A COMPARATIVE STUDY OF SOME MATHEMATICAL MODELS OF THE MEAN WIND STRUCTURE AND AERODYNAMIC DRAG OF PLANT CANOPIES

WILLIAM MASSMAN*

Laboratory for Atmospheres, NASA/Goddard Space Flight Center, Greenbelt, MD 20771, U.S.A.

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Abstract. A semi-analytical method for describing the mean wind profile and shear stress within plant canopies and for estimating the roughness length and the displacement height is presented. This method incorporates density and vertical structure of the canopy and includes simple parameterizations of the roughness sublayer and shelter factor. Some of the wind profiles examined are consistent with first-order closure techniques while others are consistent with second-order closure techniques. Some profiles show a shearless region near the base of the canopy; however, none displays a secondary maximum there. Comparing several different analytical expressions for the canopy wind profile against observations suggests that one particular type of profile (an Airy function which is associated with the triangular foliage surface area density distribution) is superior to the others. Because of the numerical simplicity of the methods outlined, it is suggested that they may be profitably used in large-scale models of plant-atmosphere exchanges.

1. Introduction

One-dimensional turbulent diffusion methods are presently being employed in both satellite-based observations of land surface processes (e.g., Taconet *et al.*, 1986) and global atmospheric circulation models (e.g., Sellers *et al.*, 1986) for describing the exchange of momentum, heat, and moisture between the atmosphere and vegetated surfaces. Important components of these large-scale models are the aerodynamic resistance terms which are determined from the mean wind speed above the canopy, the zero-plane displacement height, and the roughness length. In general, these parameters are functions of foliage structure and density, modulated by the mean wind speed and shear stress profiles within the canopies. Although the deficiencies of these models are well known (e.g., Shaw, 1977 and Finnigan, 1985), they do have the advantage of being computationally much simpler than many of the more recent second-order closure models, e.g. (Yamada, 1982; Meyers and Paw U., 1986) or some of the more realistic first-order closure models, e.g. (Li *et al.*, 1985). The purposes of this study are (1) to examine and compare against data analytical expressions for the within-canopy profiles of mean wind speed and shear stress as derived from both first- and second-order closure methods and (2) to estimate roughness lengths and displacement heights in a unified manner consistent with these profiles. The results may help in parameterizing bulk formulation of aerodynamic resistances for use within large-scale plant-atmosphere exchange models.

* Present address: USDA/US Forest Service, Rocky Mountain Forest and Range Experiment Station, 240 W. Prospect, Fort Collins, Colorado 80526, U.S.A.

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2. First-Order Closure

The method is adapted from Seller *et al.* (1986). Momentum transfer within a canopy for the simpler first-order closure methods is described using a turbulent diffusivity, K, and a drag coefficient, C_d. Following the convention of Seginer (1974), K and C_d are defined as:

$$\tau = \rho K \frac{\mathrm{d}u}{\mathrm{d}z}, \tag{1}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}z} = \rho C_d a(z) u^2, \tag{2}$$

where τ is the shear stress within the canopy, ρ is the density of air, u is the mean horizontal wind speed, z is the height above the ground surface, and $a(z)$ is the foliage distribution or foliage area density (the one-sided leaf area per unit volume of the canopy) here considered as a function of height. In this study, C_d is assumed to be constant throughout the depth of the canopy following den Hartog and Shaw (1975) and Raupach and Thom (1981). This assumption will be discussed in the closing section.

Assuming that profiles of horizontal wind speed and eddy diffusivity are similar within the plant canopy, Cowan (1968) showed that (1) and (2) could be solved for a constant foliage distribution to yield the following profile for the mean wind: $u/u_h = [(\sinh \beta\xi)/\sinh \beta]^{1/2}$. Here u_h is the mean horizontal wind speed at the top of the canopy, $\xi = z/h$ with h being the height of the canopy, and β is the profile extinction coefficient. The exact expression for β arises from decoupling (1) and (2) and normalizing the resulting equation for the canopy mean horizontal wind speed. Therefore, Equation (2) can be written:

$$\frac{\mathrm{d}^2\chi}{\mathrm{d}\xi^2} = \beta^2 f(\xi)\chi, \tag{3}$$

where $\chi = u^2/u_h^2$, $f(\xi)$ is $a(z)$ normalized by the maximum value of the foliage area density and the extinction coefficient is given as:

$$\beta = \left(\frac{2C_d \mathrm{LAI}}{\sigma\mu}\right)^{1/2}, \tag{4}$$

where $\mu = \int_0^1 f(\xi)\,\mathrm{d}\xi$ and σ expresses Cowan's (1968) similarity condition between the mean wind speed profile and turbulent diffusivity; i.e., $\sigma = K/hu = K_h/hu_h$ with $K_h = K(h)$. Here σ is taken as an unknown; like β it will be computed as a function of C_dLAI and foliage distribution. LAI denotes the leaf area index.

In addition to Cowan's (1968) solution for the canopy mean wind speed profile, another solution is given by $u/u_h = e^{-\beta(1-\xi)/2}$ which was first proposed by Inoue (1963) and Cionco (1965) and results from a slightly different lower boundary condition on u.

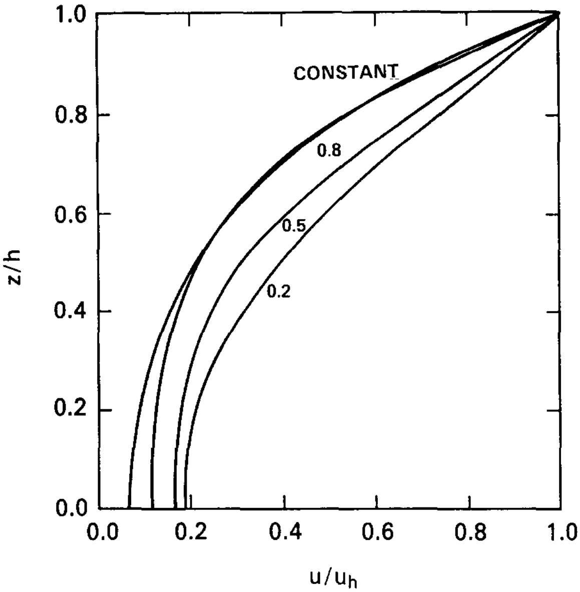

Fig. 1. Normalized hyperbolic cosine-like canopy wind speed profiles for C_dLAI = 0.6 for constant and three triangular foliage distribution functions. The uppermost curve is for the constant foliage distribution; below it are the triangular distributions with the ratio of the height of the maximum foliage density to the height of the canopy equal to 0.8, 0.5, and 0.2, respectively.

However, since these two forms are not consistent with the frequency observed zero wind gradient within the lower region of the canopy (e.g., Shaw, 1977), a more appropriate profile is $u/u_h = [(\cosh \beta\varepsilon)/\cosh \beta]^{1/2}$ which results from imposing a lower boundary condition of zero shear at $z = 0$ on u (all other assumptions remaining the same). In addition to these wind profiles, associated with a constant foliage distribution, there are direct analogs to each for the case of a triangular foliage distribution. These profiles are related to Airy functions (e.g., Abramowitz and Stegun, 1964) with a slightly different Airy's equation being valid for each region above and below the point of maximum foliage density. Therefore, in order to compute a complete profile throughout the depth of the canopy, it is necessary to match these two solutions and their first derivatives at the point of maximum foliage density. Figure 1 shows an example of the cosh wind profile associated with constant foliage distribution and three examples of the Airy-cosh wind profiles associated with the triangular distribution with heights of the maximum foliage density at 0.8, 0.5, and 0.2 h, respectively. All profiles shown in this figure assume that C_dLAI = 0.6.

The corresponding within-canopy shear stress profile can be found from (1) and (2) for each of the wind profiles and foliage distributions discussed above given appropriate boundary conditions. In this work the lower boundary condition on τ is chosen similar to that of Wilson and Shaw (1977) and Shaw and Pereira (1982). The upper boundary condition on τ, like the extinction coefficient, β, is a model unknown which will be determined as a function of C_dLAI by matching τ above the canopy to τ within the

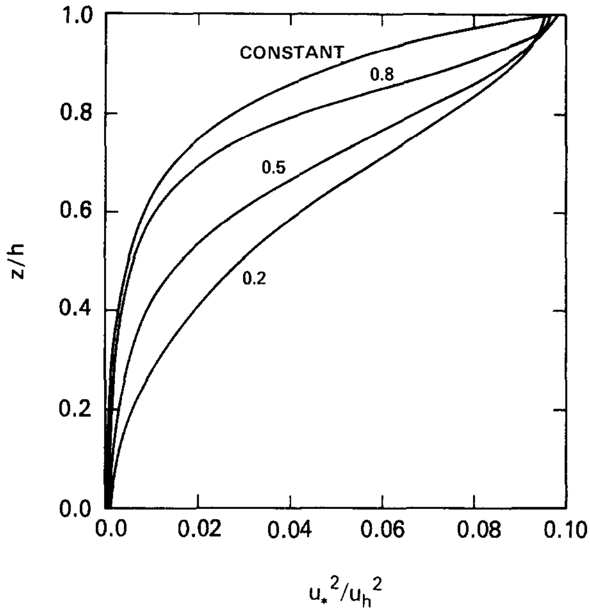

Fig. 2. Canopy shear stress profiles associated with hyperbolic cosine-like wind profiles corresponding to those shown in figure 1.

canopy. Figure 2 shows the shear stress profiles corresponding to the wind speed profiles given in Figure 1.

The model equations are now closed by assuming: (a) that the displacement height corresponds to the effective level of mean drag upon the canopy elements (Thom, 1971) and (b) that the turbulent diffusivity at the top of the canopy, K_h, is greater than it would be if the inertial sublayer were joined directly at h to the flow within the canopy (Raupach and Thom, 1981). Therefore, K_h is given as,

$$K_h = \alpha k u_*(h - d), \tag{5}$$

where α is a constant between about 1.0 and 2.0 which accounts for the presence of the roughness sublayer (Raupach and Thom, 1981); in the following, the roughness sublayer disappears entirely if $\alpha = 1.0$ and it becomes progressively deeper as α increases (see Appendix). The friction velocity, $u_* = \sqrt{\tau/\rho}$, is assumed to be constant above the canopy, k is the von Kármán constant (taken to be 0.41), and d is the displacement height. A value of $\alpha = 1.5$ used throughout the discussion of the first-order closure model was estimated from observed wind profiles within several different canopies.

Therefore, given the lower boundary conditions on wind speed and shear stress profiles for a specified foliage distribution and a value for α, Equations (2) through (5) are solved by iteration to determine, β, $(u_*^2/u_h^2)_{z=h}$, d/h, and σ as functions of C_dLAI. Figure 3 shows the extinction coefficient, β, for the cosh wind profile as a function of C_dLAI; to a very close approximation $\beta = 2C_d\mathrm{LAI}/C_L$, where $C_L = \alpha^2k^2/2$. Both Cowan (1968) and Pereira and Shaw (1980) show a similar monotonic increase in β with

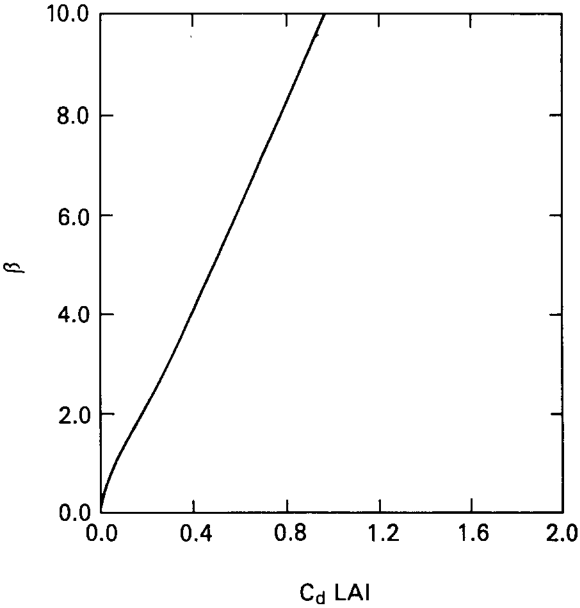

Fig. 3. Extinction coefficient, β, associated with the hyperbolic cosine wind profile for constant foliage distribution as a function of C_dLAI.

C_dLAI. The extinction coefficients for the Airy-cosh wind profiles associated with the triangular foliage distribution are similar to Figure 3 and hence are not shown. Figure 4 shows the stand drag coefficient $C_f = 2(u_*^2/u_h^2)_{z=h}$ for the hyperbolic-cosine-like wind profiles. The uppermost curve is for the constant foliage case and the other three curves are associated with the triangular distribution. All solutions reach a plateau at a value very nearly equal to C_L.

Figure 5 shows the normalized displacement height, d/h, as a function of C_dLAI for the same wind speed profiles and foliage distributions as used in the previous figure.

Once C_f and d/h have been determined as functions of C_dLAI, the normalized roughness length, z_0/h, is estimated from the following relation:

$$z_0/h = \lambda_1(1 - d/h)\, e^{-k\sqrt{(C_f/2)}}, \tag{6}$$

where $\lambda_1 = 1.07$. The parameter λ_1, discussed in the Appendix, arises from the roughness sublayer; in general, it is determined by α and height dependence of the turbulent diffusivity within this region. For example, Raupach *et al.* (1980) assume that the diffusivity is constant throughout the roughness sublayer whereas Garratt (1980) suggests that it is linearly related to height. For the purposes of this work, the distinction is unimportant because either choice results in similar values for λ_1 (see Appendix).

Figure 6a shows the normalized roughness length, z_0/h, as a function of C_dLAI for the foliage distribution discussed in the previous figures. The roughness lengths presented here resemble those of Shaw and Pereira (1982) except that they do not decrease as fast with decreasing C_dLAI as do those of Shaw and Pereira (1982).

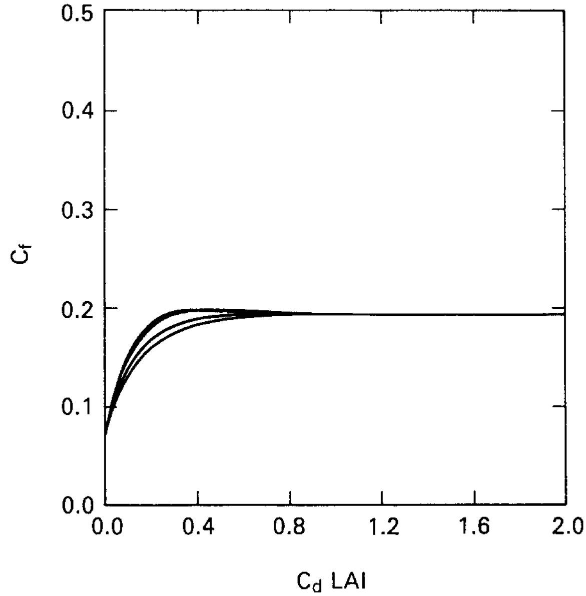

Fig. 4. Stand drag coefficient, $C_f = 2(u_{*h}/u_h)^2$ as a function of C_dLAI for the hyperbolic cosine-like wind profiles. The uppermost curve is associated with constant foliage distribution and the lower three curves are associated with the triangular foliage distribution with C_f curves decreasing with decreasing positions of the maximum foliage density.

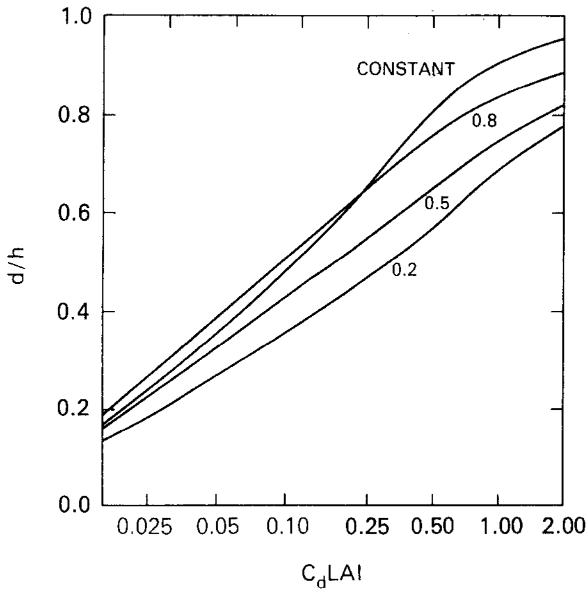

Fig. 5. Normalized displacement height, d/h, as derived from hyperbolic cosine-like wind profiles as a function of C_dLAI for the constant and three triangular foliage distributions discussed in Figure 1.

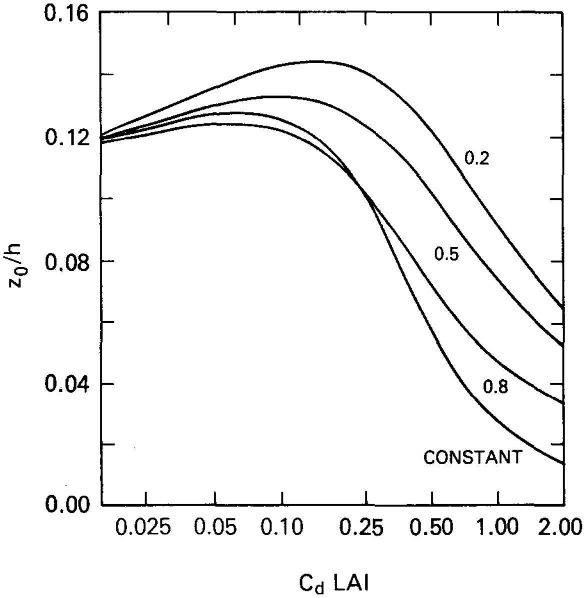

Fig. 6a. Normalized roughness height, z_0/h, as derived from the hyperbolic cosine-like wind profiles as a function of C_dLAI for the constant and the three triangular foliage distributions discussed in Figure 1.

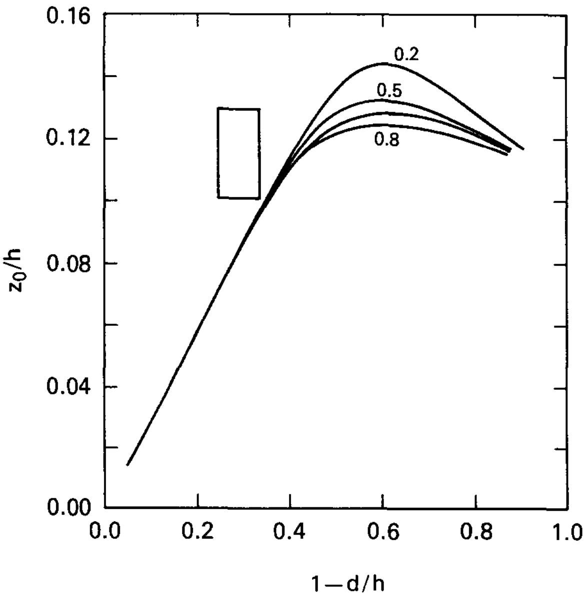

Fig. 6b. Normalized roughness height, z_0/h, vs $(1 - d/h)$ as derived from the hyperbolic cosine-like wind profiles for the constant and the three triangular foliage distributions discussed in Figure 1. The box in the upper left-hand corner is the region which typically characterizes full canopy cover: $0.10 \leq z_0/h \leq 0.13$ and $0.67 \leq (1 - d/h) \leq 0.75$.

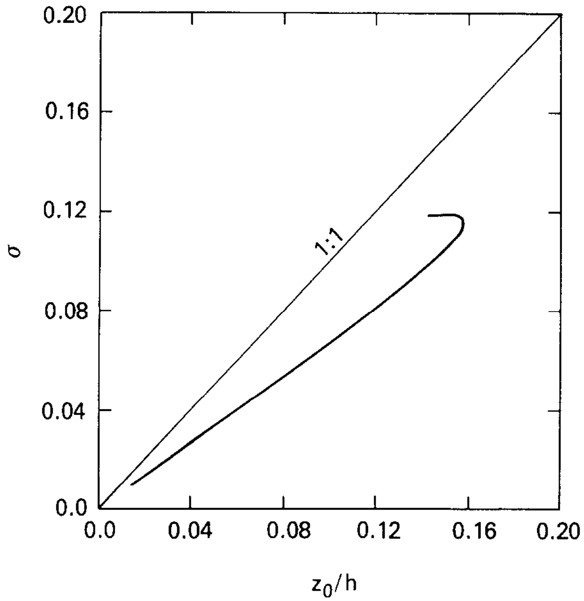

Fig. 7. $\sigma = K_h/hu_h$ vs z_0/h for the hyperbolic cosine wind profile.

However, recently Dolman (1986) found that the roughness lengths associated with a foliated and non-foliated oak forest were nearly identical, which qualitatively agrees with Figure 6a. Figure 6b shows z_0/h vs $(1 - d/h)$. The box in the upper left-hand corner gives the range of expected values for a full canopy, i.e., $0.10 \leq z_0/h \leq 0.13$ and $0.67 < d/h \leq 0.75$. The slope of the line where all curves merge is about 0.28, which is similar to Shaw and Pereira's (1982) result. However, as α increases, the slope also increases as do the maximum values of the peaks. Hence another choice of α or λ_1 would cause several of the solutions to pass directly through the box and produce a greater slope in the region where $z_0/h = \lambda(1 - d/h)$ is most appropriate.

Results associated with the hyperbolic sine-like wind profiles are not shown because they do not always give reasonable results; for the exponential-like wind profiles, the results are quite similar to those presented above. Although the results presented here are computed for small values of C_dLAI, more research is necessary on sparse canopies in order to characterize the lower limit of C_dLAI at which this model fails due to the inappropriateness of the lower boundary conditions.

Finally, Figure 7 shows σ vs z_0/h for the hyperbolic cosine wind profile and is typical of all wind profiles: except for the nonlinearities associated with small values of C_dLAI, which vary somewhat with the various profiles, σ and z_0/h are linearly related.

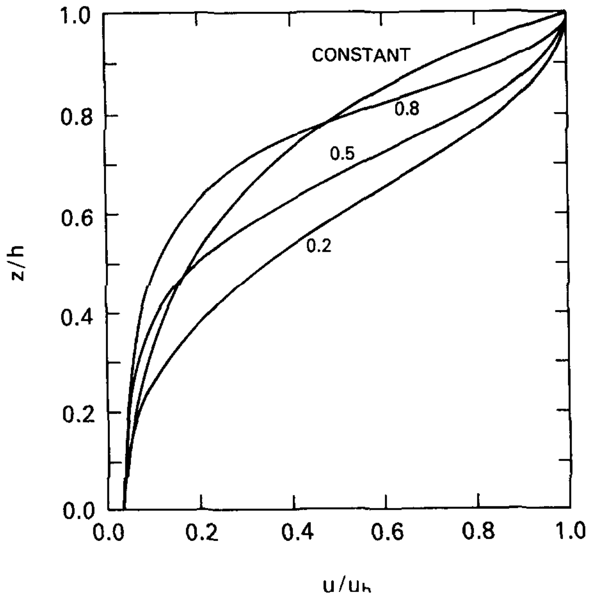

Fig. 8. Same as 1 except for the Albini model.

3. A Quasi-Second-Order Closure Model

In this section the model of Albini (1981) is extended in two important ways in order to compare it to the first-order closure models outlined earlier. Both extensions involve simple methods of parameterizing the influence of the shear stress at the ground for small values of C_dLAI. The original Albini model assumes that the ground shear stress is zero, which is a good approximation for full canopies, i.e., large values of C_dLAI. For sparse canopies, however, the ground shear stress becomes increasingly important and hence its influence must be included; but a generalization of the Albini model to a non-zero shear stress at the ground is not possible if the simplicity of the original model is to be preserved. Therefore, the extensions used in this work are heuristic; nevertheless, they do give surprisingly good results; because the ground shear stress is assumed small in all cases, these extensions do not introduce any significant inconsistencies into the equations. The first extension involves defining an appropriate stand drag coefficient, C_f, and the second involves including the effect of a non-zero shear stress at the ground on estimates of the displacement height.

The canopy wind profile for the Albini model is a generalization of the exponential wind profile discussed earlier and is given as follows (again assuming C_d is a constant): $u/u_h = e^{-\beta' C_d \mathrm{LAI}(1-\zeta)}$, where the vertical coordinate ζ is the cumulative leaf area normalized by the total LAI and β' is determined from the parameterization of the stand drag coefficient. By requiring that the canopy shear stress of the Albini model when

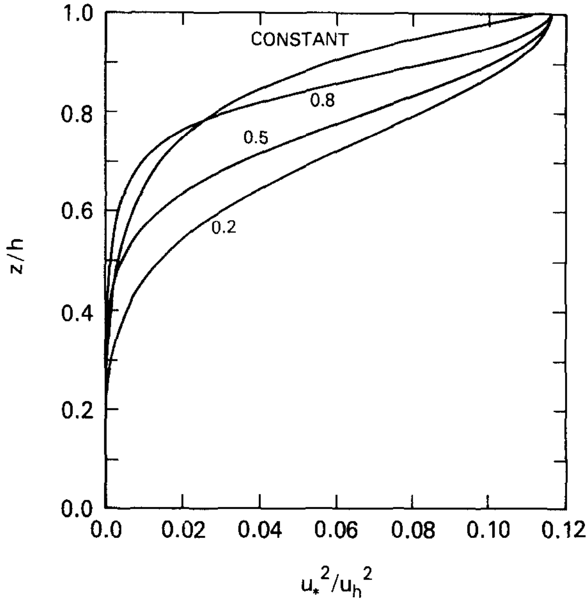

Fig. 9. Same as 2 except for the Albini model.

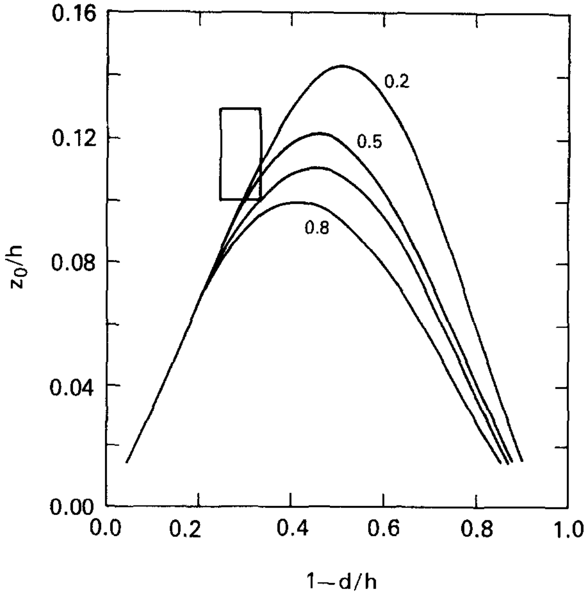

Fig. 10. Same as 6b except for the Albini model.

evaluated at the top of the canopy have a form similar to that shown in Figure 4 for the stand drag coefficient as a function of C_dLAI, it is straightforward to show that $\beta' = 4/(3C_L)$. With this new extinction coefficient, it was found that a better choice for α was 1.67 rather than 1.50 used earlier.

Figures 8 and 9 show the wind and shear stress profiles for this modified Albini model for C_dLAI = 0.6 and for the constant foliage distribution as well as for the three triangular distributions with $\xi_{max} = 0.8$, 0.5, and 0.2. The similarity with Pereira and Shaw (1980) for both the wind and shear profiles is evident. The tendency to produce a wind profile with no shear within the lower portions of the canopy is also clearly shown. Finally, for the non-constant foliage distributions, the Albini model qualitatively reproduces the observations of Shaw *et al.* (1974) and Wilson *et al.* (1982) in that near the top of the canopy the wind profile is characterized by $d^2u/dz^2 < 0$. The first-order closure models do not display this feature.

The displacement height for the Albini model is calculated in the same manner as in the previous section, except a nonzero stress at the ground is used directly rather than assuming it to be zero in accordance with the original Albini model. Again the ground-level stress was chosen similar to Wilson and Shaw (1977) and Shaw and Periera (1982). The roughness length for the Albini model used Equation (6) above.

Figure 10 shows z_0/h vs $(1 - d/h)$ for the Albini model. The influence of a slightly different choice of α is most notable when compared to the corresponding figure in the previous section. As α increases, the estimates of d/h decrease slightly while those of z_0/h increase slightly; the curves of z_0/h vs $(1 - d/h)$ in Figure 10 pass through the expected ranges of z_0/h and d/h. Unlike the cosh-like solutions, this extended Albini model shows that the roughness length decreases quite rapidly with decreasing C_dLAI.

The Albini wind profile given above, in addition to being consistent with second-order closure methods, can also be derived within the context of the first-order closure methods (Stewart and Lemon, 1969), recasting (1) and (2) appropriately. However, Legg and Long (1975) found that this approach was not consistent with observations. It is likewise possible to assume the Albini wind profile and to formulate the method outlined in a previous section consistent with it, but there is then a very serious conceptual problem. If the wind speed is characterized by the functional dependence suggested by the Albini model, then when the turbulent diffusivity is introduced (Equation (1)), it is easily shown that for the triangular foliage distribution the shear stress must vanish at $z = h$ because the foliage distribution vanishes there. As a result, a discontinuity is introduced when attempting to match the roughness sublayer flow to the canopy flow. It is possible to eliminate the discontinuity by assuming that $a(h)$ is small and non-zero similar to Perrier (1975); however, when this was done and a consistent solution was sought in the manner outlined in Section 1, significant numerical problems resulted in the computations which could only be eliminated if $a(h)$ was assumed to be large. Given these serious problems encountered when trying to formulate a wind profile similar to the Albini profile within the first-order closure methods, one concludes that wind profiles which are functions of the cumulative leaf area may be inconsistent with first-order closure methods.

The next section uses a variety of observed wind profiles to determine optimal extinction coefficients associated with the exponential, the cosh, the Albini, and the Airy cosh wind profiles and, therefore, to estimate the parameter α, as well as to explore the influence that the shelter factor (Thom, 1971) has upon these extinction coefficients.

4. Phenomenology

The wind profiles used in this section are taken from: (1) Inoue (1963) – a rice canopy; (2) Thom (1971) – a snap bean crop; (3) Legg (1975) – a wheat crop; (4) Shaw (1977) – a maize crop; (5) Wilson *et al.* (1987) – a maize crop using the 3-dimensional wind speed as determined with split film anemometers; (6) Li *et al.* (1985) – a Ponderosa pine canopy from Raupach and Thom (1981); (7) Halldin and Lindroth (1986) – a mature Scots pine canopy. Table I gives a summary of the data along with the root-mean-square

TABLE I

Canopy characteristics and mean wind profile error statistics

Researchers	LAI	C_d	C_dLAI	R.m.s. error Exponential	R.m.s. error Cosh	R.m.s. error Albini	R.m.s. error Airy cosh	Shelter factor
Inoue (rice)	–	–	0.34[a]	0.072	0.053	0.157	0.037	–
Thom (nap bean)	6.25	0.15	0.93	0.030	0.029	0.062	0.017	3.5
Legg (wheat)	7.00	0.25[b]	1.75	0.076	0.075	0.076	0.069	4.0[d]
Shaw *et al.* (maize)	3.00	0.17	0.51	0.053	0.039	0.031	0.031	–
Wilson *et al.* (maize)	2.90	0.17	0.49	0.058	0.051	0.047	0.024	–
Li *et al.* (pine)	10.0	0.30[c]	3.00	0.098	0.066	0.127	0.039	5.6[d]
Halldin and Lindroth (pine)	2.5	0.30[c]	0.75	0.060	0.047	0.101	0.031	–

[a] Estimated from Inoue's (1963) equation (7) and Table I.
[b] From Finnigan and Mulhearn (1978a).
[c] Estimated from Landsberg and Thom (1971).
[d] Estimated by linearly extrapolating with respect to LAI from Thom's value of 3.5 for LAI = 6.25.

error associated with the best fit found for each of the different types of analytical wind profiles. In the case of the Albini profile, the accumulated leaf area was estimated from the data provided by each researcher, except for Inoue (1963) and Li *et al.* (1985). For Inoue's rice canopy, the foliage distribution was assumed to be constant and C_dLAI was estimated indirectly from his Equation (7) and the data from his Table I. For the work of Li *et al.* (1985), the foliage distribution was taken from Gary (1976) for a lodgepole pine and the drag coefficient (assumed to be 0.30) was taken from Landsberg and Thom (1971) for a twig of blue spruce composed of several needles. For Legg's data, C_d was assumed to be 0.25 from observations on a wheat canopy made by Finnigan and Mulhearn (1978a). Although the optimal fits are far from perfect, it should be noted that the Airy-cosh wind profile uniformly produces the best fit and is frequently significantly better than any or all of the other simple profiles. Figure 11 shows the optimal wind profiles for the pine canopy of Li *et al.* (1985) for each of the four different types of wind profiles.

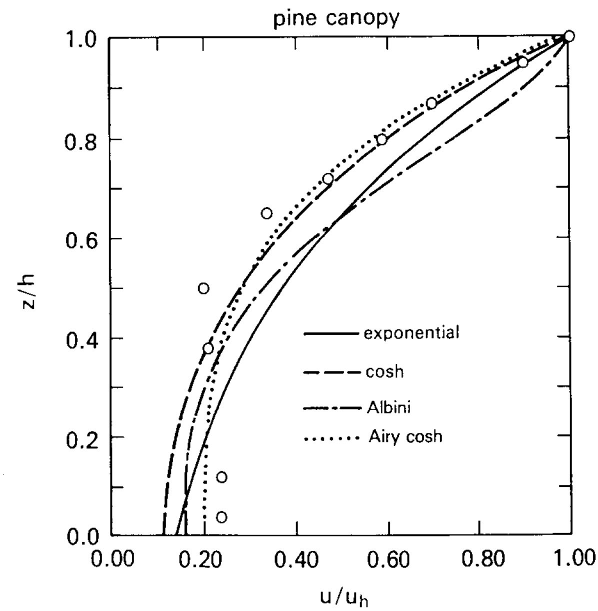

Fig. 11. Optimal (least-squares) fits of observed wind profile from a Ponderosa pine canopy. Observations are denoted by an open circle and are taken from Raupach and Thom (1981). Curves are for the exponential, cosh, and Airy cosh, and Albini profiles.

Figure 12 is a comparison between the extinction coefficients for the cosh profile and those predicted from first-order closure methods for $\alpha = 1.50$ and $\alpha = 2.00$. Those

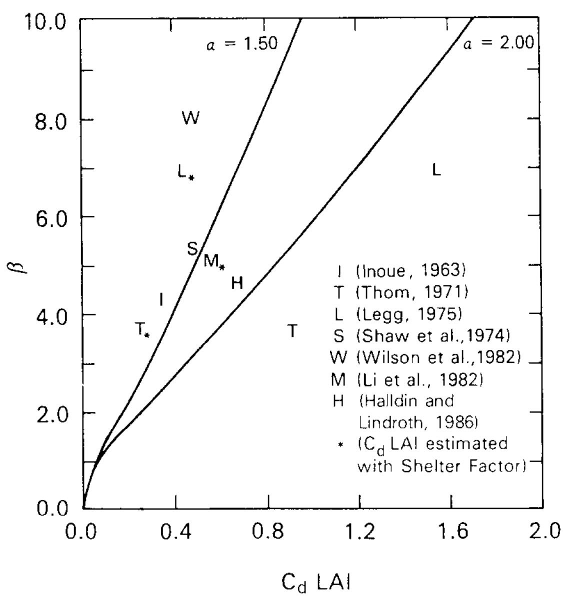

Fig. 12. Comparison of predicted (lines denoted by $\alpha = 1.50$ and $\alpha = 2.00$) and computed extinction coefficients (denoted by letters) for the hyperbolic cosine wind profile. For full explanation, see text.

alphabetic characters subscripted with an asteriks (*) include the effects of the shelter factor which tends to reduce the effective value of C_dLAI below the value computed in Table I. The shelter factor was used only for values of LAI greater than Thom's reported value of 6.25 for a mature snap-bean canopy and was estimated by linearly extrapolating with respect to LAI from his corresponding value of 3.5 for the shelter factor. The extinction coefficient for C_dLAI = 3.0 for the pine canopy of Li *et al.* is not shown on Figure 12 because it is off scale. However, the effective value for C_dLAI (which includes the shelter effect) is shown and is denoted by M_*. All other results are denoted by the first initial of the researchers. If the 'cup' wind speed of Wilson *et al.* (1982) had been used, it would almost exactly coincide with Shaw's data. It is also possible to estimate a shelter factor for canopies with lower values of LAI than Thom's snap bean crop, i.e., for the corn canopies of Shaw and Wilson *et al.* and the pine canopy of Halldin and Lindroth. Again linearly extrapolating from Thom's value suggests that the effective C_dLAI should be about 0.3 for these two corn canopies and about 0.54 for the pine canopy. However, these new values of C_dLAI do not alter the general conclusions of this section.

Although this method of comparison is admittedly crude, it does suggest that a variety of data for several different types of plant canopies are broadly consistent with $\alpha \approx 1.50$ or perhaps slightly less. Furthermore, the comparison made in Figure 12 broadly suggests that mature canopies have effective values of C_dLAI that fall into a fairly narrow range, leading, therefore, to the following hypothesis: all full canopies can be characterized by the inequality $0.25 \leq C_d\text{LAI (effective)} \leq 0.50$. Within this region, the analytical prediction for the extinction coefficient associated with the exponential wind

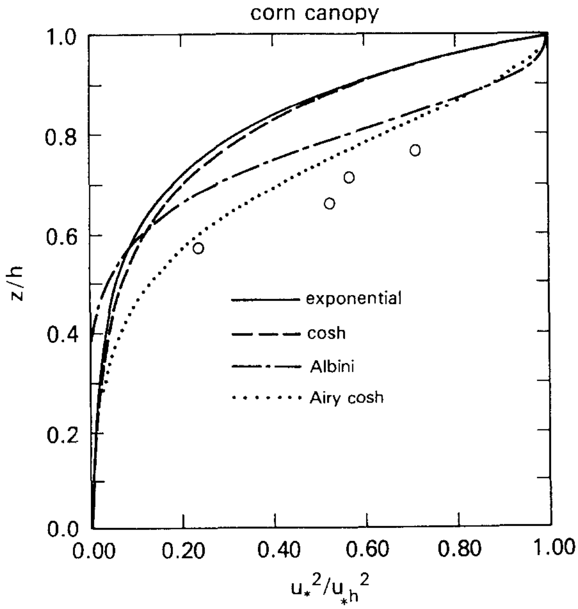

Fig. 13. Shear stress profiles associated with the exponential, cosh, Airy cosh, and Albini wind profiles as compared with the observations of Shaw (1977) taken within a maize canopy.

profile is $2.6 \leq \beta \leq 5.2$, which agrees well with Cionco's (1978) compilation of observed extinction coefficients for a variety of full canopies. A comparison similar to Figure 12 was also made for the exponential and the Albini's wind profiles. For the exponential wind profile, $\alpha \approx 1.50$ was likewise suggested; for the Albini profile, $\alpha \approx 1.67$ seemed more appropriate.

In addition to fitting the wind profiles, it is also possible to examine the shear stress profiles for three of the canopies. Using the shear stress profiles corresponding to the optimal wind profile for each of the four types of profiles considered in Table I, the predicted shear stress profiles were compared to those observed by Legg (1975), Shaw (1977) and Wilson *et al.* (1982). Figure 13 gives an example of the results for Shaw's data and Table II summarizes the three cases. Again the Airy-cosh profile is significantly better than any of the other simple expressions.

TABLE II

Canopy mean shear stress error statistics

Researchers	R.m.s. error Exponential	R.m.s. error Cosh	R.m.s. error Albini	R.m.s. error Airy cosh
Legg (wheat)	0.105	0.138	0.062	0.059
Shaw *et al.* (maize)	0.356	0.333	0.242	0.130
Wilson *et al.* (maize)	0.199	0.195	0.135	0.089

5. Summary and Conclusions

First-order closure techniques have been used to derive simple analytical expressions for the mean wind speed and shear stress profiles within plant canopies for both the constant and triangular foliage distributions. Using some of the results from the first-order methods, another analytical model for these profiles based upon second-order closure techniques is outlined. Both closure methods produce wind speed profiles which display shearless regions within the lower 25% of the canopy. By adapting the method of Sellers *et al.* (1986) for use with these profiles, it was possible to estimate the displacement height and roughness lengths as functions of C_dLAI and foliage structure. All the profiles examined in detail in this work produce the expected unimodal structure for the roughness length and the monotonically increasing behavior of the displacement height. In general, the second-order closure techniques produced results in better agreement with those of Shaw and Pereira (1982) and Pereira and Shaw (1980) than did the first-order methods. However, the first-order methods are in better qualitative agreement with Dolman's (1986) observations which suggest that the roughness length may be relatively insensitive to C_dLAI – at least for a certain region of C_dLAI values.

Comparing the different types of profiles shows that one particular wind profile – the Airy hyperbolic cosine – and its associated shear stress profile fit the data much better than any of the other wind profiles. Furthermore, comparisons between optimally fitted values of the extinction coefficients for different wind profiles within a variety of canopies and those predicted by the methods outlined in this work suggest that the shelter factor acts in a way to restrict the range of effective values for C_dLAI which characterize full canopies. Tentatively it is suggested that $0.25 \leq C_d\text{LAI} \leq 0.50$ may be sufficient to characterize full canopies. Before this can be definitely established, however, it is necessary to have a better quantitative understanding of the shelter factor. In particular, the shelter factor should also be modeled as a function of the foliage distribution of the canopy; thus the shelter factor will modulate the effects that the vertical distribution of the foliage has upon the canopy wind profile. In fact, assuming that the shelter factor is directly related to $a(z)$ will tend to produce a foliage distribution which is aerodynamically more uniform than the observed $a(z)$.

Fundamental to all results presented here is the assumption that C_d is independent of wind speed within the canopy. Without this assumption, analytical expressions for the wind speed and shear stress profiles may no longer be possible. On the other hand, because the methods outlined do not directly incorporate canopy flexibility and the related phenomenon of fluttering, streaming, and honami (Monteith, 1963; Finnegan and Mulhearn, 1978b; Cionco, 1978; Raupach and Thom, 1981), model estimates for the roughness lengths and displacement heights are not influenced by the wind speed. However, if C_d varies abruptly with the wind speed from one flow regime to another, remaining relatively constant throughout a given regime – similar to what Grant (1983, 1985) has suggested for spruce, then the methods outlined here can still be used. Thus, for a full canopy, low wind speeds would suggest that the effective C_dLAI would be lower than at high wind speeds. Furthermore, low wind speeds would be associated with lower extinction coefficients for the wind profile within the canopy, with greater values of the roughness length and with smaller values of the displacement height than those associated with higher wind speeds. Here the shelter factor is assumed to be independent of wind speed in accordance with Thom (1971) and Grant (1984).

In summary, within certain limits, the methods outlined seems broadly consistent internally, as well as consistent with previous modeling and observational studies, especially for full canopies. Because of their computational simplicity, these methods can profitably be used for estimating resistances to transfer for use within large-scale plant-atmosphere exchange models.

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Appendix

The purpose of this appendix is to derive Equation (6) and explore how λ_1 varies with different parameterizations of the roughness sublayer.

Raupach *et al.* (1980) assume that the diffusivity within the roughness sublayer is constant with the height; they show that the mean wind speed at the top of the roughness sublayer where it joins the inertial sublayer is as follows:

$$u_h = u(z_m) - \frac{u_*}{k}\, g(x_h)\,, \tag{A1}$$

where $x_h = (h - d)/(z_m - d)$ and $g(x_h) = (1 - x_h)$ and z_m is the top of the roughness sublayer. Substituting the identity $u(z_m) = u_*/k \ln(z_m - d)/z_0$ into (A1) and solving for z_0/h yields Equation (6) of the main text with $\lambda_1 = e^{-g(x_h)}/x_h$. Furthermore, from the definition of K_h, it follows that $x_h = 1/\alpha$. Therefore, for a given value of α, both x_h and λ_1 can easily be evaluated as is shown in Table IA. In this and the following table, $\lambda \equiv \lambda_1\, e^{-k/\sqrt{C_L/2}}$.

TABLE IA

Values of λ using Raupach *et al.* assumption

α	x_h	λ_1	λ
1.20	0.83	1.02	0.19
1.30	0.77	1.03	0.22
1.40	0.71	1.05	0.25
1.50	0.67	1.07	0.29
1.60	0.63	1.10	0.32
1.70	0.59	1.13	0.35
1.80	0.56	1.15	0.38
1.90	0.53	1.18	0.41
2.00	0.50	1.21	0.45

On the other hand, Garratt (1980) assumes that the diffusivity within the roughness sublayer is linearly related to height and for a neutrally stable atmosphere uses an influence function, ϕ_m, to account for the presence of the sublayer. This assumption results in the following expression for $g(x_h)$:

$$g(x_h) = a \ln x_h - a a_1 (1 - x_h) - \frac{a a_1^2}{4}(1 - x_h^2) - \frac{a a_1^3}{18}(1 - x_h^3) - \ldots$$

where $a_1 = 0.7$ and $a = e^{-a_1} \cong 0.5$. For all practical purposes, the series can be truncated after the $(1 - x_h^3)$ term. Likewise, the relationship between x_h and α is different than for the case of Raupach *et al.* Specifically $x_h = 1 - \ln \alpha / a_1$ for Garratt's model. Therefore, given a_1 and α, it is again possible to compute x_h and λ_1 for his model; the results are shown in Table IIA.

TABLE IIA

Values of λ using Garratt assumption

α	x_h	λ_1	λ
1.20	0.74	1.03	0.19
1.30	0.63	1.06	0.23
1.40	0.52	1.12	0.27
1.50	0.42	1.19	0.31
1.60	0.33	1.30	0.37
1.70	0.24	1.47	0.45
1.80	0.16	1.75	0.58
1.90	0.083	2.37	0.83
2.00	0.0098	6.77	2.49

Comparing these two tables shows that within the range for α suggested by the results of Section 4 (i.e., $1.40 \leq \alpha \leq 1.70$), the two formulations are quite similar with x_h smaller for Garratt's aproach (i.e., the depth of the roughness sublayer is greater) and that λ_1 and λ are somewhat larger than given by Raupach's *et al.* method. The difficulty with Garratt's approach at higher values of α results from the choice of $a_1 = 0.7$. Had a_1 been chosen as 0.8, the two tables would be closer for all values of α. It is also worth noting that for any given $u(z_m)$, the corresponding u_h of Raupach's *et al.* approach will always exceed that associated with Garratt's method by about 20%.

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