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Short Communication

A general equation for the evaluation of the error that affects the value of the maximum specific growth rate

W. Borzani

A general equation is proposed to evaluate the absolute error that affects the maximum specific growth rate calculated from batch or continuous experiments. This error depends on the relative errors of the cell concentration measurements and on the duration of the test.

Key words: Error evaluation, maximum specific growth rate.

A simple method (Borzani 1980) quantified the absolute error that affects the value of the maximum specific growth rate (μ) when the relative error of the cell concentration measurements is constant during all the tests. When α, the relative error that affects X (the cell biomass), is <0.20, then:

$$\Delta\mu \cong \frac{2\alpha}{\Delta t} \quad (1)$$

The above equation, however, cannot be applied if great variations occur in the cell concentration during the test because, in some cases, the value of α may decrease as the cell concentration increases (Hiss 1979).

The purpose of the present communication is to present a general method, similar to that of Borzani (1980), to evaluate the absolute error that affects μ when the relative error of X depends on X.

Evaluation of Δμ

With the only purpose to evaluate Δμ, let us consider the first (X_1) and the last (X_2) experimental points of the exponential growth phase of a batch test [Figure 1 (A)] or of the washing-out stage of a continuous experiment [Figure 1 (B)]. Figure 1 (A) permits the equations, 2, 3 and 4, to be written:

$$\mu_1 = \frac{1}{\Delta t}.ln\frac{X_2}{X_1} \quad (2)$$

$$\mu_1 = \frac{1}{\Delta t}.ln\frac{X_2'}{X_1''} \quad (3)$$

and:

$$\mu_2 = \frac{1}{\Delta t}.ln\frac{X_2''}{X_1'} \quad (4)$$

Consideriing that $X_1'' = X_1(1+\alpha)$, $X_1' = X_1(1-\alpha)$, $X_2'' = X_2(1+\beta)$ and $X_2' = X_2(1-b)$, equations (2) to (4) lead to:

$$\mu_1 = \mu + \frac{1}{\Delta t}.ln\frac{1-\beta}{1+\alpha} \quad (5)$$

and:

$$\mu_2 = \mu + \frac{1}{\Delta t}.ln\frac{1+\beta}{1-\alpha} \quad (6)$$

where β is the relative error that affects X_2. Consequently

$$\bar{\mu} = \frac{1}{2}(\mu_1+\mu_2) = \mu + \frac{1}{2.\Delta t}.ln\frac{1-\beta^2}{1-a^2} \quad (7)$$

Equation (7) shows that when α = β, $\bar{\mu}$ will be equal to μ (Borzani 1980). Depending on the values of α, β and Δt, $\bar{\mu}$ will be practically equal to μIf, for instance, α = 0.05, β = 0.02 and Δt = 5 h, the difference $\bar{\mu} - \mu$ will be 0.0002 h^{-1}. Equations (5) to (7) lead to:

$$\Delta\mu = \bar{\mu} - \mu_1 = \mu_2 - \bar{\mu} = \frac{1}{2.\Delta t}.ln\frac{(1+\alpha)(1+\beta)}{(1-\alpha)(1-\beta)} \quad (8)$$

The author is with the Instituto Mauá de Technologia, Estrada das Lágrimas 2035, 09580-900, São Caetano do Sul, SP, Brazil; fax: 55 11 743 0707.

If the relative errors of the cell concentrations measurements are smaller than 20% ($\alpha < 0.20$ and $\beta < 0.20$), we may use the following approximate equation:

$$\Delta\mu \cong \frac{\alpha+\beta}{\Delta t} \qquad (9)$$

When $\alpha = \beta$, equation (9) leads to equation (1). In the case of a washing-out phase, Figure 1 (B) leads to:

$$\mu = D - \frac{1}{\Delta t}.ln\frac{X_1}{X_2} \qquad (10)$$

$$\mu_1 = D - \frac{1}{\Delta t}.ln\frac{X_1''}{X_2'} \qquad (11)$$

and:

$$\mu_2 = D - \frac{1}{\Delta t}.ln\frac{X_1'}{X_2''} \qquad (12)$$

Combining equations (10) to (12) with the values of α and β we will obtain equations (7), (8) and (9) and their consequences.

It must be pointed out that the absolute error of μ depends not only on the relative errors of the cell concentration values, but also on the duration of the test.

Nomenclature

- X_1 — Cell concentration at the beginning of the exponential growth phase or of the washing-out period
- X_1' — Lowest value of X_1 due to experimental errors
- X_1'' — Highest value of X_1 due to experimental errors
- X_2 — Cell concentration at the end of the exponential growth phase or of the washing-out period
- X_2' — Lowest value of X_2 due to experimental errors
- X_2'' — Highest value of X_2 due to experimental errors
- α — Relative error that affects X_1
- β — Relative error that affects X_2
- Δt — Exponential growth stage or washing-out duration
- $\Delta\mu$ — Absolute error that affects μ
- μ — Maximum specific growth rate
- μ_1 — Lowest value of μ due to experimental errors
- μ_2 — Highest value of μ due to experimental errors.

References

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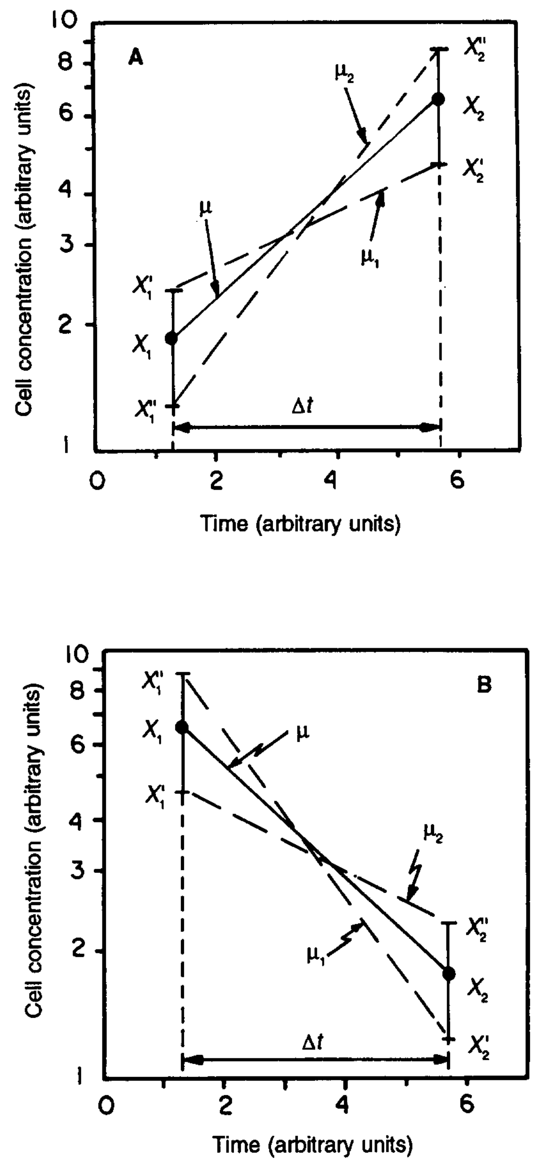

Figure 1. Schematic representation of the exponential growth phase of a batch culture test (A) and of the washing-out phase of a continuous experiment (B). The absolute errors of X_1 and X_2 are represented.

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