ray spectra over the 20–100 keV energy range in channels of order 10 keV wide at intervals of order 1 s (e.g. Frost, 1969). In this energy range there is little doubt that the dominant emission mechanism in flare conditions is collisional bremsstrahlung of high energy electrons (Korchak, 1967). Three distinct opinions exist, however, regarding the energy spectrum of these electrons and its time dependence. Quasi-thermal conditions have been favoured by Chubb *et al.* (1966) and recently reconsidered by Chubb (1970) but such models will not be considered here as the recent X-ray polarisation observations of Tindo *et al.* (1970) certainly indicate the presence of non-thermal electron streams in flares. A non-thermal model with *impulsive* injection of (non-thermal) electrons into a magnetic trap high in the chromosphere has been proposed by Takakura and Kai (1966) and elaborated by Takakura (1969) and by Holt and Ramaty (1969). This model has been found by Acton (1968), by Arnoldy *et al.* (1968), by Kane and Anderson (1970), and by the present author in a separate study, to be inconsistent with observations. These authors favour a dense X-ray emission region with high energy electrons injected by a continuous acceleration mechanism whose modulations determine the time dependence of the hard X-ray emission.

On the impulsive model, the hard X-ray time dependence subsequent to electron

Solar Physics **18** (1971) 489–502. *All Rights Reserved*

injection is determined by the degradation of the non-thermal electrons in the trap by interaction with the ambient plasma and is independent of the acceleration mechanism.

Observation of the dynamic X-ray spectrum thus provides a test of the model and a means of interpreting ambient conditions, once the X-ray spectrum is related to the electron spectrum producing the X-ray burst. On a continuous injection model, on the other hand, the X-ray time dependence is determined by modulation of the injection rate of high energy electrons at their source and turns out to be independent of the conditions in the (dense) X-ray emission region. The dynamic electron spectrum thus yields important information concerning the electron acceleration region.

Whichever model is nearer to actual flare conditions, it is therefore desirable to have a ready means of deducing the high energy electron spectrum present in the flare. This paper does not primarily attempt to compare the above non-thermal interpretations of the X-ray data, but provides an analytic method for the deduction of the effective electron spectrum in the X-ray emitting region on any model and applies this result to considerations of the energetics of the non-thermal electrons in each model. Previous approaches to the problem of deducing the electron spectrum have been to substitute trial functions for the electron spectrum into the integral Equation (4) below until a suitable fit to the observed X-ray spectrum was obtained. With the Bethe-Heitler approximation to the electron–proton bremsstrahlung cross-section over the 20–100 keV range of electron energies, Equation (4) may, however, be inverted and a general analytic solution for the electron spectrum obtained.

The electron spectrum so deduced is the effective spectrum of the high energy electrons within the X-ray emitting region, regardless of the model chosen. At the moment of electron injection, on impulsive models, this deduced spectrum is necessarily the same as the actual spectrum at injection, as injection occurs *in situ*. However, if the non-thermal electrons are continuously injected into a dense plasma, their effective spectrum in the entire X-ray emitting region is, at any instant, different from their spectrum at the electron acceleration source at that instant. This is due to (collisional) energy losses of the electrons in traversing the X-ray emission region.

As regards the properties of the accelerating mechanism and the energetics of the non-thermal electrons on continuous injection models, it is this latter spectrum (at injection), and not the spectrum in the X-ray region, which is important. The relationship between this 'injection spectrum' and the resulting X-ray spectrum is found to be independent of the actual (high) ambient density in continuous models. For a given observed X-ray spectrum the deduced injection spectrum of electrons is found to be much steeper than the electron spectrum within the X-ray emitting region.

2. Collisional Bremsstrahlung Cross-Sections

Bremsstrahlung data for all conditions have been reviewed in convenient form by Koch and Motz (1959). Recent observations of the directionality (Ohki, 1969, and Pintér, 1969), and of the polarisation (Tindo *et al.*, 1970) of hard X-ray bursts indicate that the high energy electrons present are directed (Korchak, 1967) making it necessary, to be strictly correct, to adopt a bremsstrahlung cross-section differential in photon emission direction. The observed polarisation and directionality are not, however, very great so an integrated cross-section will suffice, the results obtained referring strictly to the X-ray intensity averaged over solid angle.

X-ray energies considered here lie in the interval 20–100 keV and it will be seen below that deduced electron spectra are characteristically of steep negative gradient. Examination of Equation (4) shows that, in such a case, X-rays of energy ε are not contributed significantly, in the electron–proton bremsstrahlung process, by electrons of kinetic energy E much greater than ε. Thus a non-relativistic cross-section will be adequate and, furthermore, neglect of electron–electron bremsstrahlung will not cause substantial error at these energies (Koch and Motz, 1959). The appropriate electron–proton bremsstrahlung cross-section in this energy range is given by the Bethe-Heitler formula (Heitler, 1954; also Koch and Motz formula 3BN (a)). This has been adopted by Takakura (1969), by Kane and Anderson (1970), and by Holt and Cline (1968) who include the Elwert factor (Elwert, 1939). This correction factor is, however, effectively unity for nuclei of small atomic number at non-relativistic energies except very close to the cut-off at $\varepsilon = E$ where it is invalid in any case (Koch and Motz, 1959).

The Bethe-Heitler formula for hydrogen is

$$Q_\varepsilon(E) = \frac{8}{3}\frac{r_0^2}{137}\frac{mc^2}{\varepsilon E}\log\frac{1+\sqrt{1-\varepsilon/E}}{1-\sqrt{1-\varepsilon/E}} \tag{1}$$

where $Q_\varepsilon(E)$ is the required cross-section differential in photon energy ε, m and r_0 are the rest mass and classical radius of the electron respectively, and c is the velocity of light, cgs units being used throughout. Further simplifications of (1) by expansion in ε/E or by assuming constancy of the logarithm are invalid here since ε is not $\ll E$. This $Q_\varepsilon(E)$ will apply equally well to ionised and neutral hydrogen atoms since screening effects are small at these energies.

3. Determination of the Effective Electron Spectrum in the X-ray Emission Region, from the X-Ray Spectrum

Let the number density of ambient protons (free and in atoms) at some point in the X-ray emitting region be $n_P(\text{cm}^{-3})$ and let the number of non-thermal electrons per cm^3 per unit E range (in erg) be $n(E)$ at that point, both n_P and $n(E)$ being instantaneous values. The observed emission, being spatially unresolved, is the integral, over the emitting volume, of the X-ray emission function (photons/cm^3 sec^{-1} per unit ε range) which depends on the product $n_P\, n(E)$. In general, n_P and $n(E)$ will be spatially non-uniform and the total X-ray emission from the emitting volume V will be

$$\int_\varepsilon^\infty Q_\varepsilon(E)\, v(E)\left(\int_V n_P n(E)\,\mathrm{d}V\right)\mathrm{d}E \quad \text{(photons/sec per unit } \varepsilon)$$

where contributions from ambient atoms other than hydrogen have been neglected and v is the electron velocity corresponding to E.

If n_P is uniform through V (the case considered by previous authors), then the total X-ray emission is

$$n_P \int_{\varepsilon}^{\infty} Q_{\varepsilon}(E)\, v(E)\, N_T(E)\, \mathrm{d}E \quad \text{(photons/sec per unit } \varepsilon)$$

where $N_T(E)$ is the energy distribution function for all the non-thermal electrons in the volume.

If n_P is non-uniform, the total electron spectrum effective in producing the X-rays is not $N_T(E)$ but $N(E)$ given by

$$N(E)\, n_0 = \int_V n(E)\, n_P\, \mathrm{d}V \tag{2}$$

where n_0 is the mean value of n_P in V, i.e.

$$n_0 = \frac{1}{V}\int_V n_P\, \mathrm{d}V\,. \tag{3}$$

Thus $N(E)$ is the integral of $n(E)$ over V weighted with respect to n_P.

At the Earth's distance R, the mean photon count rate per unit ε range is then, using (1), (2) and (3),

$$I(\varepsilon) = \frac{\beta}{\varepsilon}\int_{\varepsilon}^{\infty} \frac{N(E)}{\sqrt{E}} \log\frac{1+\sqrt{1-\varepsilon/E}}{1-\sqrt{1-\varepsilon/E}}\, \mathrm{d}E \quad (\text{photons/cm}^2\, \text{sec erg}) \tag{4}$$

where

$$\beta = \frac{2}{3}\frac{n_0}{137}\left(\frac{r_0}{R}\right)^2 \mathrm{mc}^2 \sqrt{\frac{2}{m}}\,.$$

Setting $J(\varepsilon) = \varepsilon I(\varepsilon)$, (4) may be written as

$$\int_{\varepsilon}^{\infty} \frac{N(E)}{\sqrt{E}} \log\frac{1+\sqrt{1-\varepsilon/E}}{1-\sqrt{1-\varepsilon/E}}\, \mathrm{d}E = \frac{1}{\beta} J(\varepsilon)\,. \tag{5}$$

Differentiating with respect to ε,

$$-\int_{\varepsilon}^{\infty} \frac{N(E)}{\sqrt{E}} \frac{\mathrm{d}E}{\varepsilon\sqrt{1-\varepsilon/E}} - Lt_{\varepsilon\to E}\left[\frac{N(E)}{\sqrt{E}} \log\frac{1+\sqrt{1-\varepsilon/E}}{1-\sqrt{1-\varepsilon/E}}\right] = \frac{1}{\beta}\frac{\mathrm{d}J}{\mathrm{d}\varepsilon}$$

i.e.

$$\int_{\varepsilon}^{\infty} \frac{N(E)\,\mathrm{d}E}{\sqrt{E-\varepsilon}} = \psi(\varepsilon)$$

where $\psi(\varepsilon) = -\varepsilon/\beta\, J'(\varepsilon)$.

This is Abel's integral equation with solution

$$N(E) = -\frac{1}{2\pi E}\int_{E}^{\infty} \frac{\psi(\varepsilon) + 2\varepsilon\psi'(\varepsilon)}{\sqrt{\varepsilon - E}}\,\mathrm{d}\varepsilon$$

(Courant and Hilbert, 1953)

i.e.

$$N(E) = \frac{1}{2\pi\beta E}\int_{E}^{\infty} \frac{3\varepsilon J'(\varepsilon) + 2\varepsilon^2 J''(\varepsilon)}{\sqrt{\varepsilon - E}}\,\mathrm{d}\varepsilon = \tag{6}$$

$$= \frac{1}{2\pi\beta E}\int_{E}^{\infty} \frac{\varepsilon(3I(\varepsilon) + 7\varepsilon I'(\varepsilon) + 2\varepsilon^2 I''(\varepsilon))}{\sqrt{\varepsilon - E}}\,\mathrm{d}\varepsilon\,. \tag{7}$$

$I(\varepsilon)$ may conveniently be expressed as the fraction f per unit ε of all the photon flux $\mathscr{I}(\varepsilon_1)$ (photons/cm^2 sec) at energies $\varepsilon \geqslant \varepsilon_1$ for an ε_1 to be adopted later. Setting $\xi = \varepsilon/E$ the solution may then be written

$$N(E) = \frac{1}{2\pi\beta}\mathscr{I}(\varepsilon_1)\sqrt{E}\int_{1}^{\infty} \frac{\xi}{\sqrt{\xi - 1}} \times$$
$$\times\,[3f(\xi E) + 7\xi E f'(\xi E) + 2\xi^2 E^2 f''(\xi E)]\,\mathrm{d}\xi\,. \tag{8}$$

The most convenient form for computation is obtained via the change of variable $x = \sqrt{(\xi - 1)}$ and integrating the first two terms in the integrand by parts, giving the form

$$N(E) = \frac{2}{3\pi\beta}\mathscr{I}(\varepsilon_1)\,E^{5/2}\int_{0}^{\infty} f''((1 + x^2)\,E)\,[3 - 12x^2 + x^4]\,\mathrm{d}x$$

where $f(\varepsilon)$ has been assumed to have the property $f(\varepsilon)\to 0$ more rapidly than ε^{-2} as $\varepsilon \to \infty$; this will be seen in Section 5 to be valid in practical cases.

$N(E)$ is most useful, numerically, in electrons per keV. The appropriate numerical form of the above solution is

$$N(E) = 1.20 \times 10^{41}\,\frac{\mathscr{I}(\varepsilon_1)}{n_0}\,E^{5/2}\int_{0}^{\infty} f''((1 + x^2)\,E) \times$$
$$\times\,[3 - 12x^2 + x^4]\,\mathrm{d}x \tag{9}$$

where E is now expressed in keV, f is in keV^{-1}, n_0 in cm^{-3} and $\mathscr{I}$ in $\text{cm}^{-2}\ \text{s}^{-1}$.

This $N(E)$ is the instantaneous effective electron spectrum present in the X-ray emitting region subject to the definitions of $N(E)$ and n_0 in (2) and (3), regardless of the model concerned. Numerical integration of (9) at each instant t for any observed $I(\varepsilon, t)$ gives the dynamic electron spectrum $N(E, t)$.

The actual shape of the instantaneous $N(E)$ is determined uniquely from the observations by (9) but the numerical value of $N(E)$ (i.e. the scale) depends on the adoption of a value for n_0. This will depend on the location of the X-ray region in the model concerned. In an impulsive injection model n_0 is determined by the rate of decay of the burst which is, by hypothesis, due to degradation of the non-thermal electrons in the ambient plasma. n_0 is then roughly the density required to give an e-folding energy decay time, for an electron of, say, 50 keV, equal to the observed decay time of the X-ray burst at this X-ray energy. In a continuous injection model, the decay of the burst is not a reflection of ambient conditions at all and so does not determine n_0. If $N(E)$ is required then n_0 has to be estimated from the presumed location of the X-ray region in the model, taking into account the fact that n_P is highly non-uniform as will be seen. However, in continuous injection models it is not $N(E)$ but the injection spectrum $F(E)$ discussed in the next section which is important and $F(E)$ is in fact independent of n_0 and of non-uniformities in n_P.

4. Determination of the Electron Injection Spectrum in Continuous Injection Models

As was pointed out in the Introduction, if it were assumed that the non-thermal electrons were impulsively injected *in situ*, the spectrum $N(E)$ deduced in Section 3 would necessarily be the same, at the moment of injection, as the effective spectrum of the injected electrons at that moment. But, in continuous injections model, a high energy electron stream is continuously injected into a dense region where the electrons are rapidly stopped by interaction with the ambient plasma, simultaneously producing bremsstrahlung X-rays. The instantaneous effective spectrum $N(E)$ is then determined by the instantaneous flux spectrum $F(E)$ (electrons/unit E range/s) emerging from the acceleration region and by the modification of this latter distribution by the stopping processes within the X-ray region. Thus, in the usual bremsstrahlung terminology, the target is 'thick'. It is then $F(E)$, rather than $N(E)$, which is important in this type of model.

Since the X-ray emitting region is *optically* thin to hard X-rays for reasonable values of $n_0\,(\lesssim 10^{17}\ \text{cm}^{-3})$ – (Ohki, 1969), the only information needed for analysis of this thick target problem is a knowledge of the elementary stopping and scattering processes for the high energy electrons. At non-relativistic energies, collisional processes certainly dominate over synchrotron losses and bremsstrahlung emission itself for the conditions (i.e. high ambient density) prevailing in continuous injection models as is shown by, e.g., Takakura and Kai (1966). Generation of cooperative plasma waves in the ambient plasma is the only remaining candidate for domination of the (non-thermal) electron energy loss process.

None of the authors previously mentioned has considered this process in connection with the passage of directed electron streams through flare plasma despite the fact that electron beams may be entirely prevented from penetrating a plasma by just such wave generation (Langmuir, 1925; Tsytovich, 1966). However, for a *dilute* beam of high energy electrons of number density equal to a fraction λ of the ambient density, the effect of the instability is to considerably randomise the velocities in the beam, after which the electrons may pass stably through the ambient plasma, having lost only a fraction $\lambda^{1/3}$ of their energy in heating the ambient plasma (Sweet, 1970). The electron stream then emerging from the acceleration region and passing into the (denser) X-ray emitting region, on continuous models, has thus the spectrum produced by the accelerating mechanism modified by the above interaction with the ambient plasma in the acceleration region. It is this emergent spectrum which may be deduced from the X-ray observations as discussed below.

In the passage of the stream of electrons through the X-ray emission region, individual particle collisions, rather than plasma interaction, thus dominate the stopping process, giving the thick target calculation an especially simple form as will be seen. The randomisation of the electron velocities on emerging from the acceleration region as just mentioned largely removes directional effects in the X-ray emission. With the idealisation that the degree of ionisation x is uniform in the X-ray emitting region, it is now shown that the X-ray emission for a prescribed continuous injection spectrum of high energy electrons is independent of the value of n_0, of the spatial distribution of the ambient plasma comprising n_0 and of the paths of the electrons within the X-ray emitting region. The whole flare plasma will be near total ionisation down to the Hα flare region where $n_0 \approx 10^{13}$ cm^{-3}, (Fritzová-Švestková and Švestka, 1967), which is about the maximum depth to which a 50 keV electron may penetrate the solar atmosphere. Thus a uniform degree of ionisation set equal to unity will be a good approximation.

Collisional losses of high energy electrons on ambient protons are a factor of order 10^3 less important than losses on ambient electrons, both free and bound (e.g. Schatzman, 1965). Again neglecting contributions from elements other than hydrogen in the flare plasma, the energy loss rate for an average electron of energy E, (neglecting the statistical spread in this rate for a given E), may thus be written

$$\frac{\mathrm{d}E}{\mathrm{d}t} = - n_p v E \left[x Q_{ee} + (1 - x) Q_{eH} \right]$$

where, it will be recalled, n_p is the total number density of ambient protons, both free and in atoms, at the point considered.

Q_{ee} and Q_{eH} are the energy loss cross-sections for electrons incident on free and hydrogen-bound electrons respectively. With $x = 1$ everywhere only Q_{ee} is required and the original treatment by Bohr (1915) is appropriate, giving

$$Q_{ee} = \frac{2\pi e^4}{E^2} \Lambda_{ee} \quad \text{where} \quad \Lambda_{ee} = \log \left(\frac{E}{e^2} b_0 \right)$$

where e is the electronic charge (esu) and b_0 is the maximum impact parameter, usually set equal to the Debye length. For highly supersonic electrons, however, the appropriate value of b_0 is of order v/ν_0, v being the supersonic electron velocity and ν_0 the ambient plasma frequency (Landau and Lifshitz, 1960), while if a magnetic field is present b_0 is of the order of the Larmor radius (of the beam electron) if this is smaller than v/ν_0. For reasonable values of n_0 and of the magnetic field in the ranges involved these b_0 values are both of order 1 cm while e^2/E is of order 10^{-11} cm so that Λ_{ee} is insensitive to the b_0 chosen in any case. Taking Λ_{ee} as effectively constant over the 20–100 keV range at its value for $E = 50$ keV then the average energy loss rate is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{55.7\pi e^4}{E} n_P v . \tag{10}$$

A decelerated electron of initial kinetic energy E_0 produces X-rays of energy ε so long as its energy remains greater than ε, the total number of such photons, per unit ε, emitted by an average electron during this braking being

$$\nu(\varepsilon, E_0) = \int_{E=E_0}^{E=\varepsilon} Q_\varepsilon(E)\, n_P v(E)\, \mathrm{d}t$$

where $E = E(t)$ and $n_P = n_P(t)$ along the electron path i.e., using (10),

$$\nu(\varepsilon, E_0) = \frac{1}{55.7\pi e^4} \int_\varepsilon^{E_0} E Q_\varepsilon(E)\, \mathrm{d}E .$$

Thus $\nu(\varepsilon, E_0)$ is clearly independent of the ambient plasma density distribution n_P and of the electron path through this distribution as stated above.

If $F(E_0)$ electrons per unit E_0 are being injected into the X-ray emitting region per second at some instant and if the ambient plasma is dense enough that the emission of ν photons by a single electron is effectively instantaneous compared to the time scale for variation in F, then the total photon emission rate from the region at that instant is

$$\int_\varepsilon^\infty F(E_0)\, \nu(\varepsilon, E_0)\, \mathrm{d}E_0 \ \text{(photons/sec per unit } \varepsilon \text{ range)}$$

where the statistical spread of $\nu(\varepsilon, E_0)$ for a given E_0 is neglected.

Setting $55.7\pi e^4 = K =$ const and substituting for ν and for $Q_\varepsilon(E)$, the mean photon count rate at the Earth's distance is

$$I(\varepsilon) = \frac{\beta}{n_0} \sqrt{\frac{m}{2}} \frac{1}{K\varepsilon} \int_\varepsilon^\infty F(E_0) \left(\int_\varepsilon^{E_0} \log \frac{1 + \sqrt{1 - \varepsilon/E}}{1 - \sqrt{1 - \varepsilon/E}}\, \mathrm{d}E \right) \mathrm{d}E_0$$

where β is as before and I is in photons/cm^2 sec per unit ε range, the relationship being independent of n_0 since $\beta \sim n_0$.

Reversing the order of integration and setting $\varphi(E) = \int_E^\infty F(E_0)\,\mathrm{d}E_0$ this equation may be written as

$$\int_\varepsilon^\infty \varphi(E) \log \frac{1 + \sqrt{1 - \varepsilon/E}}{1 - \sqrt{1 - \varepsilon/E}}\,\mathrm{d}E = \sqrt{\frac{2}{m}}\,\frac{Kn_0}{\beta}\,J(E)$$

which is identical to (5) with $\varphi(E)$ replacing $N(E)/\sqrt{E}$ and $(\sqrt{(m/2)})\,(\beta/Kn_0)$ replacing β.

By (6) the solution is then

$$\sqrt{E}\varphi(E) = \sqrt{\frac{2}{m}}\,\frac{Kn_0}{2\pi\beta}\,\frac{1}{E}\int_E^\infty \frac{\varepsilon H(\varepsilon)}{\sqrt{\varepsilon - E}}\,\mathrm{d}\varepsilon$$

where $H(\varepsilon) = 3J'(\varepsilon) + 2\varepsilon J^\circ(\varepsilon)$
i.e.

$$\varphi(E) = \int_E^\infty F(E_0)\,\mathrm{d}E_0 = \frac{Kn_0}{2\pi\beta}\sqrt{\frac{2}{m}}\int_1^\infty \frac{\xi H(\xi E)\,\mathrm{d}\xi}{\sqrt{\xi - 1}}$$

where $\xi = \varepsilon/E$.

Differentiating with respect to E,

$$F(E) = -\varphi'(E) = -\frac{Kn_0}{2\pi\beta}\sqrt{\frac{2}{m}}\int_1^\infty \frac{\xi^2 H'(\xi E)}{\sqrt{\xi - 1}}\,\mathrm{d}\xi$$

Thus

$$F(E) = -\frac{Kn_0}{2\pi\beta}\sqrt{\frac{2}{m}}\,\mathscr{I}(\varepsilon_1)\int_1^\infty \frac{\xi^2}{\sqrt{\xi - 1}} \times$$
$$\times \left[10f'(\xi E) + 11\xi E f''(\xi E) + 2\xi^2 E^2 f'''(\xi E)\right]\mathrm{d}\xi \qquad (11)$$

in the same units as before, f being defined as for (8). Reducing the integral in the same manner as for Equation (8), the solution is then, numerically,

$$F(E) = -1.78 \times 10^{32}\,\mathscr{I}(\varepsilon_1)\,E^2 \int_0^\infty f'''((1 + x^2)\,E) \times$$
$$\times \left[15 - 105x^2 + 25x^4 + x^6\right]\mathrm{d}x \qquad (12)$$

where $F(E)$ is in electrons per keV per second when f is in keV^{-1} and $\mathscr{I}$ is in photons/cm^2 sec.

Thus $F(E, t)$ may be calculated, on a continuous injection model, from the observed dynamic spectrum $I(\varepsilon, t)$, the relationship being independent of the ambient plasma distribution. As $F(E, t)$ is a direct reflection of the temporal behaviour of conditions in the electron acceleration region, its calculation is essential in the study of the accelerating mechanism.

Certain important differences between F and N, also of relevance to more general flare properties than the hard X-ray burst alone, are illustrated in the next section by means of a particular example.

5. Comparison of $F(E)$ and $N(E)$ for a Power Law X-Ray Spectrum

In the 20–100 keV range, $I(\varepsilon)$ can be adequately represented near the time of maximum burst intensity by a power law $\sim \varepsilon^{-\gamma}$ (e.g. Frost, 1969) where $\gamma \approx 2$–4. The spectrum steepens greatly above 100 keV but examination of Equation (7) shows that no great error in $N(E)$ for $E < 100$ keV will be incurred by extrapolating the same $\varepsilon^{-\gamma}$ law above 100 keV in the integrand, due to the steepness of $I(\varepsilon)$.

$f(\varepsilon)$ is then given by

$$f(\varepsilon) = \frac{(\gamma - 1)}{\varepsilon_1} \left(\frac{\varepsilon_1}{\varepsilon}\right)^{\gamma}. \tag{13}$$

Substituting for $f(\varepsilon)$ into Equations (8) and (11), then changing the variable from ξ to $u = 1/\xi$, N and F may, in this case, be simplified to

$$N(E) = 3.61 \times 10^{41} \gamma (\gamma - 1)^3 \, B(\gamma - \tfrac{1}{2}, \tfrac{3}{2}) \frac{\mathscr{I}(\varepsilon_1)}{n_0 \varepsilon_1} \left(\frac{\varepsilon_1}{E}\right)^{\gamma} \sqrt{E} \tag{14}$$

and

$$F(E) = 2.68 \times 10^{33} \gamma^2 (\gamma - 1)^3 \, B(\gamma - \tfrac{1}{2}, \tfrac{3}{2}) \frac{\mathscr{I}(\varepsilon_1)}{\varepsilon_1^2} \left(\frac{\varepsilon_1}{E}\right)^{\gamma + 1} \tag{15}$$

where the units are the same as in (8) and (11), ε_1 is in keV and B is the Beta function

$$\text{viz. } B(p, q) = \int_0^1 u^{p-1} (1 - u)^{q-1} \, \mathrm{d}u \, .$$

Let ε_1 be the observational threshold for a particular set of X-ray burst observations. Then it is only possible to determine, from the observations, the numbers of non-thermal electrons of energies $E \geqslant \varepsilon_1$, though the non-thermal electron component may in fact extend below $E = \varepsilon_1$ as discussed below. Let the total number of non-thermal electrons of energy $\geqslant \varepsilon_1$ be $\eta(\varepsilon_1)$. For the power law spectrum of Equation (14) the number of non-thermal electrons of energy $\geqslant E$, i.e. $\eta(E)$, may readily be expressed in terms of $N(E)$, for

$$\eta(E) = \int_E^{\infty} N(E') \, \mathrm{d}E' = \frac{E}{(\gamma - \frac{3}{2})} N(E)$$

Thus

$$\eta(\varepsilon_1) = \frac{\varepsilon_1}{(\gamma - \frac{3}{2})} N(\varepsilon_1).$$

Likewise the total influx rate $\mathscr{F}(E)$ (electrons per second) of electrons of energy $\geqslant E$ may be written, for continuous injection models, as

$$\mathscr{F}(E) = \int_E^\infty F(E')\,\mathrm{d}E' = \frac{E}{\gamma} F(E)$$

so that

$$\mathscr{F}(\varepsilon_1) = \frac{\varepsilon_1}{\gamma} F(\varepsilon_1).$$

The differential and integral electron spectra determined above are then conveniently described as fractional forms as follows

$$f_1 = \frac{N(E)}{\eta(\varepsilon_1)} = \frac{\gamma - \frac{3}{2}}{\varepsilon_1} \left(\frac{\varepsilon_1}{E}\right)^{\gamma - 1/2} \tag{16a}$$

$$f_2 = \frac{F(E)}{\mathscr{F}(\varepsilon_1)} = \frac{\gamma}{\varepsilon_1} \left(\frac{\varepsilon_1}{E}\right)^{\gamma + 1} \tag{16b}$$

$$f_3 = \frac{\eta(E)}{\eta(\varepsilon_1)} = \left(\frac{\varepsilon_1}{E}\right)^{\gamma - 3/2} \quad \text{and} \quad f_4 = \frac{\mathscr{F}(E)}{\mathscr{F}(\varepsilon_1)} = \left(\frac{\varepsilon_1}{E}\right)^{\gamma}.$$

f_1 and f_2 are shown in Figure 1 for typical values $\varepsilon_1 = 20$ keV and $\gamma = 4$.

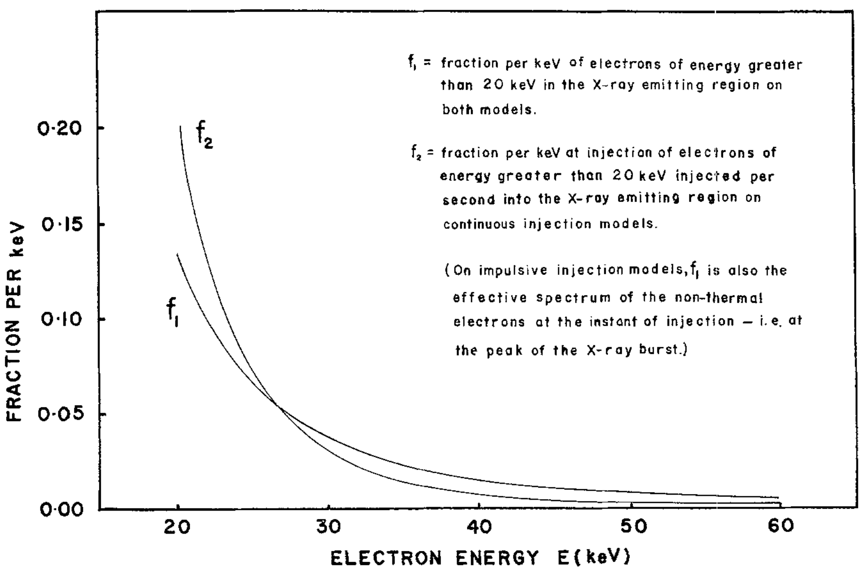

Fig. 1. Effective differential energy spectra of energetic electrons (> 20 keV) required to produce a hard X-ray burst with power law X-ray spectrum $\sim\varepsilon^{-4}$ (at photon energies $\varepsilon > 20$ keV) at the burst peak.

f_1 is the effective differential electron spectrum within the X-ray emitting volume (on either model) as a fraction of the total number of electrons in the observed energy range; f_2 is the differential spectrum, *at injection*, of the electrons injected per second into the X-ray emitting volume on continuous injection models. As has already been discussed, f_1 is also the effective spectrum of the non-thermal electrons at the instant of injection (i.e. at the burst peak) on impulsive injection models.

It is clear from Figure 1 that f_2 is a much steeper spectrum than f_1 (1.5 powers steeper in the case of a power law X-ray spectrum); the importance of this difference to the flare as a whole is considered below.

6. Discussion of the Energetics of the Non-Thermal Electrons

The greater steepness of the spectral distribution $F(E)$ of continuously injected electrons compared to the distribution $N(E)$ within the X-ray region itself is, of course, due to collisional loss of low energy electrons from $F(E)$ in traversing the X-ray region. In assessing the energy input in the form of non-thermal electrons it is thus clearly necessary to consider $F(E)$ and not $N(E)$ in continuous injection models.

Neupert (1968), Cheng (1970), and the present author have been investigating flare-heating models in which the energy deposited in the ambient flare plasma comes from collisional energy loss from the low energy end of the non-thermal electron distribution responsible for the hard X-ray burst. In support of such models, Neupert has shown that the integral over time of non-thermal emission both in microwaves (Neupert, 1968) and in hard X-rays (Neupert, 1970) up to a given instant is proportional to the thermal energy of the flare at that instant as estimated by soft X-ray line emission. In assessing whether the total energy of the non-thermal electrons is adequate for the flare, Neupert (1968) finds that it is necessary to extrapolate the non-thermal electron spectrum below the observed energy range, in fact down to less than 10 keV, to yield the required energy. This requirement, based on the electron spectrum $\sim E^{-5}$ above 250 keV deduced by Takakura and Kai (1966) from a microwave burst, is unsatisfactory on two accounts. Firstly, the numbers of electrons estimated from microwave bursts are consistently around 10^{-3} times smaller than those estimated from hard X-ray bursts. This is due to microwave absorption effects (Takakura and Kai, 1966; Holt and Ramaty, 1969) and to non-uniformity of the magnetic field (Peterson and Winckler, 1959). Secondly, electron spectra are typically much flatter below about 100 keV than at 250 keV (as noted in Section 5) thus making extrapolation to 10 keV of the spectrum at 250 keV unsound. A proper estimate of the non-thermal electron energy content of a flare must be based on the hard X-ray spectrum below the energy where the spectrum steepens, i.e. around 100 keV in most bursts.

Cline *et al.* (1968) have observed a hard X-ray burst in the 80–500 keV X-ray energy range associated with a 2+ flare. They measured the intensity to be $\mathscr{I}(\varepsilon_1) \approx 300$ photons/cm^2 sec for $\varepsilon_1 = 80$ keV at the time of burst maximum with $\mathscr{I}(\varepsilon_1)$

decaying in an *e*-folding time of about 100 s. The spectrum of this burst in the 80–136 keV range was approximately a power law with $\gamma \approx 3.5$. Integration of Equations (14) and (15) over energy E implies the total energy injected in the form of electrons of energies above 80 keV to be 10^{30} erg if an impulsive model is adopted (n_0 being inferred from the decay time) and 1.5×10^{30} erg if a continuous model is taken. Either of these energies is more than adequate to provide the Hα energy of a 2+ flare, i.e. about 5×10^{29} erg (Dizer, 1969), although marked by less than the likely total flare energy including XUV emission and mass motion. If this total is conservatively estimated at 2×10^{31} erg (cf. Bruzek, 1967) then it is necessary to extrapolate the electron spectrum below the observational 80 keV limit if the total energy is to be provided by non-thermal electrons. This extrapolation of the injection spectrum need only be taken down to 25 keV if a continuous model is adopted but, owing to the much flatter inferred electron spectrum at injection, extrapolation to less than 5 keV is needed on an impulsive model.

The safe lower bound to which such extrapolation below the observed X-ray energy range may be taken is determined by the requirement that contributions of *thermal* X-ray emission from the hot flare plasma must not be included in the total energy of the non-thermal electrons as assessed from the non-thermal X-rays they produce. A conservative estimate of this total will be obtained if the extrapolation is cut off where the thermal and non-thermal X-ray contributions are estimated to be equal. Takakura (1969) finds that thermal contributions are significant up to X-ray energies in the range 20 to 70 keV depending on the extent and temperature of the hot flare plasma. The polarisation measurements of Tindo *et al.* at 0.8 Å suggest however that a non-thermal electron component may exist down to 15–20 keV.

Comparing these estimates of the contributions of thermal and non-thermal electrons to the observed X-ray emission it is clear that, for a 2+ flare, X-ray energies below about 30 keV are in a region of increasing uncertainty as to the thermal or non-thermal nature of their source.

In summarizing the situation, it would appear from the necessary 5 keV extrapolation limit derived above that the energy available as non-thermal electron energy can scarcely be adequate for the whole flare if the hard X-ray burst is interpreted as due to impulsively injected electrons. However, with the steeper inferred injection spectrum of a continuously injected electron stream producing the same hard X-ray burst, the much safer extrapolation limit of 25 keV would provide ample energy for the entire flare in the form of injected non-thermal electrons.

Acknowledgements

I would like to express my gratitude to Professor P. A. Sweet for helpful and critical discussion of the manuscript and also to Drs C. C. Cheng, T. A. Chubb, S. R. Kane and W. M. Neupert for making their results available to me prior to their publication and to Dr E. Haug for comments on my preprint.

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