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Optimization-Based Multifrequency Test Generation for Analog Circuits

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Abstract. A robust test set for analog circuits has to detect faults under maximal masking effects due to variations of circuit parameters in their tolerance box. In this paper we propose an optimization based multifrequency test generation method for detecting parametric faults in linear analog circuits. Given a set of performances and a frequency range, our approach selects the test frequencies that maximize the observability on a circuit performance of a parameter deviation under the worst masking effects of normal variations of the other parameters. Experimental results are provided and validated by HSpice simulations to illustrate the proposed approach.

Keywords: multifrequency test generation, parametric faults, tolerance effects, fault observability maximization

1. Introduction

Analog testing is a difficult and expensive task. The difficulty stems from the fact that, unlike in digital circuits, the physical quantities of analog circuits vary over time in a continuous range. This implies a continuum of possible defects. In consequence, there is a lack of adequate fault models, since the output values of analog circuits can not be considered as either high or low levels as in the digital world where a large class of defects can be modeled by stuck-at-0/1faults. Analog circuits are traditionally tested by verifying their function, which is known to be costly. Indeed, the estimated cost of analog testing may represent 30% of the total manufacturing cost [1]. To minimize the test time and thus the cost of production testing of analog circuits, test generation techniques based on fault-models are required.

In general, faults in analog circuits can be classified into hard and parametric faults. Hard faults are caused by catastrophic variations in parameter values such shorts and opens, and usually induce a complete loss of correct functionality. Parametric faults are caused by an abnormal deviation of parameter values and result in altered performance. Both types of faults have to be detected by a test set. Milor et al. [2] reported on a test generation algorithm for detecting catastrophic faults under normal parameter variations. In [3, 4] an approach based on a statistical process fluctuation model was derived to select a subset of circuit specifications that detect parametric faults and minimize the production testing time. Test generation is formulated in [5] as a quadratic programming problem. This approach was developed for parametric faults and it determines an input stimulus $x(t)$ that maximizes the quadratic difference of responses from the good and the faulty circuits with other parameters at their nominal values. A test generation approach for hard and parametric faults based on sensitivity analysis and tolerance computation was exposed in [6]. In this approach the worst case performance was expressed in terms of sensitivity and parameter tolerance. However, frequency analysis was not considered and the model was a **linearization** obtained from first order partial derivatives.

The method presented in [7] is founded on a fault-model and sensitivity. For a given fault-list, perturbation of sensitivity with respect to frequency is used to find the direction toward the best test frequency. In [8] the authors derived a multifrequency test generation technique based upon testability analysis and a fault observability concept. The test frequencies selected are those where the output performance sensitivity is maximum with respect to faulty component deviation. In the above approaches [7–8] the masking effects due to variations of the fault-free components in their tolerance box **are not considered** and **the test frequencies may be not optimal**. A DC test generation technique for catastrophic faults was developed in [9]. This technique is fault-based and test generation is formulated as an optimization problem including the effects of parameter variations.

Any robust test set has to detect parametric and hard faults under maximal masking effects due to normal variations of parameters. Indeed, only in this case the quality of a test set may be correctly measured and guaranteed. In this paper we propose a novel test generation approach for detecting hard and parametric faults in linear analog circuits. This method is based on multifrequency testing which is, in general, more suited for subtle parameter variations than DC testing. As in [9], the test generation is formulated as an optimization problem taking into account the maximal masking effects due to normal parameter variations. In general, the resulting optimization problem is highly non-linear and is solved iteratively.

The paper is organized as follows:

In the next section, the proposed approach is outlined. A precise problem formulation is elaborated in Section 3. The test generation algorithm is presented in Section 4. Experimental results are reported in Section 5. Conclusions and a description of our future work appear in Section 6.

2. The Proposed Approach

2.1. Objectives

The purpose of our work is to generate the smallest set of robust tests that maximize the observability on a circuit performance of a parameter deviation under the worst masking effects of normal variations of the other parameters. This means that for each parameter we determine a) its smallest absolute possible deviation outside which its detectability can be guaranteed under any variation of the other parameters within their tolerances, and b) the frequency of the input signal (the test) which guarantees this detection.

A circuit is declared faulty if a test of the test set produces a performance outside its acceptance range. In this case, the parameter deviation associated with the fault is said to be observable. We consider single parametric faults here.

2.2. Problem Analysis

The number of tests depends on the input space, the output space (test points) and the performance space. First, we have to select these spaces. In this paper, the input space consists of an extended range of operating frequencies. Multifrequency testing is more appropriate for parametric faults than DC testing which is more suitable for hard faults. For instance, an AC test is needed to detect a variation in a capacitance value. The output space may be obtained by partitioning a circuit into functional blocks. Each block output can be considered as a possible test point. A performance space has to be selected depending on the selected input space. We thus assume that a set of performances such as gain, Q-factor, phase, cut-off frequency, etc., a set of test points and a frequency range are given. The goal of minimizing testing time may be viewed as minimizing the number of performances, the number of test points where these performances should be measured, and the number of frequencies at which they should be observed. This goal can be reached if we are able to answer the following questions. (1) How can we select a performance to be measured? (2) How can we choose a test point where this measurement should be performed? (3) Finally, what frequency should be selected for the best observation? Assuming that the answers to the first two questions are given, we address here the third question.

Let $T_k(f, x_1, \ldots, x_m)$, $k = 1, \ldots, n$, be a given set of n performances of the circuit under test, where $X = [x_1 x_2 \cdots x_m]^T$ is the vector of parameters of the circuit. Let T represents one of these n performances and $T_{\max}(f)$ and $T_{\min}(f)$ are the extreme values of T under the normal parameter variations that determine the acceptance range of T at frequency f. We emphasize that each component (i.e., parameter) of a circuit should be covered by at least one performance. The question now is: how can we decide if a performance T may be selected or not as a test performance? It is obvious that some performances are more sensitive to variations of a parameter than others. Those performances that are the most sensitive to parameter variations should be selected as test performances. More precisely, a performance may be selected as a test performance if it detects the smallest absolute minimum observable deviation of at least one parameter x_i $(i = 1, \ldots, m)$ of a circuit at some frequency f.

3. Problem Formulation

Let $T(f, X) = T(f, x_1, \ldots, x_m)$, be a performance to observe, function of frequency f in $[f_{\min}, f_{\max}]$, $X = [x_1 x_2 \cdots x_m]^T$ the vector of parameters of the circuit, and $X_n = [x_{1n} x_{2n} \cdots x_{mn}]^T$ the nominal value of X. Let the normal tolerance of a parameter x_i be the interval $[x_{il}, x_{iu}]$, $i = 1, \ldots, m$, and the total possible range of values x_i can be from the interval $[\underline{x}_{il}, \bar{x}_{iu}]$ such that $\underline{x}_{il} \le x_{il}$ and $\bar{x}_{iu} \ge x_{iu}$. Let $X_l = [x_{1l} x_{2l} \cdots x_{ml}]^T$ and $X_u = [x_{1u} x_{2u} \cdots x_{mu}]^T$, hence under normal circumstances $X_l \le X \le X_u$, and $X_l \le X_n \le X_u$.

3.1. *Valid Range Determination*

Let $T_{\min}(f, X)$ and $T_{\max}(f, X)$ be the extreme values of T at frequency f under the variation of $X \in [X_l, X_u]$. These extreme values can be obtained as the solution of the following optimization problems.

For a given frequency f,

$$T_{\max}(f) = \max_X T_k(f, X) \quad \text{subject to } X_l \le X \le X_u$$

and

$$T_{\min}(f) = \min_X T_k(f, X) \quad \text{subject to } X_l \le X \le X_u$$

The envelope delimited by $T_{\max}(f)$ and $T_{\min}(f)$ constitutes the acceptance range of T.

3.2. *Absolute Minimum Observable Parameter Variation*

Without loss of generality, let x_m be the parameter whose changes we wish to observe at performance T. Let $x_{m\_\min}(f) \in [\underline{x}_{ml}, x_{ml}]$ (resp., $x_{m\_\max}(f) \in [x_{mu}, \bar{x}_{mu}]$) be the smallest (resp., largest) value of x_m such that $T(f)$ is outside of the interval $[T_{\min}(f), T_{\max}(f)]$ for all x_m in $[\underline{x}_{ml}, x_{m\_\min}(f)]$ (resp., for all x_m in $[x_{m\_\max}(f), \bar{x}_{mu}]$.

Graphically, we can visualize the various intervals of values of x_m as shown in Fig. 1.

Given some frequency f, the objective is to determine the values $x_{m\_\min}(f)$ (resp., $x_{m\_\max}(f)$) regardless the values of the other x_i, $i \neq m$, within their respective normal intervals. As a result, this gives us a limit on the deviation of x_m outside its normal tolerance box, such that if x_m is smaller than $x_{m\_\min}(f)$ (larger than $x_{m\_\max}(f)$), then this (faulty) deviation is guaranteed to be detected at f (i.e., T would be outside the range $[T_{\min}(f), T_{\max}(f)]$) independently of the other parameter values within their normal variations. Furthermore, we wish to find the frequency under which we can detect the largest value $x^*_{m\_\max}$ of $x_{m\_\min}(f)$ (resp., the smallest value $x^*_{m\_\min}$ of $x_{m\_\max}(f)$), i.e.,

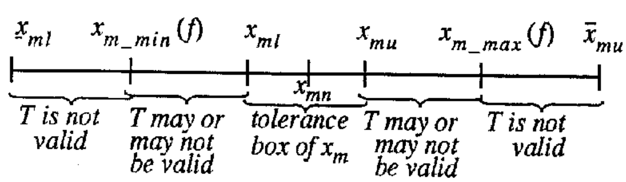


Fig. 1. Total possible range of a parameter x_m under normal variations of x_i, $i \neq m$.

the frequency which makes a faulty deviation of x_m most observable. This problem can be formulated as a max-min (resp., min-max) optimization problem as follows:

Let X_m be the parameter vector $X_m = [x_1 x_2 \cdots x_{m-1}]^T$, and let δ be the resolution of the test equipment with respect to T,

Then

$$x^*_{m\_\max} = \max_f \min_{X_m}(x_m)$$

subject to

$$\begin{aligned} T_{\min}(f) - \delta \le T(f, X) \le T_{\max}(f) + \delta \\ x_{il} \le x_i \le x_{iu} \quad i = 1, \ldots, m-1 \\ \underline{x}_{ml} \le x_m \le x_{ml} \quad \text{and} \quad f_{\min} \le f \le f_{\max} \end{aligned}$$

and

$$x^*_{m\_\min} = \min_f \max_{X_m}(x_m)$$

subject to

$$\begin{aligned} T_{\min}(f) - \delta \le T(f, X) \le T_{\max}(f) + \delta \\ x_{il} \le x_i \le x_{iu} \quad i = 1, \ldots, m-1 \\ x_{mu} \le x_m \le \bar{x}_{mu} \\ f_{\min} \le f \le f_{\max} \end{aligned}$$

In general, these are complex non-linear optimization problems. We discretize the frequency in the interval of interest $[f_{\min}, f_{\max}]$ into a set $\{f_j\}$ of p frequencies and solve the min (resp., max) problem at each f_j, using the Sequential Quadratic Programming (SQP) method from the optimization toolbox of MATLAB [10]. The principal idea behind SQP is to transform the problem into a series of QP sub-problems which are solved at each iteration. This method presents better performance (in terms of efficiency, accuracy and percentage of successful solutions) than every other method tested [11]. Although the SQP represents one of the best non-linear programming methods, T is needed to be one-dimensionally convex [12, 13] in order to guarantee that the global optimum is always found. However, if it is not the case, doing the optimization a number of times, each from different starting point may help to obtain the global optimum. When maximizing x_m, for example, the global maximum can be easily located if we start the optimization from points corresponding to a large value of x_m. Some other techniques (as scaling by variable transformation, using analytic partial derivatives instead of a finite difference approximation, etc.) are also useful for locating the optimum.

For each f_j we thus obtain the ranges $[T_{\min}(f_j), T_{\max}(f_j)]$, $[x_{m\_\min}(f_j), x_{m\_\max}(f_j)]$, $i = 1, \ldots, m$, and we must select a subset of frequencies from $\{f_j\}$ that provide the best detection of a faulty deviation for each x_i.

4. Test Generation Algorithm

A simplified form of the algorithm is shown in Fig. 2. The algorithm is given a set of performances $\{T_k\}$ and a set of frequencies $\{f_j\}$. In the first step, it finds the extreme values $T_{k\_\max}(f_j)$ and $T_{k\_\min}(f_j)$ of performance $T_k (k = 1, \ldots, n)$ at each f_j. Then, each parameter value $x_i (i = 1, \ldots, m)$ is optimized at each frequency f_j. From the set of maximal (resp., minimal) values $x_{i\_\max}(f_j)$ (resp., $x_{i\_\min}(f_j)$), the minimum (resp., maximum) observable value $x^*_{i\_\min}(k)$, i.e., $x^*_{i\_\min}(k) = \min_{f_j}(x_{i\_\max}(f_j))$, (resp., $x^*_{i\_\max}(k)$, i.e., $x^*_{i\_\max}(k) = \max_{f_j}(x_{i\_\min}(f_j))$) of x_i, is extracted with the corresponding frequency $f_i^+(k) \in \{f_j\}$ (resp., $f_i^-(k) \in \{f_j\}$) and the performance T_k. Finally, from the set of $x^*_{i\_\min}(k)$ (resp., $x^*_{i\_\max}(k)$), the smallest minimum (resp., the largest maximum) observable value x_i^S, i.e., $x_i^S = \min_k \min_{f_j} \max_{X_i}(x_i)$, (resp., x_i^G, i.e., $x_i^G = \max_k \max_{f_j} \min_{X_i}(x_i)$) is selected. At the same time, the corresponding performances T_k and the test frequencies of x_i^S and x_i^G are selected as (T_{ip}, tf_i^+) and (T_{iq}, tf_i^-), respectively. As a result, if d_i^+ and d_i^- are the smallest (resp., largest) relative observable positive (resp., negative) deviations, i.e., $d_i^+ = \frac{x_i^S - x_{in}}{x_{in}}$ (resp., $d_i^- = \frac{x_i^G - x_{in}}{x_{in}}$), each parameter x_i is characterized by two triplets (d_i^+, tf_i^+, T_{ip}) and (d_i^-, tf_i^-, T_{iq}), and for the whole circuit $D = \{(d_i^+, tf_i^+, T_{ip}), (d_i^-, tf_i^-, T_{ip}) \mid i = 1, \ldots, m\}$ defines a test set for all x_i. We can easily extract the set of test frequencies from D, while the additional information contained there identifies which x_i is tested at which T_k and frequency.

The obtained test set guarantees to detect any deviation of parameter x_i larger (resp., smaller) than the computed smallest minimum (resp., largest maximum) observable relative deviation d_i^+ (resp., d_i^-). On the other hand, the number of the observed performances and the test frequencies may be not minimal. There are always some trade-offs possible between the test quality and the testing time. This is, however, outside of the scope of this paper.

Begin
  get $\{T_k \mid k = 1, \ldots, n\}$ and $\{f_j \mid j = 1, \ldots, p\}$
  For $k = 1, \ldots, n$ do
    For all $f_j \in \{f_j\}$ do
      $T_{k\_\max}(f_j) = \max_X T_k(f_j, X)$
      $T_{k\_\min}(f_j) = \min_X T_k(f_j, X)$
    End
    For $i = 1, \ldots, m$ do
      For all $f_j \in \{f_j\}$ do
        $x_{i\_\max}(f_j) = \max_{X_i} x_i$
        $x_{i\_\min}(f_j) = \min_{X_i} x_i$
      End
      $x^*_{i\_\min}(k) = \min_{f_j} x_{i\_\max}(f_j)$
      $f_i^+(k) = f_j$ /*/ f_j is such that $x^*_{i\_\min}(k) = x_{i\_\max}(f_j)$ *\
      $x^*_{i\_\max}(k) = \max_{f_j} x_{i\_\min}(f_j)$
      $f_i^-(k) = f_j$ /*/ f_j is such that $x^*_{i\_\max}(k) = x_{i\_\min}(f_j)$ *\
    End;
  End for k;
  $D = \emptyset$
  For $i = 1, \ldots, m$ do
    $x_i^S = \min_k x^*_{i\_\min}(k)$ /*/ $x_i^S = \min_k \min_{f_j} \max_{X_i} (x_i)$*\
    $T_{ip} = T_k$ /*/ k is such that $x_i^S = x^*_{i\_\min}(k)$ *\
    $tf_i^+ = f_i^+(k)$ /*/ k is such that $x_i^S = x^*_{i\_\min}(k)$ *\
    $x_i^G = \max_k x^*_{i\_\max}(k)$ /*/ $x_i^G = \max_k \max_{f_j} \min_{X_i}(x_i)$ *\
    $T_{iq} = T_k$ /*/ k is such that $x_i^G = x^*_{i\_\max}(k)$ *\
    $tf_i^- = f_i^-(k)$ /*/ k is such that $x_i^G = x^*_{i\_\max}(k)$ *\
    $d_i^+ = \frac{(x_i^S - x_{in})100}{x_{in}}(\%)$
    $d_i^- = \frac{(x_i^G - x_{in})100}{x_{in}}(\%)$
    $D = D \cup \{(d_i^-, tf_i^+, T_{ip}), (d_i^-, tf_i^-, T_{iq})\}$
  End for m;
  Extract test frequencies and build fault dictionary from D
End of algorithm.

Fig. 2. Simplified algorithm.

5. Experimental Results

5.1. An Illustrative Example

To illustrate the above technique consider the low-pass filter shown in Fig. 3. The performance of interest is the magnitude T of the circuit transfer function:

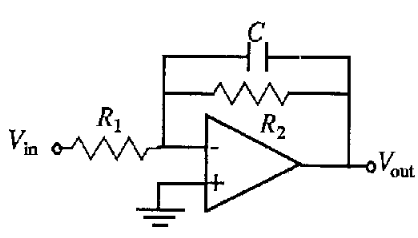


Fig. 3. Low-pass filter.

$$T = \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{R_2}{R_1}\frac{1}{\sqrt{1 + R_2^2 C^2 \omega^2}}$$

The nominal design ($R_1 = 1.6$ k, $R_2 = 16$ k, $C = 10$ nF) has the dc gain of 20 dB, with the -3 dB point at 1 kHz. Figure 4 shows the computed results of the extreme values of T under normal variations of R_1, R_2, and C in their tolerance intervals: $R_1 = [1.52, 1.68]\text{k}\Omega$ $R_2 = [15.2, 16.8]\text{k}\Omega$ and $C = [9.5, 10.5]\text{nF}$.

Figures 5, 6, and 7 show the computed maximum (minimum) values $x_{i\_\max}(f_j)$ (resp., $x_{i\_\min}(f_j)$) of R_1, R_2 and C observable at T, as functions of frequency. Table 1 shows the computed and sim-

Table 1. Computed and simulated results.

P	PV	S.O.P.D/ L.O.N.D (%)	TF (Hz)	T_{max} at TF	T_{min} at TF	S.F.G	Error
R_1	$R^*_{1\_min} = 1.858$ k $R_2 = 16.8$ k, $C = 9.5$ nF	16.2	60	11.052	9.047	9.038	−0.009
	$R^*_{1\_max} = 1.374$ k $R_2 = 15.2$ k, $C = 10.5$ nF	−14.1	40	11.052	9.047	11.063	0.011
R_2	$R^*_{2\_min} = 18.585$ k $R_1 = 1.68$ k, $C = 10.5$ nF	16.2	1	11.052	9.047	11.063	0.011
	$R^*_{2\_max} = 13.737$ k $R_1 = 1.52$ k, $C = 9.5$ nF	−14.1	1	11.052	9.047	9.038	−0.009
C	$C^*_{min} = 11.739$ nF $R_1 = 1.52$ k, $R_2 = 16.8$ k	17.4	6 k	1.812	1.483	1.473	−0.01
	$C^*_{max} = 8.499$ nF $R_1 = 1.68$ k, $R_2 = 15.2$ k	−15	8 k	1.367	1.119	1.378	0.011

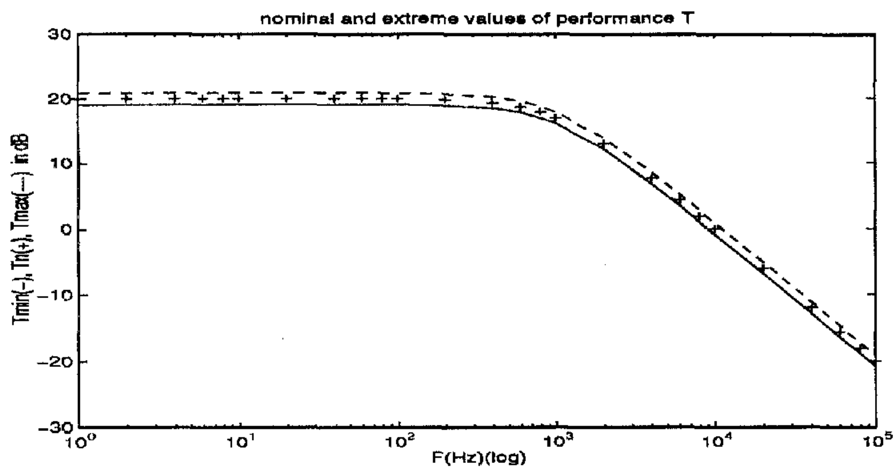


Fig. 4. Extreme values T_{max}, T_{min} of pass-low filter gain.

ulated results. Indeed, the column "PV" (parameter vector) gives the minimum (resp., maximum) observable value $x^*_{i\_min}$ (resp., $x^*_{i\_max}$) of a faulty parameter (R_1, R_2, or C), and the values of the other parameters which produce the maximum masking of the faulty parameter observed at T. The smallest (resp., largest) observable positive (negative) parameter deviations are indicated in the column "S.O.P.D/L.O.N.D". To validate these results, an HSpice simulation was performed and each computed PV was simulated. Column "SFG" indicates the simulated value of the faulty gain produced by the parameter vector "PV" observed at the test frequency "TF". The column "Error" shows that the magnitude of the errors between the simulated faulty gain and the envelope (T_{min}, T_{max}) are very close to the tester resolution which is 0.01 V. The set of tests for the low-pass filter is: TV = {1 Hz, 40 Hz, 60 Hz, 6 kHz, 8 kHz}. This set detects faulty circuit with R_1 outside]1.37, 1.86[kΩ, R_2 outside]13.74, 18.59[kΩ or C outside]8.5, 11.74[nF. From Table 1 we can see that any faulty deviation outside the range]−15, 17.4[% of any parameter is detected by the test TV. As a result TV achieves a fault coverage of 100% for deviation faults outside this range. In the range] − 15, −14.1]%∪]16.2, 17.4[% only faults in R_1 and R_2 are detected by TV and the corresponding fault coverage is 66.6%. On the other hand, faults in the range

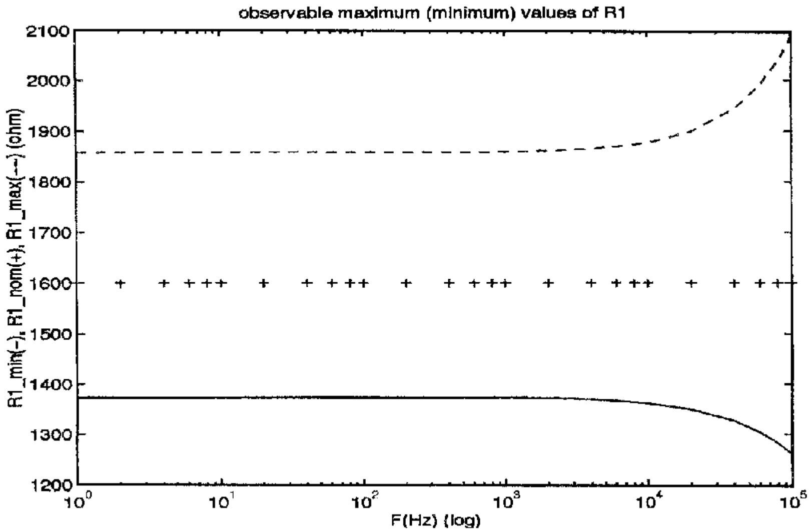


Fig. 5. Observable minimum (maximum) deviation of parameter R_1.

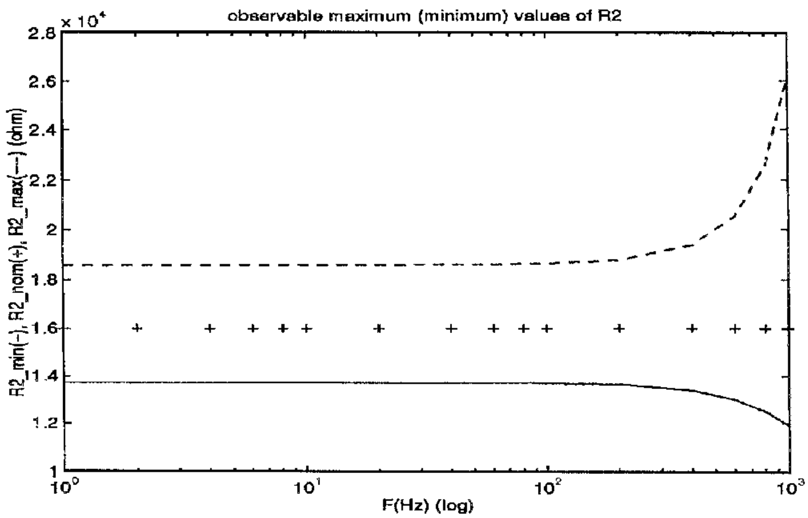


Fig. 6. Observable minimum (maximum) deviation of parameter R_2.

]−14.1, −5[∪]5, 16.2[% are not guaranteed to be detected by TV.

5.2. *A Realistic Application*

As a realistic application for our test generation method consider the biquadratic filter shown in Fig. 8. The performances (output responses) of interest are the magnitudes V_3 and V_5 of the transfer functions at nodes 3 and 5, given by the following equations:

$$V_3 = \frac{1}{R_6 C_2} \frac{\omega}{\sqrt{\left(\frac{R_8}{R_3 R_5 R_7 C_2 C_4} - \omega^2\right)^2 + \frac{\omega^2}{R_1^2 C_2^2}}}$$

$$V_5 = \frac{1}{R_3 R_6 C_2 C_4} \frac{1}{\sqrt{\left(\frac{R_8}{R_3 R_5 R_7 C_2 C_4} - \omega^2\right)^2 + \frac{\omega^2}{R_1^2 C_2^2}}}$$

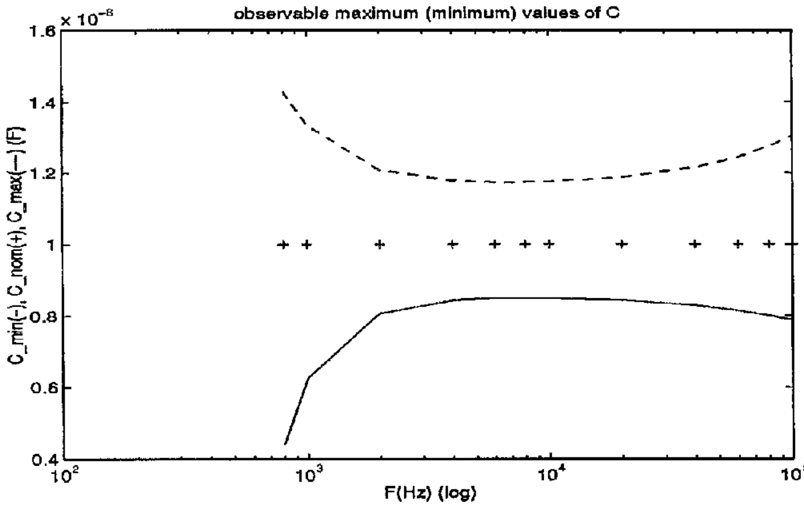


Fig. 7. Observable minimum (maximum) deviation of parameter C.

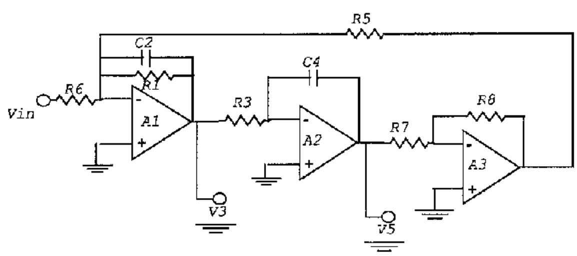


Fig. 8. Biquadratic filter.

The frequency interval of interest is [1 Hz, 20 kHz]. The smallest (largest) observable positive (negative) deviation of a parameter x_i depends on the transfer function, the tolerance intervals of the parameters, the resolution of the test equipment, the test frequency and other factors. In our case, the parameter tolerances and the tester resolution are assumed to be ±5% and 0.01 V, respectively. The results of the application of our method to the circuit are shown in Table 2. The third and the fourth columns give the smallest positive deviation "S.O.P.D" and the largest negative deviation "L.O.N.D" observed at the outputs V5 and V3 respectively at the test frequency Fr (Hz). For each parameter the final selected test frequency TF and the corresponding test performance are indicated in the last column.

The results of Table 2 were validated in a similar manner as those in the previous example. Indeed, the parameter vector consisting of the minimum (resp., maximum) observable values $x^*_{i\_\min}(k)$ (resp., $x^*_{i\_\max}(k)$) of a faulty parameter x_i, and the values of the other parameters which produce the maximum masking of the fault is injected in the model of the circuit and then simulated. The error between the simulated faulty performance and the envelope $(T_{k\_\max}, T_{k\_\min})$ of the acceptance range is compared with the tester resolution. Simulations of all computed parameter vectors corresponding to the circuit parameters were performed, and the magnitude of the errors between the obtained performances and the normal envelopes were very close to the tester resolution. For illustration, some of the simulation results are shown in Figs. 9 and 10. We can see that the envelope of the acceptance range is the region delimited by $V_{5\_\max}(f)$ and $V_{5\_\min}(f)$. The regions where the performance is observed at the selected test frequency are magnified to show the magnitude of the errors which are very close to 0.01 V (the tester resolution value), confirming the correctness of the results obtained by our method.

The set of tests, extracted from Table 2, is:

$$S = \{8.64\text{ k}, 10.3\text{ k}, 10.38\text{ k}, 11.68\text{ k}, 20\text{ k}\}_{V_3} \cup \{1\text{ Hz}, 9.6\text{ k}, 11.04\text{ k}, 11.52\text{ k}\}_{V_5}$$

S detects any fault deviation outside the following ranges (in%): $C_2 \notin\ ]-33.05, 34.93[$, $C_4 \notin\ ]-30.3, 45.3[$, $R_6 \notin\ ]-17.42, 27.4[$ $R_1 \notin\ ]-23.3, 24.9[$, $R_3 \notin\ ]-31.2, 45.7[$, $R_7 \notin\ ]-30.5, 42.9[$ $R_8 \notin\ ]-30.2, 43.5[$, $R_5 \notin\ ]-30.5, 42.9[$. The fault coverage thus

Table 2. Computed results.

Parameter	Nominal value	Output V5 S.O.P.D/ L.O.N.D	Output V5 Fr (Hz)	Output V3 S.O.P.D/ L.O.N.D	Output V3 Fr (Hz)	Final test frequency TF (Hz)
R_1	10 k	43	10 k	24.9	10.3 k	10.3 k (V3)
		−31.6	9.6 k	−23.3	10.38 k	10.38 k (V3)
C_2	1.59 nF	66.2	20 k	34.93	20 k	20 k (V3)
		−59	9.3 k	−33.05	20 k	20 k (V3)
R_3	10 k	45.7	9.6 k	92.4	4.02 k	9.6 k (V5)
		−31.2	11.52 k	−45.2	4 k	11.52 k (V5)
C_4	1.59 nF	45.3	9.6 k	92.1	4.1 k	9.6 k (V5)
		−30.3	11.04 k	−45	4 k	11.04 k (V5)
R_5	10 k	42.9	1	92.1	4 k	1 (V5)
		−30.5	1	−45.2	4 k	1 (V5)
R_6	10 k	40.4	8.140 k	27.4	11.68 k	11.680 k(V3)
		−29.1	1	−17.42	8.64 k	8.640 k(V3)
R_7	10 k	42.9	1	92.1	4 k	1 (V5)
		−30.5	1	−45.2	4 k	1 (V5)
R_8	10 k	43.5	1	82.2	4 k	1 (V5)
		−30.2	1	−48	4 k	1 (V5)

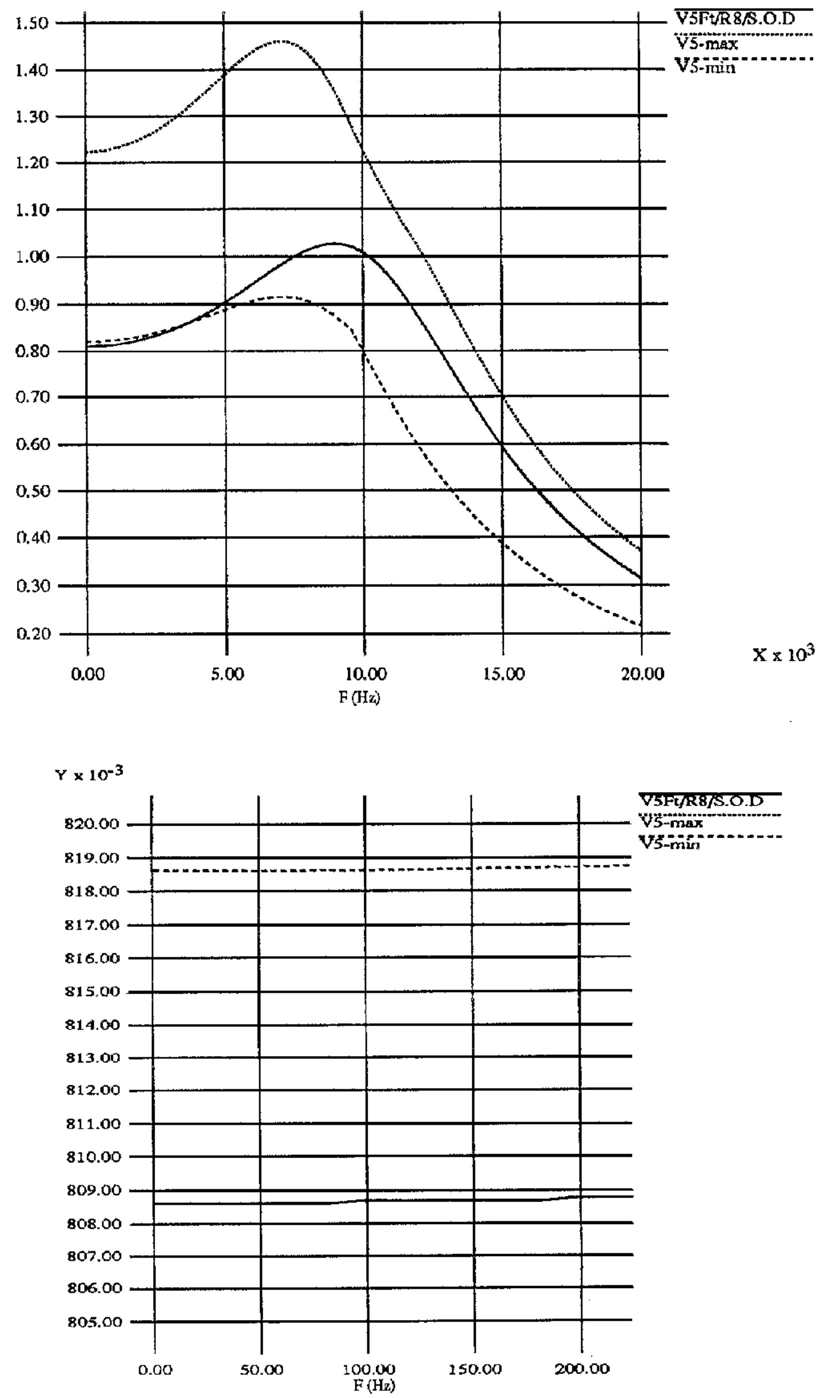
reaches 100% for any fault deviation outside the range]−33.05, 45.7[% of any parameter. On other hand, in the range]27.4, 45.7[% only faulty deviations of R_1 and R_6 are guaranteed to be detected. As a result, the guaranteed fault coverage is only 25% which is poor. Consequently, for improving the fault coverage the effect of the parameters $R_3, C_4, R_5, C_2, R_7, R_8$ should be observed at more sensitive performances. Furthermore, for selective filters, positive deviations of the parameter R_1 become undetectable under other parameter variations. Even if we also observe the signal phase, only very large deviations of R_1 can be detected. A general solution to such a problem of undetectability is the subject of on-going research. As mentioned earlier in Section 2 and illustrated here by this example, an appropriate selection of performance and test point spaces combined with an adequate selection of test frequencies are the key to a complete solution for the test generation problem.

It is important to note that the size of the test set S may be greatly reduced by using "fault dropping". For example the faulty deviations of the parameter R_3 (see Fig. 11 which is described below) can be detected at frequency 20 kHz which is already associated with the parameter C_2. As a result, the frequencies 9.6 kHz and 11.52 kHz associated with R_3 can be eliminated. Similarly, one of the frequencies 10.3 kHz or 10.380 kHz can be dropped without any effect on the detectability of the faulty deviations of R_1 (see Fig. 13). Obviously, "fault dropping" may also reduce the fault coverage since the limits S.O.P.D and L.O.N.D may be altered with the reduced test set. So, by carefully selection of the final test we can overcome this problem.

5.3. *Test Set Validation*

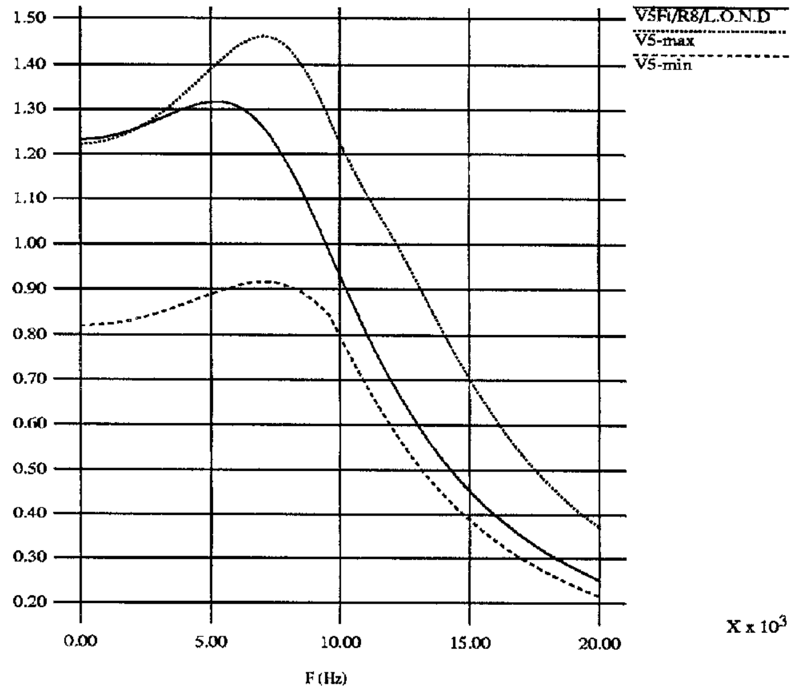
In order to validate the test set generated by our approach, we propose a fault simulation method based on faulty parameter deviations under the effects produced by fault-free parameter variations. Only single fault is considered at a time. The fault is injected by changing each parameter by −50%, the computed largest observable negative deviation, the computed smallest observable positive deviation, +50%, +100% and +1000%. Catastrophic faults (opens and shorts) are also considered.

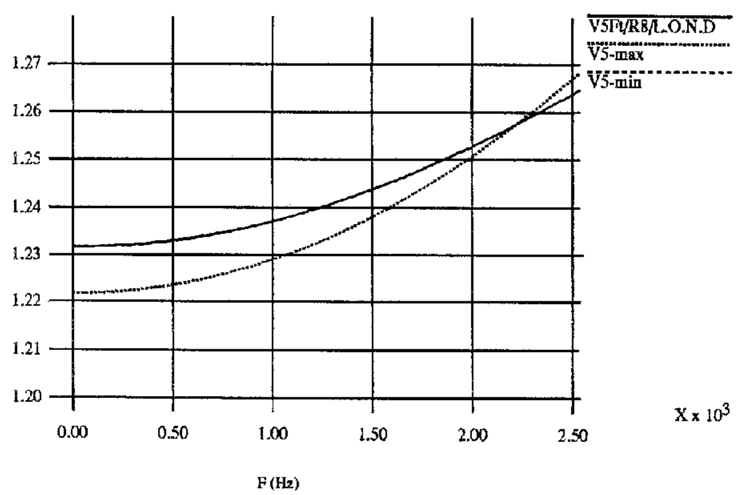
The validation process consists of generating a family of curves of the good output responses (performances) at the observed point under random normal variations of the parameters, and another family of curves for the output responses under the faulty parameter and the other random normal variations of



Magnified region where S.O.P.D of R_8 is observed on V5Ft at the test frequency TF=1Hz.

Fig. 9. Faulty response V5Ft observed under the smallest observable positive deviation (S.O.P.D) of R_8, compared to the envelope of the normal range (V5-min, V5-max).





Magnified region where L.O.N.D of R_8 is observed on V5Ft at the test frequency TF=1Hz.

Fig. 10. Faulty response V5Ft observed under the largest observable negative deviation (L.O.N.D) of R_8, compared to the envelope of the normal range (V5-min, V5-max).

the fault-free parameters. The normal and the faulty families are compared to see if they are disjoint for at least one frequency from the test set for the injected fault.

The parameter variations are generated randomly. The distribution function associated with each parameter is assumed to be uniform and 100 iterations are performed in each HSpice simulation. Due to the large number of simulated curves, we show only some of them (samples of simulated faults for parameters R_3, R_8 and R_1 are given in Figs. 11, 12 and 13, respectively). The simulation results show that all injected faults are detected by the test set, as predicted.

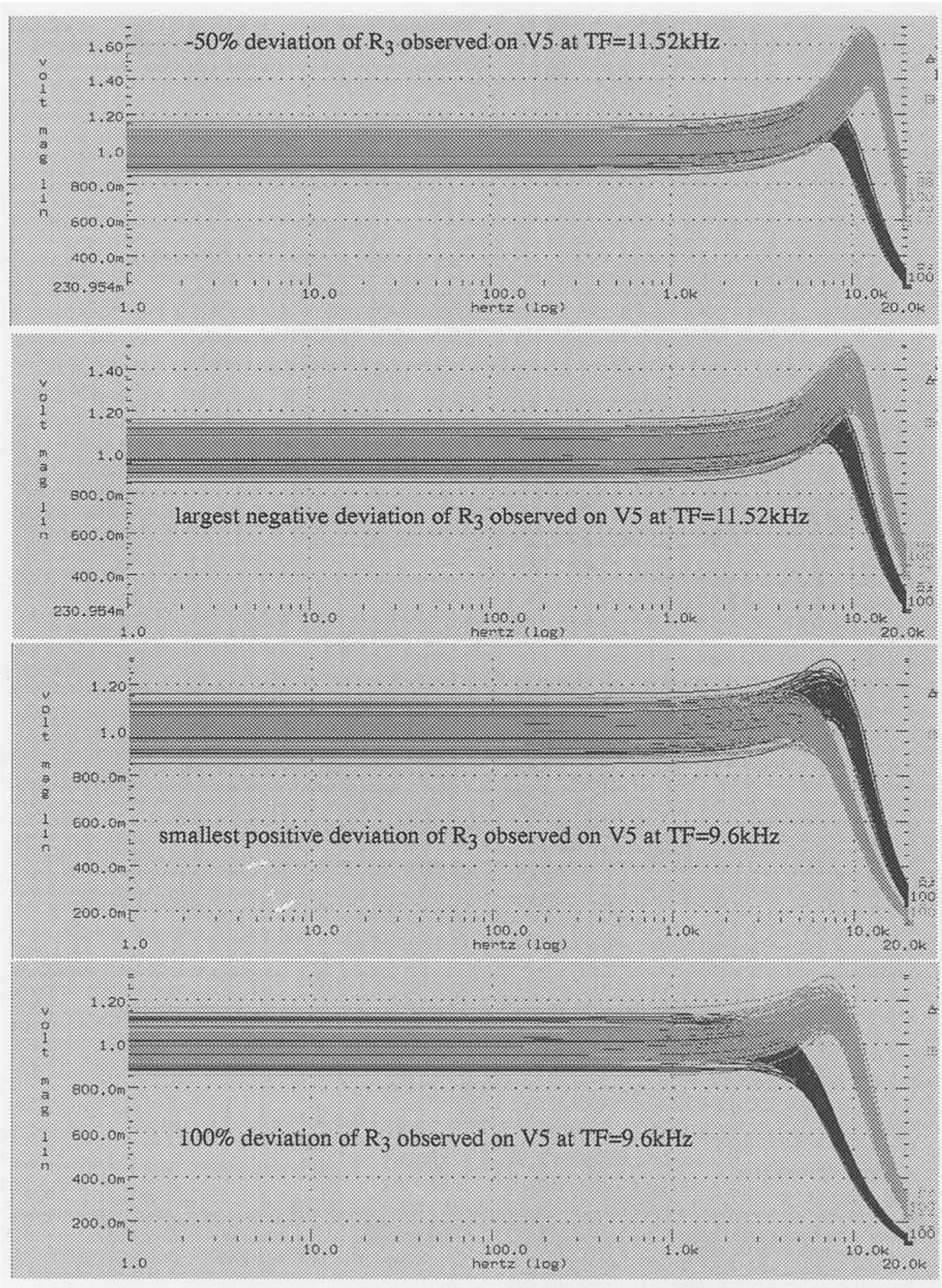


Fig. 11. Faulty and good response families under faulty R_3 and the other parameters varied randomly.

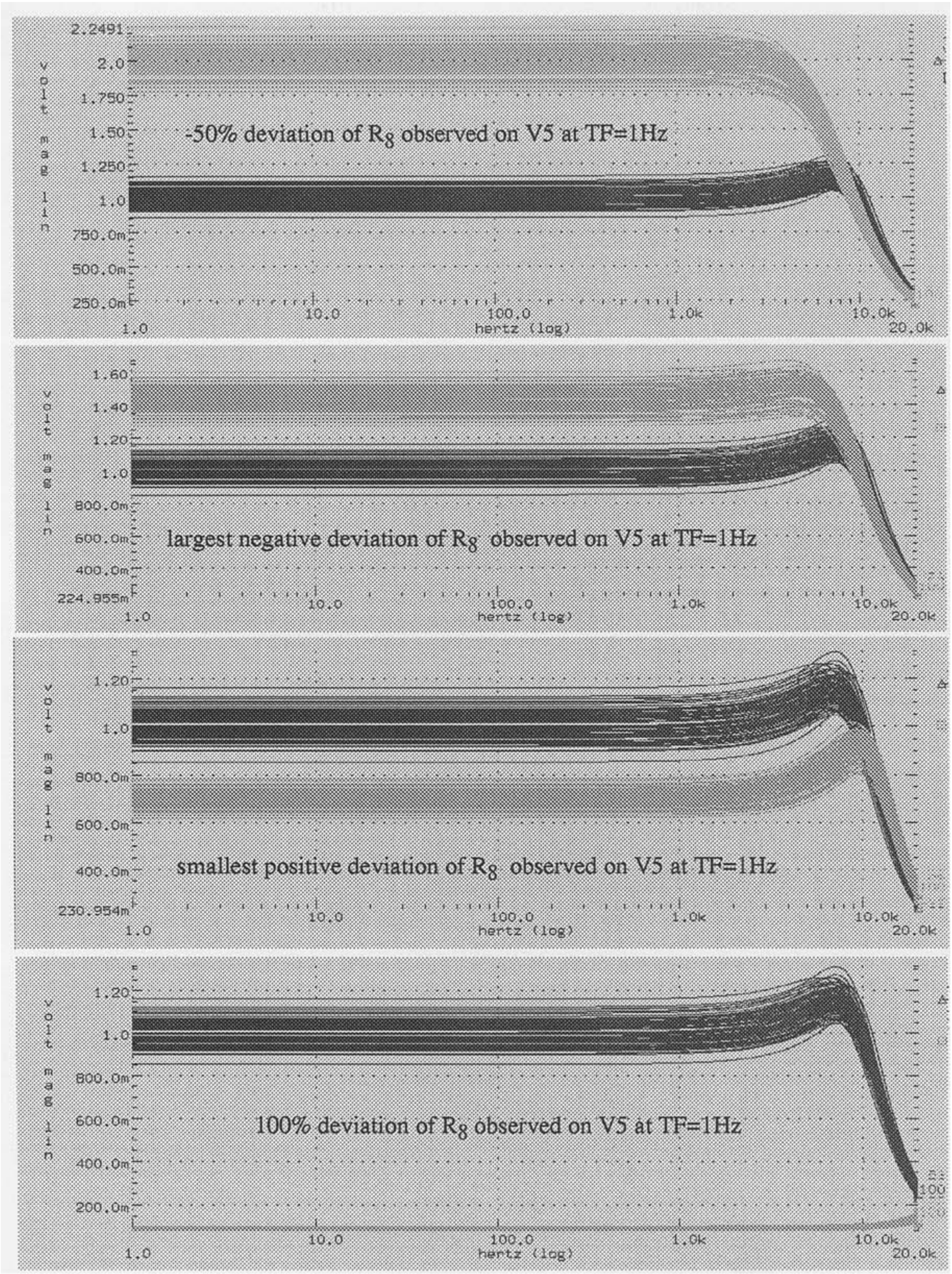


Fig. 12. Faulty and good response families under faulty R_8 and the other parameters varied randomly.

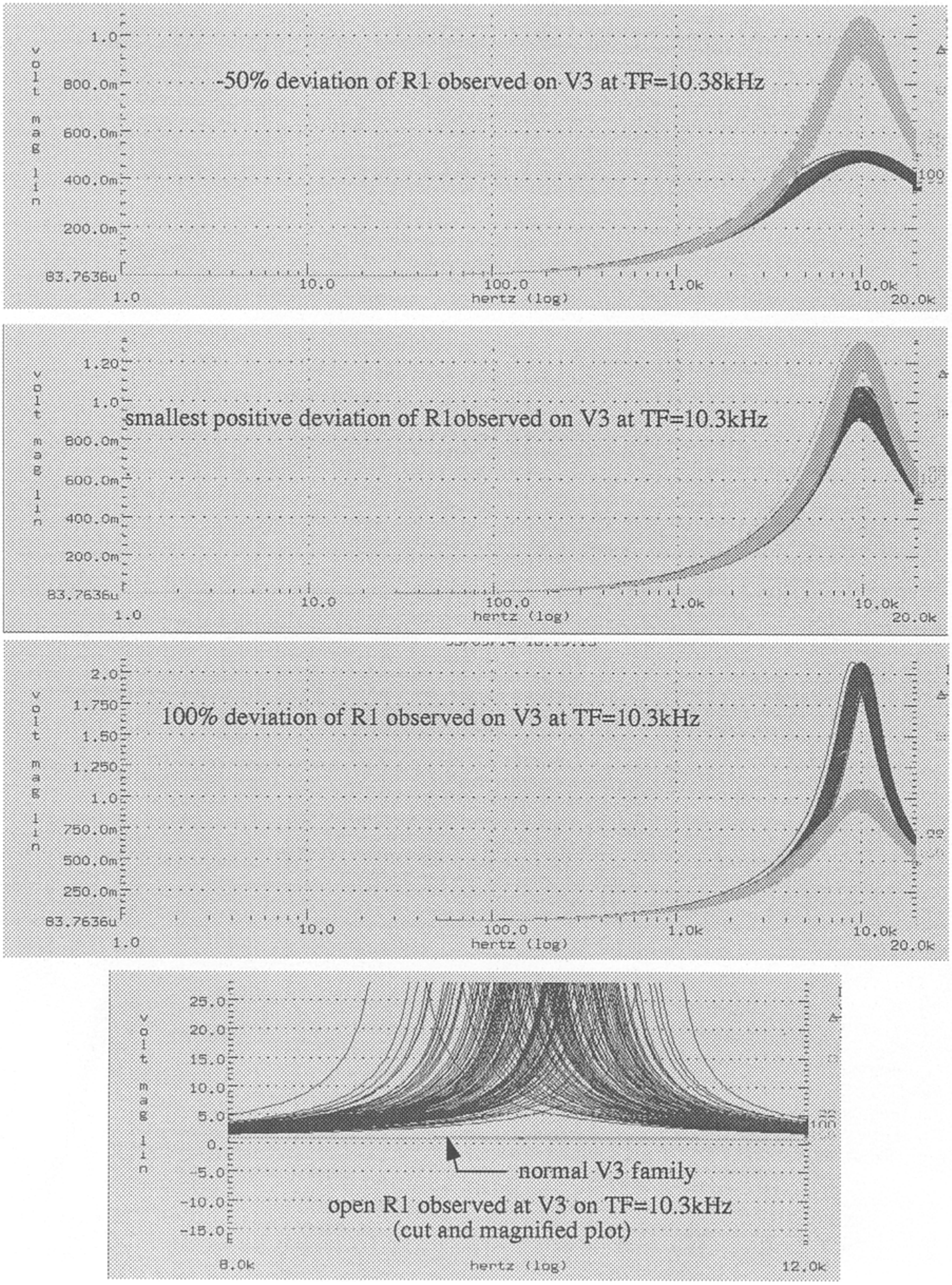


Fig. 13. Faulty and good response families under faulty R_1 and the other parameters varied randomly.

6. Conclusions

In this paper, we proposed a novel multifrequency test generation for detecting parametric and catastrophic failures in linear analog circuits. The test generation problem was formulated as an optimization problem. The method generates a robust test set that detects faults under maximal masking effects due to variations of parameters in their tolerance boxes. The proposed approach was illustrated on two examples. The computed smallest (largest) observable positive (negative) parameter deviations were validated using HSpice simulations of the observed performances which were then compared with the computed normal range (T_{k_min}, T_{k_max}). The magnitude of the errors obtained from these comparisons were very close to the resolution of the test equipment, thus confirming the accuracy of our method on these examples. Besides an adequate selection of test frequencies, it was shown that a complete solution to test generation problem needs an appropriate selection of performance and test point spaces.

In our future work, we aim at improving the approach to guarantee that the global optimum is always found regardless of properties of the performance functions. Also, we are elaborating a technique that allows to detect parameter variations that are difficult to observe (e.g., faulty deviations of R_1 in the selective biquadratic filter).

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