USE OF THE MAPPING SENTENCE FOR COORDINATING THEORY AND RESEARCH: A CROSS-CULTURAL EXAMPLE *

SHLOMIT LEVY

The Israel Institute of Applied Social Research

Introduction

Formalization of the research process may be worthwhile only within the more general context of the formalization of substantive theory. Otherwise, the formalization – and the research itself – may become mere busywork. The need for joint formalization of theory and research is emphasized in Guttman's definition of theory: "A theory is an hypothesis of a correspondence between a definitional system for a universe of observations and an aspect of the empirical structure of those observations, together with a rationale for such an hypothesis" (in Gratch, 1973, p. 35). Such a definition not only puts sharp focus on the necessity of defining the universe of observations to be researched, but also implies that the definitional system should be in a form that facilitates perceiving correspondences with aspects of the empirical data. Guttman's mapping sentence idea is intended to promote these two purposes (as well as many more): (a) definition of the universe of observations and (b) in a form that aids perception of systematic relationships with the data.

In effect, use of mapping sentences is a basic technique of facet theory (on which a textbook is in preparation). Facet theory provides general strategies for developing fruitful specific theories in the above sense of "theory". In this paper, I shall present but one simple example of the use of facets and the power of the mapping sentence approach. For some previously published examples see Aranya et al. (1976), Elizur (1970), Guttman (1959, 1970, 1971), Guttman and Levy (1975), Kernberg et al. (1972), Levy and Guttman (1974), Schlesinger and Guttman (1969), Yalan, et al. (1972).

---

* Paper presented to the symposium on "Formalization of the Research Process", at the Eighth World Congress of Sociology, Toronto, August 1974.

TABLE I

Interrelationships (Pearson Coefficients) among fifteen variables of satisfaction with life areas in the United States*

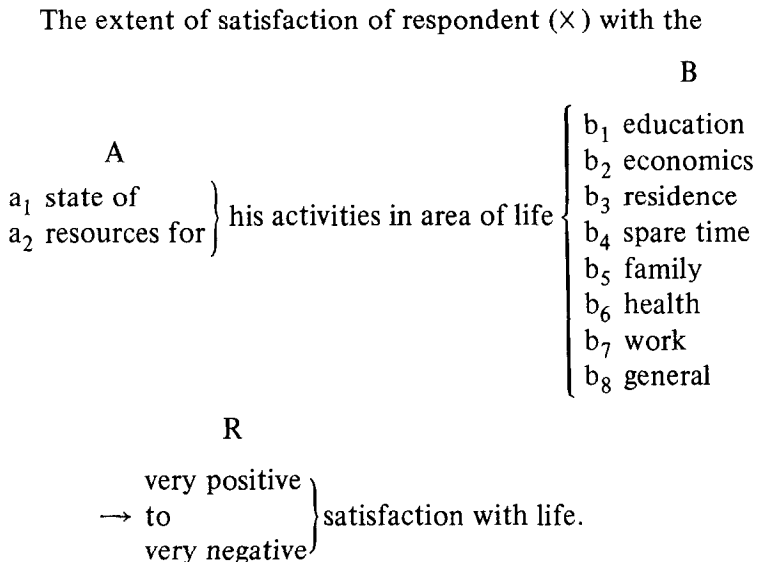|	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	–	54	44	33	19	14	22	22	05	33	25	24	14	24	28	1 City as place to live
2	54	–	49	28	18	14	21	19	00	32	23	19	13	19	23	2 Neighborhood
3	44	49	–	29	23	19	26	27	06	45	29	23	21	23	30	3 Housing
4	33	28	29	–	12	15	23	23	06	24	19	21	13	21	24	4 Life in the U.S.
5	19	18	23	12	–	54	25	26	18	32	28	16	09	18	28	5 Amount of education
6	14	14	19	15	54	–	24	23	17	24	20	17	12	18	24	6 Useful education
7	22	21	26	23	25	24	–	33	13	35	27	25	25	27	34	7 Job
8	22	19	27	23	26	23	33	–	21	37	32	40	30	40	50	8 Spending of spare time
9	05	00	06	06	18	17	13	21	–	17	17	09	12	14	26	9 Health
10	33	32	45	24	32	24	35	37	17	–	59	25	25	32	45	10 Standard of living
11	25	23	29	19	28	20	27	32	17	59	–	24	23	25	36	11 Savings and investments
12	24	19	23	21	16	17	25	40	09	25	24	–	21	31	32	12 Friendships
13	14	13	21	13	09	12	25	30	12	25	23	21	–	48	38	13 Marriage
14	24	19	23	21	18	18	27	40	14	32	25	31	48	–	50	14 Family life
15	28	23	30	24	28	24	34	50	26	45	36	32	38	50	–	15 Life in general

* Special analysis of data gathered for the study "Quality of Life" by the University of Michigan Survey Research Center on a national U.S. sample of 2164 respondents, in the summer of 1971.

The Mapping Sentence

Two surveys on certain aspects of the quality of life were conducted independently of each other, and about the same time (spring and summer of 1971), one in the U.S.* and one in Israel. Each contained some items asking about satisfaction with different aspects of one's life. The U.S. questionnaire contained fifteen items of this variety, as listed in Table I. The Israeli questionnaire had ten such items, as listed in Table II.

The aspect of the empirical data to be discussed here is the matrix of intercorrelations among the variables. What is the structure of this matrix for the U.S., and what is the structure for Israel? To what extent are the two structures similar to each other?

Inspection of the two lists of items shows some overlap of content, according to their brief titles: housing, job, health, spending of spare time. Otherwise, these and the other items have different wordings in the respective surveys. Clearly, to compare the two structures requires establishing a common definitional framework which will transcend the particular wordings. Indeed, such a framework is needed to help study the structure for each country separately. A suitable framework is proposed in terms of the following mapping sentence:

The extent of satisfaction of respondent ($\times$) with the

$$
\begin{array}{l}
\text{A} \\
\left.\begin{array}{l} a_1 \text{ state of} \\ a_2 \text{ resources for} \end{array}\right\} \text{his activities in area of life}
\left\{\begin{array}{l}
\text{B} \\
b_1 \text{ education} \\
b_2 \text{ economics} \\
b_3 \text{ residence} \\
b_4 \text{ spare time} \\
b_5 \text{ family} \\
b_6 \text{ health} \\
b_7 \text{ work} \\
b_8 \text{ general}
\end{array}\right.
\end{array}
$$

$$
\xrightarrow{\text{R}} \left.\begin{array}{l} \text{very positive} \\ \text{to} \\ \text{very negative} \end{array}\right\} \text{satisfaction with life.}
$$

TABLE II

Interrelationships (weak monotonicity coefficients) among ten variables of satisfaction with life areas in Israel*

	1	2	3	4	5	6	7	8	9	10		
1	–	50	17	17	35	65	38	38	49	42	1	Income
2	50	–	-08	11	21	37	14	10	29	31	2	Housing
3	17	-08	--	49	57	39	27	26	58	26	3	Health
4	17	11	49	–	57	35	20	29	31	33	4	Nervousness
5	35	21	57	57	–	55	35	34	57	51	5	Mood
6	65	37	39	35	55	--	47	42	66	49	6	General situation
7	38	14	27	20	35	47	–	83	49	42	7	Job
8	38	10	26	29	34	42	83	–	48	42	8	Place of work
9	49	29	58	31	57	66	49	48	–	51	9	Personal life
10	42	31	26	33	51	49	42	42	51	–	10	Spending of spare time

* Special analysis of data from the Continuing Survey for March–April 1971 of the Israel Institute of Applied Social Research and the Communications Institute of the Hebrew University, on a sample of 1,620 Jewish urban adults.

This particular sentence has four explicit facets. The first facet — symbolized by "X" — designates the population of respondents being researched. The next two facets — labelled "A" and "B" — are for classifying the content of the items. Since the first of these content facets has two elements and the other has eight elements, together they suffice to define 16 (=2 X 8) varieties of items of satisfaction. The fourth and final facet — labelled "R" — expresses a common range for the response categories of the universe of items. This is also the range of the mapping sentence. Elements of the four facets are designated by small letters: x, a, b, and r respectively.

Abstraction and Substance

The research design expressed by the mapping sentence as a whole calls for assigning to each respondent (x) a value of the range (r) for each item (ab) classified by the two content facets of the domain. Abstractly, this set of assignments can be expressed by the mapping

XAB → R.

The left member, or domain, of the mapping is the Cartesian set XAB, giving all possible combinations (structuples) of the form xab, that is, of each member of the population X with each of the 16

content varieties of the Cartesian set AB. The arrow indicates the mapping into the set R of possible responses: each structuple xab of the domain has one and only one response in R.

While such an abstract formula for a mapping may suffice for mathematical purposes, it is insufficient for substantive theory and for empirical research design. Flesh is needed to cover this skeletal structure for purposes of substantive thinking and practical usage. Guttman's proposal is to add verbal connectives and further literary additions to make the mapping readable in ordinary language. Such a substantive elaboration of a mapping is what is called a *mapping sentence*. It is actually a set of sentences of ordinary speech which have common connectives and which differ according to their facet elements.

Strategies of Modification

The emphasis of the present paper is methodological, although we shall present an actual substantive theory. The latter can be improved on, especially with respect to facet A. Indeed, in related work, instead of dichotomous facet A we have used a facet "environmental framework" which has two subsets of elements: *primary* environment (internal, social, resource) and *secondary* environment (neighborhood, town, state, world). The present facet A is a collapsing of this, since we have too few variables in the present data to document a theory about the more complete facet design. Let us just remark here that suggesting strategies for extension and intension of theories is an important feature of facet theory. Fruitful strategies are made possible by use of mapping sentences, since the latter lend themselves easily to correction, deletion, extension, and intension.

A Common Attitudinal Object: The First Law of Attitude

The fifteen U.S. and the ten Israel satisfaction items are easily classified by the two content facets A and B. Neither country's questionnaire contains all sixteen possible varieties of variables. Before going on to discuss the details of this domain classification and their implications, let us first focus on a central feature of the design: the common direction of "satisfaction" for the responses in R.

While the wording of response categories may differ from item to item, the categories of each item can be ranked from "very positive" to "very negative" expression of satisfaction. Actually each item is an

attitudinal item: each conforms to the definition of attitudinal items as formulated by Guttman (Gratch, 1973, p. 36):

"An item belongs to the universe of attitude items if and only

if its domain asks about behavior in a $\begin{cases} \text{cognitive} \\ \text{affective} \\ \text{instrumental} \end{cases}$

modality toward an object, and its range is ordered from

$\begin{cases} \text{very positive} \\ \text{to} \\ \text{very negative} \end{cases}$ towards that object."

This raises an interesting further question: do all these satisfaction items have a common object? If they do, then Guttman's First Law of Attitude should hold: the regression of any item on any other item should be monotone, and no correlation should have a negative sign (Gratch, 1973, p. 36).

Inspection of the correlation matrices of Tables I and II indeed shows that all correlations are positive or zero (the one slightly negative correlation may be regarded as a sampling error). From the wording of the domain of the mapping sentence, the immediate attitudinal object of each item is a particular area of life. However, all the items may be regarded as having a general object: life itself. According to such an interpretation, all the items do have a single attitudinal object in common as required by the First Law. This specification of a single common object is indicated by the phrase "satisfaction with life" following facet R in the above mapping sentence. Hence, we have a rationale for the First Law of Attitude to hold in this case for both the U.S. and Israel.

The Radex Theory of Satisfaction

Whether or not the First Law is appropriate, there is an apparent systematic correspondence between the two content facets and the empirical structures of the correlation matrices. Facets A and B help indicate which items should be more highly intercorrelated and which should have lower intercorrelations. This lawfulness is brought out by viewing the matrices through the eyes of Smallest Space Analysis (Lingoes, 1973), in particular SSA-I.

For each matrix, a two-dimensional geometrical space proves to give

INTERRELATIONSHIPS AMONG FIFTEEN VARIABLES* OF SATISFACTION WITH LIFE AREAS IN THE UNITED STATES
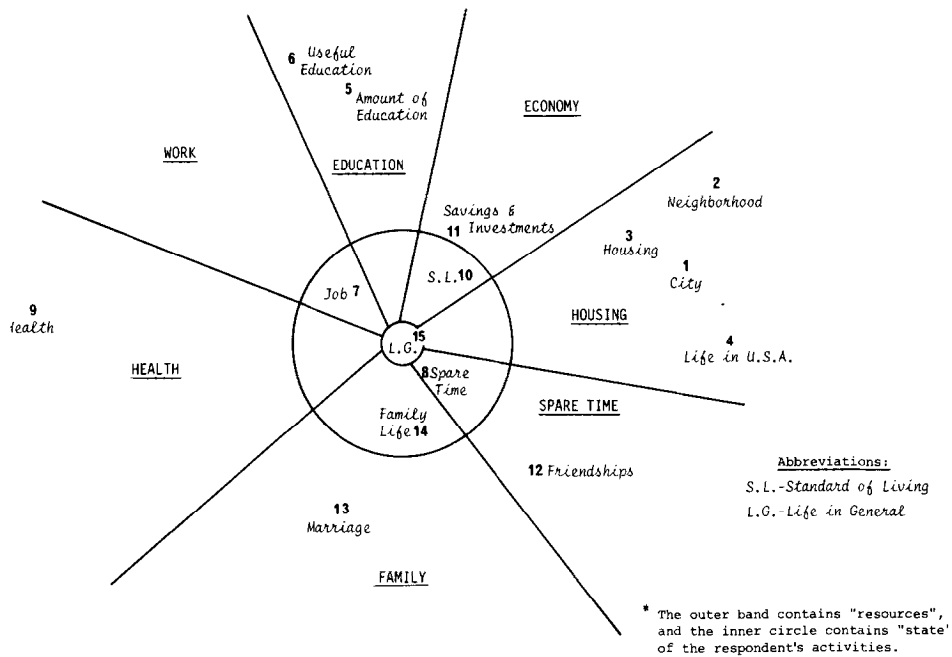

Fig. 1. Interrelationships among fifteen variables of satisfaction with life areas in the United States. The outer band contains "resources," and the inner circle contains "state" of the respondent's activities

a rather good fit. The best fitting 2-spaces for the U.S. and Israel are shown in Figures 1 and 2 respectively. In each diagram, each item appears as a point, and two points tend to be closer together as the correlation increases between the two items involved. The spread of points in the diagrams shows a clear relationship to the classification of the items by facets A and B. It turns out that the dichotomous facet A — of "state" versus "resources" — serves as a *modulating* facet, or corresponds to distance from an origin in the SSA space. The items closest to the origin, within the inner circle, assess the satisfaction with the *state* of activities, while the outer band of items assess satisfaction with *resources* for activities.

Content facet B, of eight areas of life, serves as a *polarizing* facet. Its elements correspond to regions in the SSA space emanating from the origin and radiating outward, each in its own direction. Since these directions are all contained within a two-dimensional space, they have a circular ordering amongst themselves. This circular ordering turns out
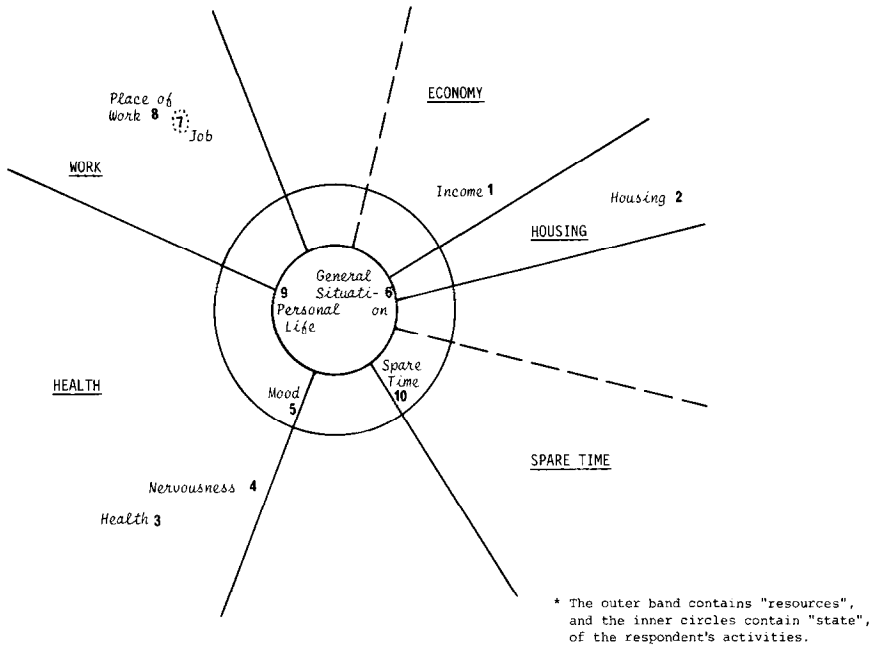
124

Fig. 2. Interrelationships among ten variables of satisfaction with life areas in Israel. The outer band contains "resources," and the inner circles contain "state" of the respondent's activities

to be the same for the U.S. and Israel. (We have found the facet "areas of life" to play a polarizing role in many other contexts as well, as reported at the 1973 annual meeting of the Israel Sociological Association.)

Having two facets correspond to modulating and polarizing partitions of an SSA space is a form of lawfulness called that of a *radex* (Guttman, 1954; Schlesinger and Guttman, 1969; Levy and Guttman, 1974). In sum, then, the content facets of the mapping sentence have the same form of correspondence with the correlation matrices for the U.S. and for Israel, namely that of a radex.

Thus we hope to have illustrated how the mapping sentence device has facilitated seeing lawfulness which would otherwise be difficult to ascertain. We have in effect provided evidence for a radex theory for satisfaction with life. Having such a toehold can be a basis for systematic extensions and intensions and other fruitful modifications of the theory.

References

Aranya, N., Jacobson, D. and Shye, S. (1976). "Organizational and occupational commitment: A facet theoretical analysis of empirical data," *Nederlands Tijdschrift voor de psychologie* (in press).

Elizur, D. (1970). *Adapting to Innovation*. Jerusalem: Jerusalem Academic Press.

Gratch, H. ed. (1973). *Twenty-Five Years of Social Research in Israel.* Jerusalem: Jerusalem Academic Press.

Guttman, L. (1954). "A new approach to factor analysis: the radex." In Lazarsfeld, P.F., ed. *Mathematical Thinking in the Social Sciences*. Glencoe, Ill.: The Free Press.

Guttman, L. (1959). "A structural theory for intergroup beliefs and action," *American Sociological Review,* 24: 318–328.

Guttman, L. (1970). "Integration of test design and analysis," *Proceedings of the 1969 Invitational Conference on Testing Problems*. Princeton: Educational Testing Service.

Guttman, L. (1971). "Social problem indicators," *The Annals of the American Academy of Political and Social Science,* 393: 40–46.

Guttman, L. and Levy, S. (1975). "Structure and Dynamics of worries," *Sociometry,* December.

Kernberg, O. et al. (1972). "Psychotherapy and Psychoanalysis: Final report of the Menninger Foundation's Psychotherapy Research Project," *Bulletin of the Menninger Clinic,* 36: 517–528.

Levy, S. and Guttman, L. (1974). *Values and Attitudes of Israeli High School Youth*. Jerusalem: The Israel Institute of Applied Social Research. (Hebrew, with summary in English).

Lingoes, J. (1973). *The Guttman-Lingoes nonmetric program series*. Ann Arbor, Michigan: Mathesis Press.

Schlesinger, I.M. and Guttman, L. (1969). "Smallest space analysis of intelligence and achievement tests," *Psychological Bulletin,* 71: 95–100.

Yalan, E., Finkel, C., Guttman, L. and Jacobsen, C. (1972). *The modernization of Traditional Agricultural Villages: Minority Villages in Israel.* Rehovot: Settlement Study Center.