MAGNETIC RECONNECTION IN A HIGH-TEMPERATURE PLASMA OF SOLAR FLARES

IV. *Resistive Tearing Mode in Non-Neutral Current Sheets*

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Abstract. Dispersion relations for the resistive tearing instability are analytically found in the hydromagnetic approximation for a current sheet with a small normal component of the magnetic field. A strong stabilizing influence of the normal component on the development of the tearing instability is shown to exist. These results are also obtained from physical considerations, and so a simple interpretation of the stabilization effect of the normal component is given. The results of the present paper are compared with those of previous works on the topic, and the previous negative results are explained.

1. Introduction

Current sheets play an important role in space physics and laboratory plasma physics (see, e.g., Priest, 1976, 1982, 1985; Syrovatskii, 1981; Hones, 1984). In particular they can be energy sources in flare-like processes in the solar atmosphere. The problem of the stability of current sheets is of special importance. On the one hand, in the fundamental work of Furth, Killen, and Rosenbluth (1963) the tendency of the sheet current to break up into a set of parallel current filaments was shown. It is the so-called tearing instability (see also Coppi, Laval, and Pellat, 1966; Drake and Lee, 1977). On the other hand, laboratory and space researches show that current sheets can be sufficiently thin and wide for a long time.

To explain this discrepancy the hypothesis that the tearing mode is stabilized by a small transverse magnetic field $B_\perp$, i.e., perpendicular to the sheet, was suggested. This effect was examined in the case of a collisionless plasma by Schindler (1974), Galeev and Zeleny (1976), Schindler and Birn (1978), Coroniti (1980); and the stabilizing effect of $B_\perp$ has been demonstrated. The hypothesis that the resistive tearing mode is stabilized by a normal component $B_\perp$ has been pointed out by Pneuman (1974) and Schindler (1976).

Later on, however, in papers of Bulanov, Sakai, and Syrovatskii (1979), Janicke (1980, 1982) the effect was asserted to be totally lacking in the hydromagnetic approximation. Moreover, it was even concluded in Bulanov, Sakai, and Syrovatskii (1979) that the transverse field $B_\perp$ results in the enhancement of instability. Thus the stabilizing effect of a transverse field was suggested to be totally lacking in the hydromagnetic approximation. This idea was supported by the phenomenon of the break-down of the totally stable states of a collisionless non-neutral (i.e., with the transverse field) sheet when collisions are allowed (Coroniti, 1980; Zeleny and Taktakishvili, 1981).

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In the present paper (and also in papers of Verneta and Somov, 1988) it is shown that, contrary to the above opinion, a transverse magnetic field exerts a strong stabilizing effect in the hydromagnetic approximation.

2. Formulation and Solution of the Problem

The present consideration is based on the equations of ideal magnetohydrodynamics for an incompressible plasma with finite conductivity:

$$\mathrm{rot}\left[\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\nabla\mathbf{V}\right)\right] = \mathrm{rot}\left[\frac{1}{c}\,\mathbf{j}\times\mathbf{B}\right], \tag{1}$$

$$\mathrm{rot}\,\mathbf{B} = \frac{4\pi}{c}\,\mathbf{j}, \tag{2}$$

$$\mathrm{div}\,\mathbf{B} = \mathrm{div}\,\mathbf{V} = 0, \tag{3}$$

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t}, \tag{4}$$

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}\,[\mathbf{B}\times\mathbf{B}]\right), \tag{5}$$

$$\frac{\partial \eta}{\partial t} + \mathbf{V}\,\mathrm{grad}\,\eta = 0. \tag{6}$$

In Equations (1)–(6) we made use of the following designations: c is the velocity of light, **E** and **B** are electric and magnetic fields correspondingly, **j** is the density of the electric current, σ is the conductivity (we will also use the quantity $\eta = c^2/\sigma$), **V** and ρ are the velocity and density of plasma. It follows from (1–6) that

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}[\mathbf{V}\times\mathbf{B}] - \mathrm{rot}\left[\frac{\eta}{4\pi}\,\mathrm{rot}\,\mathbf{B}\right], \tag{7}$$

$$\mathrm{rot}\left[\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{B}\nabla\mathbf{V}\right)\right] = \mathrm{rot}\left[\frac{1}{4\pi}\,(\mathrm{rot}\,\mathbf{B}\times\mathbf{B})\right]. \tag{8}$$

Now let us pick out the unperturbed values of the magnetic field and velocity and study the behaviour of small (strictly speaking, the infinitesimal) disturbances.

The unperturbed magnetic field,

$$B_0(y) = B_{0x}(y)\mathbf{e}_x + B_{\perp}e_y, \tag{9}$$

depends only on the y-coordinate which is perpendicular to the sheet and has a nonzero transverse (i.e., perpendicular to the sheet) component $B_\perp$ = const (see Figure 1). The choice of the function $B_{0x}(y)$ will be made below.

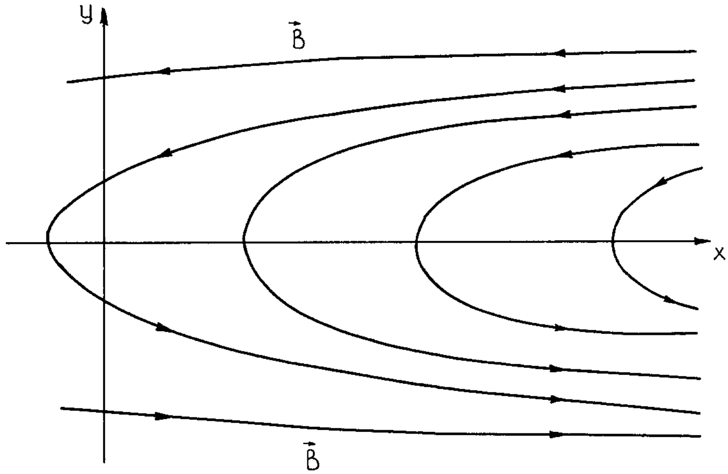

Fig. 1. A current sheet with a nonzero transverse (i.e., perpendicular to the sheet) magnetic field component $B_\perp$.

Further, we choose, as usually, a zero value for the undisturbed velocity distribution:

$$V_0 = 0\,. \tag{10}$$

The introduction of the transverse field is known to involve an additional sheet-aligned force that causes plasma to spread. On the one hand, we may formally assume that this force is neutralized by some other force of non-electromagnetic nature which does not affect the investigated phenomenon or that the time of the instability is smaller than the time of plasma outflow (see Section 5). On the other hand, the characteristic stabilizing effect which is briefly outlined in Section 5 is associated with the outflow effect. The outflow does not effect the stabilization of the transverse component (in the linear approximation), and for the sake of simplicity we put $V_0 = 0$ (as is usually done). By this assumption the unperturbed magnetic field is meant to satisfy the following condition:

$$\left\{\mathrm{rot}\left[\frac{\eta_0}{4\pi}\,\mathrm{rot}\,\mathbf{B}_0\right]\right\} = 0\,, \tag{11}$$

as in Furth, Killen, and Rosenbluth (1963). Below we shall not make use of it. Equilibria of this type have been described in detail by Soop and Schindler (1973), Birn, Sommer, and Schindler (1975), Janicke (1982).

The disturbed values are sought in the following form:

$$f_1(\mathbf{r}, t) = f_1(y)\exp(\omega t + ik_x x)\,. \tag{12}$$

In this case a set of equations for disturbed quantities, denoted by the subscript 1, is reduced to

$$\omega \,\mathrm{rot}(\rho_0 \mathbf{V}_1) = \mathrm{rot}\left(\frac{1}{4\pi}\,\{(\mathbf{B}_0\nabla)\mathbf{B}_1 + (\mathbf{B}_1\nabla)\mathbf{B}_0\}\right), \tag{13}$$

$$\omega \mathbf{B}_1 = (\mathbf{B}_0\nabla)\mathbf{V}_1 - (\mathbf{V}_1\nabla)\mathbf{B}_0 - \frac{1}{4\pi}\,\{[\mathrm{grad}\,\eta_0 \times \mathrm{rot}\,\mathbf{B}_1] - \eta_0 \Delta \mathbf{B}_1 + $$
$$+ [\mathrm{grad}\,\eta_1 \times \mathrm{rot}\,\mathbf{B}_0] - \eta_1 \Delta \mathbf{B}_0\}, \tag{14}$$

$$\mathrm{div}\,\mathbf{B}_1 = \mathrm{div}\,\mathbf{V}_1 = 0\,, \tag{15}$$

$$\omega \eta_1 + \mathbf{V}_1\,\mathrm{grad}\,\eta_0 = 0\,. \tag{16}$$

There are two equations in this set (like when $B_\perp = 0$ in Furth, Killen, and Rosenbluth) comprising terms V_{1y} and B_{1y} only. Let us write these equations down in the dimensionless form:

$$\frac{1}{\alpha^2}\,(\tilde{\rho}\,W')' = \tilde{\rho}\,W - \frac{S^2}{p}\left\{\alpha^2 F\psi + F''\psi - F\psi'' + \frac{i\xi}{\alpha}\,[\psi'' - \alpha^2\psi]'\right\}, \tag{17}$$

$$\frac{\psi''}{\alpha^2} = \psi\left(1 + \frac{p}{\tilde{\eta}\alpha^2}\right) + \frac{W}{\alpha^2}\left(\frac{F}{\tilde{\eta}} + \frac{\tilde{\eta}'F'}{p\tilde{\eta}}\right) + \frac{W'}{i\alpha^3}\,\frac{\xi}{\tilde{\eta}}\,. \tag{18}$$

Here

$$\psi = \frac{B_{1y}}{B}\,,\qquad B = |B_{0x}(a)|\,,\qquad W = -iV_{1y}k\tau_R\,,\qquad \mu = \frac{y}{a}$$

(a is thickness of the sheet),

$$F = \frac{\mathbf{k}\mathbf{B}_0}{kB}\,,\qquad k = (\mathbf{k}^2)^{1/2}\,,\qquad \alpha = ka\,, \tag{19}$$

$$\tau_R = \frac{4\pi a^2}{\langle\eta\rangle}\,,\qquad \tau_A = \frac{a(4\pi\,\langle\rho\rangle)^{1/2}}{B}\,,\qquad S = \frac{\tau_R}{\tau_A}\,,\qquad p = \omega\tau_R\,,$$

$$\tilde{\eta} = \frac{\eta_0}{\langle\eta\rangle}\,,\qquad \tilde{\rho} = \frac{\rho_0}{\langle\rho\rangle}\,,\qquad \xi = \frac{B_\perp}{B}\,.$$

Equations (17)–(19) differ from the corresponding equations in Furth, Killen, and Rosenbluth (1963) in the terms with ξ. Equation (17) is re-arranged with the help of

Equation (18) in the following way:

$$\frac{1}{\alpha^2}(\tilde{\rho}W')' = W\left[\tilde{\rho} + \frac{S^2F}{p}\left(\frac{F}{\tilde{\eta}} + \frac{\tilde{\eta}'F'}{p\tilde{\eta}}\right)\right] +$$

$$+ S^2\psi\left(\frac{F}{\tilde{\eta}} - \frac{F''}{p}\right) + \frac{S^2\xi F}{pi\alpha\tilde{\eta}}W' -$$

$$- \frac{iS^2}{\alpha p}\left\{\xi\left(\frac{p}{\tilde{\eta}}\psi + W\left(\frac{F}{\tilde{\eta}} + \frac{\tilde{\eta}'F'}{p\tilde{\eta}}\right)\right) + \frac{\xi^2 W'}{i\alpha\tilde{\eta}}\right\}'. \tag{20}$$

It may be noted that the condition (11) just as in Furth, Killen, and Rosenbluth is reduced to

$$\left(\frac{\tilde{\eta}'}{\tilde{\eta}}\right)F' = -F''. \tag{21}$$

To solve the problem of tearing instability the so-called singular perturbations method is usually employed. Two regions of a current sheet are singled out: the outer one and the inner one.

The effect of finite conductivity is negligible in the outer layer. The external solution possesses a singularity in a small internal region where this contribution is essential. The solution is sought in this inner region with finite conductivity included. Then the solutions (strictly speaking, their asymptotes) are joined. We examine separately each of these regions.

2.1. Outer Region

The magnetic field is 'frozen' to plasma in the outer region. This situation corresponds to the limit $S \to \infty$. We get from Equation (17)

$$\psi'' - \left(\alpha^2 + \frac{F''}{F}\right)\psi - \frac{i\xi}{\alpha F}(\psi'' - \alpha^2\psi)' = 0. \tag{22}$$

After that we choose the following distribution of the reconnection component of the magnetic field:

$$F = \begin{cases} -1, & \mu < -1, \\ \mu, & -1 < \mu < 1, \\ 1, & \mu > 1. \end{cases} \tag{23}$$

In this case the general solution of Equation (22) can be easily found:

$$\psi = A_1 e^{-\alpha\mu} + B_1 e^{\alpha\mu} - \frac{c_1}{\left(1 + \frac{1}{\xi^2}\right)\alpha^2} e^{i\alpha\mu/\xi}, \quad \mu < -1;$$

$$\psi = A_2 e^{-\alpha\mu} + B_2 e^{\alpha\mu} - \frac{c_2}{2\alpha} e^{-\alpha\mu} \int_0^{\mu} e^{-(i\alpha t^2/2\xi) + \alpha t}\, dt +$$

$$+ \frac{c_2}{2\alpha} e^{\alpha\mu} \int_0^{\mu} e^{-(i\alpha t^2/2\xi) - \alpha t}\, dt, \quad |\mu| < 1; \tag{22a}$$

$$\psi = A_3 e^{-\alpha\mu} + B_3 e^{\alpha\mu} - \frac{c_3}{\left(1 + \frac{1}{\xi^2}\right)\alpha^2} e^{-i\alpha\mu/\xi}, \quad \mu > 1.$$

For the chosen form of the function F we have

$$F'' = \delta(\mu + 1) - \delta(\mu - 1). \tag{24}$$

Allowing for (24) we obtain the conditions of 'joining' from Equation (22):

$$\psi \Big|_{-1-0}^{-1+0} = 0, \qquad \psi' \Big|_{-1-0}^{-1+0} = 0$$

and

$$\frac{-i\xi}{\alpha} \psi'' \Big|_{-1-0}^{-1+0} = \psi(-1), \tag{25}$$

at the point $\mu = -1$ and similar conditions at the point $\mu = 1$. Also ψ is an even function:

$$\psi(\mu) = \psi(-\mu). \tag{26}$$

Further, we make use of the condition that the solution is limited at infinity:

$$|\psi| < \text{const} \tag{27}$$

as $\mu \to \infty$. Finally we obtain from (22)–(27) for $\xi < \alpha$ that the derivative ψ has an effective jump at point $\mu = 0$.

This fact enables one to conclude that the high density current flows in the plane $\mu = 0$ associated with the instability development. In this region the dissipation effects become important. The effective jump of ψ'/ψ in the vicinity of the $\mu = 0$ equals

$$\Delta' = 2/\alpha \tag{28}$$

under the condition of $\xi < \alpha < 1$ (usually accepted in works on this topic), as in Furth, Killen, and Rosenbluth (1963).

We point out here, that the value $\Delta'/2$ is the ratio of corresponding coefficients in the asymptotics at $\mu \to \pm 0$ for the solution in the outer region. Below the ratio of the coefficients will be calculated for the asymptotic solution in the inner region.

2.2. Inner region. '$\psi =$ const' approximation

Let us examine the vicinity of the point $\mu = 0$. Within the area of the width ε_0 the 'freeze-in' condition is violated, $S < \infty$. We write down a set of Equations (17) and (18) with new designations:

$$u^{0\prime\prime} + \frac{i}{2} \tilde{\xi} \theta u^0 + \left(\frac{i}{4} \tilde{\xi} - \alpha^2 \varepsilon_0^2 \Delta\right) u^0 - \frac{\theta^2}{4} u^0 = \theta\psi - i \tilde{\xi} \psi' , \tag{29}$$

$$\psi'' - \alpha^2 \varepsilon_0^2 \psi = \varepsilon_0 \Omega_0 [4\psi + \theta u^0] + \frac{p\varepsilon_0^2}{4i} \tilde{\xi} u^{0\prime} . \tag{30}$$

Here

$$\theta = \frac{\mu}{\varepsilon_0} , \qquad u^0 = \frac{4\varepsilon_0}{p} W , \qquad \Omega_0 = \frac{p\varepsilon_0}{4} , \qquad \tilde{\xi} = \frac{\xi}{\alpha \varepsilon_0^2} , \tag{31}$$

$$\varepsilon = \left[\frac{p}{4\alpha^2 S^2}\right]^{1/4} , \qquad \Delta = \left(1 + \frac{\xi^2 S^2}{p}\right)^{-1} , \qquad \Omega = \frac{p\varepsilon}{4} , \tag{32}$$

$$\varepsilon_0 = \varepsilon \Delta^{-1/4} = \left[\frac{p}{4\alpha^2 S^2}\right]^{1/4} \left(1 + \frac{\xi^2 S^2}{p}\right)^{1/4} . \tag{33}$$

At $\xi = 0$ these equation coincide with the equations deduced in Furth, Killen, and Rosenbluth (1963). We shall search $u^0(\theta)$ in the form of a series in terms of normalized Hermite functions

$$u^0 = \sum_{n=0}^{\infty} a_n^0 u_n , \tag{34}$$

where

$$u_n'' + \left(n + \frac{1}{2} - \frac{\theta^2}{4}\right) u_n = 0 , \tag{35}$$

$$u_n(\theta) = \frac{1}{A_n} e^{-\theta^2/4} H_n\left(\frac{\theta}{\sqrt{2}}\right) , \qquad A_n = 2^{(n/2)+(1/4)} \sqrt{n!}\, \pi^{1/4} . \tag{36}$$

$H_n(x)$ are Hermite polynomials

$$H_n(x) = (-1)^n e^{x^2} \frac{\mathrm{d}^n}{\mathrm{d}x^n} e^{-x^2} . \tag{37}$$

By making use of the properties of the Hermite polynomials (Gradstein and Ryzhik, 1982; Furth, Killen, and Rosenbluth, 1963) we obtain for the coefficients of the series (34) the following recurrent relationship:

$$a_m^0 = \left\{ \frac{i}{2} \, \tilde{\xi} \left[a_{m-2}^0 \, \frac{\sqrt{(m-1)m}}{2^{m-1/2}} - a_{m+2}^0 \, \frac{\sqrt{(m+1)(m+2)}}{2^{m+3/2}} \right] + \right.$$

$$\left. + \int_{-\infty}^{+\infty} (\theta_1 \psi - i \tilde{\xi} \psi') u_m \, \mathrm{d}\theta_1 \right\} \frac{1}{-(m+\frac{1}{2}) - \alpha^2 \varepsilon_0^2 \Delta} \, . \tag{38}$$

Terms with a_{m-2}^0 and a_{m+2}^0 can be neglected as they are exponentially small with respect to m and besides they have opposite signs. It will be seen in Section 2.4 what the parameter of the expansion is in this case.

We may briefly point out that oscillations with respect to θ are not essential in this approximation. They can be taken into account if the solution of the Equation (29) is sought in the form

$$u^0 = e^{-(i/8)\tilde{\xi}\theta^2} \tilde{u}^0 \, . \tag{39}$$

Then we get for the width of the oscillation penetration layer

$$\tilde{\varepsilon} = \varepsilon \left(1 + \frac{\xi^2 S^2}{p} \right)^{1/2} . \tag{40}$$

The interval between the two nearest maxima at different sides of the point $\theta = 0$ is equal (as one can see from (39)) to

$$\Delta\theta \sim \sqrt{\frac{\alpha}{\xi}} \, \varepsilon_0 \, .$$

One can see that $\Delta\theta > \varepsilon_0$ for $\xi < \alpha$.

Thus, Equation (29) can be rewritten as

$$a_m^0 = \frac{1}{-(m+\frac{1}{2}) - \Delta\alpha^2 \varepsilon_0^2} \int_{-\infty}^{+\infty} u_m (\theta_1 \psi - i \tilde{\xi} \psi') \, \mathrm{d}\theta_1 \, ,$$

$$u^0 = \sum_{m=0}^{+\infty} a_m^0 u_m \, .$$

Following Furth, Killen, and Rosenbluth (1963), we calculate Δ' for the inner region in the approximation $\psi = \text{const}$. The condition of applicability for this approximation is as follows:

$$\frac{\psi'_\mu \varepsilon_0}{\psi} < 1 \, . \tag{42}$$

If we use (28), the above inequality is reduced to

$$\varepsilon_0 < \alpha\,. \tag{43}$$

Allowing for

$$\psi'_\mu = \frac{\Delta^{1/4}}{\varepsilon}\,\psi'_\theta\,, \tag{44}$$

we get from (30) and (41)

$$\Delta' = \Delta^{-1/4}\Omega \sum_{n=0}^{\infty}\left\{4\left[\int_{-\infty}^{+\infty} \mathrm{d}\theta_1 u_n\right]^2 - \frac{1}{(n+\frac{1}{2}) + \Delta^{1/2}\alpha^2\varepsilon^2}\left[\int_{-\infty}^{+\infty} u_n\theta_1\,\mathrm{d}\theta_1\right]^2\right\}. \tag{45}$$

By substituting the values of integrals (cf. Furth, Killen, and Rosenbluth, 1963) and using $\Delta^{1/2}\alpha^2\varepsilon^2 < 1$, we obtain:

$$\Delta' = 12\Delta^{-1/4}\Omega\,. \tag{46}$$

Unlike Furth, Killen, and Rosenbluth (1963), there is an additional multiplier $\Delta^{-1/4}$ in (43)–(46) associated with the transverse magnetic field.

2.3. The dispersion equation in the short wave-length regime

By comparing the values of Δ' derived for the outer (28) and inner (46) regions we find the dispersion equation

$$p^5 = \left(\frac{8S}{9\alpha}\right)^2 - \xi^2 S^2 p^4\,. \tag{47}$$

The condition (43) with (47) is written down as

$$p\alpha^2 > \tfrac{4}{3}\,. \tag{48}$$

For very large wavelengths the inequality (48) is violated.

One can easily see, as follows, that the second term of the right-hand side of Equation (47) describes the stabilizing effect of the transverse field.

The graphical solution of Equation (47) is given in Figure 2, where four curves are represented: the curve of the function

$$f_1 = p^5$$

and three curves of functions

$$f_2 = \left(\frac{8S}{9\alpha}\right)^2 - \xi^2 S^2 p^4$$

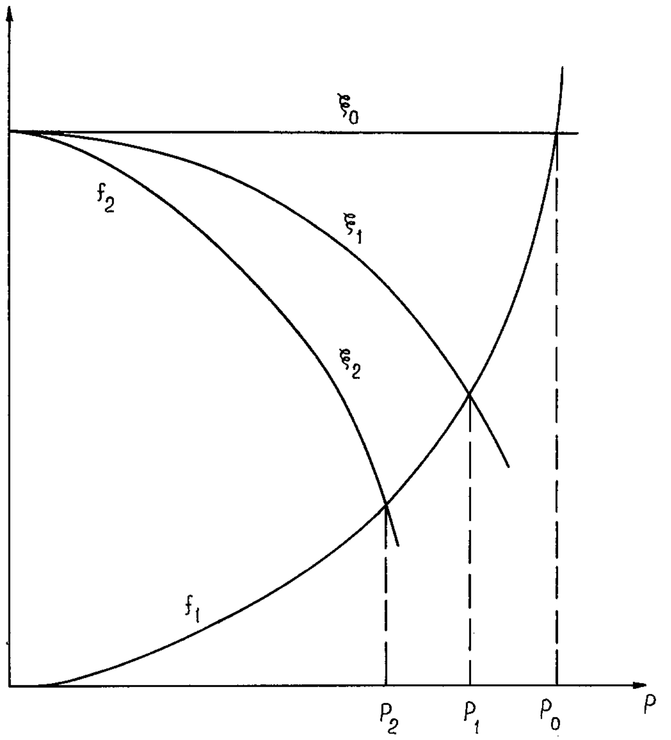

Fig. 2. Graphical solution of the dispersion equation (47).

for three values of ξ_0, ξ_1, and ξ_2 of the parameter ξ:

$$|\xi_2| > |\xi_1| > |\xi_0| = 0 \, .$$

The x-coordinate p_i ($i = 0, 1, 2$) of the intersection point of these curves is the root of Equation (47) for $\xi = \xi_i$ and

$$p_2 < p_1 < p_0 = \left(\frac{8S}{9\alpha}\right)^{2/5} .$$

The value of p_0 ($\xi = 0$) corresponds to Furth, Killen, and Rosenbluth (1963). The root decreases monotonical with the increase of ξ.

It should be noted that, generally speaking, Equation (47) has complex roots as well. In the general case we find from (47)

$$\frac{\mathrm{d}\,\mathrm{Re}\,p}{\mathrm{d}\xi} = \frac{-2\xi S^2\{5\,|p|^2 + 4\xi^2 S^2\,\mathrm{Re}\,p\}}{(5\,\mathrm{Re}\,p + 4\xi^2 S^2)^2 + (5\,\mathrm{Im}\,p)^2} \, ,$$

$$\mathrm{Re}\,p(-\xi) = \mathrm{Re}\,p(\xi) \, .$$

Therefore,

$$\frac{\mathrm{d}\,\mathrm{Re}\,p}{\mathrm{d}\xi} < 0$$

at $\xi > 0$; $\mathrm{Re}\,p > 0$, i.e. when there is instability. Hence, the value of $\mathrm{Re}\,p > 0$ decreases with the increase of $|\xi|$. This fact confines the stabilizing effect of the transverse component of the magnetic field.

2.4. The general dispersion equation

Let us study the behaviour of the growth rate of instability throughout the entire wave-length band. In Equations (17), (18), (20) the term $\alpha^2 W$ is neglected as compared with W'' in the inner region. We, therefore, obtain

$$\psi'' = \alpha^2 \psi + p\psi + \mu W + \frac{\xi}{i\alpha} W' , \tag{49}$$

$$W'' = \frac{S^2 \alpha^2}{p} \mu^2 W + S^2 \alpha^2 \mu \psi + \frac{S^2 \alpha \xi}{p^i} \mu W' - \frac{iS^2 \alpha \xi}{p} \psi''' . \tag{50}$$

By substituting

$$z = \psi'' , \qquad \tilde{\theta} = \mu \left(\frac{\alpha^2 S^2}{p} \right)^{1/4} \Delta^{1/4} , \tag{51}$$

we derive from (49) and (50) the equation for z:

$$z''' = [\nu + \tilde{\theta}^2] z' + 4\tilde{\theta} z - iM(2\tilde{\theta} z'' + 5z') . \tag{52}$$

Here

$$\nu = (\alpha^2 + p) \frac{p^{1/2}}{\alpha S} \Delta^{1/2} , \qquad M = \frac{S\xi}{(p + S^2 \xi^2)^{1/2}} \tag{53}$$

(cf. Furth, Killen, and Rosenbluth, 1963). ψ is an even function, hence, z is also an even function, therefore, $z'(0) = 0$.

The value of z is normalized with the condition

$$z(0) = 1$$

without loss of generality.

It follows from the expression for M in (53) that $|M| < 1$ when $\mathrm{Re}\,p > 0$ (instability). Below it will be shown that $|\nu - 1| < 1$. Thus, the solution of Equation (52) is to be sought in the form of an expansion in terms of parameters M and $\nu - 1$. In the zero expansion order we obtain

$$z^{(0)} = e^{-\tilde{\theta}^2/2} . \tag{54}$$

It is seen that no oscillations originate with respect to $\tilde{\theta}$.

By making use of the known formulae for the function asymptotes at infinity we have

$$\Delta' = 2 \lim_{\tilde{\theta} \to \infty} \left(\frac{\psi'}{\psi - \mu\psi'} \right) \tag{55}$$

(see also Furth, Killen, and Rosenbluth, 1963). The value Δ' is expressed in terms of variables $\tilde{\theta}$ and z in the following way (when $\xi < \alpha$):

$$\Delta' = \frac{2p\left(\Delta \dfrac{\alpha^2 S^2}{p}\right)^{-1/4} \displaystyle\int_0^{+\infty} z \, \mathrm{d}\tilde{\theta}}{1 - p\left(\Delta \dfrac{\alpha^2 S^2}{p}\right)^{-1/2} \displaystyle\int_0^{+\infty} \theta z \, \mathrm{d}\tilde{\theta}} \,. \tag{56}$$

In our problem, the consideration of the integrals in (56) can be confined to the zero approximation with respect to parameters $\nu - 1$ and M. For the case $\xi < \alpha$ we ultimately derive from (28), (54), (56) the dispersion equation

$$\Delta^{1/4} \left(\frac{\alpha^2 S^2}{p} \right)^{1/4} \left(1 - \frac{p^{3/2}}{\alpha S} \Delta^{-1/2} \right) - p\alpha \sqrt{\frac{\pi}{2}} = 0 \,. \tag{57}$$

At $\xi = 0$ Equation (57) goes over into the dispersion equation derived in Janicke (1980, 1982). But when $\xi \neq 0$ in (57) (unlike Janicke, 1980, 1982) there are multipliers associated with ξ which control the stabilizing effect of the transverse field.

In the first place let us consider the case when

$$\frac{p^{3/2}}{\alpha S} \Delta^{-1/2} < 1 \,. \tag{58}$$

From (57) we get

$$p^5 = \left(\frac{2}{\pi} \frac{S}{\alpha} \right)^2 - \xi^2 S^2 p^4 \tag{59}$$

with (58) being equivalent to the condition

$$p\alpha^2 > \frac{2}{\pi} \,. \tag{60}$$

Equation (59) and the condition (60) agree with (47) and (48) to within a numerical coefficient of order unit.

Within the limit

$$p\alpha \sqrt{\frac{\pi}{2}} < \Delta^{1/4} \left(\frac{\alpha^2 S^2}{p} \right)^{1/4} \tag{61}$$

the dispersion equation is reduced to

$$p^3 = \alpha^2 S^2 - \xi^2 S^2 p^2 . \tag{62}$$

The condition (61) is equivalent to

$$p\alpha^2 < \frac{2}{\pi} . \tag{63}$$

It is seen that the formula (59) describes the region of short wavelengths and formula (62) that of the long wavelengths (cf. Furth, Killen, and Rosenbluth, 1963; Artsimovich and Sagdeev, 1979 for $\xi = 0$). In the intermediate region both formulae produce virtually the same result. To see this, from (59) we get at $p\alpha^2 = 2/\pi$

$$\left(\frac{2}{\pi}\right)^3 = \alpha^8 S^2 - \left(\frac{2}{\pi}\right)^2 \alpha^2 S^2 \xi^2 \tag{64}$$

and from (62) we get the same result too.

It may be shown from (33) and (64) that the condition $p\alpha^2 = 2/\pi$ is equivalent to

$$\varepsilon_0 \sim \alpha , \tag{64a}$$

the limit (60) is equivalent to

$$\varepsilon_0 < \alpha \tag{64b}$$

(see also (43)), and the limit (63) corresponds to

$$\varepsilon_0 > \alpha . \tag{64c}$$

At short wavelengths the growth rate is growing with wavelength, at long ones it is falling. Hence, when the wavelength is given by the Equations (64) ($\alpha = (2\pi L)/\lambda$) it runs into its maximum. The same analysis of the Equations (57), (59), (62), and (64) as that of (47) reveals that the transverse component of the magnetic field reduces both the value of the growth rate over the entire wavelength band and the wavelength at which the growth rate maximises.

Figure 3 gives plots of the growth rate of instability $p = \omega\tau_R$ as a function of the wavelength λ/L for $S = 10^8$ and for the three values of ξ: $\xi_0 = 0$, $\xi_1 = 10^{-4}$, $\xi_2 = 10^{-3}$. Solid lines correspond to numerical solutions of Equation (57), dash-and-dash lines correspond to the solutions of the asymptotic equations (59), (62) with the account of (60), (63), and (64). An increase of the transverse magnetic field is evidently accompanied by a growth rate decrease, with its maximum shifting to the shorter wavelengths.

Now let us check the validity of the approximation

$$|\nu - 1| < 1 .$$

Neglecting α^2 in (53) (this is equivalent to the neglect of the member $\alpha^2 \psi$ as compared

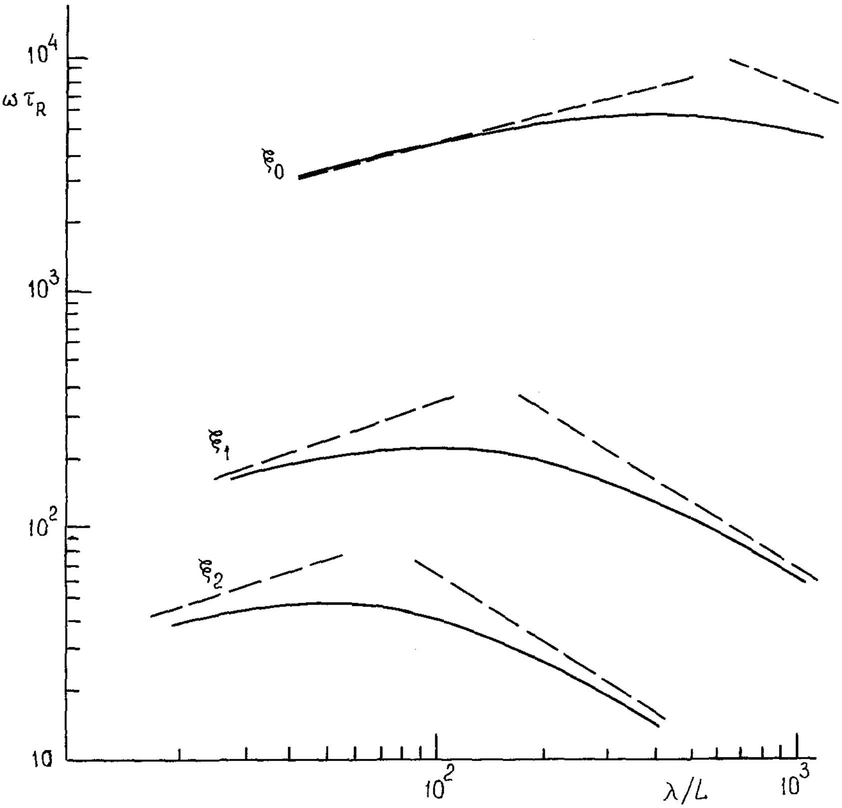

Fig. 3. Plots of the instability growth rate dependence on the wavelength for $S = 10^8$ and $\xi_0 = 0, \xi_1 = 10^{-4}$, $\xi_2 = 10^{-3}$ ($\xi = B_\perp/B_0$).

with ψ in Equation (49)) we get

$$v = \frac{p^{3/2}}{\alpha S} \Delta^{1/2} .$$

For the particular case (62)

$$1 - v = \frac{S^2 \xi^2}{p + S^2 \xi^2} .$$

Then, $|v - 1| < 1$ for $p > 0$. For the case (59)

$$v = \frac{\pi p^4}{2S^2} .$$

The maximum admissible value of p attained at $\xi = 0$, $p\alpha^2 \sim 1$ equals

$$p_m \approx S^{1/2} .$$

Therefore, $|\nu - 1| < 1$. Thus, the above technique of expansion in terms of $\nu - 1$ is valid.

3. Elucidation of Physical Results

In this section, for the sake of simplicity, we put all the coefficients of order unity equal to one. At first, let us obtain (similar to Furth, Killen, and Rosenbluth, 1963) the expression for the thickness of the region of 'decoupled flow' (RDF) $\varepsilon_0 a$. During the development of the instability (see Figure 4) plasma flows into the RDF with the velocity V_{1y}. As a consequence of this, an electric current j_1 and a restraining Lorentz force F_s are created (we use absolute values of the quantities, and we also use a system of units where the velocity of light $c = 1$):

$$j_1 = |\sigma(\mathbf{V}_1 \times \mathbf{B}_0)| = \sigma V_{1y} \varepsilon_0 B_0 , \tag{65}$$

$$F_s = |\mathbf{j}_1 \times \mathbf{B}| = \sigma V_{1y} (\varepsilon_0 B_0)^2 . \tag{66}$$

We take into account that the magnetic field is equal to $\varepsilon_0 B_0$ on the boundary of the RDF.

The restraining force is directed opposite to the plasma motion. During the growth of the instability a driving force F_d (which is due to the structure of the magnetic field outside the RDF: i.e., the tendency of the sheet current to break up into a set of parallel pinches – the 'rubber-bend' argument, see Figure 4 and Furth, Killen, and Rosenbluth, 1963) dominates the restraining force F_s within the inner region, and is itself dominated by F_s outside this region. Both forces are approximately equal to each other. Therefore, the rate at which the force F_d does work on the fluid is given by

$$P = |\mathbf{V}_1 \mathbf{F}_s| = \sigma V_{1y}^2 (\varepsilon_0 B_0)^2 . \tag{67}$$

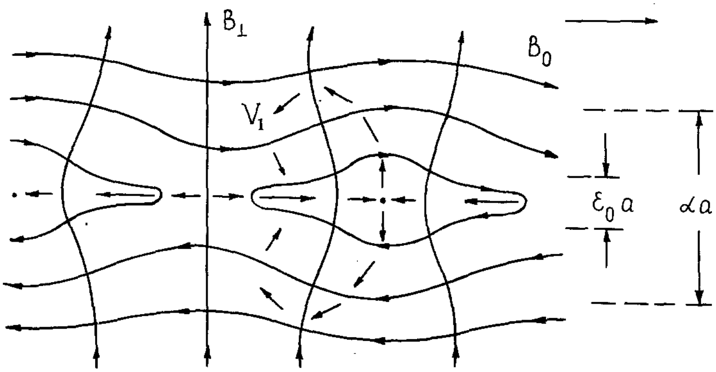

Fig. 4. Perturbed fields and velocities for a tearing mode in the presence of a transverse field. Solid arrows indicate fluid velocity.

This work per unit time is spent on: (i) the rise of the kinetic energy along the sheet, and (ii) the work done against the Lorentz force F_ξ, related with the transverse magnetic field $B_\perp$, because this field prevents plasma motion along the sheet:

$$P = T + \Pi , \tag{68}$$

where T is the rate of change of the kinetic energy and Π is the rate at which the work is done against the force F_ξ. The first term on the right-hand side of Equation (68) we may estimate (similar to Furth, Killen, and Rosenbluth, 1963) using the equation $\operatorname{div} \mathbf{V} = 0$. From this equation we have for the velocity along the sheet

$$V_{1x} = V_{1y}/k\varepsilon_0 a . \tag{69}$$

The kinetic energy that fluid receives during the time $1/\omega$ (where ω is the growth rate of the instability) is given by $\rho_0 V_{1x}^2$, hence

$$T = \omega\rho_0 V_{1x}^2 = \frac{\omega\rho_0 V_{1y}^2}{(k\varepsilon_0 a)^2} . \tag{70}$$

Now we estimate the second term on the right-hand side of Equation (68). When the plasma moves along the sheet with velocity V_{1x}, an additional electric current j_ξ is created,

$$j_\xi = \sigma V_{1x} B_\perp , \tag{71}$$

and the appropriate Lorentz force F_ξ arises. This force is directed opposite to the fluid motion and in absolute value it is equal to

$$F_\xi = j_\xi B_\perp = \sigma V_{1x} B_\perp^2 . \tag{72}$$

Then Π is equal (in absolute value) to

$$\Pi = V_{1x} F_\xi = \sigma V_{1x}^2 B_\perp^2 . \tag{73}$$

We rewrite Equation (68) with the help of (67), (69), (70), and (73) in the form

$$\sigma V_{1y}^2 (\varepsilon_0 B_0)^2 = \frac{\omega\rho_0 V_{1y}^2}{(k\varepsilon_0 a)^2} + \sigma B_\perp^2 \, \frac{V_{1y}^2}{(k\varepsilon_0 a)^2} . \tag{74}$$

From this equation we obtain the expression for the 'skin depth' ε_0:

$$\varepsilon_0^4 = \frac{\omega\rho_0}{(ka)^2 B_0^2 \sigma}\left(1 + \frac{\sigma B_\perp^2}{\omega\rho_0}\right). \tag{75}$$

It is easy to see that this expression is equivalent to the expression

$$\varepsilon_0 = \varepsilon\left(1 + \frac{\xi^2 S^2}{p}\right)^{1/4} \tag{76}$$

which we obtained in Section 2.

Now let us obtain the dispersion equations. In the region of partly decoupled flow the first term on the right side of Ohm's law,

$$\eta \mathbf{j} = \mathbf{E} + \mathbf{V} \times \mathbf{B}, \tag{77}$$

dominates over the second, and we must select $\varepsilon_0 a$ so that

$$\eta_0 j_1 = E_1 . \tag{78}$$

Even in the inner region the flow is not perfectly decoupled. The perturbed electric field E_1 is related with the perturbed magnetic field B_1 by the equation

$$E_1 \sim (\omega B_{1y})/k . \tag{79}$$

This field corresponds to the generation of the perturbation flux that links the field regions on either side of $B_0 = 0$ (Furth, Killen, and Rosenbluth, 1963).

Using

$$\operatorname{rot} \mathbf{B} = \frac{4\pi}{c} \mathbf{j}$$

and

$$\operatorname{div} \mathbf{B} = 0 ,$$

we have for $(ka) < 1$

$$j_1 \sim B''_{1y}/(4\pi k) . \tag{80}$$

Using (78), (79), and (80) we find

$$\frac{\omega B_{1y}}{\eta} \sim \frac{B''_{1y}}{4\pi} . \tag{81}$$

When the main part of the magnetic 'loop' (the field line circulated around a magnetic 'island') is frozen into the plasma, the magnetic field B_{1y} is perturbed within a region of thickness $b \sim a^2 k$ (because $\lambda b \sim a^2$). Hence, b decreases with the increase of λ. We will see from (75) and from the dispersion relations (just as we found from (33) and (59), (62)) that ε_0 grows with λ. Therefore, in the region

$$ak > \varepsilon_0 \tag{82}$$

(compare with (43), (64a)) we may use the estimate

$$B''_{1y} \sim \frac{B'_{1y}}{\varepsilon_0 a} \sim \frac{B_{1y}}{\varepsilon_0 a a^2 k} . \tag{83}$$

In the region

$$ak < \varepsilon_0 \tag{84}$$

(one may also compare with (43), (64a)) the estimate

$$B''_{1y} \sim \frac{B'_{1y}}{\varepsilon_0 a} \sim \frac{B_{1y}}{(\varepsilon_0 a)^2} \tag{85}$$

is valid.

In the first case we have from Equations (75), (81), and (83) the dispersion relation

$$\omega^5 = \frac{\eta_0^3 B_0^2}{a^{10} \rho_0} \frac{1}{k^2} - \frac{B_\perp^2}{\rho_0 \eta_0} \omega^4 \tag{86}$$

which corresponds in terms S, p, α to Equation (59).

In the second case we obtain the equation

$$\omega^3 = \frac{\eta_0 B_0^2}{a^2 \rho_0} k^2 - \frac{B_\perp^2}{\rho_0 \eta_0} \omega^2 \tag{87}$$

which corresponds to Equation (62).

In the case $ak > \varepsilon_0$ the main part of the magnetic loop is frozen to plasma which is accelerated by the tension of the loop. The magnetic tension increases with λ and consequently the growth rate ω also increases.

However, in the case $ak < \varepsilon_0$ the main part of the magnetic loop is not frozen into plasma and the 'motions' of the loop and of the plasma are 'independent' of each other. In this case plasma acceleration is inefficient, and ω decreases with the increase of λ. It is not interesting to consider the case $a\varepsilon_0 > a$ for $S \gg 1$.

4. Discussion

As was pointed out in the Introduction, in the papers by Bulanov, Sakai, and Syrovatskii (1979), Janicke (1980, 1982) the transverse component was shown to produce no stabilizing effect. This result was derived in Janicke (1980) under the condition that

$$\xi \ll \varepsilon^2 \alpha \,, \tag{88}$$

which is easily reduced to

$$\xi^2 S^2 \ll p \,. \tag{89}$$

This condition means that $\Delta \approx 1$ in formulae (31)–(33); the component $\xi^2 S^2 p^4$ is small compared with p^5 in the dispersion equation (47) or (59); $\xi^2 S^2 p^2$ is small compared with p^3 in (62). The neglect of these components justified by (89) is equivalent to the neglect of the transverse field. Meanwhile the domain where condition (88) (or (89)) is satisfied is severaly restricted: though $\xi \ll 1$, but $S \gg 1$ and $\xi^2 S^2$ can assume, generally speaking, any value. Thus the stabilizing effect is always present.

In Janicke (1982) the problem is studied for the opposite case:

$$\varphi = \left[\frac{\xi^2 S^2}{p}\right]^{1/2} \gg 1 . \tag{90}$$

It is stated that there occurs the variation of the dimension of the singular region,

$$\varepsilon_2 = \varepsilon \left[\frac{\xi^2 S^2}{p}\right]^{1/4} = \varepsilon \varphi^{1/2} , \tag{91}$$

but the growth rate value is stable.

In our paper the formulae (33), (75), (76) provide for general expression of the variation of the 'defreezing' layer width:

$$\varepsilon_0 = \varepsilon \left[1 + \frac{\xi^2 S^2}{p}\right]^{1/4} = \varepsilon \varphi^{1/2} [1 + \varphi^{-2}]^{1/4} . \tag{92}$$

In the limit (90) it is reduced to (91) when the expression $(1 + \varphi^{-2})^{1/4}$ is expanded in terms of the parameter φ^{-2} up to the term of zero order.

The formula derived in Janicke (1982) describing growth rate is independent of $B_\perp$. It is associated with the fact that the author solves the set (22a)–(22b) in Janicke (1982) with the help of an expansion in terms of the parameter φ^{-2} up to terms of first-order, and he disregards these terms in the expression describing the width of the 'defreezing' layer (the singular region). But to solve the problem correctly one should include them. This results in additional members in the sets (24), (25), (36) in Janicke (1982), and, consequently, in the change of the dispersion equation (37) in Janicke (1982).

In the paper by Bulanov, Sakai, and Syrovatskii (1979) the stabilizing effect of the transverse magnetic field is stated to be lacking. Moreover, as is seen from formulae (38) and (42) in Bulanov, Sakai, and Syrovatskii the instability growth rate found by these authors is growing with $B_\perp$. There was defined the characteristic dimension of

$$\delta_B = a \left(\frac{\xi}{\alpha}\right)^{1/2} \tag{93}$$

at which the noticeable influence of the transverse field was recorded (see formula (23) in Bulanov, Sakai, and Syrovatskii). But the value of

$$\delta_R = \frac{a p^{1/2}}{(\alpha S)^{1/2}} \tag{94}$$

is taken as a width of the region where dissipation processes are important (see (24) in Bulanov, Sakai, and Syrovatskii). In (94) we neglected the component related to the plasma's spread-over, as was done in (38) and (42) in Bulanov, Sakai, and Syrovatskii. The formulae, for the sake of simplicity, are rewritten using our symbols.

Further, it is stated (paragraphs 1 and 3 in Bulanov, Sakai, and Syrovatskii) that the necessity to allow for the normal field is equivalent to the inequality

$$\delta_B > \delta_R \,. \tag{95}$$

But in this case dissipation effect is neglected in the inner region $|y| < \delta_B$ (see Section 3 in Bulanov, Sakai, and Syrovatskii). It is seen that (95) together with (93) and (94) is equivalent to the condition of the strong field:

$$\varphi \gg 1 \,. \tag{96}$$

However, it is shown in the present paper (see formulae (33) and (76)) and in the paper by Janicke (1982) at $\varphi \gg 1$ ((19) in Janicke, 1982 or (92) in our paper) that it is in the region with dimensions

$$a\varepsilon_2 \approx \delta_B \quad (\text{when } \varphi \gg 1) \tag{97}$$

that the dissipation effects are essential. The dimension of this region depends on ξ (see (76)) and is reduced to (94) only at $\xi = 0$. At $\xi \neq 0$ it is incorrect to use expression (94) derived in Furth, Killen, and Rosenbluth (1963) at $\xi = 0$. The condition (95) shows that the dimension of the singular region ε_0 is greater than the width ε of Furth, Killen, and Rosenbluth (with $\xi = 0$). The dissipation effects are not to be neglected in the inner region since they allow the tearing instability related.

There exists a complete stabilization effect of the transverse magnetic field on the kinetic tearing instability. In Coroniti (1980), Zeleny and Taktakishvili (1981) the account of collisions is shown to result in the destabilization of these stable states. This does not compromise the suggestions of this paper which shows the strong stabilizing effect of the transverse component in the MHD-approximation, but formally speaking, the complete stabilization such as in the collisionless plasma is lacking. Besides, as was pointed out in Zeleny and Taktakishvili (1981), the transition from the collisionless limit to the MHD limit needs furhter study.

5. The Influence of the Transverse Component on HTCS Stability

Let us apply the above results to the high-temperature turbulent current sheets (HTCS) of solar flares (Somov, 1981, 1986). The main peculiarity of HTCS is that powerful heat fluxes play an essential role in the energy balance of such sheets. The presence of the slightest magnetic field transverse component in HTCS leads to an increase of the efficiency of sheet cooling. The self-consistent approach to HTCS (Somov and Titov, 1983, 1985) makes it possible for one to explain a sufficiently high energy release power in a non-neutral sheet.

It was shown in Verneta and Somov (1987), Somov and Verneta (1988), Somov, Titov, and Verneta (1987) that in the approximation of the collisionless plasma the HTCS are stable with respect to the tearing mode due to the stabilizing effect of the transverse component. However, as was pointed out, plasma turbulence can also contribute to this process. An estimation of the effect in the MHD-approximation can,

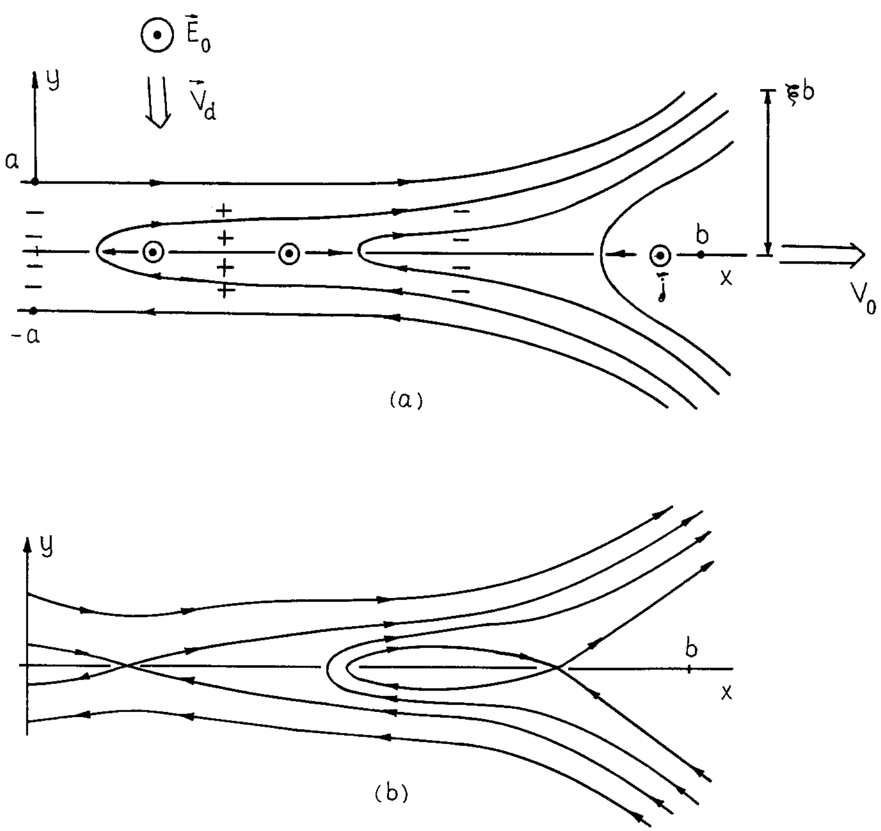

Fig. 5. (a) Unperturbed field lines in a two-dimensional HTCS. Electric current (j) redistribution related with the tearing mode creates the additional ($\pm$) energy $\delta\varepsilon_p$. (b) Perturbed field lines.

therefore, become important especially for the case of wavelengths of order of the width of the sheet $2b$ (Figure 5). If the condition $\omega \lesssim V_0/b$ is satisfied for the rate of instability growth ω, where V_0 is the velocity of plasma outflow, one can, therefore, suggest that the perturbation is emerging together with plasma without having been enhanced before leaving the current sheet. When $\omega \gg V_0/b$ the stabilization is lacking. The curves in Figure 6 representing rate of growth dependence on the wavelength are located between the solid margins with the account of the stabilization, and between the dash-and dash margins without its account. The values of V_0/b are located between the horizontal lines (see Figure 6). The calculations have been carried out with the help of the approximation of formula (57) for five states of the sheet taken from Somov (1986). It is seen that the rate of growth at the wavelengths $\lambda \sim 2b$ decreases by an order of magnitude due to stabilization as compared to that calculated without taking account of stabilization, and becomes comparable with V_0/b (horizontal lines). Hence, HTCS can be considered stable.

In case of a collisionless plasma the HTCS stability is demonstrated, but it should be noted that HTCS is stable in the range $a < \lambda < 2b$ in the MHD-approximation as

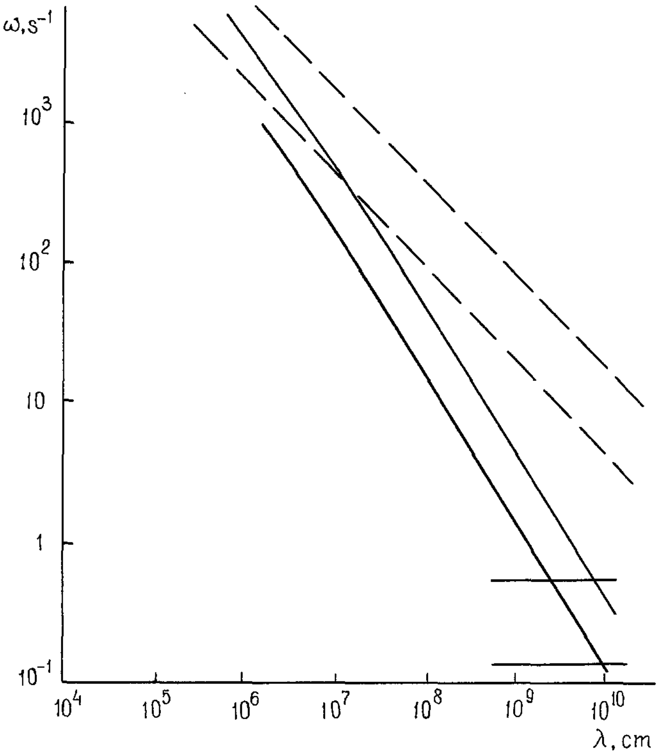

Fig. 6. The plots of the growth rate for HTCS.

well. Indeed, the rate of growth of instability satisfies the condition

$$\omega < V_d/a \sim 10^2\text{–}10^4\ \text{s}^{-1}$$

in the above range. Here V_d is the inflow drift velocity of plasma. Consequently, the gaps in the sheet are filled in with flowing plasma before they have time to originate, and the instability is suppressed.

6. Conclusion

The analysis of this paper reveals that the transverse component of the magnetic field has a strong stabilizing influence on the development of the resistive tearing MHD instability.

As all real current sheets always contain a small transverse component, their stability can be explained by this effect.

The application of the results to high-temperature turbulent current sheets reveals their stabilizing character relative to the tearing instability in the energy source of solar flares during the 'main' or 'hot' phase of their development.

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