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Private Information, Money, and Growth: Indeterminacy, Fluctuations, and the Mundell-Tobin Effect

COSTAS AZARIADIS

University of California at Los Angeles

BRUCE D. SMITH

University of Texas-Austin and Federal Reserve Bank of Minneapolis

We introduce an informational asymmetry into an otherwise standard monetary growth model and examine its implications for the determinacy of equilibrium, for endogenous economic volatility, and for the relationship between steady-state output and the rate of money growth. Some empirical evidence suggests that, for economies with low initial inflation rates, permanent increases in the money growth rate raise long-run output levels. This relationship is reversed for economies with high initial inflation rates. Our model predicts this pattern. Moreover, in economies with high enough rates of inflation, credit rationing emerges, monetary equilibria become indeterminate, and endogenous economic volatility arises.

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1. Introduction

This article examines the consequences of introducing a fairly standard informational asymmetry into a conventional monetary growth model. Typically, monetary growth models share a variety of common features. Monetary steady-state equilibria are generally unique, and unique dynamical equilibrium paths approach them. These dynamical equilibria display monotonic convergence to the steady state, ruling out any endogenous economic fluctuations. The steady-state equilibrium levels of per capita output and of the capital-labor ratio are either positively related to the steady-state rate of inflation, or else money is "superneutral," and the level of real activity (in steady-state equilibria) is unaffected by changes in the rate of inflation.[1]

These features of monetary growth models prevent them from being used to address a large set of interesting issues. First, it often has been argued that the unrestricted operation of financial markets is a source of indeterminacies, as well as enhanced economic volatility (see, for example, Keynes, 1936; Mints, 1945; Simons, 1948; or Friedman, 1960). There appears to be a variety of empirical evidence for holding this view: as Friedman and Schwartz (1963) amply document, a large number of business cycles seem to coincide with phenomena like increases in the currency-deposit ratio; more modern business cycles have often been associated with "credit crunches" (Schreft, 1990, or Schreft and Owen, 1992) or other forms of "disintermediation." The fact that most monetary models of capital accumulation deliver unique equilibria displaying monotone dynamics, of course implies that they cannot be used to discuss these kinds of possibilities.

In addition, a large body of empirical evidence suggests that, in the long run, inflation and per capita output (or its growth rate) and inflation and productivity (or its growth rate) are negatively correlated. This correlation is weakest among countries with low rates of inflation, and strongest among countries with high rates of inflation. For example, Bullard and Keating (1994), using structural vector autoregressions, demonstrate that a permanent increase in the rate of money growth leads to a permanently *higher level* of output in countries with initially low rates of inflation, while it leads to a permanently *lower level* of output in countries with initially high rates of inflation.[2] It appears to be a real challenge both to explain why inflation and measures of real activity are not negatively correlated at low rates of inflation, while they are quite negatively correlated at high rates of inflation. This is the subject of this article.

Development economists widely believe that high rates of inflation interfere with the efficient operation of capital markets. As McKinnon (1973), Shaw (1973), and other researchers in development finance have clearly documented (Tun Wai and Patrick, 1973), high inflation places considerable strain on the operation of these markets and particularly on the operation of markets extending medium- to long-term loans.[3] This literature views a stable price level as a prerequisite for the deepening of financial markets and for the extension of their role in financing capital formation.

This article constructs a neoclassical growth model with money and active credit markets in an attempt to judge the theoretical validity of these insights. In order to bring to the fore the economic mechanisms that might reverse the Mundell-Tobin effect, we choose to work with an economy that exhibits most starkly the strains that inflation places on the credit market. We therefore examine an overlapping generations framework where money is a close substitute for physical capital in private asset portfolios. We neglect technical progress and obtain steady states in which per capita income is constant.[4] In addition, we introduce heterogeneous households and postulate private information about the characteristics of potential borrowers—about their lifecycle earnings profiles and their holdings of unintermediated assets. Financial intermediaries can only observe the age and market transactions (including employment status) of borrowers but cannot detect their nonfinancial assets or measure individual consumption.

Households are assumed to have access to two distinct classes of assets: bank deposits and currency, on the one hand, and autarkic unintermediated assets (henceforth called *storage*) on the other. Currency and deposits are perfect substitutes, and of course both are subject to the inflation tax. Autarkic assets, which may be thought of as relatively unproductive investment activities—such as the accumulation of consumption inventories[5]—bear a lower rate of return that physical capital but have the advantage of privacy. Holdings of physical capital, which require production and market activity, are assumed to be publicly observable. Holdings of storage, however, involve no transactions and thus remain private information. They are also not subject to inflationary taxation.

In addition, there are two categories of borrowers. *Legitimate* or high-quality borrowers use credit to produce physical capital. *Illegitimate* or low-quality borrowers are agents who can choose either to work and save (be depositors) when young or to misrepresent their type and seek credit. The latter agents are not capable of converting current resources into future capital, however, and hence will ultimately be detected. To escape detection, counterfeit entrepreneurs simply "go underground" and convert any loans they obtain into undetectable—and unrecoverable—storage. Loans made to these borrowers are never repaid.

The solution to the implied adverse selection problem is that lenders (banks) offer loan contracts so that potential depositors cannot increase their utility by becoming counterfeit entrepreneurs.[6] However, the utility associated with being a depositor depends (positively) on the returns on bank deposits (and currency). Increases in the inflation rate reduce these returns, and—*ceteris paribus*—enhance the incentives of illegitimate borrowers to misrepresent their type. To deter false claims, binding incentive constraints on legitimate borrowers may be employed, resulting in the rationing of credit. As the inflation rate increases, incentive constraints bind more strongly, and credit rationing becomes more severe. This mechanism establishes a negative link between inflation and capital accumulation.

Equilibria in this model fall into one of two possible regimes. For low rates of money growth and low rates of inflation, incentive constraints in the credit market will be slack. In this *Walrasian regime* the slackness of incentive constraints causes competitive equilibria to behave in a manner similar to the equilibria examined by Tirole (1985). Physical capital, bank deposits, and money are perfect substitutes in asset portfolios; all three of them dominate storage in rate of return. The Mundell-Tobin effect is fully operational here: real asset yields in the steady-state drop when inflation rises. Thus, in economies with sufficiently low initial rates of inflation, inflation, and output will be positively related—in the long run. Furthermore, arbitrage conditions between capital and financial assets ensure that the monetary steady state is a saddle; dynamical equilibria that converge to that state are uniquely defined (determinate) and monotone. Neither inflation nor real balances nor the capital-labor ratio fluctuate as they converge to their stationary values.

When the rate of money creation exceeds a critical value, σ_c, a *private-information regime* prevails. The incentive constraint binds in the credit market, and credit to legitimate entrepreneurs is rationed. Capital yields more than bank deposits or currency, and, in particular, money is dominated in rate of return. The returns on financial assets are now *positively* correlated with investment; because higher returns on these assets relax incentive constraints and reduce the rationing of credit, the Mundell-Tobin effect is reversed. At high rates of inflation, further increases in the inflation rate are therefore an impediment to capital accumulation, due to the fact that higher inflation lowers yields on observable assets, lessens incentives toward truthful revelation, and hinders self-selection among borrowers.

Substantial rates of inflation will not just undo the Mundell-Tobin effect but also revoke the local uniqueness of Walrasian equilibrium. *When the rate of money creation exceeds σ_c, the monetary steady state metamorphoses from a saddle to a sink.* Since investment and rates of return on financial assets are positively related in the private-information regime, a variety of initial values for real balances can be chosen that result in convergence to the monetary steady state. Moreover, not only do dynamical equilibria become indeterminate, but for many interesting parameter values, monetary equilibria undergo damped endogenous fluctuations en route to the steady state. Along transient orbits that converge to the steady state, the rate of inflation fluctuates about a high mean that reflects the rapidity of money creation.

Our model generates several testable implications:

- For economies with initially low rates of inflation, permanent increases in the rate of inflation lead to higher long-run output levels. This relationship is reversed for economies with initially high rates of inflation.
- The rate of inflation is positively correlated with the variability of inflation, at least when inflation is high enough.
- Investment is positively correlated with yields on financial assets when incentive constraints bind.

Bullard and Keating (1994) present evidence supporting the first proposition. There is also considerable evidence for the second.[7]

Finally, McKinnon (1973) and Shaw (1973) argue that high rates of return on financial assets are conducive to investment activity in most developing countries. Thus all of these propositions are well supported by observation.

The remainder of the article proceeds as follows. Sections 2 and 3 set out the analytical framework, while Section 4 studies Walrasian equilibria with slack incentive constraints and no credit rationing. Section 5 introduces private information and examines the consequences of binding incentive constraints and the rationing of credit that these constraints require. Section 6 explores the relationship between inflation and capital accumulation along paths converging to the monetary steady state. Section 7 examines the dynamic behavior of incentive constraints, while the concluding section discusses in greater detail the economic mechanisms through which private information contributes to disinvestment, the indeterminacy of equilibrium, and cyclical fluctuations.

2. The Model

2.1. Environment

We consider a discrete-time economy populated by an infinite sequence of two-period-lived overlapping generations. Each generation is identical in size and composition, consisting of a continuum of agents with unit mass. Time is indexed by $t = 0, 1, \ldots$

Each period a single "final commodity" is produced using a constant returns to scale technology with capital and labor as inputs. A producer using K units of capital and N units of labor produces $F(K, N)$ units of the good. Let $f(k) \equiv F(k, 1)$, where $k \equiv K/N$ is the capital-labor ratio. We assume that f is a smooth, increasing, concave function such that $f(0) = 0$. Without real loss of generality, we also assume that capital depreciates completely in the process of production.

Within each generation agents are divided into two types. Type 1 agents, who comprise a fraction $\lambda \in (0, 1)$ of the population, are endowed with one unit of labor when young, and no labor when old. Young labor generates no disutility. In addition, type 1 agents are endowed with a constant returns to scale technology for storing goods between periods. One unit of the good stored at t returns $x > 0$ units of consumption at $t + 1$.

Type 2 agents, who are a fraction $1 - \lambda$ of the population, are endowed with one unit of labor (for which they have no alternative use) when old and no labor when young. They may not use the storage technology just described, but they do have access to a technology that converts one unit of the final good at t into one unit of capital at $t + 1$. Type 1 agents cannot use this technology. Any agent who owns or rents capital at t can operate the final goods production process at $t + 1$. Thus type 2 agents are "producers" in old age. We assume (again with no real loss of generality) that type 2 agents must work for themselves; their labor is not traded.

An agent's type is assumed to be known by the agent but to be private information. However, all market transactions are assumed to be observable. The information structure is quite simple: household type and input into storage are private information; age and market transactions like working and borrowing are observable. Thus, young type 2 agents who have no labor endowment cannot claim to be type 1 agents and work when young.[8] On the other hand, young type 1 agents can credibly claim to be of type 2. If they do so, they will borrow when young as type 2 agents do, and they must supply no labor. However, type 1 agents have no ability to create physical capital and therefore no ability to operate the production process when old. They would then be discovered as having misrepresented their type, and we assume that they can be punished prohibitively. Therefore, type 1 agents who borrow in youth will avoid detection only if they "abscond" with their loan. An agent who absconds never repays the bank and becomes autarkic (that is, he goes underground). An absconding agent's old-age consumption must be financed strictly by using his own storage technology. Since type 2 agents have no access to the storage technology, they choose never to abscond.[9]

Agents' preferences are also quite simple: everyone cares only about old-age consumption. Thus all youthful income is saved in some form by investing in bank deposits or in storage. The savings rate is one; this assumption is easily relaxed. In addition, we assume that all agents are risk neutral.[10] Thus if c^2 denotes period 2 consumption, utility is just c^2.

In addition to young agents, there is an initial old generation at $t = 0$. These agents are each endowed with one unit of labor and a capital stock of $K_0 > 0$. No other agents have an initial endowment of capital, nor are any agents endowed with the final good.

2.2. *Trading*

There are three types of trades that can take place in this economy. First, old producers hire at a competitive wage the labor of young type 1 agents. We let w_t be the real wage rate in period t. Second, young type 1 agents save their entire labor income, some of which is lent to young type 2 agents and possibly to dissembling type 1 agents. It will make sense to think of this lending as being intermediated. There is free entry into the activity of intermediation, and we let r_{t+1} be the gross real return offered on savings by intermediaries between t and $t + 1$. Similarly, R_{t+1} is the gross real interest rate charged by intermediaries on loans made at t and maturing at $t + 1$. Finally, we assume that this economy has a government liability like money or national debt. We let M_t denote the outstanding stock of fiat money at t and p_t denote the corresponding price level. Money intermediates trading between young and old agents.

Each initial old agent is endowed with $M_{-1} > 0$ units of currency. Thereafter, the money supply evolves according to

$$M_{t+1} = \sigma M_t, \tag{1}$$

with $\sigma > 0$ given. Any monetary injections or withdrawals are accomplished by lump-sum transfers to all young agents claiming to be of type 2. Since all capital investment is done by young type 2 agents, this transfer scheme is a government program that subsidizes capital investment by printing money.[11] Let τ_t denote the real value of the transfer received by a young type 2 agent at t. In any nontrivial equilibrium all agents will truthfully reveal their type, and the government budget constraint will be

$$(1-\lambda)\tau_t = (M_t - M_{t-1})/p_t \qquad t \geq 0. \tag{2}$$

Let $m_t \equiv M_t/p_t$ denote time t real balances and substitute (1) into (2) to obtain

$$(1-\lambda)\tau_t = [(\sigma - 1)/\sigma]m_t \qquad t \geq 0. \tag{2'}$$

We denote by b_t the real value of borrowing by (purported) young type 2 agents at t. (Young type 1 agents have no reason to borrow if their type is known so long as $R_{t+1} \geq \max(r_{t+1}, x)$. The latter condition will hold in equilibrium.) Clearly, all type 2 agents will invest in capital the resources they obtain in youth because they cannot store goods and they are not interested in consuming when young. Hence each old producer at $t+1$ will have a capital stock

$$K_{t+1} = b_t + \tau_t \tag{3}$$

that reflects the sum of loans and government transfers.

At time t each producer has an inherited capital stock of K_t, which he combines with L_t units of young type 1 labor and with his own single unit of labor. Thus his total labor input is $N_t = L_t + 1$. The producer's total real income and old-age consumption is then

$$c_t^2 = F(K_t, L_t + 1) - w_t L_t - R_t b_{t-1}$$

since the agent incurred an interest obligation of $R_t b_{t-1}$ when young. Hired labor services L_t are chosen to maximize this expression, which means that

$$F_2(K_t, L_t + 1) = w_t \tag{4}$$

must hold at any interior maximum. Then the producer's consumption is

$$\begin{aligned} c_t^2 &= F_1(\cdot)K_t + [F_2(\cdot) - w_t]L_t + F_2(\cdot) - R_t b_{t-1} \\ &= [F_1(\cdot) - R_t]b_{t-1} + w_t + F_1(\cdot)\tau_{t-1}, \end{aligned} \tag{5}$$

by Euler's law and equations (3) and (4).

Suppose that all young type 1 agents work; this is the only outcome consistent with a nontrivial equilibrium outcome, as we show below. Then at date t the supply of young labor is λ while the measure of producers is $1-\lambda$. Since all producers are identical, labor market clearing requires that

$$L_t = \lambda/(1-\lambda). \tag{6}$$

Therefore, the capital-labor ratio is

$$k_t \equiv K_t/(L_t+1) = (1-\lambda)K_t \tag{7}$$

and (4) can be rewritten as

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \tag{8a}$$

where $w(\cdot)$ is an increasing function of capital intensity. We impose the following additional assumption on technology:

$$w(k) \text{ is a strictly concave function.}^{12} \tag{a.1}$$

It is easy to see that (a.1) implies that $w(k)/k$ is a decreasing function of k and that

$$w(k)/k > w'(k). \tag{8b}$$

Finally, free entry into intermediation means that intermediaries earn zero profits in equilibrium. Let μ_t be the fraction of young type 1 agents who mimic type 2 agents at t. Then intermediaries make loans of measure $1-\lambda$ to type 2 agents at t—who repay their loans—and of measure $\mu_t\lambda$ to type 1 agents—who do not. Since young type 1 agents who mimic type 2 agents also borrow b_t at t, the zero profit condition for intermediaries requires that

$$R_{t+1} = r_{t+1}[1+\mu_t(\lambda/(1-\lambda))]. \tag{9}$$

This equation says that honest agents compensate lenders for dishonest agents who borrow and default. Of course if $\mu_t \equiv 0$ (as we have argued will hold in nontrivial equilibria), (9) reduces to

$$R_{t+1} = r_{t+1}. \tag{9'}$$

We maintain the typical assumptions of economies with adverse selection: on the loan side intermediaries are Nash competitors who announce loan contracts consisting of pairs (R_{t+1}, b_t) at t. These announcements are made by each active intermediary taking the announcements of other active intermediaries as given. On the deposit side intermediaries are assumed to be competitive, taking the cost of deposits r_{t+1} as given at t. There are no other costs of converting deposits into loans.

Before we discuss an equilibrium, we need to describe the portfolio and saving decisions of young type 1 workers. These agents earn w_t at t, all of which they save. Savings can either be deposited with an intermediary or held as real balances, or else it can be stored.

Let s_t denote storage per capita at t. Then, if $\mu_t = 0$, total savings at t is $\lambda w(k_t)$, total borrowing is $(1-\lambda)b_t$, and

$$\lambda w(k_t) = (1-\lambda)b_t + m_t + s_t \tag{10}$$

must hold. This equation simply asserts that total saving equals total borrowing plus real balances plus storage.

An absence of arbitrage opportunities in any equilibrium with positive money holdings, positive credit and nonnegative storage requires that

$$r_{t+1} = p_t/p_{t+1} \geq x. \tag{11}$$

Therefore, we also have

$$R_{t+1} \geq \max(x, r_{t+1}). \tag{12}$$

2.3. Loan Contracts

Suppose that not all young type 1 agents misrepresent their type at t and hence that $\mu_t < 1$ holds.[13] It follows that type 1 agents must do at least as well by revealing their type (working) as by mimicking type 2 agents. A young type 1 agent who works earns w_t when young, all of which is saved. This agent's life-time utility is simply $r_{t+1}w_t$. On the other hand, a young type 1 agent who misrepresents his type borrows b_t and receives a lump-sum transfer of τ_t. All of this is stored, yielding a lifetime utility of $x(b_t + \tau_t)$. Thus $\mu_t < 1$ requires that

$$r_{t+1}w_t \geq x(b_t + \tau_t). \tag{13}$$

As it turns out (see the appendix), nontrivial pooling equilibria do not exist in our economy; in what follows we focus on separating equilibria in which $\mu_t = 0$, and hence $R_{t+1} = r_{t+1}$ $\forall t$. (Conditions that guarantee the existence of a separating equilibrium are also derived in the appendix.) Moreover, competition among intermediaries for customers implies that, in any Nash equilibrium, b_t must be chosen to maximize the lifetime utility of young type 2 agents, subject to the self-selection constraint (13). Of course, from (5), the lifetime expected utility of a young type 2 agent is given by

$$\begin{aligned} c_t^2 &= [F_1(k_{t+1}, 1) - R_{t+1}]b_t + F_1(k_{t+1}, 1)\tau_t + w_{t+1} \\ &= [F_1(k_{t+1}, 1) - r_{t+1}]b_t + F_1(k_{t+1}, 1)\tau_t + w_{t+1} \end{aligned}$$

Moreover, by inverting (4) we obtain $k_{t+1} = \phi(w_{t+1})$, so that

$$F_1(k_{t+1}, 1) = F_1[\phi(w_{t+1}), 1] \equiv \psi(w_{t+1}).$$

Then the utility of a young type 2 agent at t is given by

$$c_t^2 = [\psi(w_{t+1}) - r_{t+1}]b_t + \psi(w_{t+1})\tau_t + w_{t+1}. \tag{P}$$

Intermediaries must choose b_t to maximize the expression in (P) subject to (13), taking w_{t+1}, r_{t+1}, and τ_t as given.

In any nontrivial equilibrium, the maximizing choice of b_t must be positive and finite. Therefore, both (13) and

$$f'(k_{t+1}) = \psi(w_{t+1}) \geq r_{t+1} \tag{14}$$

must hold, and at least *one of these conditions must hold with equality.* Supposing that $f'(k_{t+1}) = r_{t+1}$, the marginal product of capital equals the return on savings, which coincides with the outcome that would obtain if agents' types were fully observable. On the other hand, if $f'(k_{t+1}) > r_{t+1}$, young type 2 agents would like to borrow more than banks allow and are rationed in equilibrium. In this case, of course, (13) is an equality that determines b_t.

To summarize succinctly the properties of a separating Nash equilibrium in the loan market, we substitute equations (3), (7), and (8) into (13) to obtain an alternative expression of the incentive constraint:

$$r_{t+1}w(k_t) \geq xk_{t+1}/(1-\lambda). \tag{15}$$

Expressions (14) and (15), at least one of which is a strict equality, summarize the restrictions on the sequences $\{k_t\}$ and $\{r_{t+1}\}$ imposed by a separating Nash equilibrium in the loan market.

3. Markets

Nontrivial equilibria will satisfy three sorts of requirements at each date:

- Self-selection is observed in the credit market, and the incentive constraint (15) is satisfied.
- The arbitrage conditions (11) and (14) hold for individuals and financial intermediaries, respectively.
- Markets for loans, labor, and capital clear. In particular loans plus transfers equal investment in physical capital as per equation (3); labor supply equals demand, as in equation (6); and aggregate household wealth equals the total value of asset portfolios, as in equation (10).

To express dynamical equilibrium in a compact manner, we combine the market clearing conditions (3), (6), and (10) into

$$k_{t+1} = \lambda w(k_t) - m_t - s_t + (1-\lambda)\tau_t. \tag{16}$$

Substituting (2′) into (16), one obtains the equivalent condition

$$k_{t+1} = \lambda w(k_t) - (m_t/\sigma) - s_t. \tag{17}$$

Finally, by the definition of real balances $m_t = M_t/p_t$, we obtain

$$p_t/p_{t+1} \equiv M_t m_{t+1}/M_{t+1} m_t = (m_{t+1}/\sigma m_t). \tag{18}$$

In the remainder of this paper, we focus on equilibria in which the arbitrage condition (11) holds as an inequality—that is,

$$r_{t+1} = m_{t+1}/\sigma m_t > x, \tag{19}$$

and $s_t = 0$ for all t.[14] For equilibria of this type, the market clearing condition (17) simplifies to

$$k_{t+1} = \lambda w(k_t) - m_t/\sigma, \tag{20}$$

while the incentive constraint (15) becomes

$$k_{t+1} \leq (1-\lambda)(m_{t+1}/m_t)w(k_t)/x\sigma. \tag{21}$$

To validate (19) in stationary equilibrium, we need to assume that

$$1 > \sigma x. \tag{a.2}$$

Then the relevant equilibrium conditions are (20), (21), and the arbitrage condition for producers—that is,

$$m_{t+1}/\sigma m_t \leq f'(k_{t+1}). \tag{22}$$

Except in exceptional cases, only one of the two relations (21) and (22) will hold as an equality. When the incentive constraint (21) is binding, producers are credit rationed; we call this situation a *private-information equilibrium.* When the arbitrage condition (22) is tight, the provision of credit is competitive in the usual sense; we call this state of affairs a *Walrasian Equilibrium.* We examine each case in turn.

4. Walrasian Equilibria

Dynamical equilibria free from credit rationing are nonnegative sequences $\{k_t, m_t\}$ that satisfy equation (20), equation (22) as an equality, and (21) as a strict inequality for each $t = 0, 1, \ldots,$ given an initial condition $k_0 \geq 0$. Except for the incentive constraint (21), the dynamical system consisting of equations (20) and (22) is a slight generalization of the one studied by Tirole (1985) for the case $\sigma = 1$ and analyzed extensively in Azariadis (1993, ch. 26.2).

For values of the money growth rate σ that are not too large, the economy we are discussing has three stationary states labeled A, B, and C in Figure 1. Of these, A and C are nonmonetary equilibria[15] that bear no relation to the issues under discussion; we focus on the unique monetary steady state B and on the associated dynamical monetary equilibria.

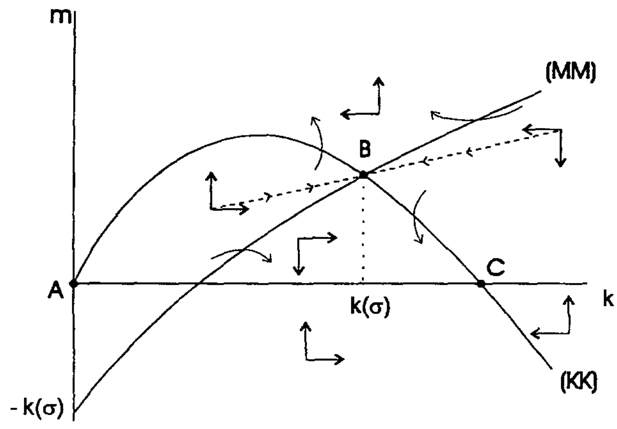

Figure 1. Full information

In Figure 1 the loci

$$(KK) = \{(k_t, m_t)\colon\ m_t \neq 0, k_{t+1} = k_t\}$$
$$(MM) = \{(k_t, m_t)\colon\ m_t \neq 0, m_{t+1} = m_t\}$$

describe combinations (k_t, m_t) such that the capital-labor ratio and per capita real balances, respectively, are constant. In particular, it is easy to show that the monetary state $\{k^*(\sigma), m^*(\sigma)\}$ of this economy satisfies the equations

$$f'(k) = 1/\sigma \tag{23a}$$
$$m = \sigma[\lambda w(k) - k] \tag{23b}$$

as well the incentive constraint (21), provided that

$$k^*(\sigma) \leq \hat{k}(\sigma), \tag{23c}$$

where the function $\hat{k}(\sigma)$ solves (21) at equality—that is,

$$\hat{k}(\sigma)/w[\hat{k}(\sigma)] \equiv (1-\lambda)/x\sigma.^{16} \tag{23d}$$

Throughout we focus on situations with $m^*(\sigma) > 0$. Also, notice that equation (23a) captures the well-known Mundell-Tobin effect: an increase in the rate of money creation (which equals the steady-state rate of inflation) raises the steady-state capital stock and output level.[17]

To study the dynamical equilibria we construct the phase diagram for this economy starting with the loci (KK) and (MM). These are defined from the equality version of the relationships

$$k_{t+1} \ \geq\ k_t \text{ iff } m_t \leq \sigma[\lambda w(k_t) - k_t] \tag{24a}$$
$$m_{t+1} \ \geq\ m_t \text{ iff } m_t \geq \sigma[\lambda w(k_t) - k^*(\sigma)]. \tag{24b}$$

The phase diagram in Figure 1 suggests, and a quick check of the relevant Jacobian matrix verifies, that the state A is a source, C is a sink, and B is a saddle with an upward sloping stable manifold. In the neighborhood of the state B, dynamical equilibrium is *determinate*: for each k_0 near $k^*(\sigma)$ there is only one value m_0 that will put this economy on the stable manifold leading to B.

The presence of the Mundell-Tobin effect and the determinacy of the monetary steady state are both features of the Walrasian economy. Each of these properties is reversed in private information economies for a wide variety of parameter values. We now consider why this reversal occurs.

5. Private Information Equilibria

Producers will be rationed in the credit market whenever the incentive constraint (21) binds and the arbitrage condition (22) holds as a strict inequality. More precisely, the nonnegative sequence $\{k_t, m_t\}$ is a private information equilibrium if, for each t:

$$k_{t+1} = \lambda w(k_t) - m_t/\sigma \quad (25a)$$

$$k_{t+1} = (1-\lambda)(m_{t+1}/m_t)w(k_t)/x\sigma \quad (25b)$$

$$m_{t+1}/\sigma m_t < f'(k_{t+1}). \quad (25c)$$

This economy has at most one monetary steady state $(\hat{k}(\sigma), \hat{m}(\sigma))$, which satisfies

$$\hat{k}/w(\hat{k}) = (1-\lambda)/x\sigma \quad (26a)$$

$$\hat{m} = \sigma[\lambda w(\hat{k}) - \hat{k}] = \sigma\hat{k}[x\sigma\lambda/(1-\lambda) - 1] \quad (26b)$$

$$f'(\hat{k}) > 1/\sigma. \quad (26c)$$

The existence of a unique solution to equations (26a) and (26b) with $\hat{m} > 0$ is implied by the following two assumptions:

$$x\sigma\lambda > 1-\lambda \quad (a.3)$$

$$w'(0) > \sigma x/(1-\lambda).^{18} \quad (a.4)$$

Moreover, this solution satisfies (26c) iff

$$\hat{k}(\sigma) < k^*(\sigma). \quad (27)$$

When (27) holds, the private information monetary steady state differs from its Walrasian counterpart in one key aspect: the capital-labor ratio $\hat{k}(\sigma)$ defined in (26a) is a *decreasing* function of the money growth rate σ. In particular, (26a) requires that an increase in σ be matched by a decline in $k/w(k)$; (a.1) implies that this must be accomplished by a reduction in k itself. The steady-state capital stock is lowered by an increase in the rate of monetary expansion because higher inflation reduces the attractiveness of bank deposits relative to unintermediated saving and hence exacerbates credit rationing.

A comparison of (23c) and (27) indicates the circumstances under which the Walrasian steady state equilibrium or the private information steady state equilibrium will obtain. In

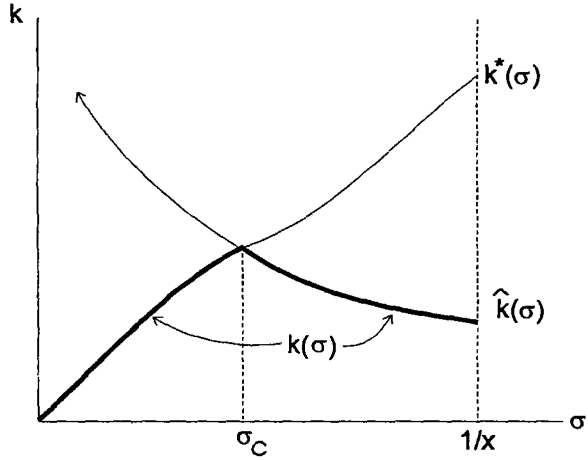

Figure 2. Inflation and the Capital Stock in the Steady State

particular, the Walrasian monetary steady-state equilibrium is the relevant one iff $\hat{k}(\sigma) \geq k^*(\sigma)$, with $\hat{k}(\sigma)$ and $k^*(\sigma)$ defined by (26a) and (23a), respectively.

The schedules $k^*(\sigma)$ and $\hat{k}(\sigma)$ are depicted in Figure 2. For values of $\sigma \leq \sigma_c$, $\hat{k}(\sigma) \geq k^*(\sigma)$ holds, and the steady-state monetary equilibrium is Walrasian. When $\sigma > \sigma_c$ on the other hand, this inequality is reversed, the incentive constraint binds, and the return on bank deposits falls short of the yield on physical capital.

As the policy parameter σ varies in the interval $(0, 1/x)$,[19] the steady-state equilibrium capital intensity is given by

$$k(\sigma) = \min\{k^*(\sigma), \hat{k}(\sigma)\}.$$

For rates of money growth $\sigma < \sigma_c$, $k(\sigma)$ will rise with the rate of inflation for the standard Mundell-Tobin reasons. When $\sigma > \sigma_c$ obtains, increases in the money growth rate result in more stringent credit rationing and reductions in the steady-state capital stock. Thus permanent increases in the rate of money creation will increase steady-state output levels in economies with initially low rates of inflation and will reduce steady-state output levels in economies with initially high rates of inflation. Such a finding accords well with the empirical results of Bullard and Keating (1994).

In order to study dynamical equilibria under private information, we combine equations (25a) and (25b) and rewrite them in the form

$$k_{t+1} = \lambda w(k_t) - m_t/\sigma \tag{25a$'$}$$

$$m_{t+1} = [x\sigma\lambda/(1-\lambda)]m_t - [x/(1-\lambda)]m_t^2/w(k_t). \tag{25b$'$}$$

Proceeding as in the previous section, we use equations (25a′) and (25b′) to derive loci of constant capital intensity (KK) and constant money balances (MM') in the state space (k, m):

$$k_{t+1} \geq k_t \text{ iff } m_t \leq \sigma[\lambda w(k_t) - k_t] \tag{28a}$$

$$m_{t+1} \geq m_t \text{ iff } m_t \leq [\lambda\sigma - (1-\lambda)/x]w(k_t). \tag{28b}$$

We know already that—under (a.3) and (a.4)—the loci (MM') and (KK) have a unique intersection in the positive orthant, as shown in Figure 3. Note that *the locus* (KK) *is the same here as in the Walrasian economy of Figure 1*, but the constant-real-balance locus (MM') is not. We also draw the local force field in Figure 3 and indicate the general shape of orbits in the neighborhood of the monetary steady state. For further reference, we compute the Jacobian matrix at that state

$$J = \begin{pmatrix} \lambda w'(\hat{k}) & -1/\sigma \\ [(1-\lambda)/x](A-1)^2 w'(\hat{k}) & 2-A \end{pmatrix}, \tag{29a}$$

where we let

$$A = x\sigma\lambda/(1-\lambda) > 1. \tag{29b}$$

The relevant trace T and determinant D are

$$T = \lambda w'(\hat{k}) + 2 - A \tag{30a}$$

$$D = \lambda w'(\hat{k})/A = \hat{k}w'(\hat{k})/w(\hat{k}), \tag{30b}$$

where the latter equality follows from (26a). Apparently assumptions (a.1) and (a.3) imply that

$$0 < D < 1 \tag{31a}$$

$$T < 1 + D. \tag{31b}$$

6. Private Information and Indeterminacy

How does the reversal of the Mundell-Tobin effect influence transitory dynamics in the neighborhood of the monetary steady state? We saw that Walrasian monetary steady states display the familiar saddlepoint property of neoclassical growth:[20] given the initial capital stock $k_0 < k^*(\sigma)$, there exists only one initial value m_0 for real balances consistent with an equilibrium sequence $\{k_t, m_t\}$ that converges to (k^*, m^*). As real balances rise ever more slowly along the stable manifold, the rate of inflation $\sigma m_t/m_{t+1}$ rises at a decreasing rate, as does capital accumulation. Key to this Walrasian dynamical process is the inverse relation between capital intensity and the rate of return embodied in the demand for capital services.

Private information reverses that relation: high yields and low rates of inflation relax the incentive constraint and permit credit to expand. An implication of the changed correlation

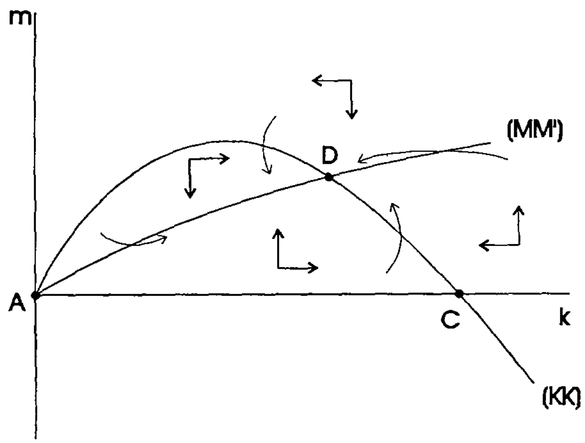

Figure 3. Private Information

between the capital stock and the rate of return is that the monetary equilibrium need no longer be unique. Under private information, the monetary state $(\hat{k}, \hat{m})$ is a sink for an open set of parameter values (λ, x, σ), and dynamical equilibria in the neighborhood of that state are indeterminate. The practical implication of indeterminacy is that long-lived changes in the policy parameter σ may cause the state variable to jump in an unpredictable manner causing unnecessary volatility in economic aggregates.

To isolate the conditions under which indeterminacy obtains, we return to the dynamical system consisting of equations (25a) and (25b). Equations (30a) and (30b) imply that the determinant D and trace T of the relevant Jacobian matrix satisfy the relation

$$T = AD + 2 - A, \tag{32}$$

where D can lie anywhere in the unit interval and where the parameter $A > 1$ is defined by equation (29b).

Combinations (T, D) for which the monetary stationary state is asymptotically stable lie inside the bold-sided triangle (ABC) in Figure 4.[21] Since $D \in (0, 1)$, equation (32) is depicted by the broken line joining the vertex B with the point $(0, 2 - A)$. The latter point can lie inside or outside the triangle; it lies outside if $A > 3$. Finally, the locus defined by (32) has slope $1/A \in (0, 1)$.

The actual configuration of equation (32), and the equilibrium value of D, potentially depends on the government's choice of σ, on the values of the exogenous parameters λ and x, and on the properties of the production function f. However, it is clear that it is possible for D to take on any value in the interior of the unit interval; for example, if $f(k) = Bk^{\beta}$, then $kw'(k)/w(k) = \beta$.[22]

In the neighborhood of $(\hat{k}, \hat{m})$, the qualitative properties of transient equilibria depend heavily on D and T. We now briefly elaborate on the kinds of dynamical equilibria that are possible. Appendix B of Azariadis and Smith (1994a) provides a detailed analysis of local

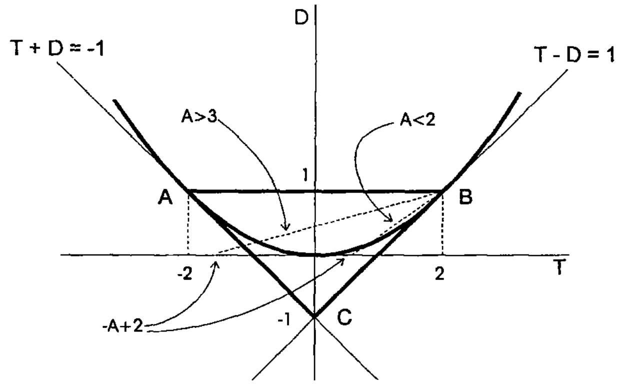

Figure 4. Indeterminacy of Private Information Equilibrium

dynamics, together with examples illustrating how indeterminate convergent equilibria can be observed. These equilibria can display either damped oscillations or monotonically approach the steady state.

- *Case 1*: $A \in (1, 2/(1 + D^{.5})]$. In this case the steady state $(\hat{k}, \hat{m})$ is a sink, and paths approaching it are monotonic. This situation obtains for "small enough" values of A or, equivalently, for relatively low rates of money growth.[23]

- *Case 2*: $A \in (2/(1 + D^{.5}), (3 + D)/(1 - D))$. Here the steady state continues to be a sink, but paths approaching the steady state display damped oscillations. This case emerges for "intermediate" values of A and hence for "intermediate" values of σ. For such values the presence of private information is a source of both indeterminacy and cyclical fluctuations.

- *Case 3*: $A > (3+D)/(1-D)$. The steady state now becomes a saddle. Paths approaching the steady state display damped oscillations. Thus, for "large" A (and σ) monetary equilibrium is determinate, but private information causes cyclical fluctuations.

To summarize: for a large range of money growth rates, private information is a source both of indeterminacy of monetary equilibrium and endogenous fluctuations.

7. The Dynamics of Incentive Constraints

We explore here the dynamic analog of the question we asked in Section 5: what values of the state variable (k, m) are consistent with a binding incentive constraint in the credit market and hence a private-information equilibrium? We know that all dynamical equilibria, with or without binding incentive constraints, must satisfy (20), (21), and (22). From (21) and (22) we obtain

$$H(k_{t+1}) \equiv k_{t+1}/f'(k_{t+1}) \leq [(1 - \lambda)/x]w(k_t). \tag{33}$$

Since H is an increasing function, we may rewrite (33) in inverse form

$$k_{t+1} \leq H^{-1}\{[(1-\lambda)/x]w(k_t)\}. \tag{34}$$

Finally, substituting (20) into (34) and rearranging terms gives

$$\sigma\lambda w(k_t) - \sigma H^{-1}\{[(1-\lambda)/x]w(k_t)\} \leq m_t, \tag{IC}$$

a condition that must be satisfied by any equilibrium sequence $\{k_t, m_t\}$.

It will now be convenient to display the relationships between three loci: (MM), (MM'), and (IC) at equality. For $m_t > 0$, these loci satisfy

$$m_t = \sigma\lambda w(k_t) - \sigma H^{-1}\{[(1-\lambda)/x]w(k_t)\} \tag{IC}$$

$$m_t = \sigma\lambda w(k_t) - \sigma k^*(\sigma) \tag{MM}$$

and

$$m_t = \sigma\lambda w(k_t) - [(1-\lambda)/x]w(k_t). \tag{MM'}$$

The locus defined by (IC) at equality lies above the locus defined by (MM) iff

$$H^{-1}\{[(1-\lambda)/x]w(k_t)\} \leq k^*(\sigma). \tag{35}$$

We rewrite (35) as

$$[(1-\lambda)/x]w(k_t) \leq H[k^*(\sigma)] \equiv k^*(\sigma)/f'[k^*(\sigma)] \equiv \sigma k^*(\sigma). \tag{36}$$

Equation (36) is satisfied for all $k_t \leq \bar{k}$ where $\bar{k}$ is uniquely defined by

$$[(1-\lambda)/x]w(\bar{k}) \equiv \sigma k^*(\sigma). \tag{37}$$

Similarly the locus defined by (IC) at equality lies above the locus defined by (MM') iff

$$H^{-1}\{[(1-\lambda)/x]w(k_t)\} \leq [(1-\lambda)/x\sigma]w(k_t). \tag{38}$$

Again, we can rewrite (38) in the form

$$\begin{aligned}[(1-\lambda)/x]w(k_t) &\leq H\{[(1-\lambda)/x\sigma]w(k_t)\} \\ &\equiv [(1-\lambda)/x\sigma]w(k_t)/f'\{[(1-\lambda)/x\sigma]w(k_t)\},\end{aligned} \tag{39}$$

which reduces to

$$[(1-\lambda)/x]w(k_t) \geq \sigma k^*(\sigma). \tag{40}$$

Thus the locus defined by (IC) at equality lies above the locus defined by (MM') at equality iff $k_t \geq \bar{k}$. The situation is shown in Figure 5. Points on or above the locus labeled IC in the figure satisfy the condition (IC). Only such points are consistent with the existence of a (dynamical) equilibrium where fiat money has value.

Figure 6 depicts an economy where incentive constraints bind in a steady-state equilibrium with valued fiat money. Points below the locus IC are *not* consistent with equilibrium

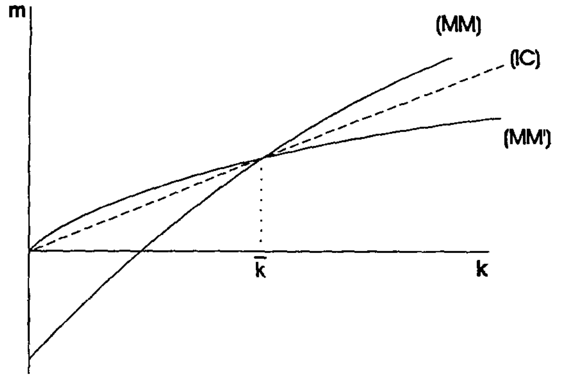

Figure 5. Dynamics of Incentive Constraints

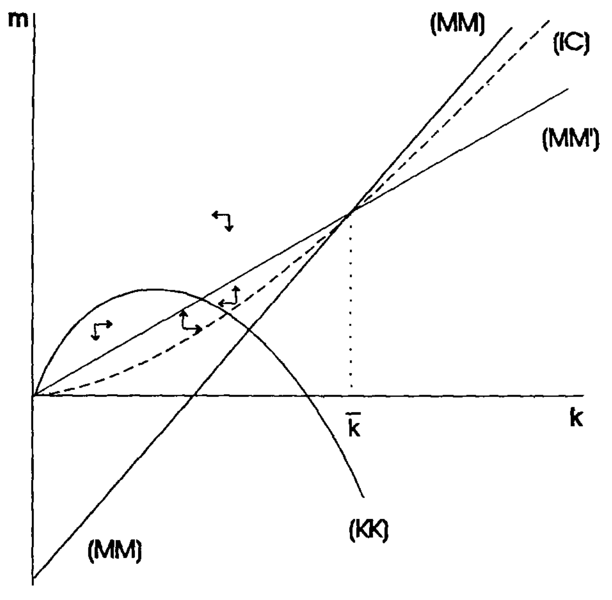

Figure 6. Private Information Equilibrium

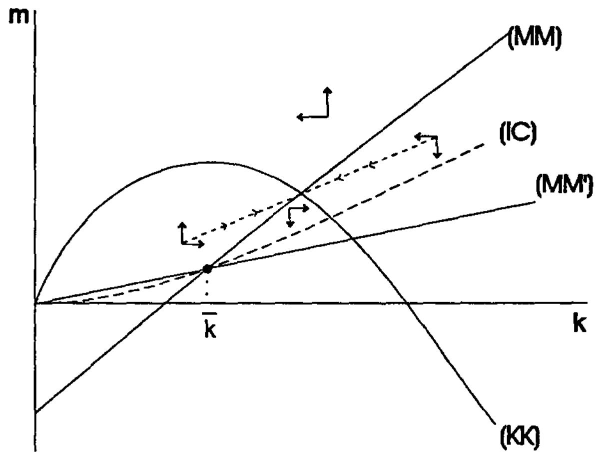

Figure 7. Walrasian Equilibrium

(except for points on the horizontal axis), while points on or above it are. Equilibrium sequences $\{k_t, m_t\}$ have the properties of the dynamical private information equilibria described in Sections 5 and 6.

Finally, Figure 7 describes a Walrasian economy in which incentive constraints are not binding on credit markets in equilibrium. Again, points below IC are inconsistent with equilibrium except for points on the horizontal axis. These equilibria are studied in Section 4.

8. Conclusions

Simple one-sector paradigms of growth in closed monetary economies (for example, the overlapping generations, Sidrauski, and cash-in-advance models) possess monetary steady states with two counterfactual properties:

- Arbitrarily rapid money creation (or price inflation) does not retard and may actually promote capital accumulation, and
- Inflation converges to its steady state monotonically with no temporal fluctuations whatsoever.

The first of these predictions is based on the superneutrality of money in representative-agent economies and on the Mundell-Tobin effect in lifecycle settings; the second one reflects the arbitrage conditions that closely tie asset yields in conventional monetary economies. Neither seems to agree with a wealth of postwar data. Nor does the class of monetary growth models described permit an examination of how inflation affects the operation of the financial system.

We have examined how adverse selection in the credit markets affects perfect foresight equilibria in an otherwise standard, one-sector overlapping generations model of monetary growth in a closed economy. Newly issued money pays for lump-sum transfers to owners of capital. When the credit market suffers from adverse selection, equilibrium loan contracts separate the different types of potential borrowers by imposing incentive constraints. These constraints are meant to induce borrowers to reveal their true characteristics and function by restricting credit to purchasers of physical capital. When incentive constraints start to bind, producers are unable to obtain all the credit they desire at the prevailing cost of capital, and capital and financial assets cease to be close substitutes whose returns are linked by simple arbitrage conditions.

The key element in this analysis is that the severity of financial market frictions depends on endogenous variables, such as the rate of inflation. In monetary economies with sufficiently high rates of inflation, more rapid inflation tightens incentive constraints and leads to increased credit rationing by raising the tax on all financial assets—including bank deposits. As a result, regimes with high inflation lead to lower asset yields and weaker investment activity. Capital accumulation is therefore negatively correlated with the rate of inflation and positively correlated with the real yields on financial assets.

Binding incentive constraints undo the arbitrage conditions that relate the yield on financial assets with the net marginal product of capital. One consequence is that the monetary steady state can lose its saddlepoint property, monetary equilibria can become indeterminate, and transitional dynamics can display damped oscillations. The reason for this dramatic change in behavior is that arbitrage equations are replaced by lifecycle incentive constraints that depend not only on asset returns but on lifetime incomes as well. Income terms endow incentive constraints with added degrees of freedom, which increase the asymptotic stability and reduce the determinacy of monetary equilibria.

Interestingly, except for those aspects of the analysis involving the rate of inflation, few of these results require the presence of money or monetary assets. In a companion piece (Azariadis and Smith, 1994b), we examine the economy of this article, but without money. Even in the absence of money, the severity of the credit market friction depends on endogenous variables, and the economy can transit between regimes where credit constraints do and do not bind purely as a result of changing depositor beliefs about the relative returns on various assets. For example, pessimism about the return on intermediated assets drives depositors away from banks, shrinks the pool of funds available for capital investment, and leads to credit rationing. This credit rationing, in turn, leads to low returns on intermediary assets, thereby validating the original beliefs of depositors.

To summarize, the endogeneity of the financial market friction can lead to the indeterminacy of equilibrium and to enhanced economic volatility in exactly the manner suggested by Keynes (1936), Friedman (1960), and many others. This can occur in the presence or absence of money. The resulting fluctuations are associated with the transfer of funds into and out of intermediated assets. The monetary history of the United States (Friedman and Schwartz, 1963) suggests that such transfers are, in fact, an important component of observed economic variability.

Appendix: Pooling Equilibria

We wish to state conditions under which the separating loan contracts derived in the text—denoted (r_{t+1}, b_t^*)—are genuine Nash equilibrium loan contracts. To do so, it is sufficient to derive conditions implying that—in the presence of the contract (r_{t+1}, b_t^*)—no intermediary has an incentive to offer an alternative contract $(\tilde{R}_{t+1}, \tilde{b}_t)$. We show that there cannot be such an incentive if $f'(k_{t+1}) = r_{t+1}$ and also that there is no such incentive if $f'(k_{t+1}) > r_{t+1}$ and λ is sufficiently large. Having demonstrated this, we go on to show that any nontrivial equilibrium ($k_t > 0\ \forall t$) must be associated with complete self-selection ($\mu_t = 0\ \forall t$) in the credit market.

First, suppose that all active intermediaries are announcing the separating contracts (r_{t+1}, b_t^*) at t. We now ask whether any (potential) intermediary has an incentive to offer an alternative contract $(\tilde{R}_{t+1}, \tilde{b}_t) \neq (r_{t+1}, b_t^*)$.

Clearly, the incentive does not exist if $\tilde{R}_{t+1} = r_{t+1}$. Then suppose $\tilde{R}_{t+1} > r_{t+1}$. If $f'(k_{t+1}) = \psi(w_{t+1}) = r_{t+1}$, such a contract will not be accepted by any young type 2 agents and hence will not be offered. Therefore, (r_{t+1}, b_t^*) is a Nash equilibrium contract if $f'(k_{t+1}) = r_{t+1}$.

Suppose now that $f'(k_{t+1}) = \psi(w_{t+1}) > r_{t+1}$. If $\tilde{R}_{t+1} \in (r_{t+1}, r_{t+1}/(1-\lambda))$, then $\mu_t \in (0, 1)$. In this case (13) must hold, and so will $\tilde{b}_t \leq b_t^*$. Then type 2 agents taking the contract $(\tilde{R}_{t+1}, \tilde{b}_t)$ would pay a higher interest rate and receive no more credit than agents taking the contract (r_{t+1}, b_t^*). Thus no type 2 agents would accept the contract $(\tilde{R}_{t+1}, \tilde{b}_t)$, and there is no incentive to offer it.

Any contract offer that attracts type 2 agents, then, is a pooling contract with $\mu_t = 1$ and $\tilde{R}_{t+1} \geq r_{t+1}/(1-\lambda)$. Since $\mu_t = 1$, the choice of $\tilde{b}_t$ is no longer constrained by (13). Thus there is *no* attractive (complete) pooling contract for young type 2 agents if

$$f'(k_{t+1}) \leq r_{t+1}/(1-\lambda), \tag{A1}$$

that is, if λ is large enough. When (A1) fails, there does exist a pooling contract that attracts all young agents.

To summarize, a separating equilibrium exists in the credit market if

$$r_{t+1} \leq f'(k_{t+1}) \leq r_{t+1}/(1-\lambda) \tag{A2}$$

It remains to demonstrate that pooling occurs only in a trivial equilibrium. This can be shown in two steps. First, suppose there is an equilibrium at t with the contract $(\tilde{R}_{t+1}, \tilde{b}_t)$ being offered, and with $\tilde{R}_{t+1} \in (r_{t+1}, r_{t+1}/(1-\lambda))$, so that $\mu_t \in (0, 1)$. Then an intermediary announcing a contract $(\hat{R}_{t+1}, \hat{b}_t)$, with $\hat{R}_{t+1} \in (r_{t+1}, \tilde{R}_{t+1})$ and $\hat{b}_t$ less than, but arbitrarily close to, $\tilde{b}_t$ will attract all young type 2 agents. The same intermediary will attract no type 1 agents, since those agents care only about the loan quantity dimension of the contract. Thus the contract $(\tilde{R}_{t+1}, \tilde{b}_t)$ cannot constitute a Nash equilibrium contract, resulting in a contradiction.

Suppose, then, that $\mu_t = 1$ at t. All type 1 agents borrow and abscond. Hence, no saving is supplied to the market, and $m_t = b_t = k_{t+1} = 0$. It follows that $k_{t+s} = 0\ \forall s \geq 1$; pooling occurs only in a trivial equilibrium.

Acknowledgments

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Notes

1. Examples of the kinds of monetary growth models we have in mind include the descriptive growth models of Mundell (1965), Tobin (1965), and Shell, Sidrauski, and Stiglitz (1969) or the Diamond (1965) and Tirole (1985) variant of the overlapping generations model. All of these models have the features described in the text, along with the property that the steady-state output level and capital-labor ratio are positively related to the steady-state inflation rate. Monetary growth models like those of Sidrauski (1967) and Brock (1974, 1975) differ in that the steady-state output level and capital-labor ratio are unaffected by the inflation rate.

   Models with variable labor supply, described by Brock (1974), Danthine (1985), or Cooley and Hansen (1989), allow inflation to reduce labor supply and therefore to reduce steady-state output levels. However, in such models the capital-labor ratio, and hence productivity, are both unaffected by the rate of inflation. This is a counterfactual implication. Nor are we aware of any empirical evidence that labor supply is significantly affected by variations in the rate of inflation.

   Other frameworks that eliminate the nonnegative relationship between inflation and real activity include overlapping generations models with money-in-the-utility function in which the nature of monetary transfer payments matters (Drazen, 1981) and models with cash-in-advance constraints applied to capital investments (Stockman, 1981). Boyd and Smith (1994) and Schreft and Smith (1994) exhibit models with multiple monetary steady states, where one of the steady states exhibits a negative correlation between inflation and real activity.
2. For further evidence on the empirical relationship between inflation and measures of real output or productivity see Fischer (1991), Levine and Renelt (1992), Backus and Kehoe (1992), de Gregorio (1992), Cooper (1993), or Wynne (1993).
3. Fry (1988, chs. 1–3) surveys the financial development literature on growth and inflation. The thinning out of markets in a regime of persistent high inflation has been a constant theme in the work of Heymann and Leijonhufvud (1992).
4. It is straightforward to convert these steady states into balanced growth paths if we introduce labor-augmenting technical progress in an economy with homothetic preferences and technology, as in Uzawa (1965) or Lucas (1988).
5. Patrick (1966) discusses the importance of consumption inventories as investments in developing countries.
6. We adopt a formulation of the adverse selection problem that follows Rothschild and Stiglitz (1976). The result is an equilibrium that displays credit rationing; see Stiglitz and Weiss (1981) for any early application of this idea.
7. For substantiation of this claim see Jaffee and Kleiman (1977), Fischer (1981), or Friedman (1992).
8. This is the purpose of assuming that type 2 agents have no labor endowment when young. In particular, this assumption implies that there can never be a binding incentive constraint requiring type 2 agents not to want to mimic type 1 agents. In addition, the assumption that type 2 agents have no young period labor endowment frees us from having to worry about issues related to the internal financing of capital investments.
9. The essential feature of any model of credit rationing based on moral hazard or adverse selection is that different agents have different probabilities of loan repayment and hence have differing attitudes toward the magnitude of the interest rate charged on a loan. Ours is the simplest version of such a scenario: type 1 agents repay loans with probability zero, while type 2 agents repay with probability one. It is straightforward to modify the analysis to allow each type to repay with a probability strictly between zero and one, but this adds complication without introducing any substantive issues.
10. Having risk-neutral agents removes any potential gains from the use of lotteries in allocating credit.

11. As we will demonstrate, this is actually a fairly innocuous method of injecting money into the economy. How money is injected into this economy is not at all central to the results we obtain. However, it should be apparent that by using the money created to subsidize capital accumulation, we are making the strongest possible case for money growth to have a *positive* effect on the capital stock.
12. This assumption ensures that our economy admits a unique positive nonmonetary steady state (see Azariadis, 1993, ch. 13.2). It is satisfied by all CES production functions with elasticity of substitution no less than one.
13. If $\mu_t = 1$, no young agents work, and there is no saving supplied in the marketplace at t. Hence $k_{t+1} = 0$, and it follows that $k_{t+s} = 0$, $\forall s \geq 1$. Thus, if $\mu_t = 1$ at any date, the economy jumps to the trivial steady-state equilibrium with $k = 0$.
14. The possibility of active storage is considered by Azariadis and Smith (1994a).
15. That there is only one steady-state equilibrium with $k > 0 = m$ follows from assumption (a.1).
16. To understand why (23c) implies that the incentive constraint is satisfied in the monetary steady state, recall that $k/w(k)$ is an increasing function of k by assumption (a.1).
17. We note also, in passing, that very large values of σ will lead to nonexistence of equilibrium: money balances in the steady state will be negative if $k/w(k) > \lambda$.
18. In particular, since

$$\lim_{k\to\infty} k/w(k) = \infty,$$

(a.4) implies that (26a) has a solution $\hat{k}$. This solution is unique, by (a.1). Evidently (a.3) then implies that the value $\hat{m}$ given by (26b) is positive.

19. If $\sigma > 1/x$, then storage dominates money in rate of return at any steady-state equilibrium and drives currency out of private asset portfolios.
20. This property holds for the overlapping generations model (in the Samuelson case) as well as for the optimum growth model with money (see Brock, 1975, and Azariadis, 1993, pp. 309–409).
21. See, for instance, Azariadis (1993, ch. 6.4).
22. If the production function has the more general CES form

$$f(k) = [ak^{-\rho} + (1-a)]^{-1/\rho}$$

with $a \in (0, 1)$ and $\rho \geq -1$, then

$$D = \hat{k}w'(\hat{k})/w(\hat{k}) = a(1+\rho)/[a + (1-a)\hat{k}^{\rho}].$$

This is a decreasing function of $\hat{k}$ (and hence an increasing function of σ) if $\rho > 0$, while it is an increasing function of $\hat{k}$ if $\rho < 0$.

23. But, of course, ones that exceed σ_c.

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