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# A Systematical Approach to Leibniz's Theory of Relations and Relational Sentences\*

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0. This paper is devoted to a systematic investigation of Leibniz's logical treatment of relations and relational sentences. I hope that the discussion of such a topic will contribute to the solution of the difficult question concerning Leibniz's attitude toward the ontology of relations.

In order to understand what in effect his intentions are concerning relational sentences, our investigation will proceed on three fronts. We must first look at the so-called "oblique terms" (*termini obliqui*) and at the inferences "from the direct to the oblique" (*a rectis ad obliqua*); secondly we will tackle the problem of the sentences with explicitly "subject-relation-subject" form; and thirdly we will look at the "inversion of relation" (*inversio relationis*). Finally some conclusions are drawn about Leibniz's general ideas on the nature of relations.

## A. Oblique terms and inferences "a rectis ad obliqua"

1. In the scholastic tradition the name "oblique terms" was given to those terms which are not in the nominative case. In the statement "Quercus est arbor", for example, both "quercus" and "arbor" are in the nominative case, whereas in the statement "Arbor habet radicem", "radicem" is an oblique term. A syllogism was called oblique if at least one of its terms, as in the following example, was in the oblique position:<sup>1</sup>

Magnum beneficium obligat,  
Rectae institutionis magnum est beneficium,  
Ergo Recta institutio obligat.<sup>2</sup>

"A rectis ad obliqua" inferences, on the other hand, were those in which an assertion in which the terms appeared in the nominative case was followed by another assertion in which one or both of the terms of the first appeared in the oblique case. A standard example of the "a rectis ad obliqua" inference was given

by the proposition: "Grammatica est ars, Ergo qui discit Grammaticam discit artem".<sup>3</sup>

In his *Logica Hamburgensis*, Joachim Jungius considered the "a rectis ad obliqua" inference as a *consequentia simplex* — that is as a *consequentia* in which the consequent follows from the antecedent without intervention of any middle term. Jungius further showed that he was perfectly aware of the fact that relations (*notiones respectivae*) were in most cases implicit in the *obliquitates*:

Frequens denique et illa e consequentiis simplicibus est, a rectis ad obliqua procedens, quae respectivam notionem cum Praedicto Antecedentis obliquum sibi adjungente in consequente enuntiat. Ut *Omnis circulus est figura, Ergo Quicumque circulum describit, figuram describit*. Notio respectiva est *Describens*.<sup>4</sup>

Jungius's concepts were to be developed by his disciple (and the correspondent of Leibniz) Johannes Vegetius, who, in the *Admonitio*, which he prefaced to the 1661 edition of the *Logica Hamburgensis*, was to subject to systematic criticism the rules for oblique syllogisms worked out by the logician and mathematician Christophorus Scheibler, and will challenge the correctness of certain proofs in Arnauld and Nicole's *La logique*.<sup>5</sup>

The Dutens edition of Leibniz's works preserves part of the correspondence between Vegetius and Leibniz, and one of the problems we find discussed in fact concerns oblique inferences.<sup>6</sup> There exist two letters which are particularly relevant to the discussion: one from Vegetius dated 4 January 1687 from Hamburg, and a certainly subsequent one from Leibniz to Vegetius datable around the middle of the same month.<sup>7</sup> Vegetius's letter is not in the Dutens edition, but is to be found among Leibniz's manuscripts. One side of the page contains, in Vegetius's hand, his explanation of certain logical steps from Jungius's *Logica Hamburgensis*; on the other side Leibniz has written an example

corresponding to each step. The *Logica Hamburgensis* text which Vegetius sets out to explain concerns the so-called “Dianocae compositae”, that is a sort of argument resulting from the contraction and connection of simple *consequentiae* and syllogisms. Given, for example, the argument:

Omnis nux est pomum,  
Simia vescitur nuce,  
Ergo pomo vescitur.<sup>8</sup>

Jungius proposes the following analysis:

- (1) Omnis nux est pomum,
- (2) Ergo, Qui vescitur nuce vescitur pomo,
- (3) Simia vescitur nuce,
- (4) Ergo, Simia vescitur pomo.<sup>9</sup>

The explanation in the *Logica Hamburgensis* clarifies that the passage from (1) to (2) is an example of *consequentia immediata a rectis ad obliqua*, whereas the passage from (2) to (4) is a first figure syllogism.<sup>10</sup> In his letter to Leibniz, Vegetius attempts to reproduce the overall logical form of Jungius’s analysis, putting capital letters as variables in the place of concrete terms and explicitly indicating the *quantity* of the subject *and* of the predicate term.

It is clear from Vegetius’s analysis that, following his master’s teaching, he is aware of the fact that relations are implicit in inferences *a rectis ad obliqua*. For the function of the verb in the example from Jungius, Vegetius in fact uses the expression *ad aliquid*, exactly the term used in the Aristotelian-scholastic logical tradition to designate the relation. Moreover Vegetius keeps to Jungius’s conviction of the irreducibility of inferences *a rectis ad obliqua*.

Leibniz’s letter to Vegetius, on the other hand, contains a “demonstration” of the *consequentia a rectis ad obliqua*. Taking his departure from the universal affirmative proposition “Graphice est ars”, Leibniz seeks to prove the logical plausibility of the passage to “qui discit graphicen, discit artem”.<sup>11</sup> The proof is based on three assumptions.

(1) Given a universal affirmative proposition, if the subject of the said proposition acts as predicate in another affirmative proposition, it is always possible to substitute *salva veritate* the predicate of the first for the predicate of the second. If, for example, we have “graphice est ars” and “res quae est graphice”, by substitution we can obtain: “res quae est ars”. And vice versa;

(1.1) if we know that the proposition “qui discit

artem” can be obtained *salva veritate* from the proposition “qui discit graphicen”, then it will be legitimate to infer “qui discit graphicen, discit artem”.

(2) A *special* oblique term of the type “qui discit graphicen” — that is a term in which the object of the verb (of the relation, according to Jungius) is directly specified — is equivalent to a *general* oblique term — that is a term in which the object of the verb is expressed in general terms by means of the word *res*, followed by the specification in the nominative (“quae, qui, quod est . . .”). Thus “qui discit graphicen” is equivalent to “qui discit rem, quae est graphice”. And vice versa:

(2.1) for “qui discit rem quae est ars” it is always possible to substitute “qui discit artem”.

(3) If we can substitute B for A, C for B, and D for C, then in every proposition D may be substituted for A *salva veritate*.

Basing himself on these assumptions, Leibniz develops the following proof:

- (a) Graphice est ars (by hypothesis);
- (b) Qui discit graphicen, discit rem, quae est graphice (assumption 2);
- (c) Ergo qui discit rem quae est graphice, discit rem quae est ars (because of (a) and (b) and assumption 1);
- (d) Qui discit rem, quae est ars, discit artem (because of 2.1);
- (e) Ergo qui discit graphicen, discit artem (because of 3 (b), (c) (d) and 1.1).<sup>12</sup>

The key point of the proof consists in analysing the assertion “qui discit graphicen” by developing it into the equivalent assertion “qui discit rem, quae est graphicen”.

This analysis has the advantage of showing clearly — despite the implicit presence of relations (dyadic predicates) in the reasoning — that the conclusion follows from the premisses with no need to refer to any specific property concerning relations. As can be seen, Leibniz has no difficulty in accepting expressions in the form: “qui discit graphicen”. This shows that he distinguishes between the possibility of analysing the logical argument in question and the possibility of further analysing the relation implicit in it, and that he is aware of the fact that, in this case, analysis of the former does not imply analysis of the latter.

Almost twenty years after his letter to Vegetius, Leibniz will return to the inferences *a rectis ad obliqua* in a short but interesting passage in the *Nouveaux essais*:

De plus il faut scavoit qu’il y a des consequences asylogistiques bonnes et qu’on ne scauroit demonstrier à la rigueur pour aucune syllogisme sans en changer un peu les termes; et ce changement même fait la consequence asylogistique. Il y en a plusieurs,

comme entre autres, *a recto ad obliquum*. . . . Et ces conséquences ne laissent pas d'être démontrables par des vérités dont les syllogismes vulgaires mêmes dependent.<sup>13</sup>

Here Leibniz is considering inferences *a rectis ad obliqua* as inferences which are *not demonstrable* by means of ordinary syllogisms, and he mentions "a modification of terms" which would be required to carry out the proof. We cannot exclude that such a "modification of terms" concerns first of all the equivalence mode in assumption (2) in the proof sent to Vegetius ("qui discit  $a =$  qui discit rem quae est  $a$ "). It can be argued, that is, that in his universal project Leibniz was thinking of prescribing the transformation implicit in assumption (2) as a necessary step in the proof of inferences *a rectis ad obliqua*. As we shall see, this hypothesis is corroborated by the analysis Leibniz makes of oblique terms in his *Generales inquisitiones*.

2. If it appears relatively easy to understand Leibniz's intentions regarding inferences *a rectis ad obliqua*, the same cannot be said for the oblique terms. Regarding inferences *a rectis ad obliqua* we have seen that Leibniz believes they should not be considered as primitive, and puts forward a completely correct analysis of them. But his attitude to oblique terms is more complex. In his reflections on "oblique terms" Leibniz constantly expresses the conviction that, in a certain sense, they imply complex sentences, and therefore cannot be fully "explained" without reference to several interconnected propositions. He holds that it is an error therefore to think it possible to present the list of "non-complex" terms (concepts) before the list of the "complex" ones (propositions).<sup>14</sup>

Another opinion constantly met with in Leibniz's notes on the "*grammatica rationalis*" is that the oblique forms are created by the presence of cases or of prepositions (or of both).<sup>15</sup> In fact, Leibniz considers grammatical cases as results of a sort of contraction and absorption of prepositions into categorematic terms: "casus est enim quasi praepositionis contractio".<sup>16</sup> And on several occasions he repeats that in "philosophical language" the presence of prepositions means that cases can be dispensed with (and vice versa). At the same time, he repeatedly insists that prepositions express relations.<sup>17</sup> It follows that he is well aware of the fact that the *obliquitates* conceal relations, and this is precisely why it is necessary to investigate the nature of Leibniz's analysis of oblique terms if we are to clarify his attitude to relations.

As is well known, a famous text concerning reflections on grammar contains Leibniz's peremptory assertion that all oblique terms and *flexiones*, as well as the abstractions, must be "banned" from the future *characteristica*.<sup>18</sup> This point of view is perfectly in line with an ontology of nominalist inspiration. However, other texts exist in which Leibniz moderates his position and seems to be content with prescribing that in philosophical language one should try as far as possible to avoid abstractions and oblique terms.<sup>19</sup> It is very probable that it was a progressive deepening of his research into rational grammar which brought him to more moderate positions, and that, having started with a strict nominalist programme, he was forced subsequently to revise his intransigence. Unless we are to suppose that there are fractures or sudden changes in the development of Leibniz's thought, this seems the most plausible hypothesis to explain how he passes from the clear prohibition we have mentioned to the following affirmations, which are just as explicit:

. . . Prepositiones et casus adhibentur ad multos obliquitatis respectus inter se discernendos, praesertim cum inter se concurrunt, ex.gr. "Mitto tibi pecuniam Johannis cambio". Hoc est: "sum missor pecuniae Johannis cambiatae cuius tu eris acceptor". Cambiatam autem voco pecuniam, ut evitem abstractum: cambium. Missor cambiati potest una etiam voce appellari campsor.

Abstractis interdum non admodum commode careri potest, ut cum dico "Agelastus semel risit", non possum commode eliminare adverbium remque ad sola nomina reducere, nisi adhibitis abstractis.<sup>20</sup>

Similarly, in a marginal note in an essay on calculus of circa 1680, Leibniz observes that in logical calculus one is forced to admit "oblique enunciations".<sup>21</sup> This is a particularly important observation, especially in relation to its context, since the note is concerned with simple primitive terms, which is to say those terms which cannot be further analysed (by us, or absolutely), and from which all the other concepts derive, by composition. In this case Leibniz recognizes that there cannot be demonstrable propositions with simple concepts as their object — that is that there cannot be the proof of a proposition which asserts: "A is thus and thus", where A is the name of a simple concept. At the same time, Leibniz appears to realize that, precisely because of their simple nature, primitive concepts can be united to form propositions which include relations (of the type: "A is compatible with B", "A is similar to C", etc.). The most important aspect of these considera-

tions consists in the implicit recognition that relations cannot be completely excluded from the “philosophical language”.

Yet, it cannot be claimed that Leibniz is clear on this point. In his *Analysis particularum* for example, immediately after asserting that one can comfortably leave prepositions aside, he maintains that in the universal language moods and cases can be neglected, though only if they are replaced by *particulae*. Thus, to the proposition: “*Petrus est doctus cum Johanne*” there should correspond in the universal language a proposition of the type: “*Petrus est doctus cum Johannes*”, in which, on analogy with French and Italian usage, the function of *cum* is preserved, without the case form being made explicit. Leibniz observes, however, that the function of *cum* could equally well be carried out by a locution of the *ut-et* type, and expresses the hope that a suitable notation be worked out so as to render the sense of *particulae* such as *cum* itself and *ut-et*.<sup>22</sup> From the *Analysis particularum*, then, there emerge both the proposal to replace prepositions and cases with special *particulae* and the conviction that the *particulae*, and in a subordinate way the cases, may be replaced by complex propositions in which neither cases nor prepositions are present.

In an essay on universal grammar which can be confidently dated circa 1686, Leibniz returns to this argument and clearly expresses the intention to replace cases with *particulae* in which the relation implicit in cases becomes evident:

Satis ergo erit a nobis non curari casus linguarum usitatarum, sed pro generali obliquo tam respectu nominis per se positi quam nominis cum verbo coalescentis adhiberi particulam, ut si diceremus: *manus relata ad Petrum; virtus relata ad Alexandrum; homo relatus ad virtutem; sacerdos relatus ad Phoebum; sacrificans relatus ad Phoebum; qui amat relatus ad homines*, et hic erit casus generalissimus nulli satis respondens casui in linguis notis. Cum dico *manus Petri, vir mortis, causa hominis, Deus fortitudinis* intelligo: pertinere ad Petrum, ad mortem, ad fortitudinem.<sup>23</sup>

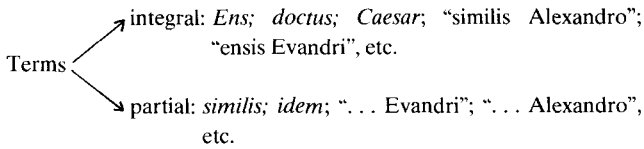
A little later on, Leibniz actually proposes to render certain forms of the genitive (possessive genitive) by means of adjectives, so that for example “*ensis Evandri*” would, in the universal language, become “*ensis Evandrius*”.<sup>24</sup> This sufficiently demonstrates that his writings on the universal language (which, it must be remembered, are little more than notes and not intended for publication) represent work in progress. With this comment in mind, we will wish to be cautious about

certain of the solutions he advances. In particular, in the case of *oblique terms* and *prepositions*, we can only advance hypotheses to reconcile the two apparently contradictory solutions just considered. It is possible for example that Leibniz was thinking of the use of special characters (special *particulae*) for the functions of prepositions and cases, while recognizing that these characters would only be a sort of shorthand. On this hypothesis, the *particulae* replacing the cases and prepositions in philosophical language would therefore be equivalent to abbreviations of complex sentences which, for brevity, it would be better to neglect. But if this really was his intention, it would be equivalent to admitting that prepositions, cases and oblique terms in general are useful in philosophical language, but not necessary, and this would contrast with the conviction, quite clearly expressed, as we have seen, that certain *obliquitates* cannot be avoided.

It would undoubtedly help greatly here if we were able to date accurately the texts and fragments on language and *grammatica rationalis*. However, we must allow that the admission of the inevitability of oblique terms gives rise to two interpretations: that *obliquitates* cannot be avoided may in fact mean (1) that relations cannot be eliminated, or, on a more superficial reading, (2) that certain grammatical forms (the cases) are necessary.

Before concluding this part, devoted to the oblique terms, mention should be made of the analysis given in the *Generales inquisitiones*. This is Leibniz’s only large-scale text on logic, and it represents one of the high points of his logical investigations. The oblique terms are here introduced in relation to an overall division of terms into integral and partial.<sup>25</sup> Leibniz uses “integral” to describe a term which may act as subject or predicate (or both) in a proposition. “Homo” and “Caesar” are integral terms, whereas “idem” and “similis” are typically partial terms — that is terms which, without integration, cannot themselves act as subject or predicate in propositions which make sense. Of course it can be pointed out that sentences of the type: “similar is an adjective” or “identical is a relation” make sense, and that the function of subject is carried out in them by partial terms. The terms of such propositions are considered to be in *suppositio materialis*, however — to be taken at a second level and not according to their meaning. Given the peculiarities of the Latin language, Leibniz holds that nouns (both proper and common) are also to be considered partial when taken on their own in

the genitive or dative or other cases — such as “Evandri” or “Caesari” — which presuppose other nouns or verbs in order to be able to be considered genuine parts of propositions. The core of Leibniz’s distinction is given schematically as:<sup>26</sup>



Leibniz considers “similis Alexandro” a term which can carry out the predicate function, while “similis” on its own — in contexts which are not second level or “material” — cannot properly carry out either the subject or the predicate function.

Writing of “partial terms” in his *Generales inquisitiones* Leibniz lays down that they cannot enter into the logical calculus unless they are “saturated”, i.e. brought back to the integral form.<sup>27</sup> He therefore proposes that bringing partial terms back to integral status should follow a standard procedure, using an ad hoc linguistic structure. Given the proposition “Caesar est similis Alexandro”, in which the saturated partial term “similis Alexandro” appears, Leibniz suggests reducing it to the form “Caesar est similis τϕ A qui est Alexander”, or to the equivalent form (explaining “τϕ A”): “Caesar est similis rei quae est Alexander”.<sup>28</sup> In the *Generales inquisitiones*, therefore, Leibniz presents in systematic form the logical-linguistic analysis of oblique terms proposed in the letter to Vegetius (which postdates or is at the earliest contemporary with that work). In the letter to Vegetius he suggests transforming the proposition “qui discit graphicen” into the equivalent proposition “qui discit rem quae est graphicen”. To transcribe his proposal symbolically, given a proposition in the form “*Sab*” this changes to “*Sax & Bx*” where *S* symbolizes a dyadic relation, *x* is equivalent to the generic term “res” in Leibniz’s original example, and *B* is the symbol for a property characterizing *x*.

The transformation he proposes can be interpreted in two ways, however. Let us consider a proposition containing a relation between two proper names, as in the example given in the *Generales inquisitiones*: “Caesar est similis rei quae est Alexander”. If my symbolic transcription is correct, it would follow that Leibniz intended to resolve the proper name *Alexander* into a sort of predicate to be attribute to a variable. In

ordinary language the transcription would thus read: “Caesar is similar to *x* and *x* Alexandrizes (or *x* is Alexander)”. In this case it must be supposed that Leibniz intended to use proper nouns with the predicate function. This may seem too bold an hypothesis and suggest a more plausible interpretation as follows: “*Sax & x = b*”, in which the second term of the conjunction is simply an identity. Yet textual evidence exists which would support the thesis that Leibniz intended proper nouns to be used attributively.

He intended to apply both to “integrated” relational terms and to functional terms the same formula for transcribing integrated terms “in general” into standard form. Given the functional expression “*Ensis Evandri*. . .” (Evander’s sword), Leibniz proposes to transform it into “*Ensis qui est res Evandri*”, and in his essays on grammar and on universal language he proposes to eliminate the oblique case contained in the functional expression, reducing “*Ensis Evandri*” to “*ensis Evandrius*”.<sup>29</sup> This leads to the possibility of obtaining from the oblique term “*ensis Evandri*” the integral term: “*ensis qui est res Evandria*”, perfectly in agreement with the fundamental tenet of his logical-grammatical research: not to differentiate between nouns and adjectives.

It is interesting to observe, finally, that he believes that the proposition “Caesar est similis rei quae est Alexander” can be analyzed into *three* integral terms: “Caesar”, “similis rei”, “quae est Alexander”, while the expression “*ensis qui est res Evandri*” is made up of the integral terms “*Ensis*” and “*qui est res Evandri (res Evandria)*”.

## B. Explicitly relational sentences

1. By the expression “explicitly relational sentences” I mean those sentences which explicitly take up the logical form “*a R b*”. These include sentences of the type: “Arthur is similar to John”, “Socrates is older than Plato”, “A and B are the same age”, and so on. The problem to be tackled now is how Leibniz analyses these propositions, what treatment he intends to subject them to within the logical calculus (the universal language).

As I pointed out in a previous paper, there is at least one text in which Leibniz classifies relations on the basis of the *logical structure* of the sentences which express them.<sup>30</sup> Here he maintains that there are two types of

relation — by *comparison* and by *connection*. If *a* and *b* are names of things (of real objects), and if *p* is a sentence which affirms that a certain relation exists between *a* and *b* (or rather, between the objects named *a* and *b*), then the relation in question will be by *comparison* if it is possible to find two sentences *q* and *r* having *a* and *b*, respectively, as their subjects, such that their conjunction is equivalent to *p*. If, for example, *a* is similar to *b*, the sentence “*a* is similar to *b*” can be divided into two sentences, each of which attributes the same property respectively to *a* and *b*: “*a* is *C*” and “*b* is *C*”.<sup>31</sup> A relation by *connection* will exist between *a* and *b*, on the other hand, when this relation can only be expressed by means of a sentence including both *a* and *b*. In the text we are considering, Leibniz provides no example of this second type of relation, but it is clear that he intends such relations as “father”, “son”, “owner”, etc. — relations which he elsewhere denominates relations “by influence” (*influxus*).

In the case of relations by connection, Leibniz brings a given sentence of the logical form “*a R b*” to another sentence of the type: “*a* is *P*, insofar as *b* is *Q*” (or: “*a* is *P* and for that reason (*et eo ipso*) *b* is *Q*”). The letters *P* and *Q* each indicate one term of a pair of correlated terms, such as *subditus-dominus*; *amans-amata*; *occidens-occisus*; *superior-inferior*, etc.<sup>32</sup> The expressions “insofar as” (*quatenus*), “for that reason” (*et eo ipso*) are syncategorematical terms connecting the propositions “*a* is *P*” and “*b* is *Q*”. Regarding the interpretation of these expressions, Leibniz sometimes states that the *quatenus* refers to a relationship of *causality* and sometimes he is content to conceive of it as a term which, placed between two sentences *p* and *q*, gives rise to the sentence: “*p* is true in consideration of the truth of *q*”.<sup>33</sup> In any event it appears that *quatenus*, *et eo ipso* etc. are to be interpreted as *conditionals* of some sort (though evidently not as material conditionals). There is also some evidence of the fact that Leibniz was aware of the tradition of the reduplicative sentences and their *expositio*.<sup>34</sup> It is not unlikely therefore that he intended to make “technical” use of the *quatenus* and similar terms — a use corresponding, in effect, to that described in the *expositiones* of the reduplicatives.<sup>35</sup> This however, is only a hypothesis, about which there can be no absolute certainty.

In his analysis of relations by *connection*, Leibniz proposes to “dissolve” the relations “*hors des sujets*” into pairs of relations *ut accidentes*.<sup>36</sup> In natural lan-

guages, the cases and the prepositions express relations interconnecting otherwise unrelated terms. If I state that “*a* is the father of *b*”, the expression “father of” connects *a* and *b* and represents a relation conceived of as a “bridge” between the two subjects. The same is true if I state that “Paris loves Helen” (where “Helen” is in the accusative case: *Paris amat Helenam*, or in the genitive: *Paris est amator Helenae*). Given his intention to create an artificial language free of cases and prepositions, Leibniz automatically proposes to eliminate relations as “accidents with one leg in one subject and one in another”.<sup>37</sup> To do this with a relational sentence such as “Paris loves Helen”, he divides the relation expressed by the verb “loves” into the two *accidentes* relations: *amans* and *amata*, and attributes them to Paris and to Helen respectively. Now to attribute the property of *loving* to Paris and of *being loved* to Helen is in no way the same as stating that Paris loves Helen. Both Paris and Helen have a property that refers to an outside term, but that term is not identified in the property in question. To establish a link between the loving of Paris and the being loved of Helen it is necessary, in fact, bring in the *quatenus*, or *et eo ipso* — it is necessary to use conjunctions to link the two subjects, Paris and Helen, which would otherwise not refer one to the other. Clearly Leibniz attempts to reduce the relations expressed as a direct link between two subjects to a logical connection between two sentences in subject-predicate form. This seems to fit quite well with his reiterate statement that relations *are truths*.<sup>38</sup>

Predicates such as *amans*, *amata*, *occidens*, *occisus*, etc. are relational in the sense that they have as subject, let us say, *a*, and refer to a term *x* which bears the corresponding related predicate. If the subject loved by Paris remains unspecified, the predicate *amans* is relational only in a general sense, and its relational function is weakened as a result. In distinguishing between relations “by comparison” and relations “by connection”, Leibniz appears to be perfectly aware of the peculiar nature of the latter and the fact that they are not able to be reduced to the former. In his *De termino, praedicato, relatione* he states that relations by connection cannot be subjected to the same analysis as relations by comparison.<sup>39</sup>

### C. The *inversio relationis*

1. In his *Logica Hamburgensis*, Jungius defines the *inversio relationis* as follows:

Inter aequipollentias *in partibus* Enuntiationis *per se significativis* consistens, frequens est illa, quae fit *per inversionem relationis*, ubi nimirum Subiectum unius, est pars consequens in praedicato alterius, et contra; pars princeps autem Praedicati in una ut correlatum se habet ad partem principem Praedicati in altera  
...<sup>40</sup>

Among the examples of *inversio relationis* which Jungius adopts are: “David est pater Salomonis, et Salomon est filius Davidis”; “Christus redemit omnes peccatores, et Omnes peccatores a Christo sunt redempti”, etc. As has correctly been observed, Jungius did not invent this particular type of inference, which, like the “*a rectis ad obliqua*” inference, was already well known to the scholastic logicians.<sup>41</sup> But he differs from his predecessors in the dominant role which he gives to the *inversio relationis* within his logical essays. For Jungius (and for his disciple Vegetius) the *inversio relationis* is a simple inference, not capable of demonstration, and very useful in the proof or disproof of more complex inferences. In the letter mentioned above which Vegetius sent to Leibniz in January 1687 he declares that he believes it impossible “for anyone with the benefit of reason” to reject or question the *inversio relationis* from the point of view of logical validity, and follows his master’s teaching in maintaining that in order to accept such inferences it is sufficient to recognize the validity of the principle that correlated terms “are mutual statements”.<sup>42</sup>

Faced with the official doctrine of Jungius’s “school”, Leibniz — as we would expect — does not set out to maintain that the *inversio relationis* inference is logically invalid; he simply does not agree that it is incapable of demonstration. In his published work and in such unpublished papers as I have been able to consult, Leibniz does not often mention the *inversio relationis*: however, on the few occasions when he deals with it he considers it a valid inference. In a series of notes on a Jungius’s text Leibniz actually makes his own proposals for a notational system better than Jungius’s in order to express the *inversio*:

*Inversio relationis*. Quadrangulum Laterum AEqualium Q<sup>c</sup>L<sup>ca</sup>AE<sup>ca</sup>. Latera AEqualia Quadranguli L<sup>ca</sup>AE<sup>ca</sup>Q<sup>ca</sup>.<sup>43</sup>

And again, in a short fragment in which he sets out to illustrate the structure of the *characteristica*, Leibniz explicitly includes the *inversio relationis* among the *enuntiationes generales* of the calculus:

Sunt et generales enuntiationes tales circa *est et non*; item *inversio relationis* ut A○—B. Ergo B○—A.<sup>44</sup>

The appearance in this fragment of the “○—” sign, borrowed from Jungius, not only brings the two last-mentioned texts closer in time, but also shows clearly enough how, at the end of the 1680s, Leibniz’s reflections on the *inversio relationis* are still under the influence of Jungius’s writings. This influence probably accounts for the pre-eminent role Leibniz assigns to the *inversio* in his short programme on the *characteristica*. In his essays on the *grammatica rationalis* written at about the same time, Leibniz clearly shows that he believes all “non syllogistic” consequences — and therefore both the *inversio* and the *a rectis ad obliqua* inference — must undergo the same formal treatment in order to be admitted to the logical calculus. From his sporadic declarations, we infer that Leibniz did not believe that the *inversio relationis* and the *a rectis ad obliqua* inference were demonstrable by means of syllogism. He speaks in this case of “consequentiae quae nullis syllogismis aliisque logicis artibus probari possunt, quas Jungius notavit” and holds that these *consequentiae* must be dealt with in the *characteristica grammatica*.<sup>45</sup>

Leibniz believes that grammatical analysis should precede logical analysis of language.<sup>46</sup> Grammatical analysis should have two functions: (1) to expose the basic structure of natural language; (2) to identify standard forms for the different kinds of linguistic expressions so as to build a solid basis for the working out of a “universal” logical calculus, free from the limitations and conditioning which natural language imposes on the reasoning process. One of the many problems to be solved by the grammatical analysis would in fact be that of the oblique cases: the grammatical *characteristica* should supply a satisfactory analysis of the various sorts of *obliquitates*, indicating an *ad hoc* logical and grammatical mechanism for dealing with them in the calculus. Since the *inversio relationis* implies various sorts of *obliquitates* depending on the type of relation which is “inverted”, it seems quite natural that Leibniz should assign treatment of the *inversio* to the *grammatica rationalis*. As we have seen, however,

Leibniz appears basically ambiguous about the oblique cases, at one moment proposing to eliminate them altogether from the “philosophical language”, at another proposing to leave them, and to give special *particulae* the task of expressing the relations implicit in them. His ambiguity affects the *inversio* problem as well. Thus, for example, in the *Grammaticae cogitationes* he maintains that the “new ways of arguing invented by Jungius” can be avoided if the oblique cases are rendered into more than one proposition.<sup>47</sup>

When, in a project (circa 1679) to construct the encyclopedia, he turns to the relationship between logic and grammar, Leibniz repeats that “in logic there are very many inferences which cannot be demonstrated on the basis of logical principles, but by means of principles derived from grammar”.<sup>48</sup> If, then, in writings which may reasonably be assumed to have been composed under the influence of Jungius’s logical conceptions, it appears that Leibniz unreservedly admits the *inversio relationis*, indeed counting it among the *generales enuntiationes* of the logical calculus, in other texts he makes entry of the *inversio* into the calculus dependent on grammatical transformations, whose nature he does not specify. Apropos of this, Leibniz speaks of “inferences” which can be demonstrated “by means of principles derivable from grammar”, but he does not explain what form such a demonstration takes, nor how it can make itself plausible. Nonetheless he calls both the *inversio* and the *a rectis ad obliqua* inference “non syllogistic inferences”, that is, not demonstrable by reference to traditional logic. He maintains this judgement in the *Nouveaux essais*: here Leibniz is admitting the validity of non-syllogistic consequences such as the *inversio*, recognizing that the syllogistic method is not able to demonstrate them.<sup>49</sup> He maintains, however, that “by changing their structure a little” they become demonstrable, bringing their demonstration back to the principles of a higher logic, to which syllogisms themselves are subordinate.<sup>50</sup> As I observed earlier, it is quite probable that when he speaks of modification of the terms in relation to the *a recto ad obliqua* inference Leibniz refers to modifications analogous to those suggested in the letter to Vegetius published in the Dutens collection.<sup>51</sup> I cannot see, however, how such suggestions — which are plausible for the passage *a recto ad obliquum* — can be adapted to the *inversio relationis* to make it demonstrable. The main difficulty with Leibniz’s convictions regarding the *inversio rela-*

*tionis* is precisely this: how did he believe the *inversio* could be demonstrated?

Referring to the *inversio relationis*, we have seen that on at least one occasion Leibniz speaks of inferences demonstrable, not “on the basis of logical principles, but by means of principles derivable from grammar”; whereas in the *Nouveaux essais* he maintains that the *inversio*, like other types of inference, is demonstrable by reference to a sort of superior logic, on which “ordinary syllogisms themselves depend”.

2. Leibniz makes few explicit references to the *inversio relationis*, but contexts exist in which he discusses it, so to say, implicitly. One of the most interesting of these is the following, from a short essay entitled *De lingua rationali*, published in part by Couturat,<sup>52</sup> and now available in its entirety in the first volume of the *Vorausedition*:<sup>53</sup>

Lingua rationalis ita utiliter constituetur, ut cuilibet vocabulo aliarum linguarum respondens possit, si velimus constitui, v.g. *Titius est magis doctus Caius*. Sensus est: quatenus Titius est doctus, et Caius est doctus, eatenus Titius est superior et Caius est inferior. Haec analysis optima quidem est, sed non exprimitur vis singulorum verborum. Quod ut assequamur, dicendum erit: Titius est doctus et qua talis est superior, quatenus inferior qua doctus est Caius. Cumque si *doctior* sit idem quod *magis doctus* patet in explicationem huius: *doctior Caius*, id est *doctior aliquo qui est Caius*, ingredi hoc ipsum. Et quidem *doctus* est quasi radicale, *magis aliquo qui est hic*, scil. *Caius* esse servilia. Et haec servilia debent posse ita esse constituta, ut apparet regula commutandi seu ex hoc *Titius est doctus magis aliquo qui est Caius* debet etiam fieri posse *Caius est doctus minus aliquo qui est Titius* item *doctrina Caii minor est quam Titii*, etc.

Here Leibniz is tackling the classic problem of the scholastic and late scholastic *summulae*: the analysis of comparatives. This is a problem which was usually discussed under the section of exponible propositions. The *expositiones* finally put forward — with the due exceptions — were only two: (1) *expositiones* in which the locution “ita sicut” was used; (2) *expositiones* in which the term *magis* or *minus* were present. For example, the comparative “*Petrus est doctior Paulo*” was open to the following two *expositiones*:

- (a) (1) *Petrus est doctus et (2) Paulus est doctus et (3) Paulus non est ita doctus sicut Petrus;*
- (b) (1) *Petrus est doctus et (2) Paulus est doctus et (3) Petrus est magis doctus Paulo.*



The (a) type *expositio* is documented in Paul of Venice's *Logica parva* and Paul of Pergula's *Compendium*;<sup>54</sup> the (b) type is found in the *Summulae* of Domingo De Soto, who attributes preference for the (a) type of exposition to the "moderns".<sup>55</sup> According to De Soto, the "moderns" prefer the *expositio* by means of the "ita sicut" locution (in proposition (3) above) because otherwise, since *magis* is an adverbial form with the same function as a comparative, the *expositio* would merely be a repetition of the *explanandum*. But De Soto says he does not share this position and prefers the (b) type *expositio*, in agreement with the *antiqui*. He argues that the "magis" adverb, together with an adjective in the positive form, more clearly expresses the nature of the comparative, thus making clear the meaning of the proposition;<sup>56</sup> it is interesting to note that, in the passage in question, Leibniz takes his start from a comparative which has already been set out in the standard form, according to the method of the *antiqui*: "Titius est magis doctus Caio", and he attempts a fresh *expositio* by means of the *particulae*: *quatenus*, *eatenus*. He recognizes the analysis thus obtained "optima quidem", but believes it is not capable of expressing the "force" of individual words. He therefore proposes a further *expositio*, which he now holds to be satisfactory:

Titius est doctus et qua talis est superior, quatenus inferior qua doctus est Caius.

For the *expositio* of a relational sentence, Leibniz turns once again to reduplicative expressions (*qua*) and to the use of correlated terms (*inferior* — *superior*) so as to avoid the cases. At this point he takes a step back, however, and poses the problem of how to make evident the logical equivalence of the two expressions "doctior" and "magis doctus".<sup>57</sup> By elliptical reasoning he therefore establishes the equivalence:

Titius est doctior Caio = Titius est doctus magis aliquo qui est Caius.

Leibniz then examines the expression "doctus magis aliquo qui est Caius" and identifies two component parts: a root term, *doctus*, and the auxiliary expression "magis aliquo qui est hic", which, given the variable function of the denominative pronoun, can be represented as "magis aliquo qui est *x*". Leibniz ends this passage with the hope that such auxiliary expressions shall be so expressed as to make obvious "at a glance"

the logical passage from a given relation to its converse.

On this occasion, Leibniz attempts to achieve co-existence of the two different analyses of relational sentences which I pointed to above. On the one hand he mentions the analysis according to which sentences of the type "*a* is similar to *b*", "*a* is wiser than *b*" etc. are reduced to the standard form: "*a* is similar to an *x* which is *b*" ("*a* est similis rei quae est *b*"), "*a* is wiser than an *x* which is *b*" ("*a* est doctus magis aliquo qui est *b*") etc.; on the other hand he mentions an analysis by means of reduplicative terms. Here too, it is the latter which is considered the more satisfactory and "profound" analysis. At the same time, however, Leibniz prefigures an aspect of the *lingua rationalis* which requires the division of linguistic expressions into root expressions — substantives, proper nouns and adjectives — and auxiliary expressions, that is, standard locutions whose function is to clarify the meaning of other expressions. From a reading of the *De lingua rationalis* it appears that Leibniz intended the construction of an artificial language composed of root terms, auxiliary terms and copula, and that he did indeed consider the *expositio* by means of reduplicatives as more profound, but also more laborious and complicated. It is therefore probable that, once the logical criteria had been established for expressing relational sentences in terms of reduplicative propositions, the rational language was to develop mainly by the use of auxiliary expressions, so as to "loosen" the relations. As far as the *inversio relationis* is concerned, Leibniz, as we have seen, clearly states that the auxiliary expressions (*servilia*) will require to be so constructed that the *inversio* becomes quite evident. The same requirement is stated in other texts, in which it is hoped that *ad hoc* linguistic expressions be constructed to allow the reader to derive, from the simple consideration of complex signs, all the logical consequences derivable from the propositions expressed by means of these signs. There is a passage in the *Analysis particularum* (1683–87) — which I have already had occasion to mention — where Leibniz makes a further, implicit reference to the *inversio relationis*:

Ipsius autem *ut-et* vel *cum* talis adhibenda est expressio, ut consequentiae quae inde duci debent, haberi possint, verbi gratia ex eo quod dicitur *Petrus est doctus cum Johanne* sequitur etiam *Petrus est doctus*, item *Johannes est doctus*, item *Johannes est doctus cum Petro*. Itaque hoc etiam ex ipsis characteribus debet esse cognoscibile, scilicet ex characteribus cognosci debet,

quando salva veritate locum habeant transpositiones, omissiones, additiones, substitutiones . . .<sup>58</sup>

And again, in a text written almost certainly in the same period, he returns to the necessity of distinguishing within the *lingua rationalis* between root characters and auxiliary characters, and to the necessity of choosing and constructing the characters in such a way that the transformations and the logical inferences — among which the *inversio relationis* — are quite evident:

. . . debent esse quidem characteres qui significant aliquid alteri substitui posse, item alii qui significant inverti posse totam dispositionem, et similes. Verbi gratia si sit: *Petrus est doctus ut et Johannes* debet ipsum *ut-et* ita esse formatum, ut inde colligi possit inverti posse, seu posse *Johannem* substitui pro *Petro* et contra in hac ipsa propositione . . .

Distinguendi characteres serviles a radicalibus.<sup>59</sup>

Regarding oblique terms, relational sentences, and the *inversio relationis*, Leibniz's attitude is consistent, therefore: the *obliquitates*, whether they correspond to what today we would call “functional expressions” (“Evander's sword” etc.) or to the cases of nouns (“to Alexander” in “. . . similar to Alexander”), indicate the presence of relations. If then the “translation” of the oblique terms into the artificial language must reflect the ontological nature of relations, all the *obliquitates* will require to be expressed by means of formulae in which have to appear only terms in nominative case, the copula, and special *particulae* or suitable locutions.<sup>60</sup> Thus reduced to its essentials, the artificial language will consist of two categories of terms: root terms (the traditional categorematic terms), and auxiliary terms to be applied to the former, in order to obtain new terms and propositions (with the use of the copula). In his project for the *characteristica* the “weight” of expressing the relations would be devolved to *particulae* and auxiliary terms. In this way any sort of *obliquitas* would undergo a standard treatment which would clarify its real nature: *obliquitates* implicit in the inversion of relation would also be suitably transcribed and, after further manipulation, the *inversio* itself would be demonstrated.

It is therefore a view concerning the ontological nature of relations which brings Leibniz to engage in a work of transcribing oblique terms and relational sentences — an option of the nominalistic type which, in certain cases, actually brings him to demand the complete exclusion of oblique terms from the

*characteristica*. His essays on the *characteristica* and on the *grammatica rationalis* show that he tempers his more radical reductionistic proposals, however, and — for sentences where there are asymmetric relations — is content to express their character of second intentions, that is of “purely ideal things”, proper to relations.<sup>61</sup>

As will be seen, one of the effects of Jungius's ideas on Leibniz was to stimulate him actually to propose a symbolism, which he considers more suitable than Jungius's to represent sentences in which relations enter. Finally he works out a genuine hierarchy of relations, to found an abstract treatment of geometry. Once he has clarified the ontological nature of relations, “playing down” their existential value, Leibniz is at liberty to make the freest use of them. So much so that — in the case of geometry, for example — relations exist between pure beings of reason, remaining, that is, within the scope of products of the imagination and of the intellect.

3. From our analysis so far, it appears that, when working out his own theories on relations, Leibniz had in mind the positions of Jungius. We should not be surprised, then, that his only attempts to devise a suitable symbolism for a logical calculus of relations are to be found precisely in texts commenting on *excerpta* from writings by Jungius. Following Jungius's example, Leibniz proposes to transpose symbolically the expressions: “Quadrangulum habens latera aequalia” and “Angulus triangulo et quadrangulo communis”.<sup>62</sup> In order to give the reader a sufficiently clear idea of Leibniz's symbolism I give a transcription of these two expressions, with the appropriate letter symbol above each categorematic term. The complete transcription will be obtained by considering the series of capital and small (index) letters above each term in each of the expressions as one single formula:

- (a) “Q<sup>s</sup>H<sup>s</sup>L<sup>a</sup>AE<sup>a</sup>”
- (1) “Quadrangulus habens latera aequalia”<sup>63</sup>
- (b) “A<sup>2</sup>M<sup>23</sup>B<sup>34</sup>Q<sup>35</sup>”
- (c) “A<sup>s</sup>M<sup>sr</sup>B<sup>rb</sup>Q<sup>rc</sup>”
- (2) “Angulus communis triangulo et quadrangulo”.<sup>64</sup>

Thus (a) is the symbolic transcription of (1), while (b) and (c) are alternative transcriptions of (2), differing only in that in (b) the indices are represented by numbers and in (c) by small letters. The function of the

indices is to show the nature of the relationship linking the symbols for the categorematic terms. In the last analysis, therefore, it is the indices that express of the relations. In this regard, (a) must be considered a symbolic transcription still in the embryonic state: the indices designate the grammatical case in which each term appears in the proposition and are limited to supplying the rather imprecise indication that the last two terms, corresponding to expressions "L" and "AE" are in the accusative (plural), dependent, that is, on the preceding terms in the nominative case. The expression of more complex relations is attempted in (b) and (c). In (b), for example, the index number 23 indicates that the categorematic term designated by "M" is connected to A, B and Q, in such a way that the concepts denoted by B and Q have concept A among their components. Reflecting on the structure of symbolic expressions (b), and (c), Leibniz further discovers that he can then simplify, expressing the concept of "common" ("A is common to B and Q") by means of the indices:<sup>65</sup> (d)  $A^{26}B^{67}C^{69}$ . In the new expression (d) the presence of the number 6 in all three indices reveals that the categorematic terms designated by the letters A, B and Q have a common element. The position of the number 6 in the first index reveals that it is precisely A which is the grammatical subject of the sentence and therefore the common element. That the number 6 occurs first in the B and Q indices tells us that B and Q are dependent on A. The other numbers, 2, 7, 9 serve to differentiate the terms designated by A, B, and Q. Leibniz observes, that if B and Q "had both been in the same subject" he would have had to write " $B^{67}Q^{67}$ " instead of " $B^{67}Q^{69}$ ".<sup>66</sup>

Given the occasional and unsystematic nature of these notes it would obviously be absurd to read too much in them — Leibniz often changes his views, suggesting different types of symbolic transcription only to end up by declaring himself dissatisfied with all the solutions proposed. And the symbolism he adopts is nearly always very crude and embryonic. In the texts summarized here, he appears not to go much beyond a transcription which declares that given symbols are related by certain relationship of dependency. Leibniz still takes his inspiration from the model of natural languages: each capital letter designates a particular term, a noun in ordinary language, and the indices, whether letters or numbers, have roughly the function of the cases. The information derived from an analysis of

the strings of symbols (b), (c), and (d) is, in effect, somewhat imprecise, such as "B and Q are conceptually different and dependent on A (or A and M)". But it is interesting to observe that Leibniz does not include among the symbols a specific "character" for relations. At the end of his notes *circa schedas Jungianas* Leibniz turns again to a theme (which, as we have seen, he never abandons in the course of his reflections) and tackles the problem of the inversion of the relation.<sup>67</sup> Given the enunciation: "Quadrangulum habens aequalia latera" he proposes a symbolic transcription in the following terms: " $Q^sH^{sc}L^{ca}AE^{ca}$ " and believes that the *inversio* ("Aequalia latera habita a quadrangulo") can be expressed as: " $L^aAE^aH^{ac}Q^{cs}$ ".<sup>68</sup> The order in which the letters in the two symbolic transcriptions are presented is practically the same as that of the words in each of the corresponding enunciations: the different relations emerge from the arrangement of the capital letters and of the index letters. The latter combine with the former to suggest very general types of connection.

4. In his mathematical manuscripts Leibniz develops the project of a geometrical analysis dealing with reference to certain basic relations between geometric figures or bodies in general. As Bertrand Russell observed, Leibniz as a mathematician could not have neglected operating with relations.<sup>69</sup> However, Russell's idea suggests what we might think of as a passive relationship, as if Leibniz — the mathematician — had not been able to avoid dealing with relations. But the mathematical manuscripts I am referring to show a clear determination to construct a geometric calculus within which a certain number of relations would have a very predominant role. Leibniz is actively interested in giving certain relations a central position in the construction of the system.

In his notes on Temmik, that is, in a generally philosophical rather than specifically mathematical context, Leibniz unreservedly recognizes the legitimacy of the "relations of relations".<sup>70</sup> In contrast to Temmik himself, who holds that it is improper to assert that "similarities are similar", Leibniz declares: "indeed, similarities may be similar, as when we have the proportion of two proportions one to the other".<sup>71</sup>

The following principle, which appears several times in his mathematical writings, is further proof of Leibniz's openness on the question of relations: If there are two true propositions  $p$  and  $q$ , which differ only in

that, precisely where in  $p$  the terms  $a_1, a_2, a_3 \dots$  appear,  $b_1, b_2, b_3 \dots$  appear in  $q$ , then the same relation exists between  $a_1, a_2, a_3 \dots$  and  $b_1, b_2, b_3 \dots$ .<sup>72</sup> As is clear from the short *Monitum de characteribus algebraicis*, Leibniz is perfectly aware that this principle is connected with the principle of substitutivity, and intends to use a special character to designate the equality of relation:

Praeter aequalitatem, proportionalitatem et similitudinem occurrit interdum et eiusdem relationis consideratio quam significare licet nota ::; exempli causa si sit  $aa + ab = cc$  et simili forma  $ll + lm = nn$ , dici potest  $a, b, c$  habere inter se eandem relationem quam habent  $l, m, n$  seu  $a; b; c :: l; m; n$ , id est, datur quaedam relatio inter  $a, b, c$ , in qua si pro his respective substituas  $l, m, n$  vera manet enuntiatio.<sup>73</sup>

Similarly, in a draft entitled *Mathesis universalis*, published by Gerhardt, Leibniz feels he has to include a “nota Relationis in genere” among the characters of his calculus — that is, a symbol to designate any kind of relation, and he again proposes to adopt a symbol for the equality of relation.<sup>74</sup> Leibniz in any case explicitly considers the calculus of relations as an essential part of the combinatorial art or *speciosa generalis*, and among the projects taking up most of his attention in the mathematical field was the construction of

a calculus completely different from those in use up till now, in which the notes or *characteres* shall not signify definite and indefinite quantities or numbers, but also other things such as points, qualities and relations (*respectus*).<sup>75</sup>

The project to achieve a “qualitative” type mathematics called first of all for a definition, by Leibniz, of a series of basic relations applicable to any kind of mathematical body. As Martin Schneider justly observes, such relations “are . . . the proper subject of the *Mathesis universalis*, which . . . appears as a kind of logic of relations or structural mathematics”.<sup>76</sup> The relations which Leibniz considers basic are the following: similarity, congruence, homogeneity, equality, coincidence. Two geometrical bodies (or two objects in general) are said by Leibniz to be similar when they can only be distinguished by means of an act of comparison which sets them directly side by side.<sup>77</sup> Similarity therefore implies what Leibniz calls “qualitative equality” between two objects, or their nondistinguishability if they are examined separately.<sup>78</sup> Similar objects, that is, cannot be distinguished on the basis of a mere analytical examination which ignores their “co-presence”.<sup>79</sup> Ex-

amples of similar objects are geometrical figures or solid bodies belonging to the same species, but of different sizes: a large circle and a small one, a large cube and a small one, and so on. Leibniz is careful in his definition of the condition of non-distinguishability on the pure analytical plane — if in fact I see a circle of radius  $x$  and, after a few minutes, see another circle of radius  $10x$ , it is very probable that I will be able to notice the difference between the two circles, and it is therefore probable that, without setting the two figures side by side, I will not consider them indistinguishable.<sup>80</sup> But in this case I would still be carrying out a mental confrontation of the image, or memory, of the first circle and the direct image of the second (or indeed of the two images as evoked by the faculty of memory). Leibniz calls this psychological act of confrontation *comperceptio*: without *comperceptio* or direct confrontation of two similar objects it is impossible to grasp the concept of the difference between them.<sup>81</sup>

Leibniz considers as homogeneous all those objects or geometrical bodies which “either are similar or can be made similar by some suitable transformation”.<sup>82</sup> He defines as equal those geometrical bodies or those objects which have the same dimensions.<sup>83</sup> Similar and equal objects are congruent.<sup>84</sup> The congruence relation in fact implies that the bodies under consideration are non-distinguishable, and therefore superposable. In order to distinguish two congruent objects reference must be made to an “external” point of view — to spatial/temporal coordinates according to which, for example, two congruent triangles are distinguishable in that they are oriented or arranged differently in space. Leibniz writes of congruent objects that

they are discriminated only with reference to place; that is, they cannot be discriminated until another object external to them is assumed as point of reference and it is observed that they have different positions with reference to this third object.<sup>85</sup>

Congruent objects which perfectly superpose are coincident. Whereas congruent objects differ only in number — a prerogative exclusive to abstract or mathematical bodies, not possessed by concrete things — coincident objects “do not differ even in number”.<sup>86</sup> It follows that coincident objects will be congruent, equal, similar and homogeneous; that congruent objects will be similar, and that, therefore, while similarity relation is the weakest of those just listed, the coincidence relation is the strongest.<sup>87</sup> In his *Characteristica geometrica*

(1679) Leibniz uses special symbols to designate each of the above relations and establishes certain “consequentiae” concerning them.<sup>88</sup> If we use “ $\sim$ ” for the similarity relation, “ $\sqcap$ ” for equality, “ $\wp$ ” for congruence, “ $\infty$ ” for coincidence, we obtain:

$a \sim b$  and  $a \sqcap b$ , therefore  $a \wp b$   
 ...  
 $a \wp b$ , therefore  $a \sqcap b$   
 $a \wp b$ , therefore  $a \sim b$   
 $a \infty b$ , therefore  $a \wp b$   
 $a \wp b$ , therefore  $a \sqcap b$   
 $a \wp b$ , therefore  $a \sim b$

Each relation has in addition its own reflexive, symmetrical and transitive properties. In the *Characteristica geometrica*, again, Leibniz enunciates transitivity as follows:<sup>89</sup>

Ex his explicationibus coincidentium, congruorum, aequalium ac similium consequentiae quaedam duci possunt. Nempe quae sunt eidem aequalia, similia, congrua, coincidentia, sunt etiam inter se, ideoque

$a \infty b$  et  $b \infty c$ , ergo  $a \infty c$   
 $a \wp b$   $b \wp c$   $a \wp c$   
 $a \sim b$   $b \sim c$   $a \sim c$   
 $a \sqcap b$   $b \sqcap c$   $a \sqcap c$

Besides these, Leibniz uses another relation, which he calls “relation of determination” and which he intended should have an important role in the *analysis situs*.<sup>90</sup> In an unpublished text of circa 1679 the relation of determination is thus defined: let there be given a relation R of an object  $x$  with the objects X, and let there be a relation R' of another object  $y$  with the same objects X; let  $R = R'$ ; if it follows that  $x = y$ , then R (= R') is a relation of determination.<sup>91</sup> In such a case the objects X are called determinants, and  $x$  (=  $y$ ) the “determinate” object. The determinants are properly the conditions which establish the determinate object or cause it to exist: “the determinants are those things which, taken together, belong to one individual only”,<sup>92</sup> they are the conditions, that is, which make it possible to discriminate without error the determinate object from all others. The determinate object is unique:

*Determinatum* est, quod ex quibusdam suis conditionibus positus non nisi unicum est.<sup>93</sup>

A circle, of which the radius and the coordinates of the centre in the plane have been given, will therefore be determinate; and a segment of which the extremities are

known will be determinate.<sup>94</sup> Concerning the relation of determination, Leibniz observes that objects determined in the same way by coincident objects must in their turn be coincident.<sup>95</sup> As Martin Schneider has observed, the relationship tying the determinants to the determinate object brings to mind the “modern” concept of *function*, and it is true that through the notion of determination Leibniz does seize an important part of this concept.<sup>96</sup> It is probable, however, that the relation of determination does not possess the general characteristic of “our” concept of function, but is rather a sort of relation defined for particular geometrical or mathematical objects. It must be observed first of all that, at least as far as we learn from the texts in question, Leibniz does not indicate the *determinatio* with an *ad hoc* sign. When he requires to express the relationship connecting determinants and determinate object, Leibniz is content to consider pairs, triples, quadruples etc. of ordered objects. In this instance too, it appears that the nominalist concern not to “multiply” *entia* prevails: if several objects are in a certain relation, it is not that we have on the one hand the relation and on the other the objects — we simply have certain objects in a given order. Thus, if the objects  $x$  and  $y$  are the determinants of  $z$ , and  $z$  is their determinate object, Leibniz is content to indicate this relationship by the expression “ $x \cdot y \cdot z$ ”.

The general principle of substitutivity applies to the relation of determination, as to the other relations of similarity, congruence, etc.:

Determinantia omnia simul substitui possunt pro determinato in alia determinatione.<sup>97</sup>

Regarding determination, however, Leibniz accepts the validity of an axiom which appears not to identify the relation of determination with the general notion of function:

Si relatio aliqua sit determinatio, unumquodque inter ea omnia quorum est relatio potest assumi pro determinato, reliqua omnia pro determinantibus.<sup>98</sup>

#### D. Leibniz's general ideas on the nature of relations

1. We have seen that Leibniz works out two different systems for “reducing” relational sentences, depending on whether they include symmetrical or asymmetrical relations.

In sentences resulting from the analysis of sentences containing *asymmetrical* relations, Leibniz leaves in place such expressions as *subditus-dominus*, *amans-amata*, etc., which are of a clearly correlative nature. This has persuaded some commentators to state that he did not intend to use the transformations of the *grammatica rationis* to carry out a rigorously “reductionist” programme. They believe that he makes a distinction between *relational sentences*, as such, and sentences containing *relational predicates*.<sup>99</sup> In the last analysis, relational sentences would be those in which relations appear in their “effective” nature as properties which, to use Leibniz’s expression, “inhere in more than one subject at the same time” (that is, in at least two subjects). Relational predicates on the other hand are predicates which inhere in a given subject and refer to another (not specified) subject, different from the first: they are, properly, relational predicates “with a place free”.<sup>100</sup> Our problem therefore is to establish whether, when he uses such expressions as *amans*, *amata*, *subditus*, *dominus*, etc. in his analysis of relational sentences, Leibniz considers them as relational predicates or not. To answer this question we must take into account the traditional scholastic doctrine about the ontology of relations. In particular, we will consider St. Thomas’s ideas on this topic.

St. Thomas distinguishes between the *relatio ut accidens* and the relation *in the proper sense*.<sup>101</sup> This distinction, based on the peculiar nature of Aristotelian ontology, was fairly common among the scholastics. Since all reality — the entire “world” — is made up of individual substances and of accidents inherent in these substances, relations can be considered in two ways. On the one hand they too are *inherent* in the substance insofar as they are accidents, and on the other hand they refer to something else (*ad aliud*). What is common to relations and the other predicaments is their *being in*; what distinguishes them, and constitutes the specific essence of the “relation” predicament, is precisely this referring to something else (the *esse ad aliud*).<sup>102</sup> If David is Salomon’s father, *being a father* is a typical relation inhering in David, just as *being red* inheres in a given red object. At the same time, however, this property, since it postulates the existence of a child, is different from the property indicated by the expression *being red*. David’s being a father is something exclusively of David’s, and in itself this property implies no individual reference to Salomon. If I declare: “David is father”, I am content to indicate that in a given subject

there inhere certain properties denoted by the expression “being a father”. Among these properties there will of course be the requirement that “somewhere in the world” at least one child of David’s should exist, but I do not specify who that child is. In his commentaries on Aristotle’s logic, Toletus is fairly clear on this point:

Ad quantum est advertendum relationem posse sumi tripliciter: uno modo in specie, altero modo particulariter, tertio modo singulariter. Ut si dicamus, paternitas, aliqua paternitas, haec paternitas, haec si in specie sumatur, tunc ad terminum refertur in specie: paternitas enim est ad filiationem, et duplum ad subduplum: si particulariter vero sumatur, etiam ad terminum particularem refertur: aliquid duplum est alicuius subdupli duplum, et aliquod melius aliquo peiori melius, ut Arist. docuit.

At in singulari non dependet a singulari termino: non enim individuatur relatio, nisi a subiecto, in quo est, ut haec paternitas non est, quia est ad hoc filiationem, sed quia est in hoc patre, scilicet tali homine et hoc duplum non est ad hoc subduplo in singulari, potest enim esse hoc duplum respectu huius medietatis, et omnium aequalium . . .<sup>103</sup>

Thus relations *ut accidentes* give rise to *sentences* of the “*a* is twice”, “*b* is father”, “*c* is bigger” type, etc., postulating an *ad aliud* reference to another subject or substance which is, respectively, half, child, smaller, etc. It therefore seems in order to consider the expressions: *amans*, *amata*, *superior*, *inferior*, etc., which Leibniz uses in his transformation of relational sentences, as analogous to “relational predicates with a place free”.

As is known, St. Thomas maintains that relations are not accidents inhering in more than one subject at the same time. The relation of *paternitas* existing between *a* and *b* is not a sort of “bridge” crossing from *a* to *b*: there is not only one, but in fact *two* distinct relations involved, each based on one and only one of the two subjects *a* and *b*:

Ad secundum dicendum, quod quidam dixerunt, ut Avicenna dicit, quod eadem numero relatio est in utroque extremorum, quod non potest esse, quia unum accidens non est in duobus subiectis: et ideo dicendum, quod in utroque extremorum est una relatio differens ab alia in quibusdam secundum speciem, sicut in illis quae diversis nominibus utrinque nominantur, ut paternitas et filiatio: sed in quibusdam non differunt specie, sed numero tantum, sicut quando utrunque est unum nomen, ut in similitudine et aequalitate: et tunc relatio, quae est in uno sicut in subiecto, est in altero, sicut in termino, et e converso. . .<sup>104</sup>

Both Leibniz and Jungius express the same conviction. Both state that a relation does not inhere in more than one subject and speak rather of *two* distinct relations into which the relation “outside the subjects” divides.<sup>105</sup> In his fifth letter to Clarke, Leibniz refers to the example

of the two lines L and M and examines two ways of considering their relation. He states that, in the first way of considering them, L, the greater, is the subject; in the second, M, the lesser, is the subject "of that accident which philosophers call 'relation'".<sup>106</sup>

At this point we must ask how we are properly to understand Leibniz's programme of transcription and "reduction" of relational sentences. If relations outside the subject are purely mental entities, what are relations when they are accidents? what is the ontological reality corresponding to relations as accidents if they are to be distinguished from relations as *mental things*? These questions take us to a central point of the traditional ontology concerning relations — a point on which the nominalist and realist schools divide, giving rise to a great number of very different positions, all of them finding it more or less difficult to explain the nature of a relational property (of a property, that is, that corresponds to a relational term).

For example, though St. Thomas denies the possibility of accidents inhering simultaneously in more than one subject, he believes that relations, insofar as they are properties of single individuals (relations as relational accidents) exist really outside the mind (*extra animam*).<sup>107</sup> This shows, beyond any doubt, that refusal of relations *hors des sujets*, to use Leibniz's expression, leaves open the problem of the nature of relations insofar as they are inherent in a single subject. It is evidence that the real problem — the problem of the reality of relations — was connected to relations understood as accidents. This was a subject of much discussion among writers in Leibniz's time who still drew inspiration from the scholastic tradition, and was normally tackled by tempering Thomist realism with suggestions derived from Scotism and the doctrine of the *nominales*. A characteristic example of this attitude, tending to modify Thomist ontology itself with moderate nominalism, and therefore avoiding excesses of any kind, is to be found in the thesis of Francisco Suarez.

2. One of the writers most read in the 17th century, and one of Leibniz's sources, Suarez establishes a position on relations which is very close to that of our author. For example, in his *Metaphysicae disputationes* he states that relations cannot inhere in more than one subject and affirms that when real, they inhere in one subject only.<sup>108</sup> He then gives the following opinion about the possibility of distinguishing a relation from its own foundation:

Nihilominus est alia sententia extreme his opposita, quae negat relationem distingui in re aliqua distinctione actuali a suo fundamento absoluto, sed tantum aliqua distinctione rationis habente in rebus aliquid fundamentum.<sup>109</sup>

Suarez attributes this concept to Ockham, Gregory of Rimini, and the *nominales* in general, and he illustrates its consequences with reference to Hervaeus's opinion, according to which "relative denominations are derived from a body of several absolute things, and not from peculiar entities or distinct modes . . . added to the absolute things".<sup>110</sup> As we shall see, Suarez's summary of this position, though correct, weakens its more radical aspects. He declares himself in favour of nominalism, in fact, but believes it should not be interpreted as if declaring that the "*ratio formalis* of the relation is null or the relative denomination is merely extrinsic, derived from some absolute form".<sup>111</sup> For Suarez, as for Leibniz, therefore, relations are not "merely extrinsic denominations".<sup>112</sup> In fact, if relations were added totally extrinsically to the foundations (*fundamenta*) "the sense of the *praedicamentum ad aliquid* would be completely perverted and indeed the predicament itself would be annulled".<sup>113</sup> According to Suarez, the relation expresses a given *real form* which "intrinsically denominates its own relative";<sup>114</sup> however this form is not "a thing or mode distinct *ex natura rei* from every absolute form", but must be conceived of as an "absolute form which is within the thing, but which is not considered as absolute, but as regarding (*respiciens*) another form included or connoted by the *denominatio relativa*".<sup>115</sup> In a certain sense a given *modus considerandi* contributes to the genesis of the relation, or rather renders it explicit, bringing it forth from the absolute forms which alone are present in the individuals. Suarez gives an example in order to clarify this concept. Consider the similarity relation between two objects, both of which are white. The similarity of one object to the other is not something really distinct from whiteness, rather it is simply "the whiteness itself insofar as it relates to another whiteness" of similar intensity. The relative denomination therefore originates in some intrinsic form, which however "includes" or "connotes" some other "intrinsic form in an intrinsic term".<sup>116</sup> If *a* is greater than *b*, no *accidens* exists in *a* which denominates itself "being great": *a* will be of a certain proportional size and it is sufficient that somewhere in the world there exists the individual *b*, also of certain particular dimensions, for anyone who becomes aware of *a* and *b* in a single act of thought to conceive

automatically the relation “*a* is greater than *b*”. According to this concept, relations are an integral part of the “modes of being” of things. If an individual is of a certain size it will automatically be greater than all the individuals of lesser dimensions, and the property of “being greater” is thus enjoyed by this individual as soon as a smaller individual exists — or, to use the scholastic expression, as soon as the term of the relation emerges (*consurgit terminus*). With this typically conciliatory approach between nominalism and moderate realism, Suarez states of relations:

(1) the essence of the relation is something *real*, intrinsic within the correlated objects;

(2) despite being something real, the relation is not distinct nor really separable from the foundation on which it is founded;

(2.1) the relation can be distinguished from its foundation only by means of a *distinction of reason*. The following statement is an exemplary summary of Suarez’s convictions:

Nam licet relatio non sit aliquid in re distinctum ab absolutis, potest esse aliquid ratione distinctum; et ideo non sequitur, quod sit simpliciter nihil.<sup>117</sup>

The relation, in other words, contains nothing real other than the foundation, and in order to have a relation it is enough that there be a “consort or coexistence” of more than one absolute thing, together with their properties. Suarez admits however that certain absolute properties “allude” to others: in the very nature of a colour or of a certain size is included the reference to all possible colours or sizes of the same species which are more or less similar, more or less “large”. At the very moment that two things are thought of together this reference is brought to light and the mind seizes the relationship between the sizes or the colours. Linguistic expressions such as “greater”, “similar”, “father”, etc. refer to certain properties which allude to correlative properties (“smaller”, “similar”, “son”, etc.) in another subject (and postulate therefore the existence of a term, which is the bearer of given accidents).

Better to understand Suarez’s position, let us imagine that for two individuals *a* and *b* it is possible to set down two lists, one containing all (and only) the properties of *a* and the other all (and only) the properties of *b*. In neither of the lists, taken on its own, will it be possible to find properties such as “greater than”, “more learned than”, “father of”, “similar to *a*”, etc. Rather, such predicates will be “results” — properties which emerge

from the comparison of list *a* with list *b*. In *a* for example we will find a set of accidents which — when we postulate the existence of *b* as the bearer of other accidents of a particular structure — will cause *a* to be considered the father of *b*. This obviously creates a problem: given an accident A of *a*, why should it necessarily be connected with a particular accident B of *b*? What makes us connect A with B (the fundamental properties of the accident “being a father” with the fundamental properties of the accident “being a son”) rather than A with, for example, C (“being a father” with “being red”)? Suarez answers that the connection takes place because in the properties characterizing “being a father” there is a special “cross-reference” to other given properties which are distinctive of what we call “being a son”.<sup>118</sup>

Suarez’s fine analyses illustrate the major difficulty in the scholastic concept of relations: to determine the nature of the properties which correspond to the *denominationes relativae*, that is to the linguistic expressions which designate relations, while maintaining an ontology founded on the concept of *substance* and *accident*. According to the “classical” doctrine of Aristotelian origin, accidents in fact inhere as individual properties in substances, which constitute their *support*. In this sense, all accidents and properties are relative insofar as they depend on a given subject (the substance). There are however particular accidents which, besides depending on their subject, also refer to a term — to another subject, external to the one on which they are founded. These are the relations. But what is their ontological equivalent? How are such accidents in fact made up? In their attempts to answer these questions the opposed parties of realists and nominalists end up by involving themselves in further difficulties. If the realists admit the existence of relations *hors des sujets* or recognise the reality of properties which correspond to relative expressions, their problem is to characterize the nature of these properties. Furthermore, they must work out a strategy to avoid the *regressus ad infinitum* (there exist relations of relations). Finally, they must admit the effective existence of properties such as “being greater”, “being smaller”, etc. The problem for the nominalists, on the other hand — when they reduce the nature of relations to the following conditions: (1) the presence of a subject and of a foundation (absolute fundamental property); (2) existence of at least one subject, situated outside the first and the bearer of a given foundation — is to explain how the relation can come about from the



simple *coexistence* of foundations which are not relative. Adding to these the condition that the foundations and the terms must be thought together, in a single act of thought, does not appear significantly to simplify the nature of the difficulty.

Insofar as they belong to the conceptualist-nominalist school, both Suarez and Leibniz must therefore explain how relations emerge once two subjects with their absolute properties are thought together. Suarez's answer, as we have seen, consists in attributing to "absolute" properties the capacity to refer "outside" themselves, without however admitting the existence of relational properties in the proper sense: relations would only be *modes of being* of absolute properties, and therefore of existing things.<sup>119</sup> From this point of view Leibniz's answer is analogous. In reality there exist only individual substances with their absolute properties: if *a* is wise and *b* is wise, the fact that *a* is wiser than *b* arises from the very nature of the quality "being wise" which is found in *a* and from the particular level of wisdom possessed by *b*. No *accidens* exists in *a* which corresponds to the expression "wiser": such expression is a linguistic term used to designate the result of a comparison between the wisdom of *a* and the wisdom of *b*. The fact that *a* is wiser than *b* arises from the act of thinking *a* and *b* and their respective "wisdoms" together. At the logical-linguistic level, Leibniz resolves the sentence "*a* is wiser than *b*" by dividing it into two sentences: "as to wisdom *a* is superior", "as to wisdom *b* is inferior", and by considering *superior* and *inferior* as correlated terms, each making reference to the other.<sup>120</sup> *Superior* and *inferior* are relational predicates founded on the *fundamentum* of wisdom and as relational predicates are, so to say, "open". The fact that *a* is superior does not imply that he must be superior precisely to *b*: to express the idea that "*a* is superior to *b*" the two sentences "*a* is superior", "*b* is inferior" would require to be linked by *particulae* such as *quatenus*, *eatenus*, etc. Leibniz therefore divides the sentence "*a* occidit *b*" into the two sentences: "*a* est occidens", "*b* est occisus", in which the expressions *occidens* and *occisus* are relative denominations which are connected by the *particula quatenus*: "*b* est occisus quatenus *a* est occidens".<sup>121</sup> What corresponds within the subject *a* to the term *occidens* is an individual accident which in its mode of being does not include a reference to something else.

Leibniz's reductionistic programme thus consists in resolving those relational sentences which contain

*symmetrical* relations into pairs of sentences in subject-predicate form, in which the predicate is not a relational accident. On the other hand, relational sentences where *asymmetrical* relations appears are reduced to sentences containing correlative terms (and hence relational predicates) which are without oblique expressions or prepositions, so that the relation *hors des sujets* is reduced to pairs of relations *ut accidentes*. *These, in their turn, are purely mental concepts, generated by comparing and thinking together the subjects to which they refer, with their fundamental properties.*<sup>122</sup>

3. From his *juvenalia* right through to his mature writings, Leibniz remained faithful to a conceptualist ontology of relations: relations arise spontaneously at the moment when at least two objects, with their modifications or properties, are thought together. In commenting on the text by the Jesuit Aloys Temmik (1706), he not only states that relations "result" as soon as the single objects are produced, but that they have a reality which is independent of our intelligence (*habent realitatem, citra intelligentiam nostram*).<sup>123</sup> By maintaining that relations "inhere (*insunt*) even if no-one thinks them", Leibniz appears here to contradict his own conceptualism. What he wants to assert, however, is that the reality of relations does not merely depend on individual thought. His object is to assign to relations a reality which is mental, but *objective*. Relations "receive reality from the divine intellect, without which nothing would be true".<sup>124</sup>

Modum putem proprie esse accidens determinans seu limites quosdam adiciens ei quod perpetuum est et modificatur. Relationi autem adeoque omni accidenti nolim hoc tribuere. Relatio autem ex substantia et modis resultat nulla propria mutatione, sed consequentia tantum, et aliquo modo Ens rationis dici potest, etsi simul reale sit quia ipsae res omnes vi summi intellecti constituuntur, quae causa quoque est ut possibilitates et veritates sint aeternae, etiam cum existentia abest.<sup>125</sup>

Thus, if in a certain sense (*aliquo modo*) relations can be defined as *entia rationis*, they must at the same time (*simul*) be considered as "real".

Relations have their proper foundation *in mente Dei* insofar as God, by creating or thinking single individuals together in a determined world, connects them and articulates their reciprocal relationships. The resulting relations are not the fruit of divine arbitrariness, but are a *consequence* of the non-relational properties, of the modes of being of the individual substances. Paragraphs 50 to 52 of the *Monadology* provide further confirma-

tion of what we may call the “metaphysical intuition” which lies at the heart of Leibniz’s theory of relations. In these paragraphs Leibniz sets out the problem of the influence monads or individual substances bring to bear one upon the other, and asserts that these influences can be only *ideal*. When God puts several individuals substances together in a single world he regulates the mechanism of each in relation to those of the others, and it is in this arrangement, Leibniz maintains, that their reciprocal influence lies:

Car Dieu, comparant deux substances simples, trouve en chacune des raisons, qui l’obligent à y accomoder l’autre, et par consequent ce qui est actif à certain égard, est passif suivant un autre point de consideration: *actif* en tant que ce qu’on connoist distinctement en luy, sert à rendre raison de ce qui se passe dans un autre, et *passif* en tant, que la raison de ce qui se passe en luy, se trouve dans ce qui se connoist distinctement dans un autre.<sup>126</sup>

In gathering together several individual substances in a single world, God is “obliged” — Leibniz’s term — to “accommodate” each individual with respect to the others. This seems to amount to saying that God establishes a relation between two given individuals on the basis of their intrinsic modifications only. Therefore the problem remains open whether such “intrinsic modifications” are or are not identical with “absolute properties” in the same sense in which modifications like “being red”, “being rational” etc., can be considered, from the logical point of view, “absolute properties” of a given individual. Properly speaking, conceived ideas (concepts) and representations are *parts* of the intrinsic states of monads: they are internal to the mind and cannot be considered *terms* of a real relation whatsoever. Therefore, *intramonadic* relations (i.e. relations internal to each monad), being intelligible without any appeal to something external to a given subject, should be considered “qualities” and not *extrinsic denominations*. Accordingly, the intimate structure of perceptions becomes one of the most relevant causes which oblige God to adapt each monad to all the others. If the individual substance corresponding to the name of Paris is characterized at the time  $t_1, \dots, t_k$  by a series of internal states  $S_1, \dots, S_k$  (including representations), then God cannot put in the same world another substance different from Helen: he is “obliged” from the nature of Paris’s “modifications-with-representation”, to adapt Paris’s internal states  $S_1, \dots, S_k$  to Helen’s internal states  $S_1^*, \dots, S_k^*$ . Thus the same sentence: “Paris loves Helen” expresses two different sides of the same fact: a subjective side and an objective one. Insofar

as “Paris loves Helen” affirms something which “happens” in the world, then to this sentence there corresponds an individual named Paris with some internal modification  $S_i$  and an individual named Helen with another internal modification  $S_i^*$ . All this can be expressed through the sentence: “Paris loves and *eo ipso* Helen is beloved”. But the subjective side of the relation — i.e. the connection between Paris’s mind and the representation of Helen — cannot be further analysed. In this case the name “Helen” denotes not an individual substance, but a representation in Paris’s mind — something which should properly be denoted by means of a description, but not by means of a proper name. If Paris loves Helen, then the sentence which expresses “the subjective side” of this fact has the form: “Paris loves A”, where A cannot be replaced by a proper name, and “loves A” — from a logical-linguistic point of view — is a relational term to which there corresponds an *absolute* psychological state, i.e. a state in itself perfectly intelligible without any appeal to something external to a given subject (Paris).

Leibniz repeatedly states that monads are always active and that they do not cease to have perceptions and appetite; therefore in the complete description of a particular monad it is impossible to leave out relational expressions such as “*a* perceives X”, etc. In agreement with Leibniz’s metaphysical ideas, the perception of X, internal to a given monad, has to be carefully distinguished from the individual (if any) corresponding to X and *external* to the perceiving monad. The strong connection which subsists between the relations that a given monad maintains with all the other monads of the same world, and the perceptions internal to that monad are concisely expressed by Leibniz in a letter to Des Bosses:

... nam cum Monas semper intra se exprimat suas ad caetera omnia relationes, longe alia percipiet cum in equo erit, quam cum in cane.<sup>127</sup>

Thus representations and internal states of the individual substances become the most relevant “reasons” which determine God’s reciprocal “accommodation” of monads in a world.

## Notes

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In the notes I have employed the following abbreviations:

LH = *Leibniz' Handschriften* . . .

PA = G. W. Leibniz, *Sämtliche Schriften und Briefe*, Herg.von der Deutschen Akademie der Wissensch. zu Berlin, 1950ff.

VE = G. W. Leibniz, *Voraußedition*, zur Reihe VI – Philosophische Schriften – in der Ausgabe der Akademie der DDR.

Dutens = G. W. Leibniz, *Opera omnia*, nunc primum collecta . . . Ed. Ludovicus Dutens, Vols. I–VI, Geneva, 1768.

PG = G. W. Leibniz, *Die philosophischen Schriften*, Hrsg. von C. I. Gerhardt, Berlin 1857–90, vols. I–VII.

MG = G. W. Leibniz, *Mathematische Schriften*, Hrsg. von C. I. Gerhardt, Halle 1849–63, vols. I–VII.

*Opuscules* = G. W. Leibniz, *Opuscules et fragments inédits*, ed. L. Couturat, Paris, 1903.

GI = G. W. Leibniz, *Generales Inquisitiones – Allgemeine Untersuchungen über die Analyse der Begriffe und Wahrheiten*, Herg.von F. Schupp, Hamburg, 1982.

Leibniz, *Logical Papers* = G. W. Leibniz, *Logical Papers. A Selection*, ed. by G. H. R. Parkinson, Oxford, 1966.

Vagetius, *Admonitio* = J. Vagetius, *De Logica Hamburgensi Admonitio*, in J. Jungius, *Logicae Hamburgensis Additamenta*, ed. W. Risse, Göttingen, 1977.

Jungius, J., *Logica Hamburgensis* = J. Jungius, *Logica Hamburgensis*, Hamburg, 1638.

Mates, B., *Leibniz* = B. Mates, *The Philosophy of Leibniz*, New York-Oxford, 1986.

Schneider, M., *Mathesis Universalis* = M. Schneider, "Funktion und Grundlegung der Mathesis Universalis im Leibnizschen Wissenschaftssystem", in *Leibniz: Questions de Logique, Studia Leibnitiana*, Sonderheft 15, Wiesbaden 1988, pp. 162–82.

Suarez, F., *Disputationes Metaphysicae* = F. Suarez, *Disputationes Metaphysicae*, Paris, 1866 – Reprint, ed. Olms, Hildesheim 1965.

<sup>1</sup> Cf. J. Vagetius, *Admonitio*, p. 267: "Syllogismus ex obliquis definitur is esse in quo aliquis terminus ponitur in casu obliquo, veluti Genitivo, Dativo, Accusativo, Ablativo".

<sup>2</sup> J. Vagetius, *Admonitio*, p. 268.

<sup>3</sup> Cf. J. Jungius, *Logica Hamburgensis*, p. 116.

<sup>4</sup> J. Jungius, *Logica Hamburgensis*, pp. 122–23.

<sup>5</sup> Cf. J. Vagetius, *Admonitio*, p. 254ff. Vagetius makes use of the Latin translation of *La logique* (*Logica sive ars dirigendi cogitationes* . . . traducta per Corn. ab Ackersdyck, Traiecti, 1666).

<sup>6</sup> Cf. *Dutens*, VI, i, pp. 30ff.

<sup>7</sup> For Vagetius's letter cf. LH IV, 7c, Bl. 163 r; Leibniz's letter in *Dutens*, VI, i, pp. 36–39 (in particular the English translation in Leibniz, *Logical Papers*, pp. 88–89).

<sup>8</sup> J. Jungius, *Logica Hamburgensis*, p. 192.

<sup>9</sup> *Ibid.* To be precise, Jungius in the above-mentioned passage, proposes two different types of analysis, the second of which I have quoted in the text.

<sup>10</sup> *Ibid.*

<sup>11</sup> *Dutens*, VI, i, pp. 36–39 (Leibniz, *Logical Papers*, pp. 88–89).

<sup>12</sup> *Dutens*, VI, i, p. 39 (Leibniz, *Logical Papers*, p. 89).

<sup>13</sup> PA, VI, vi, p. 479.

<sup>14</sup> VE, p. 178: "Obliquitates in terminis incomplexis satis exponi non possunt nisi explicatis quibusdam Enuntiationibus. Peccant scilicet *in methodo* qui putant omnia Terminorum incomplexorum tradi posse ante complexos, uti peccant qui putant omnia linearum rectarum posse tradi ante circulos, seu omnia linearum ante superficies".

<sup>15</sup> Cf. VE, pp. 315–16; 339; 380; 354–55.

<sup>16</sup> VE, p. 315.

<sup>17</sup> VE, p. 351: "Omnes praepositiones proprie significant relationem Loci, translate aliam relationem quamcunque"; cf. also VE, pp. 361; 514.

<sup>18</sup> VE, p. 350.

<sup>19</sup> Cf. VE, p. 102: "Sed in lingua rationali videndum an non abstractis abstinere possit, aut saltem quousque possit".

<sup>20</sup> VE, p. 927.

<sup>21</sup> VE, p. 91: "Verum tamen necessario superesse quaedam enuntiationes in obliquo".

<sup>22</sup> VE, p. 521.

<sup>23</sup> VE, p. 1063.

<sup>24</sup> *Ibid.*

<sup>25</sup> GI, p. 4 (Leibniz, *Logical Papers*, p. 47).

<sup>26</sup> GI, pp. 4–6 (Leibniz, *Logical Papers*, pp. 47–48).

<sup>27</sup> *Ibid.*

<sup>28</sup> GI, p. 6 (Leibniz, *Logical Papers*, p. 48).

<sup>29</sup> VE, p. 1063.

<sup>30</sup> Cf. M. Mugnai, "Bemerkungen zu Leibniz' Theorie der Relationen", *Studia Leibnitiana* 10 (1978), 19–21; the text – entitled by Leibniz *De termino, praedicato, relatione* – is now published in VE, pp. 379–81.

<sup>31</sup> VE, p. 380. The same analysis is present in W. Ockham, *Summa logicae*, II, ch. 11 (N.Y., St Bonaventure, 1974, p. 281): "Similiter quando ponitur aliquod relativum in propositione, requiruntur veritates plurium propositionum. Sicut ad veritatem istius 'Sortes est similis Platoni' requiritur quod Sortes habeat aliquam qualitatem et quod qualitatem eiusdem speciei habeat Plato. Unde ex hoc ipso quod Sortes est albus et Plato est albus, Sortes est similis Platoni et e converso."

<sup>32</sup> Cf. VE, pp. 178; 349–50; *Opuscules*, pp. 244; 280 and the list of texts quoted by B. Mates, *Leibniz*, pp. 179–81.

<sup>33</sup> Cf. VE, p. 350: "Imo ipsum quatenus adhuc amplius explicari deberet hoc modo: *si Ensis patitur, quia Evander agit, non ideo Evander est iniustus*. Imo opus erit recurrere usque ad definitionem causae et effectus"; and VE, p. 351: "*Quatenus generaliter idem significat quod respectu habito ad hanc propositionem quae sequitur . . .*"

<sup>34</sup> VE, p. 1082: "Et sane revera *Socrates qua Musicus bene canit* est enuntiatio praegnans constans ex his *Socrates canit, Socrates est Musicus, et Nisi Socrates esset Musicus non bene caneret*."

<sup>35</sup> Cf. M. Mugnai, "Intensionale Kontexte und 'Termini reduplicativi' in der Grammatica Rationalis von Leibniz", *Studia Leibnitiana*, Sonderheft 8 (1979), pp. 82–92.

<sup>36</sup> A relation *ut accidens* is a relative accident which inheres in a singular substance and refers to another correlated accident of another substance.

<sup>37</sup> Cf. *Correspondance*, Leibniz-Clarke, ed. A. Robinet, Paris, 1957, p. 145.

<sup>38</sup> "Relationes ipsae nihil aliud videntur esse quam veritates."

<sup>39</sup> VE, p. 380.

- <sup>40</sup> J. Jungius, *Logica Hamburgensis*, p. 89.
- <sup>41</sup> Cf. E. J. Ashworth, "Joachim Jungius and the Logic of Relations", *Archiv für Geschichte der Philosophie* 49, 72–85; I. M. Bocheński, "Non-Analytical Laws and Rules in Aristotle", *Methodos*, 1951, pp. 70–80.
- <sup>42</sup> LH IV, 7c, Bl. 163 r: "... utique per inversionem relationis (rejectionem impossibilem omni illi, qui novit et fatetur, correlata se mutuo ponere, quod faciunt, qui ratione utuntur, omnes)".
- <sup>43</sup> VE, p. 1633 (*Opuscules*, p. 426).
- <sup>44</sup> VE, p. 931 (*Opuscules*, p. 327).
- <sup>45</sup> VE, p. 1007 (*Opuscules*, p. 406).
- <sup>46</sup> VE, p. 813 (*Opuscules*, p. 353): "Hac analysi grammatica absoluta, sequitur analysis Logica . . ."; cf. H. Burkhardt, *Logik und Semiotik in der Philosophie von Leibniz*, München, 1980, pp. 480ff.
- <sup>47</sup> VE, pp. 349–50 (*Opuscules*, p. 287).
- <sup>48</sup> VE, p. 470 (*Opuscules*, p. 36).
- <sup>49</sup> PA, VI, vi, p. 479.
- <sup>50</sup> *Ibid.*
- <sup>51</sup> Dutens, VI, i, p. 39.
- <sup>52</sup> *Opuscules*, p. 280.
- <sup>53</sup> VE, pp. 143–44.
- <sup>54</sup> Paulus Venetus, *Logica parva*, VI, 7 (English transl. by A. Perreiah, München, 1984, pp. 194–95).
- <sup>55</sup> Domingo de Soto, *Summulae*, Salamanca 1554 (Hildesheim, 1980), p. 116.
- <sup>56</sup> *Ibid.*
- <sup>57</sup> VE, p. 143 (*Opuscules*, p. 280).
- <sup>58</sup> VE, p. 521.
- <sup>59</sup> VE, p. 928.
- <sup>60</sup> VE, p. 357 (*Opuscules*, p. 289): "Omnia in oratione resolvi possunt in Nomen substantivum *Ens* seu *Res*, copulam seu verbum substantivum *est*, nomina adjectiva, et particulas formales"; cf. H. Burkhardt, *Logik und Semiotik*, p. 410.
- <sup>61</sup> On this point cf. B. Mates, *Leibniz*, pp. 171ff; B. Mates, "Nominalism and Evander's Sword", *Studia Leibnitiana Supplementa* XXI (1980), 213–25; M. Mugnai, "On Leibniz's Theory of Relations", *Studia Leibnitiana*, Sonderheft 15, 1988, pp. 145–61.
- <sup>62</sup> VE, p. 1014.
- <sup>63</sup> *Ibid.*
- <sup>64</sup> VE, p. 1630.
- <sup>65</sup> *Ibid.*
- <sup>66</sup> VE, p. 1631.
- <sup>67</sup> VE, p. 1015.
- <sup>68</sup> *Ibid.*
- <sup>69</sup> B. Russell, *A Critical Exposition of the Philosophy of Leibniz*, Cambridge, 1900, p. 13.
- <sup>70</sup> Cf. Leibniz's marginal note to p. 83 of *Philosophia vera Theologiae et Medicinae Ministra* . . . auctore R. D. Aloysio Temmik, Coloniae, 1706: "Imo similitudines possunt esse similes, ut datur ratio duarum rationum inter se, seu  $a:b$  ad  $b:c$ , ut  $ac$  ad  $bb$  [.]  $a:b$ , ut  $aa$  ad  $bb$ ".
- <sup>71</sup> VE, p. 1087.
- <sup>72</sup> LH, XXXV, I, 14, 47 v: "Si sint duae propositiones, quae in hoc tantum differunt, quod in una ponitur A, B, C etc. ubi in altera ponitur L, M, N etc. respective, tunc aliqua dicitur esse relatio eadem inter A, B, C et inter L, M, N etc.". The same principle is reaffirmed in LH, XXXV, xii, 1, 47 r (English translation in B. Mates, *Leibniz*, p. 226).
- <sup>73</sup> MG VII, p. 222.
- <sup>74</sup> MG VII, p. 57.
- <sup>75</sup> MG VII, p. 267.
- <sup>76</sup> M. Schneider, *Mathesis Universalis*, p. 172.
- <sup>77</sup> MG VII, p. 30; M. Schneider, *Mathesis Universalis*, pp. 176–78.
- <sup>78</sup> MG VII, p. 19: "*Similia sunt eiusdem qualitatis*".
- <sup>79</sup> MG VII, p. 30; M. Schneider, *Mathesis Universalis*, p. 176.
- <sup>80</sup> MG V, pp. 153–54.
- <sup>81</sup> MG V, p. 153: "Itaque dicere soleo, *similia* non discerni nisi per comperceptionem".
- <sup>82</sup> MG VII, p. 30.
- <sup>83</sup> MG V, p. 153.
- <sup>84</sup> MG V, p. 154. A large list of definitions from Leibniz's works concerning *similarity*, *homogeneity*, *equality* and *congruity* is included in M. Schneider, *Mathesis Universalis*, pp. 176–78.
- <sup>85</sup> MG V, p. 155.
- <sup>86</sup> MG VII, p. 265: "Et in universum quicquid de uno congruorum fieri dicere potest, id de altero quoque fieri potest et dici, hoc uno excepto, quod ea quae in uno adhibentur, numero differunt seu positione ab iis quae in alio adhibentur."
- <sup>87</sup> Cf. M. Schneider, *Mathesis Universalis*, pp. 172ff.
- <sup>88</sup> MG V, p. 154.
- <sup>89</sup> MG V, p. 155.
- <sup>90</sup> MG VII, pp. 261ff.
- <sup>91</sup> LH XXXV, I, 5 Bl.47 v; cf. also LH, XXXV, I, 14, Bl. 5v.
- <sup>92</sup> MG VII, p. 29; M. Schneider, *Mathesis Universalis*, p. 181.
- <sup>93</sup> LH, XXXV, I, 14 Bl. 5v.
- <sup>94</sup> MG VII, p. 29.
- <sup>95</sup> MG VII, p. 34.
- <sup>96</sup> Cf. M. Schneider, *Mathesis Universalis*, pp. 181ff.
- <sup>97</sup> LH, XXXV, I, 5, Bl. 48r.
- <sup>98</sup> LH, XXXV, I, 5, Bl. 47v.
- <sup>99</sup> Cf. J. Hintikka, "Leibniz on Plenitude, Relations, and the 'Reign of Law'", in *Leibniz*, ed. H. Frankfurt, New York, 1972, pp. 160–62; H. Ishiguro, "Leibniz's Theory of the Ideality of Relations", in *Leibniz*, ed. H. Frankfurt, New York, 1972, pp. 208–9; D. Mertz, "Leibniz's Monadic Treatment of Relations", *Auslegung*, a Journal of Philosophy, 7 (1980), 261ff; D. Wong, "Leibniz's Theory of Relations", *The Philosophical Review* 89 (1980), 244; M. A. Kulstad, "A Closer Look at Leibniz's Alleged Reduction of Relations", *Southern Journal of Philosophy* 18 (1980), 428.
- <sup>100</sup> Cf. H. Ishiguro, "Leibniz's Theory of Ideality of Relations", pp. 208–9.
- <sup>101</sup> St. Thomas, *De potentia*, q. 7, a. 9: ". . . ipsa relatio . . . aliud habet in quantum est accidens et aliud in quantum est relatio . . .".
- <sup>102</sup> Cf. A. Krempel, *La doctrine de la relation chez Saint Thomas*, Paris, 1952, pp. 50ff.
- <sup>103</sup> Franciscus Toletus, *Commentaria . . . in Universam Aristotelis Logicam*, Lugduni, 1579, p. 135.
- <sup>104</sup> St. Thomas, *In quatuor libros Sententiarum*, Liber I, Dist. xxvi, q. 2, ar. 3.
- <sup>105</sup> Cf. PG II, p. 486; J. Jungius, *Logica Hamburgensis*, p. 42.
- <sup>106</sup> PG VII, p. 401.
- <sup>107</sup> St. Thomas, *De potentia*, q. vii, ar. 9.
- <sup>108</sup> Cf. F. Suarez, *Disputationes Metaphysicae*, II, Disp. XLVII (pp. 782–866).
- <sup>109</sup> F. Suarez, *Disputationes Metaphysicae*, II, XLVII, ii, 12 (p. 789a).

<sup>110</sup> *Ibid.*

<sup>111</sup> F. Suarez, *Disputationes Metaphysicae*, II, XLVII, ii, 22–23 (pp. 792b–793a).

<sup>112</sup> *Ibid.*

<sup>113</sup> *Ibid.* (p. 792).

<sup>114</sup> *Ibid.*

<sup>115</sup> *Ibid.* (p. 793).

<sup>116</sup> *Ibid.*; cf. Ockham, *Reportatio* II, q. ii.

<sup>117</sup> *Ibid.*

<sup>118</sup> F. Suarez, *Disputationes Metaphysicae*, II, XLVII, xii, 7–8 (pp. 832–33).

<sup>119</sup> F. Suarez, *Disputationes Metaphysicae*, II, XLVII, ii, 8ff (pp. 784ff).

<sup>120</sup> Cf. what Leibniz writes in the unpublished text LH IV, 7, B, iii,

Bl. 56v: "... omne correlatum constituit aliquod praedicatum alterius correlati, ut pater constituit praedicatum hoc quod alter est eius filius."

<sup>121</sup> Cf. VE, p. 518: "*A Titio occisus est Caius*; id est quatenus Titius est occidens eatenus Caius est occisus."

<sup>122</sup> Thus properties like "father", "son", "to be similar" and so forth, inhere in the complete concept of a given substance, but not in the substance itself.

<sup>123</sup> VE, p. 1083.

<sup>124</sup> *Ibid.*

<sup>125</sup> LH IV, 3, 5c, Bl. 2r (*Notationes ad schedam Hamaxariam*).

<sup>126</sup> PG VI, p. 615.

<sup>127</sup> PG II, p. 457.