AN EARLY ESTIMATE FOR THE SIZE OF CYCLE 23

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(Received 1 November, 1991; in revised form 21 January, 1992)

Abstract. Two features are found in the modern era sunspot record (cycles 10–22: ca. 1850–present) that may prove useful for gauging the size of cycle 23, the next sunspot cycle, several years ahead of its actual onset. These features include an inferred long-term increase against time of maximum amplitude (*RM*, the maximum value of smoothed sunspot number for a cycle) and the apparently inherent differing natures of even- and odd-numbered sunspot cycles, especially when grouped consecutively as 'even–odd' cycle pairs. Concerning the first feature, one finds that 6 out of the last 6 sunspot cycles have had $RM \geq 110.6$ (the median value for the modern era record) and that 4 out of 6 have had $RM > 150$. Presuming this trend to continue, one anticipates that cycle 23 will likewise have $RM \geq 110.6$ and, perhaps, $RM > 150$. Concerning the second feature, one finds that, when one groups sunspot cycles into consecutively paired even–odd cycles, the odd-following cycle has *always* been the larger cycle, 6 out of 6 times. Because cycle 22 had $RM = 158.5$, one anticipates that cycle 23 will have $RM > 158.5$. Additionally, because the average 'difference' between *RM*(odd) and *RM*(even) for consecutively paired even–odd cycles is 40.3 units (sd = 14.2), one expects cycle 23 to have $RM \geq 162.3$ ($RM = 198.8 \pm 36.5$ at the 95% level of confidence). Further, because of the rather strong linear correlation ($r = 0.959$, se = 13.5) found between *RM*(odd) and *RM*(even) for consecutively paired even–odd cycles, one infers that cycle 23 should have $RM \geq 176.4$ ($RM = 213.9 \pm 37.5$ at the 95% level of confidence). Since large values of *RM* tend to be associated with fast rising cycles of short ascent duration and high levels of 10.7-cm solar radio flux, cycle 23 is envisioned to be potentially one of the greatest cycles of the modern era, if not *the* greatest.

1. Introduction

The size of a sunspot cycle is a crucial parameter for estimating a variety of physical effects on or near Earth. Some of these effects include the variation of the Sun's luminosity over the solar cycle (Willson and Hudson, 1991), the influence of solar forcing on global climate (Wilson, 1989; Kelly and Wigley, 1990; Reid, 1991; Friis-Christensen and Lassen, 1991), the modulation of solar neutrinos (Wilson, 1987c; Krauss, 1990; Bahcall and Press, 1991), and the determination of the near-Earth space environment, especially as related to satellite drag (Vampola, 1989; Gorney, 1989, 1990; Walterscheid, 1989). In particular, for the decade of the nineties and extending into the next century, the importance of accurate long-range sunspot cycle prediction has become even more paramount, owing to the realization of the great observatories (e.g., Hubble and the Gamma-Ray Observatory), preparations for the deployment and on-orbit maintenance of Space Station Freedom, and the planned return to the Moon and venture beyond to Mars. So, in an effort to describe the variation with time of *RM* (the maximum amplitude of a sunspot cycle, based on smoothed sunspot number; Howard, 1977) and assess inferred trends and associations found in *RM* during the modern era of sunspot observations (ca. 1850–present), this study was undertaken.

Accurate prediction for the size of a sunspot cycle several years ahead of its actual onset remains a goal for solar forecasters. While it remains a long-term goal, some progress towards attaining it has been made in recent years. For example, on the basis of selected 'precursor' techniques it is now possible to better quantity (with an accuracy of about 20 smoothed sunspot number units) the maximum amplitude of a sunspot cycle some 1–5 year ahead of its actual occurrence, depending upon the inferred strength of the upcoming cycle (Wilson, 1988b; 1990a, b, c; Withbroe, 1989; Layden *et al.*, 1991). In this paper, the maximum amplitude for cycle 23, the next sunspot cycle, is estimated, not on the basis of precursor methods, for cycle 23 has yet to begin, but instead from inferred statistical trends and associations found in the modern era (cycles 10–22) that seem to be important. In particular, the estimate results from an extrapolation of the inferred long-term upward trend in *RM* and the apparently inherent differences found in even- and odd-numbered sunspot cycles.

2. Results

2.1. Data

Table I gives the maximum amplitudes *RM* for the modern era of sunspot cycles (10–22). The modern era of sunspot cycles represents the most reliably determined cycles, those occurring since 1848, the year when Rudolf Wolf introduced his relative

TABLE I
RM values for the modern era of sunspot cycles (10–22)

Cycle	*RM*	Difference[a]
10	97.9	
11	140.5	42.6
12	74.6	
13	87.9	13.3
14	64.2	
15	105.4	41.2
16	78.1	
17	119.2	41.1
18	151.8	
19	201.3	49.5
20	110.6	
21	164.5	53.9
22	158.5	
23	?	?
Mean (10–22)	119.6	40.3
sd (10–22)	41.1	14.2
Mean (even)	105.1	
sd (even)	37.5	
Mean (odd)	136.5	
sd (odd)	41.5	

[a] Difference = $RM(\text{odd})_i - RM(\text{even})_{i-1}$ where i is cycle number.

sunspot number to serve as a measure of solar activity (Waldmeier, 1961; McKinnon, 1987; Withbroe, 1989). At the bottom of the table are the means and standard deviations (sd) for several groupings of the cycles: (i) the all-inclusive group (10–22), (ii) the even-numbered group, and (iii) the odd-numbered group. Also given in Table I is the 'difference' between maximum amplitudes for consecutively paired sunspot cycles, with the shown pairing being even–odd, in that order; the difference is calculated as *RM*(odd) minus *RM*(even) for each pairing, and the mean and sd for the difference also appears at the bottom of Table I. Inspection of Table I reveals that the average modern era sunspot cycle has $RM = 119.6$ (sd = 41.1 units); means (and sd) for the even- and odd-numbered cycle groupings are, repectively, 105.1 (37.5) and 136.5 (41.5); and the average difference between consecutively paired even–odd sunspot cycles is 40.3 units (sd = 14.2), with the odd-following cycle being the larger cycle.

2.2. The long-term upward trend in *RM*

Figure 1 plots the *RM* values tabulated in Table I against sunspot cycle number (bottom) and the residual (top), in units of standard error (se), based on the fit of *RM* versus cycle number. Given in the figure are the regression equation ($\hat{Y}$), the coefficient of correlation (r), the coefficient of determination (r^2), the standard error (se), the Student t-test statistic, and the confidence level (CL) for the fit. Also shown are the means and sd values for two separate cycle groupings: cycles 10–15 and 16–22, along with the t statistic and CL for the difference of the two means (Lapin, 1978, p. 486). Lastly, the probability P of obtaining the observed distribution, or one more suggestive of a departure from independence, is displayed, calculated using the Fisher's exact test for 2×2 tables (Everitt, 1977, p. 15), based on the median values for cycle number and *RM*, depicted as the thin vertical and horizontal lines; the heavy line, spanning from lower-left to upper-right, is the inferred regression.

2.3. The *RM*(odd) versus *RM*(even) correlation

Figure 2 depicts the scatter plot of *RM*(odd) versus *RM*(even) for even–odd pairings of sunspot cycles (bottom) and the residual (top), in units of standard error (se), based on the inferred fit. Also shown are the regression equation ($\hat{Y}$), the coefficient of correlation (r), the coefficient of determination (r^2), the standard error (se), the Student t statistic, the confidence level (CL), the median values of *RM*(odd) and *RM*(even), and the Fisher's exact test probability P.

3. Discussion

From Table I, it is noted that *RM*, on average, has been about 119.6 with the distribution having an sd = 41.1. Assuming that *RM* is distributed normally with no overt long-term trend (not true, as will be discussed later in the text), for the small sample of modern era cycles ($n = 13$) one can expect *RM* to always lie in the range of 30.0 to 209.2 at the 95% level of confidence. Such a range, in fact, is found to cover the observed *RM*-values tabulated in Table I (64.2–201.3). For even-numbered cycles, the range is somewhat

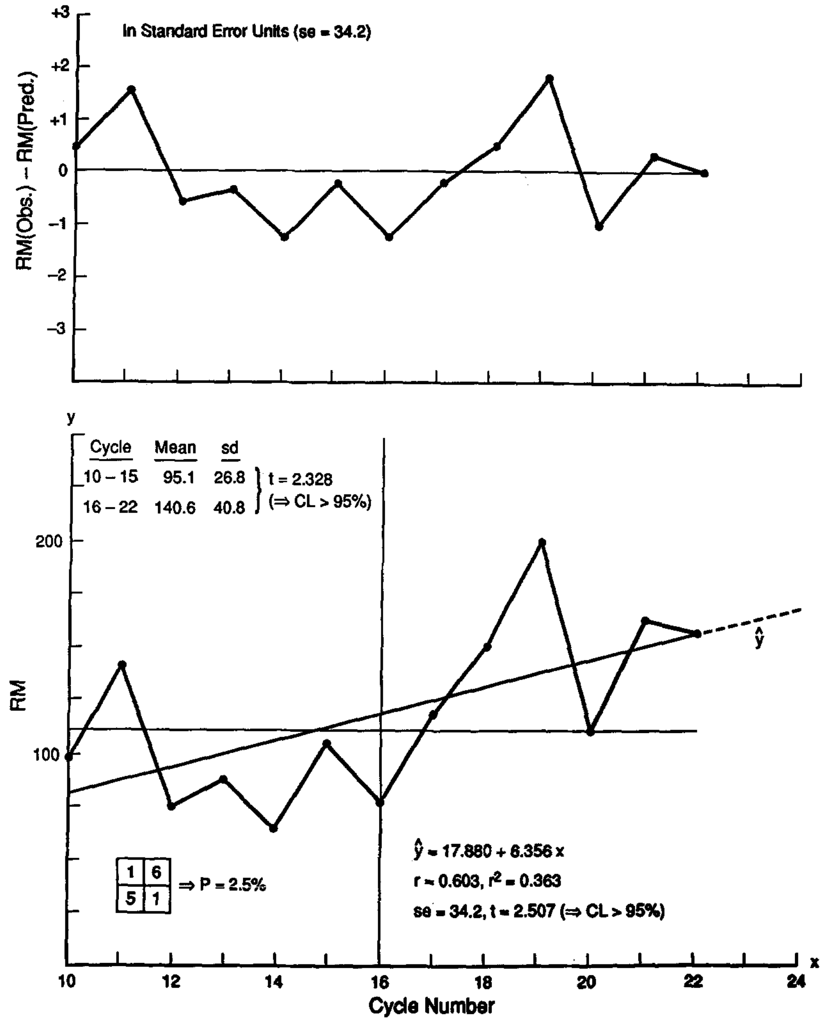

Fig. 1. (*Bottom*) *RM* vs cycle number. The thin lines are the median values and the heavy line is the linear fit. Details are explained in the text. (*Top*) Residual vs cycle number, in units of standard error.

lower (13.3–196.9), while it is higher for odd-numbered cycles (29.8–243.2). On the other hand, the 'difference' between odd-following and even-numbered cycle pairs has averaged about 40.3 with an sd = 14.2. Based on the small sample of modern era cycle pairs ($n = 6$), one expects the 'difference' to always be about 3.8 to 76.8 at the 95% level of confidence, inferring that cycle 23 should have an *RM* in the range of 162.3 to 235.3, presuming, of course, that *RM* for cycle 22 remains equal to 158.5, having occurred in July 1989.

Figure 3 charts cycle 22 through early 1991, using a variety of solar cycle related markers: sunspot number (R_0), 10.7-cm radio flux (F_0), the number of groups (G_0), and the total corrected area of sunspots ($(AT)_0$), where the latter two quantities are

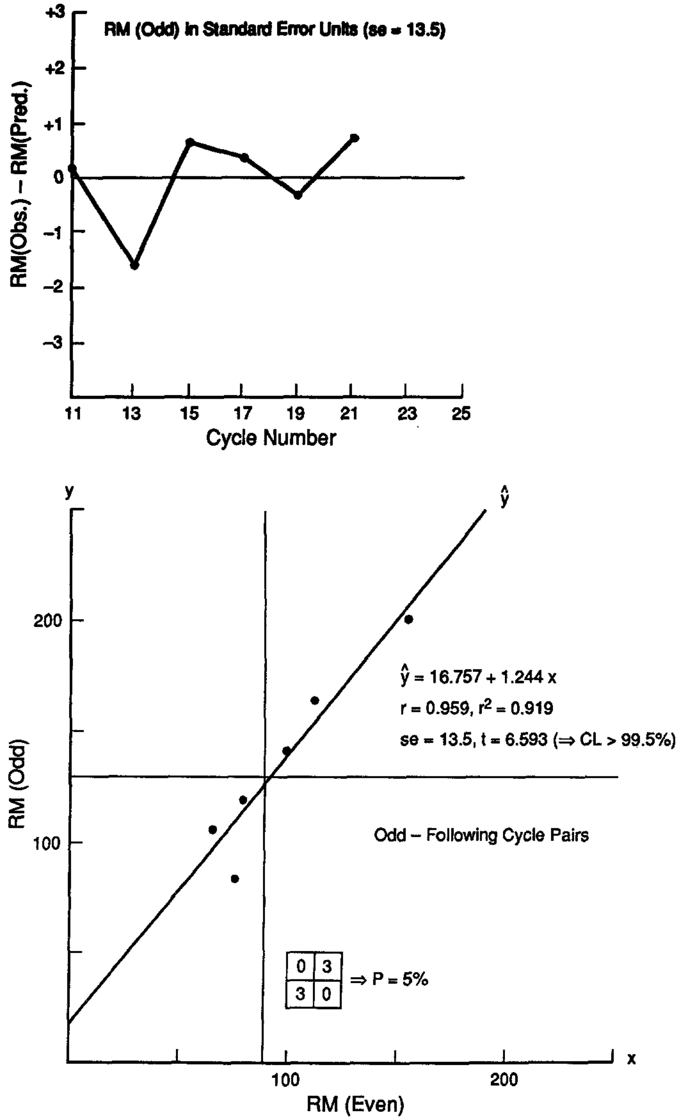

Fig. 2. (*Bottom*) *RM*(odd) vs *RM*(even) for odd-following cycle pairs. The thin lines are median values and the heavy line is the linear fit. Details are explained in the text. (*Top*) Residual vs cycle number, in units of standard error.

based on observations reported by the Space Environment Services Center of the National Oceanic and Atmospheric Administration and where the subscript '0' indicates that the numbers are smoothed in the same sense as international sunspot number (Howard, 1977). The units for F_0 and $(AT)_0$ are, respectively, solar flux units ($= 10^{-22}$ W m^{-2} Hz^{-1}) and millionths of a solar hemisphere. Figure 3 argues that, while there has so far been two major peaks of activity in cycle 22, the first peak in

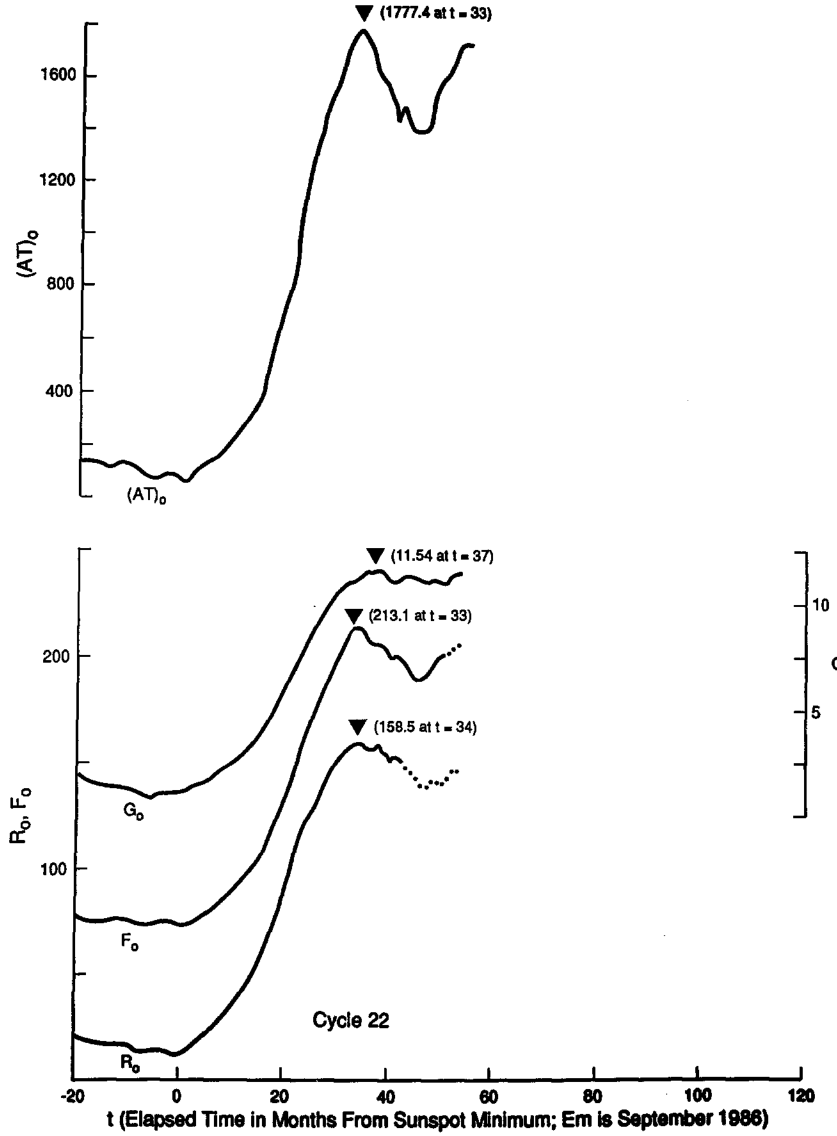

Fig. 3. The growth and development of cycle 22. Details are explained in the text. The lines connect final values for the parameters, while the dots represent preliminary values. The filled triangles denote the occurrences of the maximum values which likewise appear to the right of each parameter. The 'epoch' of sunspot minimum occurrence is denoted here as Em and is noted to have occurred in September 1986.

mid-1989 appears to represent the true (conventional) maximum for the cycle. Values for the parameters are expected to decline rather dramatically over the next year or two.

Continuing from Figure 1, it is apparent that over the course of the modern era of sunspot cycles, *RM* has increased in average value, thus, negating the reliableness of the expected range of *RM* values given earlier in this section based on a normal distribution of *RM* with *no* overt long-term trend. This is seen using any of the three test results: the t statistic for the slope from the linear regression analysis (Lapin, 1978, p. 348), the probability P from Fisher's exact test for 2×2 tables (Everitt, 1977, p. 15), or the difference of two means by dividing the modern era into two groups (cycles 10–15 and 16–22; Lapin, 1978, p. 486). All three test results suggest at >95% level of confidence that cycles of late are larger and probably part of a long-term upward trend in *RM*, with about 36.3% of the variation in *RM* being 'explained' by the long-term upward trend. Thus, cycle 23 is expected to be larger than the median value (= 110.6) and should have an *RM* in the range 88.8–239.4 at the 95% level of confidence, based on the inferred linear fit.

The inferred upward trend in Figure 1 does not include any short-term modulation (e.g., an 8-cycle variation called the 'Gleissberg' cycle; Wilson, 1988a). Certainly, within the modern era of sunspot cycles, there have been two positive excursions (where the observed value was >1.5 se higher than the predicted value). These occurred in cycles 11 and 19, which happen to be 8 cycles apart in time. Negative excursions (where the observed value was considereably below the predicted value) have not shown an 8-cycle modulation. Thus far, cycle 14 has shown the greatest negative excursion (about −1.25 se); so had the Gleissberg cycle been a strong contributor to the variation of *RM* during the modern era, one would have expected cycle 22 to have displayed a negative excursion as well. Instead, cycle 22 had an *RM* about equal to what the long-term upward trend postdicted it to be and, in fact, had a value for *RM* making it the third largest cycle of the modern era. (Based on annual averages, it is the second largest cycle of the modern era.) Thus, inclusion of a term to account for short-term modulation of *RM* to reduce the size of the prediction interval does not yet seem practical. (There is insufficient data to properly account for this short-term effect. It would require a minimum of about 240 years more of reliable data.)

It has been well established that the magnetic arrangement of sunspots alternates every cycle, with south leading polarity in the northern hemisphere during even-numbered cycles (Hale and Nicholson, 1938; Howard, 1977; Wilson, 1988c). This suggests that each solar (magnetic) cycle is comprised of two consecutive sunspot cycles, often called the 'Hale' cycle. The preferred pairing appears to be even–odd, in that order (Gnevyshev and Ohl, 1948; de Jager, 1959; Vitinskii, 1965; Wilson, 1988c). Because cycles 22 and 23 represent a 'new' Hale cycle pair and because *RM* for cycle 22 is known (= 158.5, from Figure 3), on the basis of the highly correlative fit between *RM*(odd) versus *RM*(even) shown in Figure 2, a fit that suggests that one can 'explain' about 91.9% of the variation in *RM*, one predicts cycle 23 to have an *RM* in the range 176.5–251.4 at the 95% level of confidence. Thus, cycle 23, like its predecessor, is expected to be comparable to the largest sunspot cycles on record, perhaps becoming the new second largest, or very possibly *the* largest, sunspot cycle of the modern era. Combining the results of Table I and Figures 2 and 3, one finds the overlap of the 95% prediction intervals to be 176.5–209.2, where the lower cutoff is from the *RM*(odd) versus *RM*(even) fit and the upper cutoff is from the mean fit for cycles 10–22. Because of the apparent statistically significant upward trend in *RM*, the upper cutoff is probably unrealistic and, hence, too low. Perhaps a more appropriate upper cutoff may be 239.4, from the fit itself. (It should be noted that this technique only works for even–odd cycle pairs and not for odd–even pairs. The probability due to chance of having the odd-following cycle always to be the larger cycle for 6 even–odd cycle pairs is computed from the binomial formula to be $P = 1.6\%$; Lapin, 1978, p. 163.)

Table II summarizes early estimates of *RM* for cycle 23 based on the afore-mentioned

TABLE II

Early estimates of *RM* for cycle 23

Method	Prediction	± 1 sd (se)	Fraction
Mean *RM*(odd) cycle	136.5	41.5	$\frac{4}{6}$
Difference	198.8	14.2	$\frac{5}{6}$
Upward trend	164.1	34.2	$\frac{8}{13}$
RM(odd) vs *RM*(even)	213.9	13.5	$\frac{5}{6}$

analyses, also giving the mean *RM*(odd) value for comparison. Given in the table are the prediction, the sd or se associated with the prediction, and the fraction of modern era cycles that had *RM* values within the ± 1 sd (or se) interval for that particular prediction method. Because the greatest fraction of 'successful' postdictions are those associated with the 'difference' method and the *RM*(odd) versus *RM*(even) fit, one may expect *RM* for cycle 23 to lie in the range 200.4–213.0, where the lower cutoff is from the linear fit and the upper cutoff is from the 'difference' method.

Because *RM* can be used as an estimator for other useful solar cycle related parameters (e.g., *FM* – the 'maximum' value for the smoothed 10.7-cm solar radio flux, in solar flux units; *GM* – the 'maximum' value for the smoothed number of groups; *(AT)M* – the 'maximum' value for the smoothed total corrected area of sunspots, in millionths of a hemisphere; and *ASC* – the ascent duration or number of months from sunspot minimum occurrence (the month that the minimum value of the smoothed sunspot number for a sunspot cycle occurred) to sunspot maximum occurrence (the month that the maximum value of smoothed sunspot number of a sunspot cycle occurred)), Figure 4 is included to provide this ancillary data. All regressions are found to be highly correlative, having confidence levels $> 98\%$.

Table III summarizes the findings for the various values of *RM* that have been predicted for cycle 23: 136.5 (mean of odd-numbered cycles), 198.8 (based on the mean 'difference' and known value of *RM* for cycle 22, equal to 158.5), 164.1 (from the inferred upward trend in *RM*), and 213.9 (from the *RM*(odd) versus *RM*(even) fit). As an example (and at the 95% level of confidence), using the *RM* estimate for cycle 23 equal to 213.9, one expects *FM* for cycle 23 to lie in the range 233.6–287.6 solar flux units; *GM* to lie in the range 14.96–17.99; *AT*(*M*) to lie in the range 2167.3–4029.0 millionths of a hemisphere; and *ASC* to lie in the range 24–48 months. Thus, based on the high estimate for *RM* (= 213.9), one expects cycle 23 to be near record-setting in essentially every parameter; the record values are *FM* = 245.4 solar flux units (cycle 19),

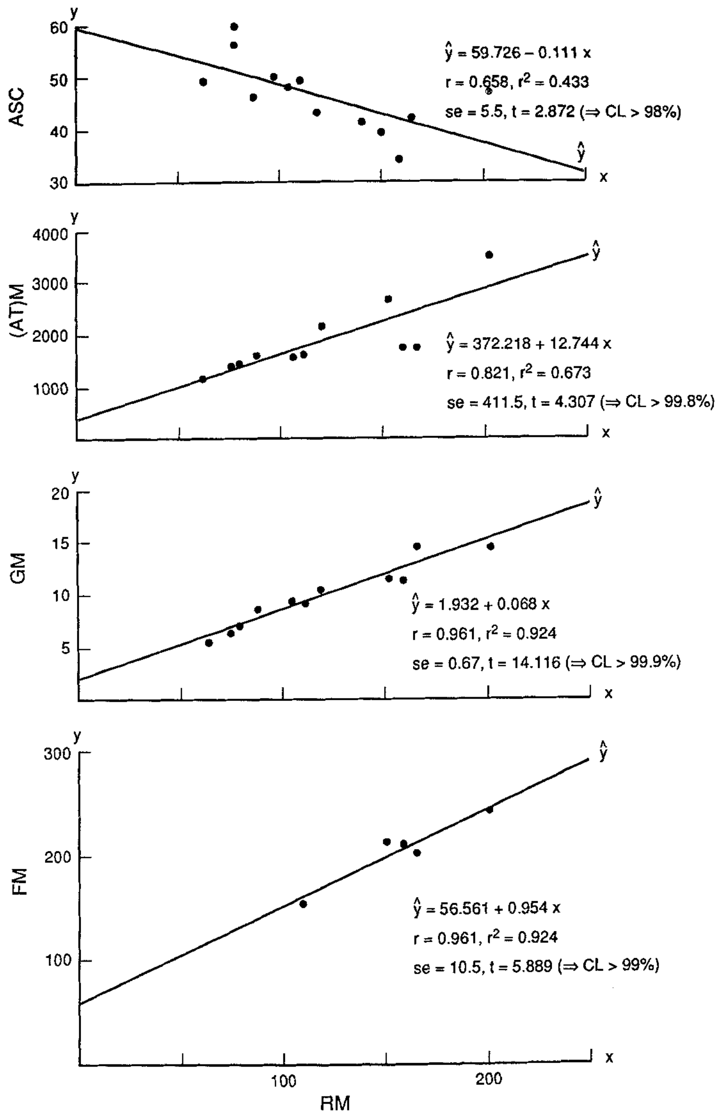

Fig. 4. Selected scatter plots against *RM*. *ASC* is the 'ascent' duration in months from sunspot cycle minimum occurrence to maximum occurrence; *(AT)M* is the maximum value for total corrected area of sunspots in millionths of a hemisphere; *GM* is the maximum number of groups; and *FM* is the maximum value for 10.7-cm solar radio flux in solar flux units. *(AT)M*, *GM*, and *FM* represent maximum values as expressed in 'smoothed' units, where the smoothing follows that used for sunspot number. See text for additional details.

GM = 14.88 (cycle 21), $(AT)M$ = 3547.7 millionths of a hemisphere (cycle 19), and ASC = 34 months (cycle 22; the fastest rise to sunspot maximum).

Another parameter of interest is the minimum-to-minimum period (PER) for a cycle. Figure 5 depicts the variation with time of PER (bottom) and the variation of PER

TABLE III

Estimates for selected cycle parameters for cycle 23

Parameter	se	Prediction			
		RM = 136.5	198.8	164.1	213.9
FM	10.5	186.8	246.2	213.1	260.6
GM	0.67	11.21	15.45	13.09	16.48
$(AT)M$	411.5	2111.8	2905.7	2463.5	3098.2
ASC	5.5	44.6	37.7	41.5	36.0

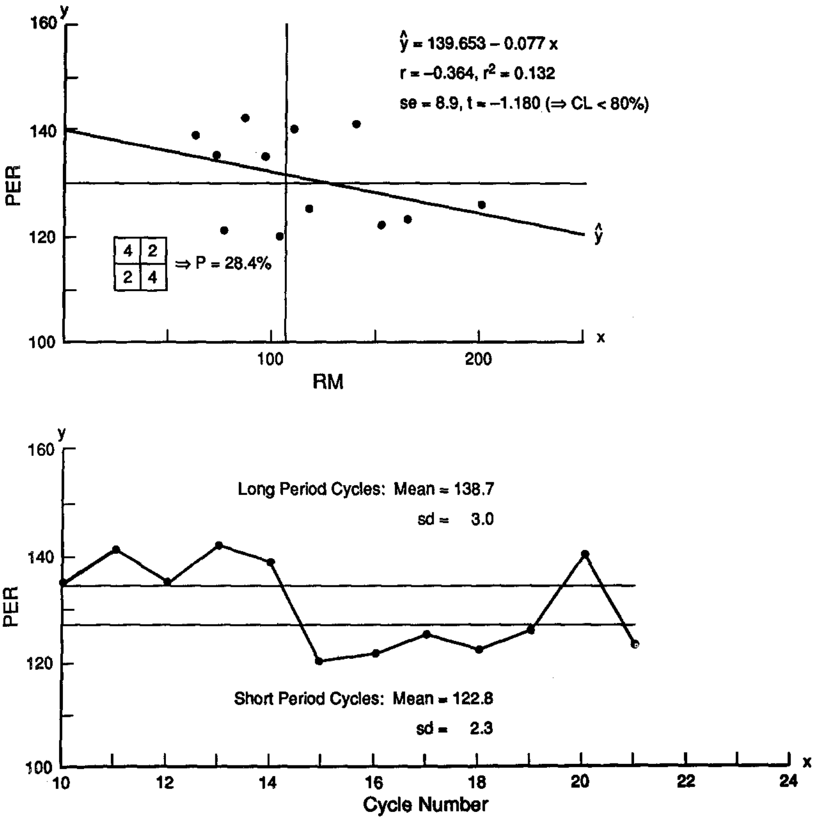

Fig. 5. (*Bottom*) PER vs cycle number. Notice the separation of PER into two cycle groupings – long-period cycles and short-period cycles. The two horizontal lines delineate the extent of the observed extremes between the two classes. (*Top*) PER vs *RM*. The thin lines are the median values and the heavy line is the linear fit. Details are explained in the text.

against *RM* (top). Unlike the afore-mentioned parameters shown in Figure 4, one finds that PER does not correlate well with *RM*. The linear fit has a confidence level $CL < 80\%$ and a Fisher's exact test probability $P = 28.4\%$, suggestive that the variation of PER against *RM* is due entirely to chance. Certainly, there have been large maximum amplitude cycles that were cycles of longer than average period (e.g., cycle 11) and, conversely, small maximum amplitude cycles that were cycles of shorter than average period (e.g., cycle 16). Inspection of the bottom-portion of Figure 5 further leads one to believe that sunspot cycle period may be 'bimodal', for PER has always been either longer than 134 months or shorter than 127 months (Wilson, 1987a, 1988d). If PER is truly bimodally distributed, then one finds that the long-period cycle mode has cycles whose length averages about 138.7 months (sd = 3.0) and the short-period cycle mode has cycles whose length averages about 122.8 months (sd = 2.3), and that 6 out of the last 7 sunspot cycles have been short-period cycles. If cycle 22 turns out to be a short-period cycle, then it would be expected to have a period in the range of 117–129 months (at the 95% level of confidence), implying that minimum for cycle 23 should occur during the interval of June 1996 to June 1997; on the other hand, if cycle 22 turns out to be a long-period cycle, then it would be expected to have a period in the range of 131–146 months (at the 95% level of confidence), implying that minimum for cycle 23 would be delayed until sometime during the the interval of August 1997 to November 1998. Only continued monitoring of the behavior of the sunspot cycle searching, in particular, for the occurrence of 'new cycle' spots at high latitudes (Wilson, 1987b; Rabin *et al.*, 1990), will reveal whether cycle 22 will be of short or long period.

4. Summary

Several methods have been described for estimating the maximum amplitude *RM* of a sunspot cycle a number of years ahead of its onset. Furthermore, each of the methods was found to be statistically important. Hence, they should prove instructive for estimating the size of cycle 23, the next sunspot cycle due to begin about mid-1996 to late-1998. The consensus of the methods is that cycle 23 will be a large amplitude, fast-rising cycle and one that is record setting or near record setting in nearly all of the usual solar cycle descriptors (sunspot number, 10.7-cm solar radio flux, number of groups, total corrected area of sunspots, and ascent duration). If the prediction runs true, then it surely will be of concern to solar modelers (in that a physical explanation must be sought for the 'preferred' even–odd cycle pairing), climate modelers (in that the Sun may influence Earth's climate more directly than previously believed), electrical power distributors (in that enhanced solar activity may produce geomagnetic storms as great or greater than those of 1989; Kurth, 1991), and space mission planners (in that enhanced solar activity may mean greater satellite drag for low-Earth orbital missions and potentially greater risks to astronauts, due to the effects of large solar flares, when they are outside the Earth's protective magnetic field while on route to the Moon or Mars).

In closing, it should be remembered that the results reported here are based on the small sample of modern era sunspot cycles that are available for detailed study, not on the reconstructed record that extends further back in time. The modern era represents the most reliable sunspot data and is based on a complete record of daily values extending back to 1849 (the year following the year of maximum for cycle 9). Data of lesser quality extend back to 1818, with the data being considered of poor quality for earlier times. Had one used the earlier data back to 1818, then one might argue that the guiding principle of odd-following cycles being the larger of the two cycles in the even–odd cycle pairing failed for cycles 8 and 9; however, it should be noted that the maximum values (based on annual averages) computed for both cycles 8 and 9 are based on a fairly incomplete daily record (the maximum annual averages for these two cycles are based on 150 and 234 days, respectively). For example (see Waldmeier, 1961, p. 25), cycle 8 had its maximum annual average in 1837 (= 138.3, sd = 28.0) and cycle 9 had its maximum in 1848 (= 124.7, sd = 20.6). Hypothesis testing for comparing these two means (Lapin, 1978, p. 486) yields a $t = 1.355$ which for 22 degress of freedom is not a statistically significant result ($CL < 90\%$), implying that the maximum value for cycle 8 is statistically no different in size from that of cycle 9. Clearly, it is difficult to reckon which of the two cycles was truly the larger. If one goes even further to include cycles −4 through 9 (ca. 1700–1850), then one finds that the odd-following cycle has been the larger cycle 10 out of 13 times, implying that a statistically significant result is still achieved ($P = 4.6\%$, based on the binomial formula, meaning that the probability due to chance of having the odd-following cycle to be the larger cycle 10 or more times in a sample of 13 cycles is $P = 4.6\%$).

Acknowledgements

The author thanks John M. Davis (NASA MSFC) for reading the manuscript and the unnamed referee for helpful suggestions. This research was supported by NASA's Solar Physics Branch of the Space Physics Division.

References

Bahcall, J. N. and Press, W. H.: 1991, *Astrophys. J.* **370**, 730.
De Jager, C.: 1959, in S. Flügge (ed.), *Encyclopedia of Physics*, Vol. LII, *Astrophysics III: The Solar System*, Springer-Verlag, Berlin, p. 150.
Everitt, B. S.: 1977, *The Analysis of Contingency Tables*, John Wiley and Sons, New York.
Friis-Christensen, E. and Lassen, K.: 1991, *Science* **254**, 698.
Gnevyshev, M. N. and Ohl, A. I.: 1948, *Astron. Zh.* **25**, 18.
Gorney, D. J.: 1989, *J. Spacecraft Rockets* **26**, 428.
Gorney, D. J.: 1990, *Rev. Geophys.* **28**, 315.
Hale, G. E. and Nicholson, S. B.: 1938, *Publ. Carnegie Inst.*, No. 498.
Howard, R.: 1977, in A. Bruzek and C. J. Durrant (eds.), *Illustrated Glossary for Solar and Solar-Terrestrial Physics*, Vol. 69, Astrophysics and Space Science Library, D. Reidel Publ. Co., Dordrecht, Holland, p. 7.
Kelly, P. M. and Wigley, T. M. L.: 1990, *Nature* **347**, 460.
Krauss, L. M.: 1990, *Nature* **348**, 403.
Kurth, W. S.: 1991, *Nature* **353**, 705.
Lapin, L. L.: 1978, *Statistics for Modern Business Decisions* (2nd ed.), Harcourt Brace Jovanovich, Inc., New York.
Layden, A. C., Fox, P. A., Howard, J. M., Sarajedini, A., Schatten, K. H., and Sofia, S.: 1991, *Solar Phys.* **132**, 1.
McKinnon, J. A.: 1987, *Report UAG-95*, World Data Center A for Solar-Terrestrial Physics, Boulder, Colorado, 112 pp.

Rabin, D. M., DeVore, C. R., Harvey, K. L., Hoeksema, J. T., and Sheeley, N. R., Jr.: 1990, NOAO Preprint No. 352, NOAO, Tucson, Arizona, 77 pp.
Reid, G. C.: 1991, *J. Geophys. Res.* **96**, 2835.
Vampola, A. L.: 1989, *J. Spacecraft Rockets* **26**, 416.
Vitinskii, Yu. I.: 1965, *Solar Activity Forecasting*, NASA TT F-289, Washington, D. C., 129 pp.
Waldmeier, M.: 1961, *The Sunspot-Activity in the Years 1610–1960*, Schultness and Co., Zürich.
Walterscheid, R. L.: 1989, *J. Spacecraft Rockets* **26**, 439.
Willson, R. C. and Hudson, H. S.: 1991, *Nature* **351**, 42.
Wilson, R. M.: 1987a, *J. Geophys. Res.* **92**, 10101.
Wilson, R. M.: 1987b, *Solar Phys.* **111**, 255.
Wilson, R. M.: 1987c, *Solar Phys.* **112**, 1.
Wilson, R. M.: 1988a, *Solar Phys.* **115**, 397.
Wilson, R. M.: 1988b, *Solar Phys.* **117**, 179.
Wilson, R. M.: 1988c, *Solar Phys.* **117**, 269.
Wilson, R. M.: 1988d, *J. Geophys. Res.* **93**, 10011.
Wilson, R. M.: 1989, *NASA Tech. Paper 2948*, NASA, Marshall Space Flight Center, Huntsville, Alabama, 54 pp.
Wilson, R. M.: 1990a, *Solar Phys.* **125**, 133.
Wilson, R. M.: 1990b, *Solar Phys.* **125**, 143.
Wilson, R. M.: 1990c, *Solar Phys.* **127**, 199.
Withbroe, G. L.: 1989, *J. Spacecraft Rockets* **26**, 394.