THE INTERPRETATION OF VELOCITY FILTERGRAMS*

I: *The Effective Depth of Line Formation*

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Abstract. The paper describes a numerical experiment in which the effect of an assumed velocity distribution in the solar atmosphere on the intensity difference between a blue- and a red-wing filtergram is derived. This results in the effective optical depth at which the velocity is measured. It is shown that this τ_{eff} strongly depends on the assumed velocity distribution.

1. Introduction

A previous publication (Beckers, 1968) described a technique which permitted simultaneous photography of two images $\frac{1}{4}$ Å apart in wavelength through a narrow-band birefringent filter. These two images can refer to the blue and red wings of the 6569.2 Å absorption line so that Doppler shifts of this line will result in intensity differences between the two images. Measurement of these intensity differences results in a value for the Doppler shift of the line. The responsible line-of-sight velocity will refer to some point in the solar atmosphere.

In this paper we attempt to determine the location of this point. This is done by assuming a monotonic line-of-sight velocity distribution V vs. the continuum optical depth τ. The resulting influence on the intensity difference between the two filtergrams is calculated from $V(\tau)$ and the velocity v inferred from this intensity difference is derived. This v is compared with $V(\tau)$ and the effective optical depth τ_{eff} at which the velocity is measured is derived from:

$$v = V(\tau_{eff}). \tag{1}$$

We feel that the effective optical depth at which a velocity is measured is better defined this way than by the line contribution function, as is often done.

2. Description of the Computations

In computing the line profile of the 6569.2 Å line we assumed LTE and the Bilderberg

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model solar atmosphere (Gingerich and De Jager, 1968). The 6569.2 Å line is due to Fe, the excitation potential being 4.7 eV. The iron abundance was taken from Goldberg *et al.* (1960); the *gf* value and the Voigt profile were adjusted to fit a photoelectrically-observed profile at $\cos\theta = 1.0$.

After this the velocity distribution $V(\tau)$ is introduced. The resulting line profile is $I(\Delta\lambda)$. After transmitting $I(\Delta\lambda)$ through the birefringent filter, one obtains blue- and red-wing intensities I_b and I_r equal to $I_b = \int_{-\infty}^{+\infty} I(\Delta\lambda)\, P_b(\Delta\lambda)\, d\Delta\lambda$ and $I_r = \int_{-\infty}^{+\infty} I(\Delta\lambda) \times P_r(\Delta\lambda)\, d\Delta\lambda$. The filter transmissions P_b and P_r are equal to:

$$P(\Delta\lambda) = A\,\frac{\sin^2(2\pi\Delta\lambda)}{(2\pi\Delta\lambda)^2}\cos^2\left[\pi(4\Delta\lambda \pm 0.5)\right]. \tag{2}$$

In Equation (2) the A is a normalization factor, $+$ stands for P_b, $-$ for P_r and $\Delta\lambda$ is in angstroms.

As a measure for the intensity difference between the two filtergrams we used

$$\Delta \equiv \frac{I_b - I_r}{I_b + I_r}. \tag{3}$$

The dependence of Δ on the Doppler velocity v can be derived observationally (Beckers, 1968) or from theory by assuming $V(\tau) = \text{constant} = v$. Both methods resulted in identical $\Delta(v)$ curves for $\cos\theta = 1$. At other $\cos\theta$ values we will use the theoretical curves. The computations at each $\cos\theta$ followed the scheme: $V(\tau) \rightarrow I(\Delta\lambda) \rightarrow (I_b, I_r) \rightarrow \Delta \rightarrow v \rightarrow \tau_{\text{eff}}$.

The τ_{eff} were derived for the following $V(\tau)$ relations:

$$V(\tau) = C_1\tau, \tag{4}$$

$$V(\tau) = C_2\tau^2, \tag{5}$$

$$V(\tau) = C_3\varrho^{-1} \quad \text{and} \tag{6}$$

$$V(\tau) = C_4\varrho^{-1/2}. \tag{7}$$

In Equations (4) to (7) the C_n are constants and ϱ is the atmospheric density. The last two relations (6) and (7) represent a constant momentum and kinetic energy with height in the solar atmosphere. The magnitude of C_n was varied. Table I, column 1 gives the values of C_n which were used; columns 2 to 6 give the resulting τ_{eff} and the corresponding height.

We also computed τ_{eff} for the line absorption coefficient without the 1.1 km/sec 'turbulent' velocities ξ (= 0.8 km/sec rms) needed to explain the line profile. Since the line profile was measured with a low spatial resolution, it includes a broadening by moving elements which may be resolved in the high-resolution filtergrams; hence, the need to study the τ_{eff} for different values of the nonthermal velocities. However, we found that the results for $\xi = 0$ were almost identical to the ones listed in Table I so that the precise shape of the absorption coefficient profile apparently does not matter in the determination of τ_{eff}.

The last two rows of Table I give the τ_{eff} for the point in the solar atmosphere

TABLE I

Effective optical depth τ_{eff} divided by $\mu = \cos\theta$ for various μ and $V(\tau)$. Turbulent velocity $\xi = 1.1$ km/sec. The value in the parentheses is the height (km) in the solar atmosphere.

$V(\tau)$ km/sec	$\mu=1.0$	0.8	0.6	0.4	0.2
$1.3\,\tau$	.36 (51)	.36 (64)	.39 (80)	.45 (96)	.51 (128)
$3.3\,\tau$	.33 (54)	.33 (71)	.36 (86)	.42 (100)	.48 (132)
$6.6\,\tau$	.29 (63)	.29 (81)	.32 (92)	.38 (105)	.44 (136)
$13.2\,\tau$	.23 (81)	.23 (95)	.26 (104)	.32 (117)	.38 (144)
$15.7\,\tau$	.21 (88)	.21 (100)	.24 (109)	.30 (120)	.36 (147)
$3.3\,\tau^2$	.45 (47)	.52 (48)	.61 (51)	.73 (63)	.87 (98)
$6.6\,\tau^2$	.37 (50)	.44 (52)	.53 (57)	.65 (72)	.79 (103)
$13.2\,\tau^2$	.27 (68)	.34 (68)	.43 (72)	.55 (84)	.69 (111)
$3.0 \times 10^{-7}\varrho^{-1}$	.06 (160)	.08 (156)	.10 (160)	.13 (170)	.20 (181)
$3.7 \times 10^{-7}\varrho^{-1}$	.08 (141)	.10 (141)	.12 (148)	.15 (160)	.22 (183)
$4.3 \times 10^{-7}\varrho^{-1}$	.11 (124)	.13 (127)	.15 (134)	.18 (147)	.25 (173)
$5.6 \times 10^{-4}\varrho^{-1/2}$	.05 (173)	.05 (191)	.06 (199)	.08 (207)	.12 (240)
$6.3 \times 10^{-4}\varrho^{-1/2}$	.06 (160)	.06 (175)	.07 (187)	.09 (199)	.13 (229)
$6.9 \times 10^{-4}\varrho^{-1/2}$	.07 (149)	.07 (165)	.08 (176)	.10 (191)	.14 (222)
	.03 (210)	.03 (220)	.03 (240)	.03 (260)	.04 (290)
	1.2 (0)	1.2 (10)	1.0 (30)	.90 (50)	.80 (100)

where the line contribution function equals half its maximum. The contribution function was computed for the steepest part of the line profile. The τ_{eff} values lie between these two widely separated τ values but the individual τ_{eff} can vary as much as an order of magnitude corresponding to a depth difference of approximately 100 km, depending on the $V(\tau)$ relationship.

3. Discussion

In order to understand the differences in τ_{eff} for the various $V(\tau)$ we analytically computed the line profile for a model atmosphere with a source function of the form $S = a + b\tau$, a constant ratio η_0 of line center-to-continuum absorption coefficient and a constant Doppler width $\Delta\lambda_D$. For small $V(\tau)$ one can then compute the profile $P_v(\Delta\lambda)$. For a Doppler shift $\Delta\lambda_v$ equal to

$$\Delta\lambda_v = [(p + q\,e^{-\delta\tau})]\,\Delta\lambda_D, \tag{8}$$

and for $\eta(\Delta\lambda) = \eta_0 \exp[-(\Delta\lambda - \Delta\lambda_v)^2/\Delta\lambda_D^2]$, the line profile becomes

$$P_v(\Delta\lambda) = P_0(\Delta\lambda) - \frac{dP_0(\Delta\lambda)}{d\Delta\lambda}\left[p + q\,\frac{\alpha}{\alpha - \delta}\right],$$
$$\alpha \equiv 1 + \eta_0 \exp(-\Delta\lambda^2/\Delta\lambda_D^2) \tag{9}$$

when higher order derivatives from the line profile $P_0(v=0)$ are neglected. The effective Doppler shift of P_v therefore equals

$$\Delta\lambda_v = p + q\,\frac{\alpha}{\alpha - \delta}, \tag{10}$$

where the value of α should be taken in the steepest part of the line wing. From (8) and (10) we obtain for τ_{eff}:

$$\tau_{\text{eff}} = \frac{1}{\delta} \ln\left(\frac{\alpha}{\alpha - \delta}\right), \tag{11}$$

independent of both p and q. In the line profile α varies between 1 and $1+\eta_0$ but in the steepest part of the line, which is most relevant, it varies between 1 and 2. From Equation (11) one derives $\tau_{\text{eff}} > 1/\alpha$ for $\delta > 0$ and $\tau_{\text{eff}} < 1/\alpha$ for $\delta < 0$. For δ approaching α or $-\infty$, τ_{eff} approaches $+\infty$ and 0, respectively, so that in principle any τ_{eff} value is possible [for $\delta > \alpha$ the relations (9) to (11) are invalid]. The τ_{eff} is a monotonic function of δ which one could call the inverse optical scale height of the velocity.

For the four $V(\tau)$ relations listed in Table I the values for δ are approximately $+0.1$, $+5$, -13 and -10 for the $C_1\tau$, $C_2\tau^2$, $C_3\varrho^{-1}$ and $C_4\varrho^{-1/2}$ relations, respectively. Qualitatively the results, as listed in Table I, are in good agreement with the predictions as derived from the simplified Milne-Eddington model atmosphere. This comparison is only valid for the smallest $V(\tau)$. From Table I it can be seen that there is an increase in τ_{eff} for δ negative when the velocity increases, and a decrease for δ positive.

From Table I it is apparent that τ_{eff}/μ is not independent of μ, as is often assumed. For all $V(\tau)$ this ratio increases with μ so that one looks at the limb deeper into the sun than would be the case if τ(:) μ were assumed.

4. Conclusion

Unless one knows the $V(\tau)$ relation for the point of the atmosphere under study, it is impossible to assign an accurate value to the height at which a line-of-sight velocity is measured. All four optical scale heights δ^{-1} discussed in the previous section are reasonable so that differences larger than 100 km are possible in this height. Larger differences are even possible for other values of δ as shown in the Milne-Eddington model computation in Section 3. The effective optical depth could therefore be far away from the main part of the line contribution function.

$V(\tau)$ curves could probably be derived from Doppler-shift measurements in a number of lines. However, no measurements exist which are accurate enough to give reliable values for all three parameters p, q and δ in Equation (8). Very likely the $V(\tau)$ curve at any point in the solar granulation pattern will vary with position and time so that the τ_{eff} may be variable.

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