BILL BARTON

MAKING SENSE OF ETHNOMATHEMATICS: ETHNOMATHEMATICS IS MAKING SENSE

ABSTRACT. There are unacknowledged difficulties within the literature of culture and mathematics, specifically in the use of the term ethnomathematics. As a step towards a more coherent approach, a framework to review the literature is proposed, and three authors examined. From this analysis, a definition of ethnomathematics is derived and elaborated. Two examples are reviewed as a test of the power of the definition and the resultant description of ethnomathematics.

INTRODUCTION

In the last decade, there has been a growing literature dealing with the relationship between culture and mathematics, and describing examples of mathematics in cultural contexts. What is not so well-recognised is the level at which contradictions exist within this literature: contradictions about the meaning of the term 'ethnomathematics' in particular, and also about its relationship to mathematics as an international discipline.

There are three dimensions to these difficulties. In one sense there is an epistemological confusion: problems with the meanings of the words used to explain ideas about culture and mathematics. For example, in the first paragraph, the phrase "international discipline" sets the reader's mind in a particular way. Some important ideas may be excluded as a consequence. Another difficulty is philosophical. There is little agreement on the extent to which mathematics is universal, and on how mathematical ideas can transcend cultures. Very little of the ethnomathematical literature is explicit about its philosophical stance. This is one of the areas which must be addressed if the subject is to gain wider legitimacy in mathematical circles.

A third level of difficulty relates to the meaning of 'mathematics'. The problem here is that one of the reasons for writing about ethnomathematics is to change what is understood by 'mathematics'. It is therefore not surprising that many writers will be at odds with each other.

Given this multi-dimensioned confusion, how can sense be made of the literature, and how can progress be made? The answer lies, not in

Educational Studies in Mathematics **31**: 201–233, 1996.

trying to identify each point of confusion and resolving it, but in creating a framework whereby the differing views can be seen in relation to each other. Then any reader can see which particular confusions are important to them and, possibly, how they might be resolved.

This paper presents one possible framework for re-viewing. The framework is based on the intentions of the writers when approaching mathematics and culture. From the analysis made possible by the framework, a definition for ethnomathematics is proposed and elaborated. It is further tested by using it to explain two examples of ethnomathematics.

A FRAMEWORK: THE INTENTIONAL MAP

Culture and mathematics is an extremely diverse field: it is written about from several contexts, and it is touched upon by writers whose main aim is to write about other ideas (e.g. the psychology of mathematics, or the politics of mathematics curriculum). A first distinction to be made is between writing about mathematics itself, and writing about mathematics education.

Culture and mathematics itself

Four general areas can be identified. One of these is philosophical. Writers like Bloor (1976, 1983), Ernest (1991) and Zheng (1994) wish to debate the ways in which mathematical knowledge is culturally based. Others who write about the nature of mathematics are part of this debate because they defend acultural, or pan-cultural, mathematics (see Barrow, 1992; Penrose, 1989).

Another area concerns the nature of mathematical thought and activity in various cultures. Well known explicit examples are the studies by Harris (1991) and Cooke (1990) in Australia, Gay and Cole (1967) in Liberia, Pinxten (1987) in North America, and Zaslavsky's (1973) work on African mathematics. Also included is anthropological work which can be regarded as mathematical, e.g. Kyselka (1987) on Pacific navigation, and Ascher (1981) on Aztec quipu. These studies we can refer to as cultural mathematics.

A third reason for writing about culture and mathematics is to describe the evolution of mathematics, what might be called the social anthropology of the subject. Notable examples are Kline (1953), Fang and Takayama (1975), Joseph (1991), Swetz (1987), Restivo (1992), and Restivo *et al.* (1993). The intention of all these writers is to show how mathematics has a cultural history which has affected the nature of the subject itself.

Other writers are debating the politics of mathematics as a cultural issue. Bishop (1990) and Fasheh (1991) are perhaps the most explicit statements to date. The intention of these writings is to explore the ways in which mathematics has affected other aspects of our society, and has changed peoples' conceptions and values.

Culture and mathematics education

The intentions of those writing about culture and mathematics education can also be categorised according to four aspects. Some writing is about mathematics education itself. It attempts to show that mathematics education can be more effective if examples are taken from culturally specific contexts. In particular, it explores the relationships between the thought processes of a cultural group, and mathematics education. Let us label this activity curriculum development. Examples include the writing on street maths vs school maths initiated by Carraher *et al.* (1985), and Mtetwa (1992).

Other writing concerns the way mathematics education in general is determined by the culture in which it is situated. In this area we may include the extensive literature on situated cognition (see Dowling, 1991; Lave, 1988; Nunes, 1992; and Saxe, 1990), mathematics and language, and bilingualism (e.g. Cocking and Mestre, 1988; Secada, 1992; Stephens, 1994).

Mathematics education also affects society, for example in the way it supports particular political systems. Gerdes analysis of pre- and post-revolutionary mathematics education in Mozambique is an example of this (Gerdes, 1981, 1985). The critical mathematics literature and its developments have similar intentions (e.g. Abraham and Bibby, 1988; Frankenstein, 1983; Mellin-Olsen, 1987).

A fourth body of literature concerns the relationship between mathematics and mathematics education. While less cultural in its emphasis, it discusses the way theoretical paradigms in the two areas are related (e.g. Borba, 1990; Pompeu, 1992; and Vithal, 1992).

These areas of intention within the literature may be illustrated thus: see Fig. 1.

MAKING SENSE OF ETHNOMATHEMATICS

Where does existing writing on ethnomathematics fit on this map? How can such a placement provide a helpful analysis of the literature? As an example of the potential of this framework, the writings of three main

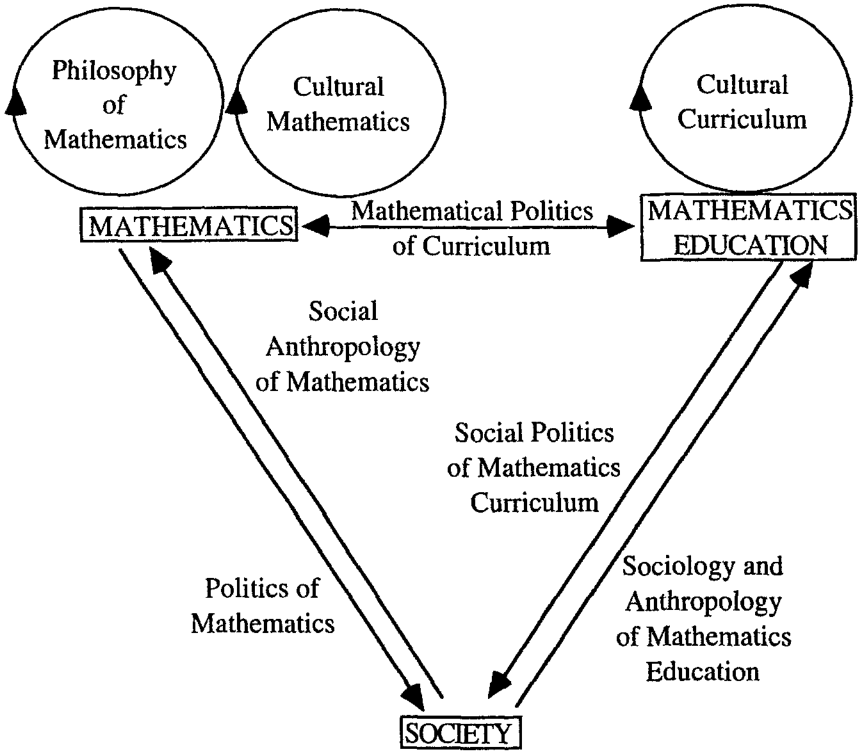

Fig. 1.

authors are briefly described in terms of the map, and two areas of confusion are analysed. Out of this comes a possible definition for the term 'ethnomathematics'.

The three writers considered in more detail are Ubiratan D'Ambrosio in Brazil, Paulus Gerdes in Mozambique, and Marcia Ascher in America. Alan Bishop is often associated with this area, however he does not write about ethnomathematics directly. His work concentrates on the nature of the culture of mathematics itself, and, recently, on how cultural conflict plays itself out within mathematics and mathematics education (Bishop, 1994).

Ubiratan D'Ambrosio

Ubiratan D'Ambrosio is the most prolific of the modern writers on ethnomathematics, in that, for ten years, he has written regularly and explicitly on the subject. His influence is detectable in almost all of the other writing in this area.

Thus is not surprising that, on the Intentional Map, something of D'Ambrosio's work can be located in almost every area. Most is located in the socio-anthropological dimension between society and mathematics. This relationship is described in his early model of human behaviour (1984), although he uses examples from cultural mathematics to illustrate this model, and, later (1985a and b), explicitly deals with the relationship between society and mathematics education. However he returns again (1985b) and again (1987) to address the socio-anthropological dimension of mathematics itself. He has increasingly used his model to analyse the way in which mathematical knowledge is colonised and how it rationalises social divisions within society and between societies, i.e. on the political aspects of the topic (1990, 1991). His later work refers to the educational implications of this analysis and to the nature of a cultural curriculum (1990, 1994).

Paulus Gerdes

Gerdes is the next most profilic writer. Based in Mozambique, Gerdes began writing soon after the revolutionary government of Samora Machel took over from the Portuguese. The realities of living and working in a country recovering from severe colonial oppression have affected both the subject of his writing and the orientation Gerdes brings to it: compared with D'Ambrosio's theoretical approach, Gerdes' work is both practical and politically explicit.

On the Intentional Map, Gerdes is located much closer to mathematics education than to mathematics. In particular, his early papers (1981, 1985, 1986a and b) are directly concerned with elaborating the importance of the social politics of mathematics education and discuss strategies by which mathematics education can serve people in a liberatory way. His 1988 papers move into the area of cultural curriculum, and he then begins to write about the mathematical politics of the curriculum and the cultural nature of mathematics itself (1989a and b). His work on Tchokwe sona drawings (1990, 1991a and c) is more mathematical still, although all his writing contains references to the underlying purpose of mathematics education in Mozambique. A later paper on the history of mathematics (1992) is located within cultural mathematics.

Thus Gerdes has written on all the dimensions of the map associated with mathematics education, moving from the links with society earlier, to more mathematical material later. His recent writing attempts to link the two, developing the term 'ethnomathematics' to describe a movement or research direction which is motivated by particular socio-political aims.

Marcia Ascher

Marcia Ascher is an American academic living and working in New York. Her book (1991) is the most comprehensive single publication on ethnomathematics. With D'Ambrosio she has edited a special issue of *For The Learning of Mathematics* on mathematics education and culture.

On the Intentional Map, Ascher is placed firmly in the area of cultural mathematics, while acknowledging the political implications of her writing for society and for education. The first sentences of her book are:

> Let us take a step toward a global, multicultural view of mathematics. To do this, we will introduce the mathematical ideas of people who have generally been excluded from discussions of mathematics (1991: p. 1).

In the final chapter Ascher presents an essay on ethnomathematics in which she is explicit about the purpose of her work. One aim is to increase understanding of diverse cultures in order to increase our understanding of our own: shedding light on the assumptions we make and illuminating that which makes our culture distinctive. Another aim is to recognise that, even within our own culture, mathematical ideas exist in different contexts and are not the exclusive property of a select few. The intention of the label 'ethnomathematics' is, for Ascher, to indicate an interest in a broader realm than just the subject mathematics: to include mathematical thinking in whatever context it occurs.

In summary, the three authors can be placed on the Intentional Map as follows: see Fig. 2.

What is ethnomathematics?

The first of two issues to be analysed is the subject of ethnomathematics. Does it refer to a body of knowledge, or to a collection of practices, or to something else?

It will be seen that the subject of ethnomathematics has shifted away from its initial conception as being the mathematics of particular cultural groups. The direction of this shift can be related to the intentions of the authors, and any congruences between them lie exactly where their intentions overlap.

D'Ambrosio's early paper (1984) defines ethnomathematics as the way different cultural groups mathematise (count, measure, relate, classify and infer). The implication is that ethnomathematics constitutes practices, although the examples given of practices from Amazonian Indian society are referred to as 'bodies of knowledge'. For example, it is unclear whether D'Ambrosio is referring to the process of building a boat, or to the set of techniques implied by the finished construction.

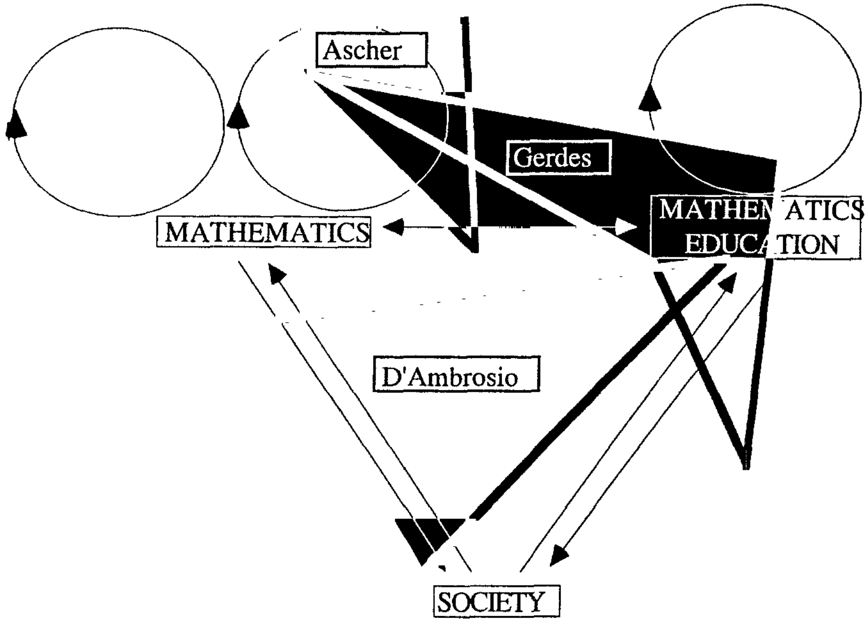

Fig. 2.

The following year, D'Ambrosio (1985b) suggests that different codes and jargons lead to different theories of knowledge. He clearly wishes to make a case for whole systems of knowledge on which culturally defined practices are based. He uses 'ethnomathematics' to refer to an evolving form of knowledge which is manifest in practices which may change over time. His programme for action is explicit:

We are collecting examples and data on the practices of culturally differentiated groups which are identifiable as mathematical practices, hence ethnomathematics, and trying to link these practices into a pattern of reasoning, a mode of thought. Using both cognitive theory and cultural anthropology we hope to trace the origin of these practices. In this way a systematic organisation of these practices into a body of knowledge may follow (p. 47).

Two years later, D'Ambrosio (1987) relates the concept of ethnomathematics to Bernstein's 'restricted code' and Illich's 'vernacular universe'. Ethnomathematics is the codification which allows a cultural group to describe, manage and understand reality. And later still D'Ambrosio (1989) is making an explicit case for ethnomathematics as a research programme which encompasses the history of mathematics. This trend towards a wider perspective for ethnomathematics continues with the use of an etymological definition:

the art of explaining, understanding and coping with the socio-cultural and natural environment ... The dynamic of this interaction [between the individual and the environment], mediated by communication and the resulting codification and symbolisation, produces structured knowledge which eventually becomes disciplines (1990; p. 22).

In several of his later writings, D'Ambrosio concentrates on this dynamic evolution of a systematic body of knowledge, rather than the knowledge itself. Thus ethnomathematics becomes the process of knowledge-making. In this sense it is encompassing of the history and philosophy "not only of mathematics, but of everything" (D'Ambrosio and Ascher, 1994: p. 43). In such an enlarged state it becomes undefinable – as he acknowledges in his most recent publication (1994: p. 449).

D'Ambrosio is concerned to claim a status for the knowledge of people in non-dominant societies. In order to do this the relationship between knowledge and society must be seen in a global way, so that there is an equivalence in this relationship for all societies. Ethnomathematics has become D'Ambrosio's tool for this task. Thus the subject of ethnomathematics has necessarily become more global as the parameters of the task have become clear.

For Gerdes, ethnomathematics was initially (1986b) the mathematics implicit in each practice. He wrote about "recognising the mathematical character" (p. 10), and identifying the "frozen mathematics" (p. 12) in production techniques. However Gerdes does not discuss the idea of a systematic body of knowledge, he only describes isolated ideas which are hidden in examples of practice. Unlike D'Ambrosio, Gerdes does not link ethnomathematics to different value systems, although he does acknowledge that they may involve different codes and conventions. Most of his examples are elaborations of Western mathematics inspired by traditional practices.

Gerdes' emancipatory, educational perspective leads him to conduct ethnomathematical explorations in the cultures of Southern Africa. There is an urgency in his work, an imperative to quickly finds means to transform, through mathematics education, a colonised culture into a modern, independent one which uses world mathematics (1988b: p. 20). This immediate political agenda leads him into agreement with D'Ambrosio on developing the concept of ethnomathematics as a research field.

Thus, by 1989, Gerdes describes ethnomathematics as a movement (1989a). It is an active reclaiming of a mathematical point of view as part of indigenous culture. This includes generating new mathematics from traditional sources and conventional mathematics combined, as in his example using Tchokwe drawings. Ethnomathematics is of the present, rather than a collection of practices from the past, and it is motivated by particular socio-political aims, e.g. to contribute to the mathematical awareness of colonised people, or to draw attention to mathematics as a cultural product. His current view, summarised in Gerdes (1994), defines this movement as a research field involving anthropological reconstruction. But his orientation is mathematical, and ethnomathematics remains within that context.

Ascher shares with Gerdes a practising mathematician's perspective, so the subject of ethnomathematics remains bounded by mathematics itself.

In 1986 Ascher defined ethnomathematics as "the study of mathematical ideas of non-literate peoples" (Ascher and Ascher, 1986). Thus she had already made the shift from the practices themselves to the study of such practices from a mathematical point of view. However the definition restricts the cultures from which examples may be drawn. Together with her co-author, an anthropologist, the intention is to draw on ethnographic work to illuminate our understanding of mathematics.

Ascher sees these mathematical ideas as models, structures and patterns which can be manipulated and discussed in the abstract. In the earlier paper is a long section (1986: pp. 137–139) giving evidence that a New Hebridean kinship system was conceived as an abstract system, and manipulated as such, i.e. it was not just a social feature given structure by anthropologists. Thus ethnomathematics implies structured knowledge, not just its practical manifestation. This is an orientation to be expected from a practising mathematician.

But, as a mathematician, Ascher is also drawn into mathematical analysis of these structures. Ethnomathematics becomes doing mathematics itself, as well as identifying the structures and how they are used. Ascher is not always clear which of these two activities is being referred to, although she does acknowledge this difference and "trusts that the reader will do so as well" (1991: p. 3).

In her work on the quipu in 1980, Ascher acknowledged that the mathematical ideas of a culture have "resonance in other parts of (that) culture" (D'Ambrosio and Ascher, 1994: p. 6). The congruence with D'Ambrosio is in their mutual recognition of the vitalising potential of ethnomathematics within mathematics education. But Ascher's intention is to vitalise mathematics; D'Ambrosio's is to vitalise education.

For all writers, the early difficulties of identifying the subject of ethnomathematics occurred because this subject is assumed to be located within another culture. This causes difficulty because there is no consideration given to the appropriateness of using the term mathematics to describe practices or concepts in a culture which may not contain mathematics as a category of knowledge.

Another difficulty is the colonial assumption that all cultures have components which can be described in conventional mathematical terms. Thus ethnomathematics developed into a research programme, with a broader referent. It now includes: a) the formation of all knowledge (D'Ambrosio); (b) mathematics in relation to society (Gerdes); and (c) mathematical ideas wherever they occur (Ascher). The problem is to reconcile these extensions, and to formally acknowledge that, through the use of the term 'mathematics', the programme ethnomathematics will remain, for the present, culturally specific.

Is ethnomathematics part of maths?

The second issue to be analysed concerns the relationship between ethnomathematics and mathematics. Is ethnomathematics a precursor, a parallel body of knowledge, or a precolonised body of knowledge with respect to mathematics?

It will be seen that there are major differences between the three writers being discussed, and that these differences can be related to the intentions behind their writing. There is, however, an evolution in each of the conceptions: a single evolution which can be applied to all three models.

Much of D'Ambrosio's writing implies that he regards ethnomathematics as a different kind of knowledge from mathematics. In his 1984 paper, for example, a very clear distinction is made between ethnomathematics (which is taught informally) and 'learned mathematics' (which is taught in schools). Learned mathematics is a closed body of knowledge and changes through the activity of mathematicians. Ethnomathematics, on the other hand, has continuous interaction with all members of society.

D'Ambrosio states that ethnomathematics and mathematics are parallel and different: "different modes of thought may lead to different forms of mathematics" (D'Ambrosio, 1985b: p. 44). He sees mathematics as deriving from the division between academic and practical mathematics. The paper goes on to say that the former is now regarded as mathematics, and the latter (which may include high-level techniques which have not been formalised or given sufficient rigor) is ethnomathematics because it can be identified with some group.

But the cumulative character of ethnomathematical knowledge is different from that of mathematics. Mathematics evolves internally, by building from one idea to the next, preserving the old idea in some codified fashion which is incorporated in the new. Ethnomathematics, on the other hand, evolves as a result of societal change, with new forms replacing the old. The experience of old forms is thus not codified within the new, rather it is part of the practice. D'Ambrosio's research programme is to systematically organise these practices, so that the structure and evolution of ethnomathematics will become obvious.

Later, (D'Ambrosio, 1987), mathematics and ethnomathematics are further distinguished epistemologically. The former is regarded as aprioristic compared to the relative, evolutionary character of ethnomathematics. This distinction highlights the psycho-emotional aspect of ethnomathematics vis à vis mathematics. Historical change in the nature of rationality takes place within a complex set of social characteristics, thus explaining the close relationship between ethnomathematics and society. Ethnomathematics is value-bound and is validated by individual's world views, whereas mathematics is rational and is validated by a hierarchy of authority.

For D'Ambrosio, ethnomathematics resides in individuals in their relationship with the environment. The structured knowledge which is produced in this interaction is expropriated by the power structure and returned to the people. This is done by codifying it in the rationalistic codes of mathematics. Thus mathematics is contained within a particular culture, but ethnomathematics relates to knowledge-making in all cultures. D'Ambrosio's etymological definition (see above) builds on this conception, and "can restore to the mathematics in every culture – that is, to ethnomathematics – its breadth" (D'Ambrosio and Ascher, 1994: p. 42).

Gerdes, on the other hand, links ethnomathematics to 'folk mathematics' and 'indigenous mathematics', thereby implying that this is distinct from 'world mathematics'. However Gerdes (1988b) goes on to say that world mathematics is the union of all possible ethnomathematics'.

The implied idea that world mathematics is an ideal, compared with the reality of ethnomathematics, is different from D'Ambrosio's conception. It also explains how ethnomathematics is a living and changing body of knowledge, and makes understandable the mechanism of the colonial effect of Western mathematics. It is the Western world view which is being promulgated, not the mathematical content itself. Gerdes' focus on political education and the reality of living in a society seeking education for urgent technological advancement, has generated a more smooth connection between mathematics and ethnomathematics.

Gerdes (1994: p. 20) retains the idea of the union of many ethnomathematics', but also acknowledges the implication that ethnomathematics, as such a collection, is thus defined at another level, i.e. as the research field of the cultural anthropology of mathematics.

Ascher views mathematics and ethnomathematics as separate fields of study. Mathematics is seen as a closely defined category of knowledge particular to Western culture. It is the province of mathematicians and has a particular history. Ethnomathematics, on the other hand, is seen as the study of mathematical ideas of cultures which do not have a category of knowledge with the label 'mathematics'. She has referred to this group as 'nonliterate' (Ascher, 1986) and 'traditional' (Ascher, 1991), but is at pains to point out that this is not to imply inferior status or a stage in a developmental process leading to real mathematics. Thus ethnomathematics is different from mathematics, the difference being defined culturally.

Where is the line to be drawn between mathematics and cultural reality? In several studies Ascher uses conventional mathematical concepts, symbolisms and analyses. She acknowledges this:

> ... in trying to convey the significance of ideas, we will do so by elaborating on our Western expressions of them. Throughout, we differentiate ... between mathematical ideas that are implicit and those that are explicit, and between Western concepts that we use to describe or explain and those concepts we attribute to people in other cultures (Ascher, 1991: p. 3).

How, then, are we to describe her analyses? Are the concepts themselves, say the kinship relations of the Warlpiri, ethnomathematics; and Ascher's diagrams and group analysis of these concepts, mathematics? Although ethnomathematics and mathematics are distinct, for Ascher there seems to be an intersection between them.

Ascher recognises that ethnomathematics cannot be universally defined as the interpretation of the mathematical concepts of one culture through the mathematical concepts of another. This attempt to be pan-cultural fails because, even though we could visualise Warlpiri elders analysing European kin relations, they may not perceive these discussions as mathematical. For them it might be an ethnogenealogical inquiry, i.e. they would use a category specific to their culture. A pan-cultural activity cannot be defined in the terms of one culture. Thus Ascher's concept of ethnomathematics is a subjective one.

Ascher does, however, come to her idea of ethnomathematics from two different points of view. There is the global 'reading' of a culture which is implicit in the mathematical ideas exhibited within it, and a recognition of the daily aspects of mathematical activity, e.g. the geometric visualisation of the weaver expressed through actions and materials. As she puts it:

> ... the carpenter definitely is dealing with a mathematical idea; the mathematician who [arbitrarily decided to trisect an angle only with ruler and compass] was dealing with an idea. They are both important, but they are different. And, they are linked (D'Ambrosio and Ascher, 1994: p. 38).

The idea of a link leads to a characterisation of Ascher's view of ethnomathematics, not as a distinct subject intersecting with mathematics, but as the intersection of mathematics and culture.

Can the conceptions of the three authors be brought together? Allowing for some licence in the way inclusion and intersection are used, and

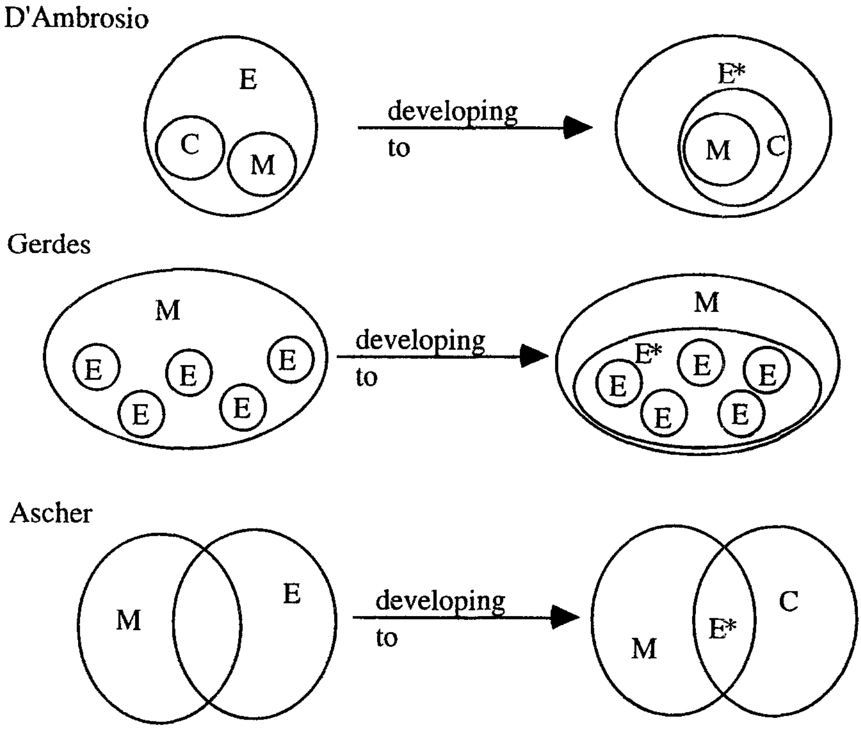

Fig. 3.

acknowledging a simplification which loses the richness of each writer's view, it is possible to diagrammatically express the relationships they imply between ethnomathematics and mathematics. These diagrams are drawn to exaggerate the difference. (E – ethnomathematics, E* – ethnomathematics as a research programme, M – mathematics, C – culture).

Having exaggerated the differences, the common thread in all models is the idea of ethnomathematics as an interpretive programme between mathematics and culture. Each model can be seen as some kind of window (Oates, 1994). For D'Ambrosio it is a window on knowledge itself; for Gerdes it is a cultural window on mathematics; and for Ascher it is the mathematical window on other cultures.

This metaphor can be used to pose the problems for a definition of ethnomathematics. The earlier analysis concerns what is being viewed through this window. The second analysis concerns the nature of the window. A further question concerns the actor(s) in this viewing process. Each of these issues is addressed in the next section.

A DEFINITION: ETHNOMATHEMATICS IS MAKING SENSE

The discussion above requires a definition of ethnomathematics which makes clear both its subject and its relationship to mathematics. Building on D'Ambrosio's and Gerdes' concept of a research programme, and Ascher's use of 'mathematical ideas', the following definition is presented:

Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics.

The statement of a definition requires three forms of elaboration. First it is necessary to define the terms used; second, any implications should be exposed; and finally the definition must be shown to be useful in characterising ethnomathematics.

Four terms are critical to the definition.

Mathematics. Refers to the concepts and practices in the work of those people who call themselves mathematicians. Mathematics with this meaning is what has been referred to as 'learned mathematics', 'school mathematics' or 'world-mathematics' (D'Ambrosio, 1984; Carraher *et al.*, 1985; and Gerdes, 1988b respectively). Mathematics in this sense is extremely widespread in schools and universities around the world. However, while there is huge common ground in this subject, there is also disagreement in practice about whether some aspects are legitimately mathematics.

Mathematical. Refers to those concepts and practices which are identified as being related in some way to mathematics. The relationship may be identified because an idea parallels some aspect of mathematics in its structure or symbolism (e.g. kinship systems which can be seen as group structures), or because an idea is linked to other ideas which are regarded as mathematics (e.g. navigation practices are linked to trigonometry and spherical geometry). Mathematical ideas may not have been accepted by mathematicians as part of their subject (e.g. the traditional Königberg bridge problem was a recreational puzzle for centuries before becoming part of mathematics as network theory). Mathematical ideas are often inextricably linked to other concepts which are clearly identified as not mathematics (e.g. kowhaiwhai designs in Maori art may be analysed mathematically (Knight, 1984), but will always be embedded in Maori symbolism).

Both *mathematics* and *mathematical* are culturally specific because their referents depend upon who is using the terms. It is possible, for example, that some mathematicians will disagree on what is legitimately mathematics.

The *we* used in the definition is a group who share an understanding of mathematics and who are interested in ethnomathematics. That group will usually include mathematicians, who are taken to be self-defined, but will also include others who have experienced mathematics as a category in their own education. When a different ethnic culture is involved, *we* refers to members of a culture which contains the category mathematics. The use of the pronoun makes the point that the ethnomathematician has a particular point of view.

Cultural. Taken to have the meaning used by D'Ambrosio, who refers to a group of people who have "developed practices, knowledge, and, in particular, jargons and codes, which clearly encompass the way they mathematise, that is the way they count, measure, relate and classify, and the way they infer" (D'Ambrosio 1984). Such a group may be an ethnic group, a national group, a historical group, or a social group within a wider culture. *Culture* refers to an identifiable shared set of communications, understandings and practices. It is not necessary to the definition of ethnomathematics that this set is exactly describable.

Having defined the terms, there are four implications of the definition:

(a) ethnomathematics is not a mathematical study, it is more like anthropology or history;
(b) the definition itself depends on who is stating it, and it is culturally specific;
(c) the practice which it describes is also culturally specific; and
(d) ethnomathematics implies some form of relativism for mathematics.

Ethnomathematics is not mathematics

Ethnomathematics does not consist of the mathematical ideas of other cultures, nor is it the representation of these ideas within mathematics. These constructs may be part of ethnomathematics, but they are not the essence of it. Ethnomathematics is an attempt to describe and understand the ways in which ideas which the ethnomathematician calls mathematical are understood, articulated and used by other people who do not share the same conception 'mathematics'. It attempts to describe the ethnomathematician's mathematical world, as others see it. Thus, like anthropology, one of the difficulties of ethnomathematics is to describe another person's world with one's own codes, language and concepts.

In this respect, ethnomathematics is more like the history of mathematics than it is like mathematics. A history of mathematics will contain a good deal of mathematics, but it is primarily about the way ideas originated and developed within mathematics, not about the ideas themselves. The history of mathematics and ethnomathematics overlap. However ethnomathematics tries to uncover how these ideas were perceived at the time, and how present cultural mathematical activities are derived from those of the past; the history of mathematics tries to uncover how these ideas developed, and how they have evolved into mathematics.

Ethnomathematics does create a bridge between mathematics and the ideas (and concepts and practices) of other cultures. Part of an ethnomathematical study will elucidate why those other ideas are regarded as mathematical, and therefore why they might be of interest to mathematicians. Such a study creates the possibility both of mathematics providing a new perspective on the concepts or practices for those within the other culture, and of mathematicians gaining a new perspective on, (and possibly new material for), their own subject.

For example, a study of the Inca quipu is not mathematics, although it may consist in describing the quipu system in numerical form, and the system may give rise to numerical systems not before considered within mathematics. Again, an ethnomathematical study of brick-laying is unlikely to uncover new mathematics, but it may provide a point of view on some geometrical concepts, and a demonstration of some geometrical results in a practical form.

Ethnomathematics is culturally specific

The definition of ethnomathematics is culturally specific: it is written from the point of view of one culture or social grouping, namely a culture or social group which has a conceptual category named 'mathematics'. One of the problems with writings on ethnomathematics is that they have tried to be universal in the scope of their definitions. And yet, such an attempt is a denial of the intent of the ethnomathematician with respect to mathematics. Part of the purpose of ethnomathematics is to challenge the universal nature of mathematics, and to expose different mathematical conceptions. If this is successful, then ethnomathematics is also specific to one particular concept of mathematics. Thus a universal definition is not possible.

A culturally specific definition implies that it does not make sense to speak of, for example, 'Maori mathematics', or 'carpenters' mathematics', *unless* the social group concerned has a category called mathematics for themselves. Since the category mathematics is not common to all cultures, then the concept ethnomathematics is not reflexive. Another implication of the subjective definition is that cultures without a category mathematics cannot have an activity called ethnomathematics.

Not only is the definition of ethnomathematics made in the terms of a specific culture, but the practice of ethnomathematics must also be culturally specific. Studying the way another culture perceives particular practices and concepts is an interpretive exercise from one culture to another. Such an activity must use the form of discourse of the interpreter. In particular the ethnomathematician will be using the concepts of mathematics.

For example, in Ascher's (1991) work on Warlpiri kinship relations, she uses the symbolisms and concepts of group algebra to elucidate the mathematical component of the relationships. It was this mathematical component which made the topic of interest in the first place. Gerdes does the same with an analysis of Tshokwe sand drawings, resorting to a matrix algebra analysis (Gerdes, 1990). Ethnomathematics includes a dialogue between the ideas of another culture and the conventional concepts of mathematics. This dialogue is likely to lead both to new areas of application for mathematics, and to new mathematics through adaptation to new ideas.

Although the ethnomathematician comes from the mathematical culture, this does not mean that the dialogue is completely biased towards that culture. If the practice of ethnomathematics is carried out with integrity, there will be cognisance of those aspects of the practices and concepts which are other-culture based and which may not, initially, be considered mathematical. It is possible that some of these aspects will become incorporated into the mathematical description and analysis of the practices, or will, at least, have some influence on them. Furthermore, if the focus of the study is contemporary, there are likely to be members of the other culture or group who are interested in the dialogue, and who will be re-interpreting, in terms of their culture, the mathematical activities of the ethnomathematician. These people, and the ethnomathematicians, will be able to understand each other, and will be able to 'see' what each other is saying with sufficient accuracy to talk about it.

Thus ethnomathematics can be seen as a process of the social construction of knowledge at a cultural level. This is the creative process of ethnomathematics: not only is this study likely to extend existing mathematics by applying it to new areas, but also mathematics is likely to be enriched by a re-examination of its concepts from the perspective of another culture.

Mathematical relativism

This definition of ethnomathematics implies two senses in which mathematics is universal, and two senses in which it is relative.

The first universal sense arises from the fact that, if you are from a mathematical culture, then it is possible to identify aspects of all other cultures which are mathematical. In other words, it is possible to see all other worlds through a mathematical perspective. For example, Bishop (1988) identifies six pre-mathematical practices which he claims are present in every culture: counting, measuring, locating, designing, playing and explaining. This leaves open the question as to whether numbers (or triangles or the continuum) exist in some real sense because everyone counts, (or designs or measures), or whether these 'objects' are merely conceptual tools with no existence beyond the conceiver.

The second sense in which mathematics is universal results from the fact that, if you come to acknowledge a category mathematics, then you acknowledge the conventional one. If you don't, then it is difficult to justify the use of the label mathematics. Mathematics exists as a knowledge category. If you call something else mathematics, then you do not understand what mathematics is. Such self-referencing universalises mathematics for those who are part of it.

These universal aspects of mathematics are balanced by two senses in which it is possible to say that mathematics is relative.

First, mathematics must be changing. This change needs to be more than just an evolutionary building on what has gone before, it must involve revolutionary change in the sense of Daubin (Gillies, 1993). There is no difficulty for any mathematician to accept that their subject is a growing one, building on what has gone before, with new conceptions including the old within a new paradigm. However ethnomathematics requires more than this. It must admit the possibility of other mathematical concepts which are not subsumable by existing ones, or by some new, overarching generalisation. This is not to say that all ethnomathematical study will generate alternative mathematics. What is necessary, is the idea that it could happen: that new ideas could transform the way mathematics is conceived.

If this possibility is not admitted, then ethnomathematics reduces to the study of particular cultural practices from the point of view of a mathematician, (as opposed to a study of the way other people conceive of their own practices). Ethnomathematics thus becomes a (not very important) part of mathematics. Unless mathematics can change in a radical way, there is no point to examining the way other people view things which we call mathematical. If there is only one view of mathematical phenomena, then why try and find another one?

The second sense in which mathematics must be relative is that there must be a recognition that mathematics is not the only way to see the world, nor is it the only way to see those aspects of the world commonly referred to as mathematical, i.e. having to do with number, shape, and relationships. What is more, there needs to be a recognition that alternative ways of seeing these phenomena are legitimate and valid. For if they are not legitimate, then there is no point in trying to study them, there would only be point in trying to finds ways to 'educate' those who do not see in the 'correct' way.

This second version of relativity does have some implications for the nature of mathematical phenomena. If it is true that numbers (triangles, groups, ...) have a reality outside those who express them, then an ethnomathematician can only be concerned with the way people from other cultures approximate these real objects. They would not be interested in others' perceptions, only in how closely those perceptions correspond to the ideal conceptions of the mathematician. The ethnomathematician must have a working assumption that a mathematician's symbols and conceptions are limited in the task they hope to perform. The ethnomathematician will be constantly trying to overcome the limitations of these mathematical tools (and may generate new tools in the process). If mathematics has a Platonic reality, then there will be no tension between the tools and ethnomathematical activity.

To clarify the universal/relative dichotomy, it may help to distinguish between historical relativity and contemporary relativity. If we are to ask whether there is, in fact, another mathematics equal in power to what is commonly understood as mathematics, then the answer is no. On the other hand, if we are to ask whether mathematics could have been different, then the answer is yes.

Historically, the line of progress of mathematics could have been otherwise. There is no way of knowing what theory of mathematics we may now have, nor whether this hypothetical theory would be more comprehensive, more sophisticated, more applicable (or 'better' by any other criterion of progress). The evidence that history could have been otherwise is necessarily circumstantial. It requires a 'what if' thought-experiment, and this is why we have the strong illusion there is only one mathematics. It is the job of the sociology of mathematics to identify the places where divergent ideas may have changed the path of history, and to trace those paths as far as possible.

The lack of more than one contemporary, sophisticated mathematics, does not imply the universality of the mathematics we know – it only contributes to our feeling of its truth. Contemporary relativity is better conceived as the potential for mathematical development.

CATEGORISING ETHNOMATHEMATICAL STUDIES

The definition can now be used to characterise ethnomathematics. This is done by classifying ethnomathematical studies, and by describing ethnomathematical activity.

A classification

One crucial characteristic of ethnomathematics is that one group is attempting to understand particular practices and conceptions which are held by another group. The way the second group is identified gives three dimensions according to which ethnomathematics can be classified. They are the dimensions of time, culture, and mathematics.

On the time dimension, ethnomathematics may be concerned with the conceptions of an ancient or a contemporary cultural group. For example it may include the way the quipu of the Incas was used and developed (Ascher, 1981), or the genealogy structures of present-day Australian indigenous people (Cooke, 1990).

The culture dimension of the definition extends from a distinct ethnic group, to a purely social or vocational group. Thus ethnomathematics may include the weaving design of Maori kete, sport statistics in the NZ sport-leisure culture, or the measuring practices of carpenters.

The mathematical dimension of ethnomathematics is determined by the relationship of the mathematical ideas to mathematics itself, i.e. ethnomathematics is a study which may be internal to mathematics, or conceptually removed from existing mathematical conventions. An example at the internal end of the spectrum is the conflicting conceptions of statistics in the Bayesian/Frequentist debate; an external example is traditional Polynesian navigation.

The various ethnomathematical studies possible can be placed on these dimensions: see Fig. 4.

Each of these dimensions is a continuum, and ethnomathematical studies may be located at any point in the space. For example, ethnomathematical questions from three points in this space are:

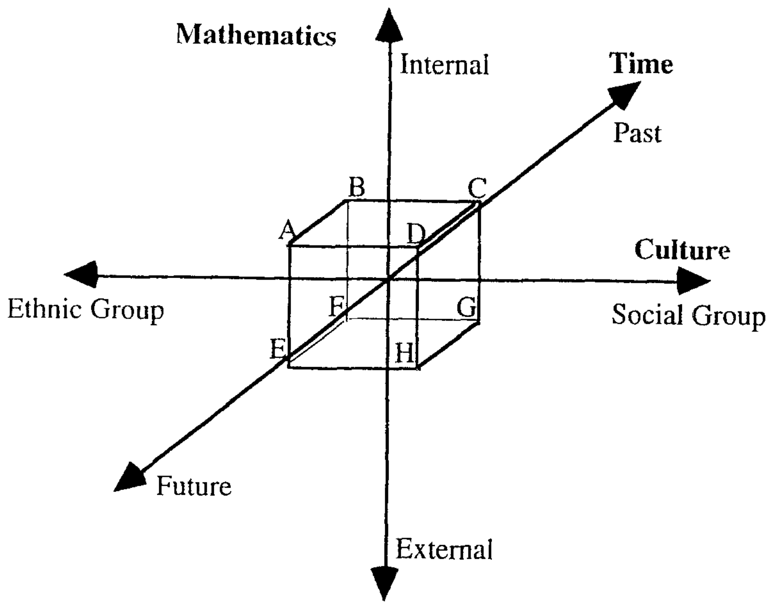

Fig. 4.

B: Historical, ethnic group, internal
What were the mathematical practices and conceptions of the ancient Hindu? or Greeks? It is generally recognised that these were two of the sources of modern day mathematics, but their view of the subject differed from today's conceptions.

H: Contemporary, social group, external
Do carpenters do mathematics? Are the diagrams in a netball manual mathematics? What about the calculations required to play Dungeons and Dragons? To what extent are knitting patterns a mathematical symbol system? There are many activities in every culture which have mathematical components which can be identified, studied and linked to mainstream mathematics.

G: Historical, social group, external
Sea-faring, whether it is Polynesian, or European, was an ancient craft practiced by navigators and captains from many nations. Their navigational systems have been the object of much study and debate concerning the extent to which they represent mathematical systems.

Ethnomathematical activity

Having classified ethnomathematical studies it is necessary to ask how these studies are undertaken. What it is that an ethnomathematician might do in order to find out how people from other cultures 'understand, articulate and use' mathematical concepts? Four types of activity are relevant: descriptive, archaeological, mathematising, and analytic activities. Any or all of these may be part of ethnomathematics.

Descriptive Activity

The first task of an ethnomathematician is to describe those practices or conceptions which are under consideration. This means a description which is, as much as possible, within the context of the other culture. It will probably use ordinary language, and will include some concepts from the other culture which are related to the subject matter. The description will not only focus on the mathematical aspects, but may draw on anthropological conventions or theory.

Examples of this activity are parts of Ascher's work (1991), Abreu and Carraher (1989) on the mathematics of Brazilian sugar cane farmers, and Kyselka's book (1987) about Caroline Island navigation systems.

Archaeological activity

Once the activity has been described there are several ways of bringing out the mathematical aspects. One of these is to trace backwards in time to uncover the mathematics which lies behind the current practice or conception.

This is more an archaeological inquiry than an historical one because the history of the practice will not usually be written in mathematical terms. Thus it is not a matter of finding documents or antecedent mathematics, but of finding the implied mathematics in the derivation of the practice.

This is the kind of activity referred to by D'Ambrosio (1985a) in his ethnomathematical programme. He specifically identifies the mathematical history of existing practices which is implied by the way previous practices have been discarded or modified over time. Uncovering this history may not be easy since earlier practices will have disappeared as they became less useful. Gerdes, too, discusses the need to 'unfreeze' the mathematics in cultural practices such as weaving and house building (1988). He notes that the present day artisans are not necessarily conscious of the mathematics implied in these activities, but that some understanding of the mathematical principles must have been present when the practices were formulated.

Mathematising activity

A second way of exposing the mathematical aspects in an ethnomathematical study is by mathematising, i.e. by translating the cultural material into mathematical terminology, and relating it to existing mathematical concepts. In this activity the ethnomathematician is consciously eschewing the context of the original practice in order to illustrate the mathematics. Such work does not imply that the other culture has such a mathematical consciousness, it just identifies and develops the mathematics implied in the activity.

Examples of this interpretive mathematising is the work of Ascher (1991) with respect to Warlpiri kin relationships and Tshokwe sand drawings. In New Zealand similar work has been done on the kowhaiwhai rafter patterns and weaving patterns (Knight 1984).

As well as interpretive mathematising, it is possible to work with the interpreted mathematics and extent it in a mathematical way. The purpose of such activity might be two-fold: to indulge in a purely mathematical, creative investigation; or to re-interpret the extended mathematics into the original context in order to gain further insight into that context.

An example of a creative mathematical investigation using Tchokwe sand drawing is referred to in Gerdes (1992: p. 12). An example of extended mathematics being reinterpreted is the work of Ascher (1991, pp. 95–109) with respect to Mu Torere, a Maori game, which exposed the idea that early Maori probably had an awareness that alternative versions of the game were not as interesting as the version which survives.

Analytic activity

Having described and developed mathematical ideas from other cultures, researchers seek to find out why the practices are like they are. If the aim is to understand the perceptions of another group, then those things which influenced the development of the phenomenon need to be considered. This activity is more historical/social than it is mathematical.

Examples of the questions asked in analytic activity are: What were the practical requirements for woven fishing nets? What statistical questions were being asked when frequentist views of probability developed? What effect did the type of craft have on the development of Pacific and European navigation systems? What are the artistic conventions under which Maori artists work? What is the hereditary effect of the kinship relations of the Warlpiri?

The four types of ethnomathematical activity described here parallel activity usually associated with history, mathematics, anthropology and sociology. Together they constitute a fascinating, useful, and creative field of study.

Mechanisms of interaction

What remains to be done in this analysis of ethnomathematics is to describe the mechanics of interaction between mathematics and ethnomathematics. At what stages does ethnomathematics become part of mathematics, and what is the status of mathematics which originates as ethnomathematical studies. At what point is ownership relevant, at what point is it lost? When is it appropriate to use words like appropriation and colonisation of knowledge, and when is it appropriate to talk about the growth and development of mathematics? Such a discussion is left for a future paper.

TWO EXAMPLES OF ETHNOMATHEMATICS

A definition and description of ethnomathematics has now been made. Can it add perspective to particular examples of ideas which could be described as mathematical, but which are not primarily regarded that way in their originating cultures? Two quite different examples are chosen. The first, triple weaving, is not new in ethnomathematical literature. The second, sports statistics, is chosen because it is contemporary and sourced in accepted statistical concepts.

Triple weaving

Triple weaving occurs in many parts of the world – Gerdes (1992) mentions examples from Mozambique, Brazil, India, Laos, China, Japan and Indonesia. Each of the patterns below is an example of a triple weaving pattern. The reader is invited to spend a few moments deciding which patterns are of the same type, i.e. to make a simple categorisation of these four designs.

The interesting feature of these patterns is that, from the point of view of weaving, three of them should be categorised together: namely A, B and D. Pattern C is quite different.

In this type of weaving it is usual that the order of colours of the strands is the same in all three directions. In each of the patterns A, B and D this order is white/white/black. The weaving is identical, it is the displacement of the white/white/black strands to the right or left which gives a different pattern. Pattern C is a pattern obtained from alternating white/black/white/black strands. It is possible to take this analysis much further, setting up theoretical structures of groups of patterns and

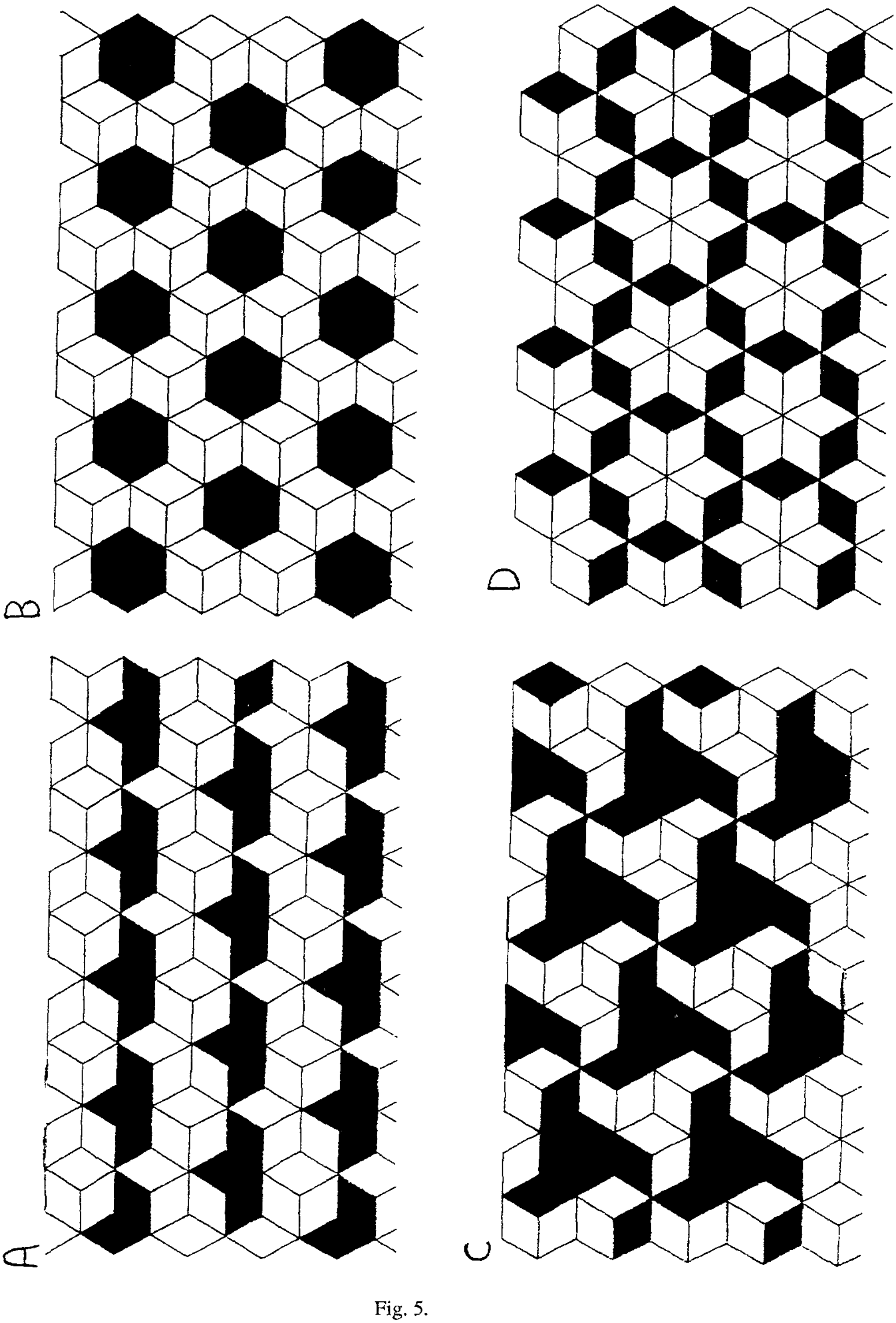

Fig. 5.

the relationships between them. These structures are readily identified as mathematical. They are henceforth referred to as strand analysis.

Strand analysis gives different results from symmetry analysis. For example, in symmetry analysis patterns A and C would be categorised together as having 3-fold rotational symmetry, and all patterns have different line symmetry. There is, however, cultural evidence that strand analysis is a recognised basis of discourse amongst weavers. New Zealand Maori weavers use the word *whakapapa* to describe the way strands are set up prior to weaving. In their basket and mat weaving several designs with the same *whakapapa* will be given the same generic name. For example, the patterns shown below are all referred to as *patikitiki*, although their similarity is not immediately obvious to non-weavers, even when one knows to look for the order of strand colours.

The point of this example is that the mathematical difference resides in the process of analysis, in the basis of the language used to describe the pattern, and in the resulting classification. This is not to suggest that strand analysis is to be equated with symmetry analysis of pattern. Symmetry has a wide sphere of application, whereas strand analysis is restricted to weaving. But strand analysis is not a subset of symmetry analysis: it is different in a significant and mathematically meaningful sense.

A view of ethnomathematics as the study of ideas as well as practices alerts us to look beyond the artifacts or practices to the concepts which are behind them. In this particular case a recognition that strand analysis is constructive, (i.e. it is based on the actions of the weavers in doing their work) raises the same question about symmetry. To what extent did symmetrical analysis arise from the act of making designs? In other words, to what extent did the origins of this branch of mathematics depend on the actions of artists and designers? Highlighting the relative nature of the way in which we analyse shape opens our minds to other possibilities, and thus increases our chances of some creative mathematical thinking.

In terms of the four ethnomathematical activities described above, the *description* of triple weaving focuses attention on pattern and the constructive process, and *archaeology* exposes the existence and depth of strand analysis as an alternative basis for pattern. *Mathematising* shows that strand analysis is a truly mathematical form of considerable richness, and *analysis* leads us to question the social roots of symmetry. This has added to the potential means of producing basket designs for weavers, as well as opening a new area for mathematical investigation.

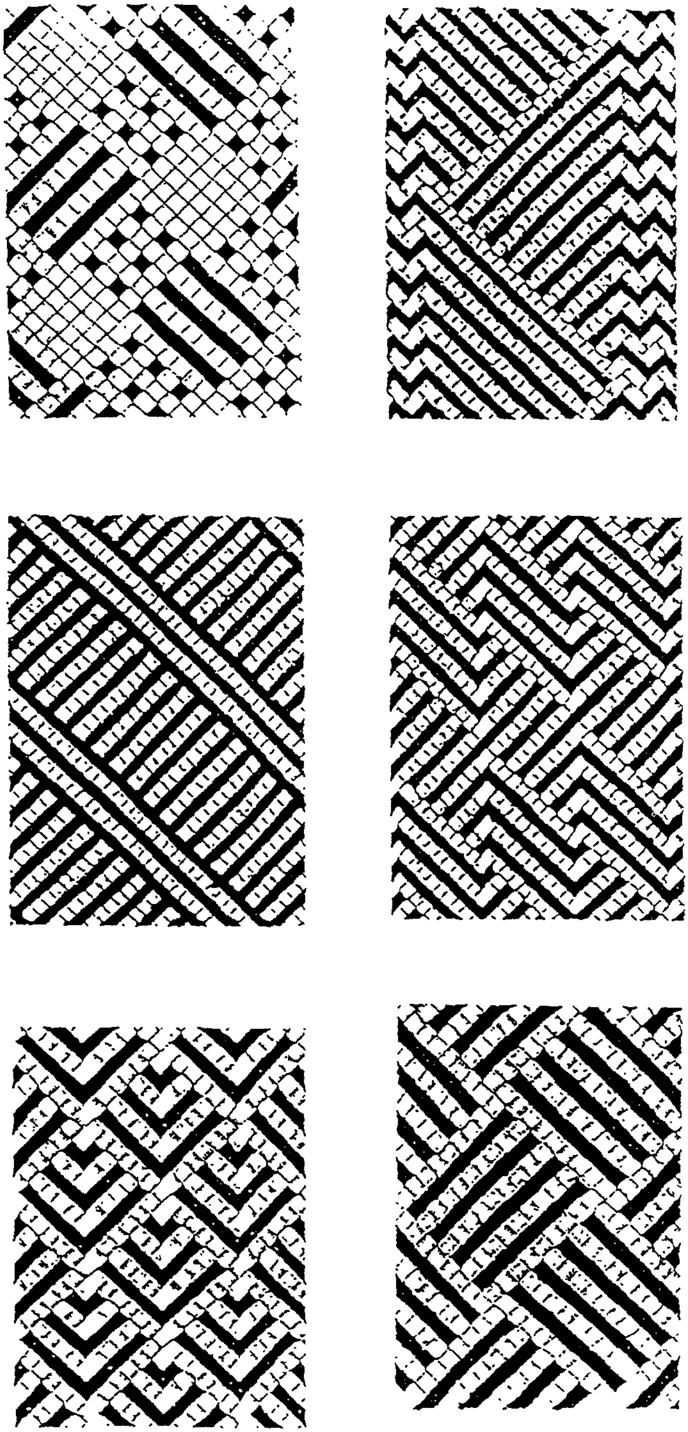

Fig. 6.

Sports statistics

An example of a different cultural conception in mathematics which originates in contemporary Western culture is the use of sporting statistics. The way that statistical ideas are being used in sport represents a different type of analysis from that which is taught in schools.

Consider the statistical analyses which are flashed across our TV screens during the progress of a game: say one-day cricket, netball, rugby league, or soccer. In one-day cricket such data includes: run-rates for batters and strike rates for bowlers. Netball data includes, possession, percentage of goals made by the goal shoot and goal attack, goal difference at various stages of the game and turnovers. League data includes: territory held, tackles in opponents' territory, number of tackles made by individual players, goal shooting rates by kickers, handling errors, and penalty and scrum counts. Soccer data similarly includes territory and possession held.

To get at the basis of the analysis it is necessary to look at the characteristics of these statistics: they are mostly rates rather than measurements; they describe a state rather than an object such as a population, they are in continuous, cumulative change rather than being static descriptors; they are temporary; and they are supposed to be predictive indicators of an event in the future.

For the television viewer the main objective is the prediction of the game outcome. Note that the prediction itself varies in importance through the game – we are aware that it is strongest in the third quarter of the game, and it is of no consequence after the event.

It is possible to characterise sports statistics using the definitions of statistics given in the dictionary or by current statisticians. However, just because you can describe sports analysis in those terms, this does not mean that that conception of statistics is appropriate for analysis. There may be an alternative description which is more accurate, or more plausible for the users of these statistics.

For example, sports statistics could be described as more Bayesian than Frequentist. This distinction lies in the foundations of statistics, and is strongly debated amongst working statisticians. The traditional, frequentist, views are behind school statistics – the only statistical background experienced by most of those in the sport leisure culture.

A frequentist interpretation would be to think of the outcome of one game as one member of a population of encounters between two teams. However this does not make intuitive sense, since the exact conditions of any game will never be repeated. On the other hand, it does make sense to regard each game as a unique event about which we have some prior information, and, as the game proceeds, the prior information is modified to give us new probabilities for the outcome. This is a Bayesian conception.

What would an ethnomathematical study of this situation reveal? This brief description points to the appropriateness of a non-standard, Bayesian account of statistics. An archaeological study may reveal more about the formation of prior probabilities by sports viewers. Prior probability is a continuing problem for Bayesian statisticians, and could be helped by this study. Mathematisation may lead to new, better methods for game analysis. For example, there appears to be an opportunity to develop a theory of the weight of a prediction based on cumulative statistics such as those used in sports.

CONCLUSION

The kaleidoscope of our world can be seen from different angles, which throw up different patterns. Thus ethnomathematics is a tool by which we may better make sense of our world, both as we see, and as others see it.

This paper has attempted to create a framework with which to talk about culture and mathematics. It does this by acknowledging our intentions as writers and speakers. The resulting map, the analysis which proceeds from it, and the definition for ethnomathematics which follows may help to categorise ethnomathematical studies and activities.

There is no doubt that mathematics is a valuable aspect of human understanding, and that it is worth pursuing. There is also no doubt that the role it currently plays in many countries and cultures is a narrow version of its potential. Mathematics education is aimed at furthering mathematics understanding for everyone. To accomplish this it is necessary to change the status and functions of mathematics in our society. An ethnomathematical conception to the mathematics education task assists this change.

In the words of D'Ambrosio "the revitalization of mathematics through ethnomathematics will be the result because ethnomathematical pedagogy is an active one" (D'Ambrosio and Ascher, 1994). Acknowledging the cultural component of mathematics will enhance our appreciation of its scope and of its potential to provide an interesting, artistic and useful view of the world.

REFERENCES

Abraham, J. and Bibby, N.: 1988, 'Mathematics and society: Ethnomathematics and a public educator curriculum', *For The Learning of Mathematics* 8(2), 2–11.

Abreu, G. and Carraher, D.W.: 1989, 'The mathematics of Brazilian sugar cane farmers', Keitel, Damerow, Bishop and Gerdes (eds.), *Mathematics, Education and Society*, UNESCO, Paris.

Ascher, M.: 1981, *Code of the Quipu: A Study in Media, Mathematics and Culture*, University of Michigan Press, Ann Arbor.

Ascher, M. and Ascher, R.: 1986, 'Ethnomathematics', *History of Science* 24, 125–144.

Ascher, M.: 1991, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Brooks/Cole Publishing Co., New York.

Barrow, J.D.: 1992, *Pi in the Sky*, Clarendon Press, London.

Bishop, A.J.: 1988, *Mathematical Enculturation: A Culturation Perspective on Mathematics Education*, Kluwer Academic Publishers, Dordrecht.

Bishop, A.J.: 1990, 'Western mathematics: The secret weapon of cultural imperialism', *Race & Class* 32(2), 51–65.

Bishop, J.: 1994, 'Cultural conflicts in mathematics education: Developing a research agenda', *For The Learning of Mathematics* 14(2), 15–18.

Bloor, D.: 1976, *Knowledge and Social Imagery*, Routledge & Kegan Paul, London.

Bloor, D.: 1983, *Wittgenstein. A Social Theory of Knowledge*, MacMillan Press, London.

Borba, M.C.: 1990, 'Ethnomathematics and education', *For The Learning of Mathematics* 10(1), 39–42.

Carraher, T.N., Carraher, D.W., and Schliemann, A.D.: 1985, 'Mathematics in the street and in schools', *British Journal of Developmental Psychology* 3, 21–29.

Christie, M.J.: 1993, 'Yolngu linguistics', *Journal of Australian Indigenous Issues*, June 1993.

Cocking, R.R. and Mestre, J.P. (eds.): 1988, *Linguistic and Cultural Influences on Learning Mathematics*, Lawrence Erlbaum, Hillsdale, NJ.

Cooke, M.: 1990, *Seeing Yolngu, Seeing Mathematics*, Batchelor College, Northern Territory, Australia.

D'Ambrosio, U.: 1984, *Socio-Cultural Bases for Mathematical Education: Proceedings of ICME-5*, Adelaide.

D'Ambrosio, U.: 1985a, 'Environmental influences', *Studies in Mathematics Education*, Vol. 4, UNESCO, Paris.

D'Ambrosio, U.: 1985b, 'Ethnomathematics and its place in the history and pedagogy of mathematics', *For The Learning of Mathematics* 5(1), 44–48.

D'Ambrosio, U.: 1987, *Etnomatematica: Raizes Socio-Culturais da Arte ou Tecnica de Explicar e Conhecer*, Campinas, Sao Paulo.

D'Ambrosio, U.: 1989, 'A research program and a course in the history of mathematics: Ethnomathematics', *Historia Mathematica* 16, 285–288.

D'Ambrosio, U.: 1990, 'The role of mathematics education in building a democratic and just society', *For The Learning of Mathematics* 10(3), 20–23.

D'Ambrosio, U.: 1991, *On Ethnoscience*, CIMEC, Campinas.

D'Ambrosio, U.: 1994, 'Cultural framing of mathematics teaching and learning', R. Biehler, R.W. Scholz, R. Sträßer, B. Winkelman (eds.), *Didactics of Mathematics as a Scientific Discipline*, Kluwer Academic Publishers, Dordrecht.

D'Ambrosio, U. and Ascher, M.: 1994, 'Ethnomathematics: A dialogue', *For The Learning of Mathematics* 14(2), 36–43.

Dowling, P.: 1991, 'The contextualising of mathematics: Towards a theoretical map', Harris, M. (ed.), *Schools, mathematics and work*, The Falmer Press, Basingstoke.

Ernest, P.: 1991, *The Philosophy of Mathematics Education*, The Falmer Press, London.

Fang, J. and Takayama, K.: 1975, *Sociology of Mathematics and Mathematicians. A Prolegomenon*, Paideia Press, Hauppauge, N.Y.

Fasheh, M.: 1991, 'Mathematics in a social context: Math within education as praxis versus math within education as hegemony', Harris, M. (ed.), *Schools, Mathematics and Work*, The Falmer Press, Basingstoke.

Frankenstein, M.: 1983, 'Critical mathematics education: An applicatino of Paulo Freire's epistemology', *Journal of Education* 165, 315–339.

Frankenstein, M. and Powell, A.B.: 1989, 'Mathematics education and society: Empowering non-traditional students', Keitel, C. *et al.* (eds.), *Mathematics, Education and Society*, UNESCO, Paris.

Gay, J. and Cole, M.: 1967, *The New Mathematics and an Old Culture: A Study of Learning among the Kpelle of Nigeria*, Holt, Rinehart & Winston, New York.

Gerdes, P.: 1981, 'Changing mathematics education in Mozambique', *Educational Studies in Mathematics* 12, 455–477.

Gerdes, P.: 1985, 'Conditions and strategies for emancipatory mathematics education in undeveloped countries', *For The Learning of Mathematics* 5(1), 15–34.

Gerdes, P.: 1986a, 'On culture, mathematics and curriculum development in Mozambique', S. Mellin-Olsen and M. Johnson Hoines (eds.), *Mathematics and Culture: A Seminar Report*, Radal, pp. 15–42.

Gerdes, P.: 1986b, 'How to recognise hidden geometrical thinking? A contribution to the development of anthropological mathematics', *For The Learning of Mathematics* 6(2), 10–12, 17.

Gerdes, P.: 1988a, 'A widespread decorative motif and the Pythagorean theorem', *For The Learning of Mathematics* 8(1), 35–39.

Gerdes, P.: 1988b, 'On possible uses of traditional angolan sand drawings in the mathematics classroom', *Educational Studies in Mathematics* 19(1), 3–22.

Gerdes, P.: 1988c, 'On culture, geometrical thinking and mathematics education', *Educational Studies in Mathematics* 19, 137–162.

Gerdes, P.: 1989a, 'The use of ethnomathematics in the classroom', *Proceedings of Politics of Mathematics Education Conference*, NECC Mathematics Commission, University of Western Cape, 26–36.

Gerdes, P.: 1989b, 'Reconstruction and extension of lost symmetries: Examples from the Tamil of South India', *Computers and Mathematics with Applications* 17(4–6), 791–813.

Gerdes, P.: 1990, 'On mathematical elements in the Tchokwe 'sona' tradition', *For The Learning of Mathematics* 10(1), 31–34.

Gerdes, P.: 1991a, *Lusona: Geometrical recreations of Africa*, Eduardo Mondlane University Press, Maputo.

Gerdes, P.: 1991b, 'Ethnomathematical research: Preparing a response to a major challenge to mathematics education in Africa', *Proceedings 3rd Pan-African Congress of Mathematics*, Nairobi.

Gerdes, P.: 1991c, 'Exploration of the mathematical potential of SONA: An example of stimulating cultural awareness in mathematics teacher education', *Proceedings 8th Symposium of the Southern Africa Mathematical Sciences Assoc.*, Maputo.

Gerdes, P.: 1992, 'On the history of mathematics in Africa South of the Sahara', *AMUCHMA Newsletter 9*, Higher Pedagogical Institute, Maputo, 3–32.

Gerdes, P.: 1993a, *Ethnomathematics and Education in Africa*, University of Stockholm Institute of International Education.

Gerdes, P.: 1993b, 'Survey of current work on ethnomathematics', Paper for Annual Meeting of the American Association for the Advancement of Science, Boston.

Gerdes, P.: 1994, 'Reflections on ethnomathematics', *For The Learning of Mathematics* 14(2), 19–22.
Gerdes, P.: in press, *Ethnomathematics and Education in Africa*, Inst. of International Education, University of Stockholm.
Joseph, G.G.: 1991, *The Crest of the Peacock: Non-European Roots of Mathematics*, Tauris, London.
Kline, M.: 1953, *Mathematics in Western Culture*, Oxford Univesity Press, Oxford.
Knight, G.H.: 1984, 'The geometry of Maori Art – rafter patterns', *NZ Mathematics Magazine* 21(2), 36–41.
Knijnik, G.: 1993, 'An ethnomathematical approach in mathematical education: A matter of political power', *For The Learning of Mathematics* 13(2), 23–26.
Kyselka, W.: 1987, *An Ocean in Mind*, University of Hawai'i Press, Honolulu.
Lave, J.: ,1988, *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*, Cambridge University Press, New York.
Mellin-Olsen, S.: 1987, *The Politics of Mathematics Education*, Kluwer Academic Publishers, Dordrecht.
Mtetwa, K.J.: 1991, *An Investigation of Zimbabwean Secondary School Students' Mathematical Beliefs and Classroom Contexts*, Unpublished Doctoral Dissertation, University of Virginia.
Nunes, T.: 1992, 'Ethnomathematics and everyday cognition', D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, MacMillan, New York.
Oates, G.: 1994, Masters seminar, The University of Auckland.
Penrose, R.: 1989, *The Emperor's New Mind: Concerning Computers, Minds and the Laws of Physics*, Oxford University Press, Oxford.
Pinxten, R., van Dooren, I., and Soberon, E.: 1987, *Towards a Navajo Indian Geometry*, K.K.I. Books, Gent.
Pompeu, G.: 1992, *Bringing Ethnomathematics into the School Curriculum*, Unpublished Doctoral Dissertation, University of Cambridge.
Restivo, S.: 1992, *Mathematics in Society and History: Sociological Enquiries*, Kluwer Academic Publishers, Dordrecht.
Restivo, S., van Bendegem, J.P., and Fischer, R. (eds.): 1993, *Math Worlds: Philosophical and Social Studies of Mathematics and Mathematics Education*, State University of New York Press, Albany, N.Y.
Saxe, G.B.: 1990, *Culture and Cognitive Development: Studies in Mathematical Understanding*, Lawrence Erlbaum Press, Hillsdale, N.J.
Secada, W.G.: 1992, 'Race, ethnicity, social class, language, and achievement in mathematics', D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, MacMillan, New York.
Stephens, M., Waywood, A., Clarke, D., and Izard, J.: 1994, *Communicating Mathematics; Perspectives from Classroom Practice and Current Research*, Australian Council for Educational Research, Hawthorn, Australia.
Swetz, F.J.: 1987, *Capitalism and Arithmetic: The New Math of the 15th Century*, Open Court, La Salle, Illinois.
Turnbull, D.: 1991, *Mapping the World in Mind: An Investigation of the Unwritten Knowledge of the Micronesian Navigators*, Deakin University Press, Geelong.
Vithal, R.: 1992, *The Construct of Ethnomathematics and Implications for Curriculum Thinking in South Africa*, Unpublished M.Phil. Dissertation, University of Cambridge.
Zaslavsky, C.: 1973, *Africa Counts: Number and Pattern in African Culture*, Prindle, Weber, and Schmidt, Boston.

Zaslavsky, C.: 1994, '"Africa Counts" and ethnomathematics', *For The Learning of Mathematics* 14(2), 3–8.

Zheng, Y.: 1994, 'Some remarks on the philosophy of mathematics education', *POME Newsletter*, February 1994.

Department of Mathematics,
The University of Auckland,
P.B. 92 019,
Auckland,
New Zealand.
b.barton@auckland.ac.nz