

THE SIGNIFICANCE OF INDEPENDENT DECISIONS IN  
UNCERTAIN DICHOTOMOUS CHOICE SITUATIONS

ABSTRACT. The main concern of this paper is the selection of optimal decision rules for groups of individuals with identical preferences but diverse and dependent decisional skills. The main result establishes that within the uncertain dichotomous choice situation independent voting is always weakly superior to any pattern of interdependence among individual decisions. For the special class of total interdependence patterns the optimal rule is explicitly identified.

1. INTRODUCTION

A number of studies have examined various aspects of the expert resolution problem employing alternative modified versions of the uncertain dichotomous choice framework originally suggested by Condorcet [1785]. The optimal decision rule for a group of individuals with known ability parameters, sharing identical preferences, and voting independently while facing two symmetric alternatives was recently shown to be a weighted majority rule. The weights defining the rule are proportional to the logarithms of the individual odds of identifying the more desirable alternative – the alternative that better suits the decision-makers' common objective, Nitzan and Paroush (1982a), Grofman *et al.* (1983). Other studies have analyzed the optimality issue as well as related suboptimality issues relaxing some of the restrictive assumptions underlying the basic uncertain dichotomous choice model. Notably, neither the analysis of optimality, nor the analysis of related suboptimality issues<sup>1</sup> has been carried out without resorting to the assumption of independent individual decisions. This observable indispensability of the independence assumption in the existing literature is possibly due to fundamental or practical considerations. That is, either one may believe that the analysis of independent decision-making yields implications that are equally valid for the interdependent situation. Or, one may simply consider the analysis of all possible special cases of interdependence a very tedious and possibly hopelessly complex task.

In this paper we deal with interdependent decision-makers. The major purpose of our work is twofold. First, it is to present a conceptual framework for distinguishing among basic patterns of interdependence. Second, while employing the suggested dependence forms, we wish to consider the effect of interdependence on the performance of the group under independent voting.

In the following section we present the basic uncertain dichotomous choice model. Section 3 introduces the general pattern of dependence on which this paper focuses and discusses some common particular cases. We proceed in Section 4 with the main result that sheds some light on the effect of explicit recognition of interdependence on the solution to the problem of selecting an optimal decision rule for a group of independent experts. The implications of the main result and its corollaries are summarized in the concluding section.

## 2. THE MODEL<sup>2</sup>

Consider a set of individuals  $N = \{1, \dots, n\}$  confronted with a dichotomous choice – the set of alternatives consists of two elements denoted alternative  $a$  and alternative  $b$ . For each individual  $i$  in  $N$  let  $x_i$  be a decision variable that summarizes his actual choice.  $x_i = 1$  and  $x_i = -1$  are interpreted as votes cast by individual  $i$  for alternative  $a$  and alternative  $b$ , respectively. An  $n$ -tuple  $x = (x_1, \dots, x_n)$  that specifies all individual choices is called a voting profile. The set of all possible voting profiles is denoted  $X$ . A decisive decision rule  $f$  is a function from the set  $X$  to the set  $\{1, -1\}$ . We interpret  $f(x_1, \dots, x_n) = 1$  or  $-1$  as the group  $N$  having selected alternative  $a$  and alternative  $b$ , respectively. Henceforth we shall deal with neutral decision rules.<sup>3</sup> A neutral decision rule satisfies  $f(-x) = -f(x)$  for any voting profile  $x$  in  $X$ . That is, reversing all individual votes results in the reversal of the collective decision. This implies that  $f$  does not discriminate against one of the alternatives on labelling grounds. Members of  $N$  share a common system of norms and therefore, they all prefer the choice of that alternative that best suits their common objective. The selection of such an alternative is referred to as a correct choice. Let  $p_i$  be the probability that individual  $i$  judges correctly when choosing independently between the two alternatives. Given some information regarding the individual probabilities of making the correct choice, the main concern of the literature inspired by Condorcet's probabilistic

approach (1785) is the design of a collective decision-making procedure which maximizes the likelihood that the group  $N$  makes the correct choice. Nitzan and Paroush (1982a) have recently established a partial solution to this problem. Assuming that alternative  $a$  and alternative  $b$  are symmetric (in particular,  $a$  and  $b$  are equi-probable), and that individual choices are independent, the following result is proven:

The optimal neutral decisive decision rule is a weighted majority rule given by  $\hat{f}(x_1, \dots, x_n) = \text{Sign}(\sum_{i=1}^n \beta_i x_i)$  where  $\beta_i = \ln(p_i/(1-p_i))$ .

The independence assumption is clearly very restrictive. Numerous interaction processes do take place, at least to some extent, in almost any conceivable decision-making group. If the independence condition is unlikely to be satisfied in most committees, panels of experts, juries, management boards, courts and other decision-making groups, then naturally one wonders what is the effect of interdependence on the above result. In order to analyze the optimality issue for situations where the independence assumption is violated, we now turn to the preliminary task of clarifying the possible meaning of interdependent choices. We shall then proceed to study the implications of dependent individuals' decisions on the optimal decision-making rule for the group  $N$ .

### 3. INTERDEPENDENT DECISIONS

Individual decisional skills are rarely statistically independent. Numerous factors account at least for some degree of interdependence of individual competences. Among the most pervasive factors are real or potential social pressures enhancing conformity, various forms of commitments, leadership effects, similar background or similar training of individuals, exchange of relevant information, threats, persuasion and, finally, false conceptions leading individuals to try and improve their group performance by adopting promising strategies such as following superior decision-makers' views.

Some patterns of interaction among decision-makers may positively affect individual skills in a manner analogous to that of a learning process. More generally, certain interactive processes may act as investment in human capital. In such cases the interaction among decision-makers alters their vector of skills. That is, pursuant to the termination of the interaction process, the competency parameter of every individual reflects his past ability as well as

his newly acquired capability. Each individual's new decisional skill is totally unconditional upon any views or decisions of other individuals. This important form of interaction is not the subject of our discussion. In this paper we consider only interdependence patterns wherein the actual decisions of some members of  $N$  are dependent on the views or choices of some other group members, and the original decisional skills of each individual are maintained. Even in this case, however, we do not intend to cover all possible patterns of interaction. In particular, we disregard situations where decisions depend on the decision rule in use or on factors that are external to the model.

In order to formalize the notion of interdependence, let us partition the set  $N$  into two subsets,  $I$  and  $D$ ,  $I \cap D = \emptyset$ ,  $I \cup D = N$ . The set  $I$  consists of all individuals acting independently and  $D$  contains the remaining individuals. Note that  $I \neq \emptyset$ . Otherwise, a perpetual process is generated and group decision-making becomes impossible. With no loss of generality, let  $I = \{1, \dots, m\}$  where  $1 \leq m \leq n$ . In turn,  $D = \{m + 1, \dots, n\}$ .

We let individual views prior to any interaction be represented by an  $n$ -tuple  $y = (y_1, \dots, y_n)$ . Here  $y_i = 1$  or  $-1$  is interpreted as individual  $i$ 's judgment supporting alternative  $a$  and alternative  $b$ , respectively. The set of all possible views is  $Y = \{1, -1\}^n = \{y = (y_1, \dots, y_n) : y_i \in \{1, -1\}, i = 1, \dots, n\}$ . The actual vote of an independent decision-maker coincides with his views. The relationship between a dependent individual's actual vote,  $x_j$ , and his and others' views, is given by the function  $g_j$  where  $g_j : (1, -1)^{m+1} \rightarrow (1, -1)$ . That is, individual  $j$ 's actual vote depends on his view as well as on the views of the independent members  $1, \dots, m$ . Furthermore, we require that the dependence scheme between  $j$ 's actual vote and any relevant  $m + 1$  tuple of views be neutral. Specifically, if all relevant views change, so does individual  $j$ 's vote, i.e.,  $g_j(-y_j, -y_1, \dots, -y_m) = -g_j(y_j, y_1, \dots, y_m)$  for any vector of views  $y$  in  $Y$ . A voting profile  $x$  is thus obtained from the vector of views  $y$  using the dependence transformations  $g_{m+1}, \dots, g_n$ . Specifically,  $\forall j \in I, x_j = y_j, \forall j \in D, x_j = g_j(y_j, y_1, \dots, y_m)$ . Denote by  $G$  the set  $\{g_{m+1}, \dots, g_n\}$ . Thus, a dependence pattern is fully specified by  $D$  and  $G$ . Any dependence pattern is a transformation  $G$  from  $Y$  to  $X$ . The identity transformation is called the independence pattern and is denoted  $G_0$ . Under this latter pattern the set  $D$  is clearly empty.

Social pressure is often considered as the cause of an interdependence or as a deviation of  $G$  from  $G_0$ . Very often, such pressure is considered as a major

force making for conformity. It is argued, for instance, that the desire to be an acceptable group member tends to silence actual disagreement and favours consensus. Majority opinions, according to the conventional wisdom, tend to be accepted. The following dependence pattern may serve as a typical example.

Let  $D = \{n\}, I = \{1, \dots, n-1\}$  and

$$g_n(y_n, y_1, \dots, y_{n-1}) = \begin{cases} \text{Sign} \left( \sum_{i=1}^{n-1} y_i \right) & \text{if } \left| \sum_{i=1}^{n-1} y_i \right| > \alpha(n-1) \\ y_n & \text{otherwise} \end{cases}$$

where  $1/2 < \alpha < 1$ .

Here individual  $n$  is the only dependent decision-maker. He follows the  $\alpha$ -majority view when such a majority view exists. Otherwise he resorts to his own judgment; that is, he chooses according to his own view. Note that  $g_n$  is a neutral dependence function, i.e.,  $g_n(-y_n, -y_1, \dots, -y_{n-1}) = -g_n(y_n, y_1, \dots, y_{n-1})$ .

The decisional skill of any individual  $i$  is represented by the probability that  $y_i = 1$ , i.e.,  $p_i = \Pr \{y_i = 1\}$ . The probability of individual  $i$  actually making the correct choice depends, in general, on his decisional skill and on the specific dependence pattern  $G$  characterizing the interaction among decisions of members in  $N$ .

Returning to the above example, if  $p_n \leq p_j$  for all  $j \in I$ , that is, if individual  $n$  is the least competent decision-maker, then clearly his probability of voting correctly can be positively affected by the dependence pattern  $G$ . In other words, given  $G$ ,  $p_n \leq \Pr \{x_n = 1\}$ . Individual  $n$ 's dependence may be attributed to different factors within the realm of social psychology. It may also reflect  $n$ 's belief that such dependence is a reasonable strategy to enable him, and in turn, the group to improve their performance. Such a belief might be induced by the seemingly desirable positive increment  $[\Pr \{x_n = 1\} - p_n]$ . A dependence pattern that seems to be individually desirable may turn out to be spurious and therefore, a key issue, then, is whether a particular interdependence pattern has a positive or a negative effect upon the probability of obtaining collectively the correct alternative. The analysis of this issue as well as some related problems constitute the main task of the remainder of the paper.

## 4. INDEPENDENT VS. INTERDEPENDENT DECISION-MAKING

The probability of obtaining collectively the correct alternative depends on three factors: first, the particular decision rule employed by the decision-makers,  $f$ ; second, the skills of the decision-makers,  $p = (p_1, \dots, p_n)$ ; third, the interdependence among the decision-makers,  $G$ . Let us denote this probability by  $\pi(f, p, G)$ . The following result establishes in a simple but rigorous manner the intuitive tenet that interaction between decision-makers is often dysfunctional. In our model any interdependence pattern  $G$  cannot be superior to the independence pattern  $G_0$ . Formally,

**THEOREM.** In a dichotomous choice situation let  $\hat{f}$  be a decisive decision rule satisfying  $\pi(\hat{f}, p, G_0) \geq \pi(f, p, G_0)$  for any neutral  $f$  and any given skills vector  $p$ . Then, for any neutral interdependence pattern  $G$  and any decisive decision rule  $f$ ,

$$\pi(\hat{f}, p, G_0) \geq \pi(f, p, G).$$

*Proof.* Given a neutral rule  $f$  and an interdependence scheme  $G$ , define a rule  $h$ ,  $h: Y \rightarrow (1, -1)$  using the pair  $f, G$ .

$$h(y_1, \dots, y_n | f, G) \equiv f(y_1, \dots, y_m, g_{m+1}(y_{m+1}, y_1, \dots, y_m), \dots, g_n(y_n, y_1, \dots, y_m))$$

By assumption,  $f$  is neutral and  $g_j, j = m + 1, \dots, n$ , are neutral. Hence,  $h$  is neutral. Under the interdependence pattern  $G_0, X = Y$ . Clearly then,  $\pi(h, p, G_0) = \pi(f, p, G)$ . By assumption,  $\pi(\hat{f}, p, G_0) \geq \pi(f, p, G_0)$  for any neutral  $f$ . In particular,  $\pi(\hat{f}, p, G_0) \geq \pi(h, p, G_0)$  and therefore,  $\pi(\hat{f}, p, G_0) \geq \pi(f, p, G)$ .

Usually, the vector of decisional competences  $p$  and the dependence pattern  $G$  are both externally given. For any pair  $(\bar{p}, G)$  there exists an optimal rule  $f^o$ . That is, the rule  $f^o$  secures the largest probability of obtaining the correct decision given both individual abilities and the particular pattern of interdependence among the choices of the decision-makers. The above result implies that, given independent decision-making, the optimal rule cannot be inferior to the optimal rule corresponding to any pattern of interdependence, provided that individual skills are held constant. This implies that any form of interdependence has an effect on the performance of the group  $N$  similar to

the introduction of an additional constraint into the classical problem  $\text{Max}_f \pi(f, p, G_0)$ . Our theorem does not provide a constructive solution to the problem the group faces under the common situations where  $p$  and  $G$  are given. Put differently, although we do know  $\hat{f}$  we cannot generally construct  $f^\circ$ . It does suggest, however, that whenever the pattern of interaction  $G$  is a control variable, the designer of the optimal decision rule should better transform it into  $G_0$ . In other words, any dependence structure within the class of structures on which this paper focuses can only adversely affect the performance of the group. Such an effect is due to the disposition of some useful decisional resources associated with any harmful pattern of interdependence.

A second immediate implication of the above theorem suggests that the employment of the optimal decision rule  $\hat{f}$  given independent voting is superior, in fact weakly superior, to using the same rule when decisions are interrelated by a pattern  $G$ ,  $G \neq G_0$ . The following corollary summarizes the above applications.

**COROLLARY 1.** For a given vector of skills  $p$  and interdependence pattern  $G$  let  $f^\circ$  be a decisive decision rule satisfying  $\pi(f^\circ, p, G) \geq \pi(f, p, G)$  for any neutral  $f$ . Then

- (i)  $\pi(\hat{f}, p, G_0) \geq \pi(f^\circ, p, G)$ .
- (ii)  $\pi(\hat{f}, p, G_0) \geq \pi(\hat{f}, p, G)$ .

Corollary 1 bears significant consequences for situations where the dependence pattern among individual decisions is a decision variable. For such situations the more independent the group members are the better.<sup>4</sup> A possible way of adhering to such a recommendation is by augmenting the optimal decision-making rule with appropriate institutions or procedures designed to maintain the independence of the decision making process. A clear example of such a procedure is secret voting.

The traditional argument in favor of secret voting typically runs as follows: A voter can be held to account only if his vote is known; therefore, without open voting, corruption or intimidation by a few powerful decision-makers or by the pressure of public opinion cannot exist. More generally, with open voting, the collective decision-making process is more vulnerable to manipulation of various sorts and, in turn, to voters' preference misrepresentation.

The case against secret voting is not so obvious and is now of interest largely because it was forcibly put forward by J. S. Mill in his advocacy of Representative Government. His arguments, which still have some relevance to the experience of compact social groups, emphasize the possibility that secret voting may hinder a kind of responsible democratic process, that of reaching general agreement by open discussion (Mill, 1861). In our context, voters, members of professional committees, experts, managers or judges have no conflict of interest, and by assumption, decisional skills are held fixed. In other words, members of  $N$  are unanimously agreed upon ends but there is a possible conflict over means (alternative  $a$  or alternative  $b$ ). Here the traditional arguments for or against secret voting are irrelevant. The issue of open vs. secret voting remains, however, of considerable significance and given our stylized decision theory model, it can be formally approached, analyzed and resolved by resorting to straightforward efficiency considerations. Specifically, we can simply compare the effects of the two possible procedures on the objective function  $\pi$ .

Corollary 1 unequivocally resolves the dilemma, provided that one accept the tenable presumption that secret voting guarantees a greater degree of independence among individual decisions. To facilitate exposition, consider the extreme case where secret voting is associated with  $G_0$  and open voting is associated with some  $G$ ,  $G \neq G_0$ . If the group  $N$  operates optimally under open voting, then by (i) of Corollary 1, one concludes that the replacement of the open voting procedure with secret voting might be beneficial to the group. If the group  $N$  operates optimally under secret voting, then using (ii) of Corollary 1, one concludes that replacing secret voting with open voting can only be harmful to the collective interest. Within our framework secret voting thus emerges as the desirable mode of voting, given that the group indeed designs the optimal decision-making apparatus. If the decision rule the group adopts under secret voting differs from  $\hat{f}$ , then it is perfectly possible that open voting improves the collective judgment. Put differently, if for some reason that group is restricted in choosing among decision rules (a not untenable assumption when the model is enriched to include, for example, cost considerations, see Nitzan and Paroush (1980), (1983)), then the formation of dependence patterns might be advantageous, serving as a vehicle to bypass institutional, organizational, technological or economic constraints inhibiting the selection of certain rules, in particular, the rule  $\hat{f}$ . Under such



circumstances the issue of open vs. secret voting cannot be *a priori* resolved without information on  $p, G$ , and the constrained subset of feasible decision rules the group confronts.

EXAMPLE.

$$N = \{1, 2, 3\}$$

$$p = (p_1, p_2, p_3), p_1 \geq p_2 \geq p_3$$

Suppose that  $G_0$  is associated with secret voting, and  $G$  is associated with open voting, where

$$I = \{1\}, D = \{2, 3\}, g_2(y_2, y_1) = y_1, g_3(y_3, y_1) = y_1.$$

For a three-member decision-making body there exist two relevant neutral decision rules (see Nitzan and Paroush, 1981). The expert rule,  $f_1(x_1, x_2, x_3) = x_1$  and simple majority rule,  $f_2(x_1, x_2, x_3) = \text{Sign}(x_1 + x_2 + x_3)$ . Suppose that the group  $N$  is restricted to using  $f_2$ . If  $N$  operates under secret voting, then

$$\begin{aligned} \pi(f_2, p, G_0) &= p_1 p_2 p_3 + p_1 p_2 (1 - p_3) + p_1 (1 - p_2) p_3 + \\ &+ (1 - p_1) p_2 p_3. \end{aligned}$$

If  $N$  operates under open voting, then  $\pi(f_2, p, G) = p_1$ .

It can be readily verified that if

$$\frac{p_1}{1 - p_1} > \frac{p_2}{(1 - p_2)} \frac{p_3}{(1 - p_3)}, \text{ then } \pi(f_2, p, G) > \pi(f_2, p, G_0).$$

This illustrates the possible superiority of open voting when the group cannot select the unconstrained optimal rule  $\hat{f}$  (in our case  $\hat{f}$  is the expert rule  $f_1$ ). Obviously,  $\pi(f_1, p, G_0) = p_1$ .

In the above example, both individual 2 and individual 3 demonstrate an extreme mode of self-denial. Such total dependence is quite commonly observed under various circumstances where the impact of leaders, the effect of collegiality or the degree of individual insecurity is extreme. A general definition of total interdependence is presented below.

An interdependence pattern  $G$  is called *total* whenever the members of  $D$  never resort to their own judgment in making an actual decision, i.e.,

$g_j(y_j, y_1, \dots, y_m) \equiv g_j(y_1, \dots, y_m)$  for every  $j = m + 1, \dots, n$ . A particular case of total interdependence occurs when a single leader is blindly followed by all other decision-makers, as in the previous example. Less extreme cases are characterized by a more balanced collegiality pattern. For example, the dependent individuals may follow the majority view held by a recognized subset of qualified decision-makers (the experts). The more common cases are, of course, those where different subsets of dependent individuals are inspired by their particular subsets of independent individuals. For example, several independent individuals may be supported by their totally dependent factions. For total interdependence patterns we can add the following constructive corollary.

**COROLLARY 2.** Let  $G$  be a total interdependence pattern with  $I = \{1, \dots, m\}$ . Then, for a given vector of skills  $p$ ,  $\pi(f^\circ, p, G) \geq \pi(f, p, G)$  for any neutral  $f$ , if  $f^\circ(x_1, \dots, x_n) = \text{Sign}(\sum_{i=1}^m \beta_i x_i)$  where  $\beta_i = \ln(p_i/(1-p_i))$ .

*Proof.* If  $G$  is total, then

$$\pi(f, p, G) = \pi(f, (p_1, \dots, p_n), G) = \pi(f, (p_1, \dots, p_m), G_0).$$

That is, the probability of choosing correctly given  $f, (p_1, \dots, p_n)$  and  $G$  is identical to the probability of obtaining a correct choice in the group  $I = \{1, \dots, m\}$  given  $f, (p_1, \dots, p_m)$  and  $G_0$ . By theorem 1 in Nitzan and Paroush (1982a) the optimal rule is indeed  $f^\circ$ .

Corollary 2 emphasizes that the nature of the optimality result under independent voting is unaltered when the interdependence pattern is total. In these latter cases the dependent members of  $N$  should become inessential and the independent members are assigned the same weights as under the rule  $\hat{f}$ .

Our concluding example illustrates the two corollaries of the main result. It also demonstrates the viability of the two corollaries presenting a situation where  $\pi(\hat{f}, p, G_0) > \pi(f^\circ, p, G) > \pi(\hat{f}, p, G)^5$

**EXAMPLE.**

$$N = \{1, 2, 3, 4\}$$

$$p = (p_1, p_2, p_3, p_4) \text{ where } \begin{cases} p_1 \geq p_2 \geq p_3 \geq p_4 \geq \text{and} \\ \frac{p_1}{(1-p_1)} < \frac{p_2}{(1-p_2)} \frac{p_3}{(1-p_3)} < \\ < \frac{p_1}{(1-p_1)} \frac{p_4}{(1-p_4)}. \end{cases}$$

In a four-member group there exist only three relevant neutral decision rules (see Nitzan and Paroush [1981]). The expert rule,  $f_1$ ; the simple majority rule applied among the three most competent individuals,  $f_2$ ; and the simple majority rule with a tie-breaking chairman,  $f_3$  (the most qualified individual being the chairman). Given our assumption on the vector  $p$ ,

$$\pi(f_3, p, G_0) > \pi(f_2, p, G_0) > \pi(f_1, p, G_0).$$

(See Nitzan and Paroush, 1983.) Now consider the total interdependence scheme  $G$  where

$$D = \{4\}, I = \{1, 2, 3\} \text{ and } g_4(y_4, y_1, y_2, y_3) = y_1.$$

By corollary 3,  $f^o(p, G) = f_2$  and hence  $\pi(f^o, p, G) = \pi(f_2, p, G)$ . By theorem 1 in Nitzan and Paroush (1982a),  $\hat{f}(p, G_0) = f_3$ , and therefore

$$\pi(\hat{f}, p, G_0) = \pi(f_3, p, G_0).$$

By definition of  $G$ ,

$$\pi(f_3, p, G) = \pi(f_1, p, G_0).$$

We thus obtain,

$$\pi(\hat{f}, p, G_0) > \pi(f^o, p, G) > \pi(\hat{f}, p, G).$$

### 5. SUMMARY

Nitzan and Paroush (1982a) and Grofman *et al.* (1983) have recently reported that in an uncertain dichotomous choice situation with symmetric alternatives the decision rule that maximizes the collective probability of making the correct choice is a weighted majority rule  $\hat{f}$  with weights that are proportional to the logarithms of the individual decision-makers' odds of identifying the correct alternative. This result hinges upon a restrictive and quite extreme

assumption. Namely, individual decisions are presumed to be independent. The current paper is concerned with three major issues: First, how can the widely recognized phenomenon of interdependence among individual decisions be explicitly incorporated into the standard dichotomous choice model? Second, replacing the dubious independence assumption with some operational dependence pattern, can we identify the optimal decision rule for the group? That is, given a particular interdependence scheme, which rule maximizes the group probability of choosing correctly? In particular, what can be salvaged from the optimality result under independent voting for the different and more realistic circumstances where choices are interrelated? Finally, assuming the second issue cannot be satisfactorily resolved, is it still possible to accomplish the more modest task of comparing the group performance under the alternative independent and interdependent patterns?

Starting with the conceptual challenge we have deliberately confined our discussion to situations where individual actual decisions are neutrally dependent on own views and on the views of others. Although such a definition is quite general, it leaves out all sorts of interactive learning processes. Interaction acting as investment in human capital is thus excluded from our model. The suggested definition of interdependence does cover, however, common interaction processes, some of which are extensively dealt with in the field of social psychology. These include patterns of interdependence attributed to social pressure, leadership effects, collegiality, timidity, status effects or variable degrees of confidence in individuals' own views. A widely observed form of interdependence which is an interesting special case is total interdependence. Under such a pattern some members of the group totally belittle themselves. Extreme self-denial is reflected in the actual choice being dependent only on the views or decisions of others.

The main intuitive result of this work and its corollaries deal with the two remaining issues. As one may expect, given the generality and richness of our suggested definition of interdependence, the derivation of the optimal decision rules corresponding to all possible interdependence patterns is an impossible task. Corollary 2 does provide a general answer to the second issue for the special class of total interdependence forms. Specifically, for such forms the optimal rule is a weighted majority rule with zero weights assigned to dependent individuals. The weights assigned to independent individuals should be proportional to the logarithms of their odds of identifying the

correct alternative. Clearly then, the optimality result under independent voting retains its viability under total patterns of interdependence.

The main theorem provides a complete answer to the third problem. It strengthens further the optimality result given independent voting by establishing that the optimal group performance under independent voting can only be adversely affected by any pattern of interaction. In particular, the optimal rule corresponding to such a pattern cannot be superior to  $\hat{f}$ . Nor can any deviation from the independent pattern  $G_0$  be fruitful (Corollary 1). In other words, elimination of interdependence, if possible, will only serve a useful purpose from the group's standpoint.

#### NOTES

<sup>1</sup> The optimality of decision rules assuming asymmetric alternatives is treated in Nitzan and Paroush (1982a, b). Suboptimality issues that were recently dealt with include the following: complete ranking of feasible decision rules for small panels of experts, Nitzan and Paroush (1983); comparison of the most common rules, simple majority rule vs. the expert rule, given full information on decisional skills, Grofman (1978), given partial information on the skill parameters, Grofman *et al.* (1983), or given variable group size, Grofman (1978); investigation of a particular rule allowing variability in decisional skills through investment in human capital, Nitzan and Paroush (1980).

<sup>2</sup> A complete description of the uncertain dichotomous choice model is available in Nitzan and Paroush (1982a). We, therefore, present only the necessary components of the model, and very briefly.

<sup>3</sup> For the justification of imposing the neutrality condition when the alternatives are symmetric, the reader is referred to Nitzan and Paroush (1982a).

<sup>4</sup> Note that this statement is somewhat more general than Theorem 1 and can be proven using a similar method. The pattern  $(G_1, D_1)$  is called more independent than  $(G_2, D_2)$  if  $D_1$  and  $G_1$  are subsets of  $D_2$  and  $G_2$ , respectively.

<sup>5</sup> Note that in the previous example we have only presented a case where  $\pi(\hat{f}, p, G_0) = \pi(f^0, p, G)$ .

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